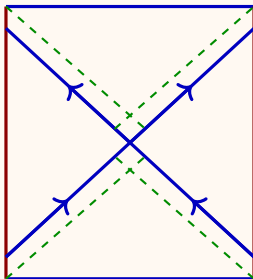


Geometry of TOCs and OTOCs in 3D de Sitter gravity

Herman Verlinde

Leinweber Network Workshop

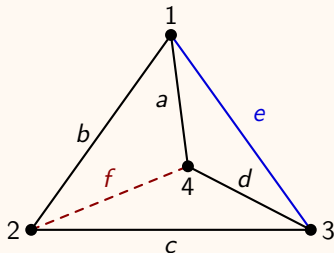
U Michigan, 10/10/2026



Based on joint work with D. Kim, T. Marini, D. Tietto and J. Xu

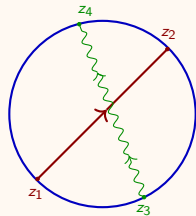
Einstein action of dS_3 OTOC evaluates to a tetrahedral volume

$$S_{EH} = \int_{\mathcal{M}} \sqrt{-g}(R - 2) = \text{Vol}(dS_3 \setminus \mathcal{T})$$

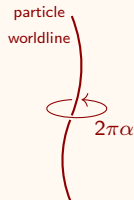
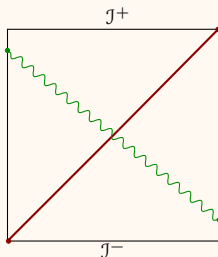


Are the SYK OTOC and the de Sitter OTOC related?

SYK OTOC = twisted two-point function =

