

# Matrix theory from Holography

Leinweber Network Workshop:  
Quantum Black Holes

Chintan Patel

Princeton University

10/11/2026

- *Matrix Theory from Holography*

With Shota Komatsu and Eunwoo Lee

arXiv:2609.13386

- *Matrix Theory from Holography: Part 2*

With Shota Komatsu, Eunwoo Lee, and Shivansh Tomar

2411.xxxxx

- Motivation and the triality conjecture
- The holographic route to triality
- Matching supersymmetric indices
- Ongoing work: ADHM theory and protected correlators
- Open directions

## **Matrix theory**

BFSS: large rank  $\leftrightarrow$   
flat-space M-theory

[Banks, Fischler, Shenker, Susskind, 1997]

## **Matrix theory**

BFSS: large rank  $\leftrightarrow$   
flat-space M-theory

Susskind: finite rank  $\leftrightarrow$  DLCQ

[Banks, Fischler, Shenker, Susskind, 1997]

[Susskind, 1997]

## Matrix theory

BFSS: large rank  $\leftrightarrow$   
flat-space M-theory

Susskind: finite rank  $\leftrightarrow$  DLCQ

[Banks, Fischler, Shenker, Susskind, 1997]  
[Susskind, 1997]

## Holography

Holographic CFTs  $\leftrightarrow$  M-theory  
in asymptotically AdS spacetimes

[Maldacena, 1997; Gubser, Klebanov, Polyakov, 1998;  
Witten, 1998; Pele, Siebelink, 1999;  
Aharony, Bergman, Jafferis, Maldacena, 2008]

# Motivation

## Matrix theory

BFSS: large rank  $\leftrightarrow$   
flat-space M-theory  
Susskind: finite rank  $\leftrightarrow$  DLCQ

[Banks, Fischler, Shenker, Susskind, 1997]  
[Susskind, 1997]



## Holography

Holographic CFTs  $\leftrightarrow$  M-theory  
in asymptotically AdS spacetimes

[Maldacena, 1997; Gubser, Klebanov, Polyakov, 1998;  
Witten, 1998; Pelc, Siebelink, 1999;  
Aharony, Bergman, Jafferis, Maldacena, 2008]



**If the bulk limit gives the same DLCQ background:**

matrix theory  $\longleftrightarrow$  a sector of the CFT

# Motivation

## Matrix theory

BFSS: large rank  $\leftrightarrow$   
flat-space M-theory  
Susskind: finite rank  $\leftrightarrow$  DLCQ

[Banks, Fischler, Shenker, Susskind, 1997]  
[Susskind, 1997]

## Holography

Holographic CFTs  $\leftrightarrow$  M-theory  
in asymptotically AdS spacetimes

[Maldacena, 1997; Gubser, Klebanov, Polyakov, 1998;  
Witten, 1998; Pelc, Siebelink, 1999;  
Aharony, Bergman, Jafferis, Maldacena, 2008]

**If the bulk limit gives the same DLCQ background:**

matrix theory  $\longleftrightarrow$  a sector of the CFT

**This work:**

BMN  $\longleftrightarrow$  sectors of ABJM and ADHM

# ABJM and BMN: a very brief overview

## ABJM theory

Three-dimensional  $\mathcal{N} = 6$   
Chern–Simons–matter theory with gauge group

$$U(N)_k \times U(N)_{-k}.$$

**Holographic dual** [Aharony, Bergman, Jafferis, Maldacena, 2008]

- M-theory on

$$\text{AdS}_4 \times S^7/\mathbb{Z}_k.$$

- The rank and level determine the curvature scale:

$$\frac{R^6}{\ell_P^6} = 32\pi^2 Nk.$$

## BMN quantum mechanics

A supersymmetric mass deformation of the BFSS matrix model with gauge group

$$U(m).$$

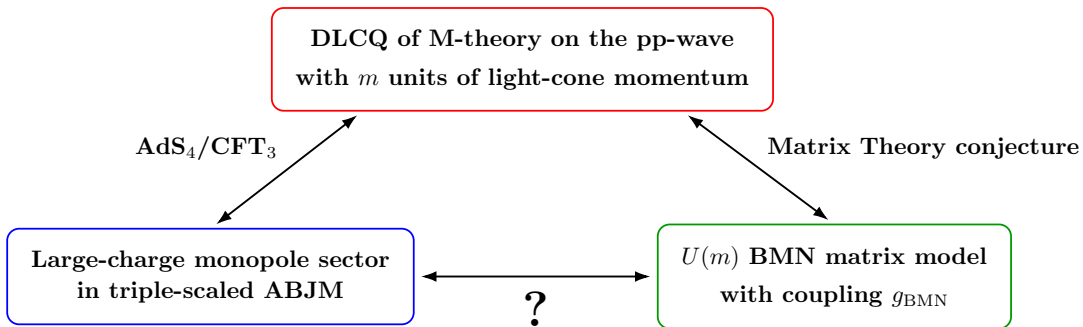
## Conjectured dual

[Berenstein, Maldacena, Nastase 2002]

- DLCQ M-theory on the maximally supersymmetric eleven-dimensional pp-wave.
- The rank counts light-cone momentum units:

$$-p_- = \frac{m}{R_-}, \quad x^- \sim x^- + 2\pi R_-.$$

# The triality



$$N, k, J \rightarrow \infty$$

$$g_{\text{BMN}}^2 = \frac{4\pi^2}{27} \frac{N}{k^2} \text{ fixed, } m = J/k \text{ fixed}$$

**Motivation from holography**

# The Penrose limit

- ABJM theory at level  $k$  is dual to M-theory on  $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ . Locally, the metric is

$$ds^2 = \frac{R^2}{4} \left( -\cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_2^2 \right) \\ + R^2 \left( d\theta^2 + \cos^2 \theta \, d\phi^2 + \sin^2 \theta \, d\Omega_5^2 \right).$$

# The Penrose limit

- ABJM theory at level  $k$  is dual to M-theory on  $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ . Locally, the metric is

$$ds^2 = \frac{R^2}{4} \left( -\cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_2^2 \right) \\ + R^2 \left( d\theta^2 + \cos^2 \theta \, d\phi^2 + \sin^2 \theta \, d\Omega_5^2 \right).$$

- Zoom in on the null geodesic  $\rho = \theta = 0$ ,  $\phi = t/2$ :

$$r = \frac{R\rho}{2}, \quad y = R\theta, \quad x^+ = \frac{3}{\mu} \left( \frac{t}{2} + \phi \right), \quad x^- = \frac{\mu R^2}{6} \left( \frac{t}{2} - \phi \right).$$

# The Penrose limit

- ABJM theory at level  $k$  is dual to M-theory on  $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ . Locally, the metric is

$$ds^2 = \frac{R^2}{4} (-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_2^2) \\ + R^2 (d\theta^2 + \cos^2 \theta d\phi^2 + \sin^2 \theta d\Omega_5^2).$$

- Zoom in on the null geodesic  $\rho = \theta = 0$ ,  $\phi = t/2$ :

$$r = \frac{R\rho}{2}, \quad y = R\theta, \quad x^+ = \frac{3}{\mu} \left( \frac{t}{2} + \phi \right), \quad x^- = \frac{\mu R^2}{6} \left( \frac{t}{2} - \phi \right).$$

- At fixed  $r, y, x^\pm$ , the limit  $R \rightarrow \infty$  gives

$$ds_{\text{pp}}^2 = -2 dx^+ dx^- - \left( \frac{\mu^2 r^2}{9} + \frac{\mu^2 y^2}{36} \right) (dx^+)^2 + d\mathbf{r}^2 + d\mathbf{y}^2.$$

$\mathbf{r} \in \mathbb{R}^3$ , and  $\mathbf{y} \in \mathbb{R}^6$ .

[Blau et al., 2002; BMN, 2002]

# The Penrose limit

- The orbifold shifts all four angles by  $\phi_I \sim \phi_I + 2\pi/k$ . Along the chosen geodesic,

$$(x^+, x^-) \sim \left( x^+ + \frac{6\pi}{\mu k}, x^- - \frac{\pi\mu R^2}{3k} \right).$$

Taking  $k \rightarrow \infty$  with  $\mu R^2/k$  fixed leaves  $x^+$  noncompact and makes  $x^-$  a compact null direction:

$$x^- \sim x^- + 2\pi R_-, \quad R_- = \frac{\mu R^2}{6k}.$$

# The Penrose limit

- The orbifold shifts all four angles by  $\phi_I \sim \phi_I + 2\pi/k$ . Along the chosen geodesic,

$$(x^+, x^-) \sim \left( x^+ + \frac{6\pi}{\mu k}, x^- - \frac{\pi\mu R^2}{3k} \right).$$

Taking  $k \rightarrow \infty$  with  $\mu R^2/k$  fixed leaves  $x^+$  noncompact and makes  $x^-$  a compact null direction:

$$x^- \sim x^- + 2\pi R_-, \quad R_- = \frac{\mu R^2}{6k}.$$

- The BMN dictionary relates the dimensionless coupling to the pp-wave parameters:  
[Berenstein, Maldacena, Nastase, 2002]

$$g_{\text{BMN}}^2 = \frac{R_-^3}{\mu^3 \ell_P^6} = \frac{4\pi^2}{27} \frac{N}{k^2}$$

# Map between Hamiltonian and lightcone momentum

- The light-cone Hamiltonian becomes the BMN Hamiltonian:

$$H_{\text{BMN}} = i\partial_{x^+} = \frac{\mu}{3} \left( \Delta - \frac{J_1}{2} \right).$$

Finite-energy states therefore have  $\Delta - J_1/2$  fixed in the limit.

# Map between Hamiltonian and lightcone momentum

- The light-cone Hamiltonian becomes the BMN Hamiltonian:

$$H_{\text{BMN}} = i\partial_{x^+} = \frac{\mu}{3} \left( \Delta - \frac{J_1}{2} \right).$$

Finite-energy states therefore have  $\Delta - J_1/2$  fixed in the limit.

- The light-cone momentum is

$$-p_- = i\partial_{x^-} = \frac{6}{\mu R^2} \left( \Delta + \frac{J_1}{2} \right).$$

Using  $R_- = \mu R^2/(6k)$ ,

$$-R_- p_- = \frac{\Delta + J_1/2}{k} = \frac{J_1}{k} + \frac{\Delta - J_1/2}{k}.$$

Thus, the surviving sector satisfies

$$J_1, k \rightarrow \infty, \quad \frac{J_1}{k} \rightarrow m = -R_- p_-, \quad \Delta - \frac{J_1}{2} = \text{fixed}.$$

# Monopole charge and the BMN rank

- The Chern–Simons Gauss law imposes the following constraint on the monopole charges:

$$m = \sum_i n_i = \sum_i \tilde{n}_i = \frac{J_1 + J_2 + J_3 + J_4}{k}.$$

# Monopole charge and the BMN rank

- The Chern–Simons Gauss law imposes the following constraint on the monopole charges:

$$m = \sum_i n_i = \sum_i \tilde{n}_i = \frac{J_1 + J_2 + J_3 + J_4}{k}.$$

- Therefore the states of interest in our limit carry monopole charge  $m = \frac{J_1}{k} = -R_- p_-$ , where  $m$  acts as the rank of BMN.

# Monopole charge and the BMN rank

- The Chern–Simons Gauss law imposes the following constraint on the monopole charges:

$$m = \sum_i n_i = \sum_i \tilde{n}_i = \frac{J_1 + J_2 + J_3 + J_4}{k}.$$

- Therefore the states of interest in our limit carry monopole charge  $m = \frac{J_1}{k} = -R_- p_-$ , where  $m$  acts as the rank of BMN.
- The index matching suggests that only sectors with aligned GNO-charges  $\{n_i\} = \{\tilde{n}_i\}$  survive in the above limit. Number of such choices for given  $m$  is given by  $p(m)$ .

## Evidences

# BMN vacua and monopole sectors

- The supersymmetric vacua of  $U(m)$  BMN quantum mechanics are labelled by partitions of  $m$ .  
[Dasgupta, Sheikh-Jabbari, Van Raamsdonk, 2002]

# BMN vacua and monopole sectors

- The supersymmetric vacua of  $U(m)$  BMN quantum mechanics are labelled by partitions of  $m$ .  
[Dasgupta, Sheikh-Jabbari, Van Raamsdonk, 2002]
- Classically, these fuzzy-sphere vacua satisfy

$$\frac{\mu}{3R_-} X^i + \frac{i}{2\ell_P^3} \epsilon_{ijk} [X^j, X^k] = 0, \quad X^a = 0.$$

$$X^i = \frac{\mu \ell_P^3}{3R_-} J^i, \quad [J^i, J^j] = i \epsilon_{ijk} J^k.$$

# BMN vacua and monopole sectors

- The supersymmetric vacua of  $U(m)$  BMN quantum mechanics are labelled by partitions of  $m$ .  
[Dasgupta, Sheikh-Jabbari, Van Raamsdonk, 2002]
- Classically, these fuzzy-sphere vacua satisfy

$$\frac{\mu}{3R_-} X^i + \frac{i}{2\ell_P^3} \epsilon_{ijk} [X^j, X^k] = 0, \quad X^a = 0.$$

$$X^i = \frac{\mu\ell_P^3}{3R_-} J^i, \quad [J^i, J^j] = i\epsilon_{ijk} J^k.$$

- The  $J^i$  are the generators of  $SU(2)$  in an  $m$  dimensional representation, these can be broken up into irreducible blocks.

# BMN vacua and monopole sectors

- The supersymmetric vacua of  $U(m)$  BMN quantum mechanics are labelled by partitions of  $m$ . [Dasgupta, Sheikh-Jabbari, Van Raamsdonk, 2002]
- Classically, these fuzzy-sphere vacua satisfy

$$\frac{\mu}{3R_-} X^i + \frac{i}{2\ell_P^3} \epsilon_{ijk} [X^j, X^k] = 0, \quad X^a = 0.$$
$$X^i = \frac{\mu \ell_P^3}{3R_-} J^i, \quad [J^i, J^j] = i \epsilon_{ijk} J^k.$$

- The  $J^i$  are the generators of  $SU(2)$  in an  $m$  dimensional representation, these can be broken up into irreducible blocks.
- The size of each irreducible block has one-to-one correspondence with the GNO charges  $n_i > 0$  in ABJM.

$$m = n_1 + \cdots + n_s, \quad n_1 \geq \cdots \geq n_s > 0.$$

[Sheikh-Jabbari, Simón, 2009; Kovacs, Sato, Shimada, 2014]

# ABJM superconformal index

- The index is a graded trace over gauge-invariant states on  $S^2$  [Kim, 2009]

$$\begin{aligned} I(p, x, y, z, w) &= \text{Tr}_{\mathcal{H}(S^2)} \left[ (-1)^F e^{-\beta\delta} p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right] \\ &= \text{Tr}_{\text{BPS}} \left[ (-1)^F p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right], \quad p = xyzw \end{aligned}$$

# ABJM superconformal index

- The index is a graded trace over gauge-invariant states on  $S^2$  [Kim, 2009]

$$\begin{aligned} I(p, x, y, z, w) &= \text{Tr}_{\mathcal{H}(S^2)} \left[ (-1)^F e^{-\beta \delta} p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right] \\ &= \text{Tr}_{\text{BPS}} \left[ (-1)^F p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right], \quad p = xyzw \end{aligned}$$

- Only states with  $\delta = 0$  contribute, where

$$\delta = \{Q_{\text{BPS}}, S_{\text{BPS}}\} = \Delta - j_3 - \frac{1}{2}(J_1 + J_2 - J_3 - J_4).$$

# ABJM superconformal index

- The index is a graded trace over gauge-invariant states on  $S^2$  [Kim, 2009]

$$\begin{aligned} I(p, x, y, z, w) &= \text{Tr}_{\mathcal{H}(S^2)} \left[ (-1)^F e^{-\beta \delta} p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right] \\ &= \text{Tr}_{\text{BPS}} \left[ (-1)^F p^{j_3} x^{J_1} y^{J_2} z^{-J_3} w^{-J_4} \right], \quad p = xyzw \end{aligned}$$

- Only states with  $\delta = 0$  contribute, where

$$\delta = \{Q_{\text{BPS}}, S_{\text{BPS}}\} = \Delta - j_3 - \frac{1}{2}(J_1 + J_2 - J_3 - J_4).$$

- $j_3$  is the angular momentum on  $S^2$ . The  $J_I$  are Cartan charges dual to rotations on  $S^7$ .

# ABJM index at large $N$

- At large  $N$ , the index factorizes into neutral, positive-monopole, and negative-monopole contributions [Kim, 2009]:

$$I_{\text{ABJM}} = I^{(0)} I^{(+)} I^{(-)}.$$

# ABJM index at large $N$

- At large  $N$ , the index factorizes into neutral, positive-monopole, and negative-monopole contributions [Kim, 2009]:

$$I_{\text{ABJM}} = I^{(0)} I^{(+)} I^{(-)}.$$

- Take  $k \rightarrow \infty$  while keeping the following fugacities fixed:

|            |            |           |           |
|------------|------------|-----------|-----------|
| $q = x^k,$ | $a = y/x,$ | $b = xz,$ | $c = xw,$ |
|------------|------------|-----------|-----------|

$$|q|, |a|, |b|, |c| < 1.$$

# ABJM index at large $N$

- At large  $N$ , the index factorizes into neutral, positive-monopole, and negative-monopole contributions [Kim, 2009]:

$$I_{\text{ABJM}} = I^{(0)} I^{(+)} I^{(-)}.$$

- Take  $k \rightarrow \infty$  while keeping the following fugacities fixed:

$$\boxed{q = x^k, \quad a = y/x, \quad b = xz, \quad c = xw,} \quad |q|, |a|, |b|, |c| < 1.$$

- Then  $x = q^{1/k} \rightarrow 1$ , and states with  $J_1 \sim km$  retain a finite weight  $q^m$ .

# ABJM index at large $N$

- At large  $N$ , the index factorizes into neutral, positive-monopole, and negative-monopole contributions [Kim, 2009]:

$$I_{\text{ABJM}} = I^{(0)} I^{(+)} I^{(-)}.$$

- Take  $k \rightarrow \infty$  while keeping the following fugacities fixed:

$$\boxed{q = x^k, \quad a = y/x, \quad b = xz, \quad c = xw,} \quad |q|, |a|, |b|, |c| < 1.$$

- Then  $x = q^{1/k} \rightarrow 1$ , and states with  $J_1 \sim km$  retain a finite weight  $q^m$ .
- Negative-monopole contributions are suppressed, so  $I^{(-)} \rightarrow 1$  and  $I_{\text{ABJM}}/I^{(0)} \rightarrow I^{(+)}$ .

# The positive-monopole sector

- After integrating out the zero-flux holonomy eigenvalues at large  $N$ , [Kim, 2009]

$$\begin{aligned} I^{(+)}(x, y, z, w) = & \sum_{M_1, M_2=0}^{\infty} \sum_{\substack{n_1 \geq \dots \geq n_{M_1} > 0 \\ \tilde{n}_1 \geq \dots \geq \tilde{n}_{M_2} > 0}} (xyzw)^{\epsilon_0/2} \\ & \times \int dU d\tilde{U} e^{ik(\sum_i n_i \alpha_i - \sum_j \tilde{n}_j \tilde{\alpha}_j)} \\ & \times \text{PE} \left( \sum_{i,j} f_{ij}^{\text{bif}} + \sum_{i,j} f_{ij}^{\text{adj}} + \sum_{i,j} \tilde{f}_{ij}^{\text{adj}} \right). \end{aligned}$$

# The positive-monopole sector

- After integrating out the zero-flux holonomy eigenvalues at large  $N$ , [Kim, 2009]

$$\begin{aligned} I^{(+)}(x, y, z, w) = & \sum_{M_1, M_2=0}^{\infty} \sum_{\substack{n_1 \geq \dots \geq n_{M_1} > 0 \\ \tilde{n}_1 \geq \dots \geq \tilde{n}_{M_2} > 0}} (xyzw)^{\epsilon_0/2} \\ & \times \int dU d\tilde{U} e^{ik(\sum_i n_i \alpha_i - \sum_j \tilde{n}_j \tilde{\alpha}_j)} \\ & \times \text{PE} \left( \sum_{i,j} f_{ij}^{\text{bif}} + \sum_{i,j} f_{ij}^{\text{adj}} + \sum_{i,j} \tilde{f}_{ij}^{\text{adj}} \right). \end{aligned}$$

- The integrals run over holonomies in the nonzero-flux sector.

# The positive-monopole sector

- After integrating out the zero-flux holonomy eigenvalues at large  $N$ , [Kim, 2009]

$$\begin{aligned} I^{(+)}(x, y, z, w) = & \sum_{M_1, M_2=0}^{\infty} \sum_{\substack{n_1 \geq \dots \geq n_{M_1} > 0 \\ \tilde{n}_1 \geq \dots \geq \tilde{n}_{M_2} > 0}} (xyzw)^{\epsilon_0/2} \\ & \times \int dU d\tilde{U} e^{ik(\sum_i n_i \alpha_i - \sum_j \tilde{n}_j \tilde{\alpha}_j)} \\ & \times \text{PE} \left( \sum_{i,j} f_{ij}^{\text{bif}} + \sum_{i,j} f_{ij}^{\text{adj}} + \sum_{i,j} \tilde{f}_{ij}^{\text{adj}} \right). \end{aligned}$$

- The integrals run over holonomies in the nonzero-flux sector.
- At large  $k$ , misaligned GNO-charge contributions are suppressed by factors of order  $y_0^{\mathcal{O}(k)}$ , where  $y_0^2 = xyzw$ .

# The positive-monopole sector

- After integrating out the zero-flux holonomy eigenvalues at large  $N$ , [Kim, 2009]

$$\begin{aligned}
 I^{(+)}(x, y, z, w) = & \sum_{M_1, M_2=0}^{\infty} \sum_{\substack{n_1 \geq \dots \geq n_{M_1} > 0 \\ \tilde{n}_1 \geq \dots \geq \tilde{n}_{M_2} > 0}} (xyzw)^{\epsilon_0/2} \\
 & \times \int dU d\tilde{U} e^{ik(\sum_i n_i \alpha_i - \sum_j \tilde{n}_j \tilde{\alpha}_j)} \\
 & \times \text{PE} \left( \sum_{i,j} f_{ij}^{\text{bif}} + \sum_{i,j} f_{ij}^{\text{adj}} + \sum_{i,j} \tilde{f}_{ij}^{\text{adj}} \right).
 \end{aligned}$$

- The integrals run over holonomies in the nonzero-flux sector.
- At large  $k$ , misaligned GNO-charge contributions are suppressed by factors of order  $y_0^{\mathcal{O}(k)}$ , where  $y_0^2 = xyzw$ .
- We evaluate the  $U$  integral by residues and retain the leading contributions in the fixed- $q, a, b, c$  limit.

# The BMN index from residues

- The leading residues lie at

$$e^{i\alpha_j} = x e^{i\tilde{\alpha}_j}.$$

# The BMN index from residues

- The leading residues lie at

$$e^{i\alpha_j} = x e^{i\tilde{\alpha}_j}.$$

- Summing over these residues,

$$\begin{aligned} I^{(+)} &\longrightarrow \sum_{m=0}^{\infty} q^m \sum_{\lambda \vdash m} \int [d\tilde{U}]_{\lambda} \text{PE} \left[ f_{\text{eff}}^{\lambda}(a, b, c; \tilde{U}) \right] \\ &= \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}(a, b, c). \end{aligned}$$

# The BMN index from residues

- The leading residues lie at

$$e^{i\alpha_j} = x e^{i\tilde{\alpha}_j}.$$

- Summing over these residues,

$$\begin{aligned} I^{(+)} &\longrightarrow \sum_{m=0}^{\infty} q^m \sum_{\lambda \vdash m} \int [d\tilde{U}]_{\lambda} \text{PE} \left[ f_{\text{eff}}^{\lambda}(a, b, c; \tilde{U}) \right] \\ &= \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}(a, b, c). \end{aligned}$$

- This matches with the integral form of the BMN index. [Chang, 2024]

# The BMN index from residues

- The leading residues lie at

$$e^{i\alpha_j} = x e^{i\tilde{\alpha}_j}.$$

- Summing over these residues,

$$\begin{aligned} I^{(+)} &\longrightarrow \sum_{m=0}^{\infty} q^m \sum_{\lambda \vdash m} \int [d\tilde{U}]_{\lambda} \text{PE} \left[ f_{\text{eff}}^{\lambda}(a, b, c; \tilde{U}) \right] \\ &= \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}(a, b, c). \end{aligned}$$

- This matches with the integral form of the BMN index. [Chang, 2024]
- Each partition  $\lambda \vdash m$  labels a BMN vacuum. The coefficient of  $q^m$  sums the index over all vacua at rank  $m$ .

# The index matching

- In the triple-scaling limit, with  $q, a, b, c$  fixed,

$$\lim_{\substack{N, k \rightarrow \infty \\ N/k^2 \text{ fixed}}} \frac{I_{\text{ABJM}}}{I_{\text{ABJM}}^{(0)}} = \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}(a, b, c).$$

# The index matching

- In the triple-scaling limit, with  $q, a, b, c$  fixed,

$$\lim_{\substack{N, k \rightarrow \infty \\ N/k^2 \text{ fixed}}} \frac{I_{\text{ABJM}}}{I_{\text{ABJM}}^{(0)}} = \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}(a, b, c).$$

- The factorization suggests a decoupling of the neutral KK sector from the positive-momentum sector.

# Large-rank behavior of the BMN index

At large  $N$ , the ABJM index matches the multiparticle graviton index on  $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ .  
[Kim, 2009] Using this, we can write a very simple formula for grand canonical BMN index.

$$\frac{I_{\text{grav}}}{I_{\text{grav}}^0} \longrightarrow \frac{1}{\prod_{n=1}^{\infty} (1 - q^n)} \prod_{n=1}^{\infty} \prod_{i=1}^{\infty} \frac{(1 - q^n (abc)^i)}{(1 - q^n a^i)(1 - q^n b^i)(1 - q^n c^i)} = \sum_{m=0}^{\infty} q^m I_{\text{BMN}}^{(m)}, \quad (1)$$

For  $a = b = c = s$ , the index entropy at excitation charge  $Q = \mathcal{O}(m^2)$  grows as  $S_{\text{index}} = \mathcal{O}(m)$ .

**Correlator match**

## ADHM theory and correlator match

- We propose an analogous triality involving three-dimensional  $\mathcal{N} = 4$  ADHM theory. This theory appears on worldvolume theory of D2 branes dissolved in D6 branes.

## ADHM theory and correlator match

- We propose an analogous triality involving three-dimensional  $\mathcal{N} = 4$  ADHM theory. This theory appears on worldvolume theory of D2 branes dissolved in D6 branes.
- The theory contains a  $U(N)$  vector multiplet, an adjoint hypermultiplet, and  $N_f$  fundamental hypermultiplets.

## ADHM theory and correlator match

- We propose an analogous triality involving three-dimensional  $\mathcal{N} = 4$  ADHM theory. This theory appears on worldvolume theory of D2 branes dissolved in D6 branes.
- The theory contains a  $U(N)$  vector multiplet, an adjoint hypermultiplet, and  $N_f$  fundamental hypermultiplets.
- Its holographic dual is M-theory on  $AdS_4 \times S^7/\mathbb{Z}'_{N_f}$ , whose appropriate Penrose limit yields the same pp-wave background. [Pelc, Siebelink, 1999]

## ADHM theory and correlator match

- We propose an analogous triality involving three-dimensional  $\mathcal{N} = 4$  ADHM theory. This theory appears on worldvolume theory of D2 branes dissolved in D6 branes.
- The theory contains a  $U(N)$  vector multiplet, an adjoint hypermultiplet, and  $N_f$  fundamental hypermultiplets.
- Its holographic dual is M-theory on  $AdS_4 \times S^7/\mathbb{Z}'_{N_f}$ , whose appropriate Penrose limit yields the same pp-wave background. [Pelc, Siebelink, 1999]
- We find that the ADHM superconformal index reproduces the BMN index in the triple-scaling limit.

# ADHM theory and correlator match

- We propose an analogous triality involving three-dimensional  $\mathcal{N} = 4$  ADHM theory. This theory appears on worldvolume theory of D2 branes dissolved in D6 branes.
- The theory contains a  $U(N)$  vector multiplet, an adjoint hypermultiplet, and  $N_f$  fundamental hypermultiplets.
- Its holographic dual is M-theory on  $AdS_4 \times S^7/\mathbb{Z}'_{N_f}$ , whose appropriate Penrose limit yields the same pp-wave background. [Pelc, Siebelink, 1999]
- We find that the ADHM superconformal index reproduces the BMN index in the triple-scaling limit.
- Exact localization results for protected Coulomb-branch correlators make ADHM theory a natural setting for testing the triality beyond the index. [Dedushenko, Fan, Pufu, Yacoby, 2018, 2019]

- On the ADHM side, we consider protected Coulomb-branch correlators involving monopole operators. [Dedushenko, Fan, Pufu, Yacoby, 2018, 2019]

- On the ADHM side, we consider protected Coulomb-branch correlators involving monopole operators. [Dedushenko, Fan, Pufu, Yacoby, 2018, 2019]
- The 'screened' monopole GNO charges specify the BMN fuzzy-sphere vacuum. Neutral operator insertions map to protected excitations around that vacuum.

- On the ADHM side, we consider protected Coulomb-branch correlators involving monopole operators. [Dedushenko, Fan, Pufu, Yacoby, 2018, 2019]
- The 'screened' monopole GNO charges specify the BMN fuzzy-sphere vacuum. Neutral operator insertions map to protected excitations around that vacuum.
- In the triple-scaling limit, both the localization measures and the localized operator insertions match. This reproduces the protected BMN correlators. [Asano, Ishiki, Shimasaki, Terashima, 2018]

- We propose an embedding of the BMN quantum mechanics at finite rank inside monopole sectors of ABJM or ADHM theory.

# Summary

- We propose an embedding of the BMN quantum mechanics at finite rank inside monopole sectors of ABJM or ADHM theory.
- The total monopole charge gives the BMN rank. The dimensionless BMN coupling satisfies  $g_{\text{BMN}}^2 \propto N/k^2$  for ABJM and  $g_{\text{BMN}}^2 \propto N/N_f^2$  for ADHM.

# Summary

- We propose an embedding of the BMN quantum mechanics at finite rank inside monopole sectors of ABJM or ADHM theory.
- The total monopole charge gives the BMN rank. The dimensionless BMN coupling satisfies  $g_{\text{BMN}}^2 \propto N/k^2$  for ABJM and  $g_{\text{BMN}}^2 \propto N/N_f^2$  for ADHM.
- The surviving GNO-charge configurations are in one-to-one correspondence with the BMN fuzzy-sphere vacua.

# Summary

- We propose an embedding of the BMN quantum mechanics at finite rank inside monopole sectors of ABJM or ADHM theory.
- The total monopole charge gives the BMN rank. The dimensionless BMN coupling satisfies  $g_{\text{BMN}}^2 \propto N/k^2$  for ABJM and  $g_{\text{BMN}}^2 \propto N/N_f^2$  for ADHM.
- The surviving GNO-charge configurations are in one-to-one correspondence with the BMN fuzzy-sphere vacua.
- The supersymmetric indices match in the triple-scaling limit, providing a protected test of the triality.

# Summary

- We propose an embedding of the BMN quantum mechanics at finite rank inside monopole sectors of ABJM or ADHM theory.
- The total monopole charge gives the BMN rank. The dimensionless BMN coupling satisfies  $g_{\text{BMN}}^2 \propto N/k^2$  for ABJM and  $g_{\text{BMN}}^2 \propto N/N_f^2$  for ADHM.
- The surviving GNO-charge configurations are in one-to-one correspondence with the BMN fuzzy-sphere vacua.
- The supersymmetric indices match in the triple-scaling limit, providing a protected test of the triality.
- In the same limit, protected Coulomb-branch correlators in ADHM theory match protected BMN correlators.

## Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .

## Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?

## Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?

# Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?
  - pp-wave matrix string theory:  $4d \mathcal{N} = 2 \leftrightarrow$  massive ABJM

# Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?
  - pp-wave matrix string theory:  $4d \mathcal{N} = 2 \leftrightarrow$  massive ABJM
  - BFSS:  $k^2 \ll N \ll k^5$

# Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?
  - pp-wave matrix string theory:  $4d \mathcal{N} = 2 \leftrightarrow$  massive ABJM
  - BFSS:  $k^2 \ll N \ll k^5$
  - pKKT:  $\leftrightarrow ??$

# Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?
  - pp-wave matrix string theory:  $4d \mathcal{N} = 2 \leftrightarrow$  massive ABJM
  - BFSS:  $k^2 \ll N \ll k^5$
  - pKKT:  $\leftrightarrow ??$
- New notions of Fortuity in ABJM at infinite  $N$ , from BMN viewpoint?

# Open directions

- Can we test this proposal beyond protected quantities: e.g. matching anomalous dimensions of unprotected operators with energy of non-BPS BMN states? Perhaps in weakly coupled BMN regime  $N \ll k^2$ .
- Can BMN emerge from a six-dimensional theory dual to  $\text{AdS}_7 \times S^4/\mathbb{Z}_k$ ?
- Can holographic theories provide embeddings of other matrix theories?
  - pp-wave matrix string theory:  $4d \mathcal{N} = 2 \leftrightarrow$  massive ABJM
  - BFSS:  $k^2 \ll N \ll k^5$
  - pKKT:  $\leftrightarrow ??$
- New notions of Fortuity in ABJM at infinite  $N$ , from BMN viewpoint?
- Can holography clarify the large-rank BFSS conjecture?

Thank you!

# ADHM correlators

- We consider protected Coulomb-branch operators inserted on a great circle of  $S^3$ .
- The correlators contain two monopole operators with opposite charges, together with neutral scalar insertions.
- Localization reduces the calculation to an integral over Hermitian matrix eigenvalues.
- The scalar insertions become functions of these eigenvalues and the monopole charges.

## Mapping to BMN correlators

- Protected BMN correlators also reduce to matrix eigenvalue integrals, with scalar insertions depending on the chosen fuzzy-sphere vacuum.
- In our scaling limit, integrating out the zero-flux ADHM eigenvalues gives a measure matching the BMN measure, before including monopole bubbling.
- We also propose a map between the protected operator insertions in the two theories.
- The comparison is simplest for the trivial BMN vacuum, where the corresponding ADHM monopoles do not bubble.
- A possible connection between monopole bubbling and BMN instantons remains to be understood.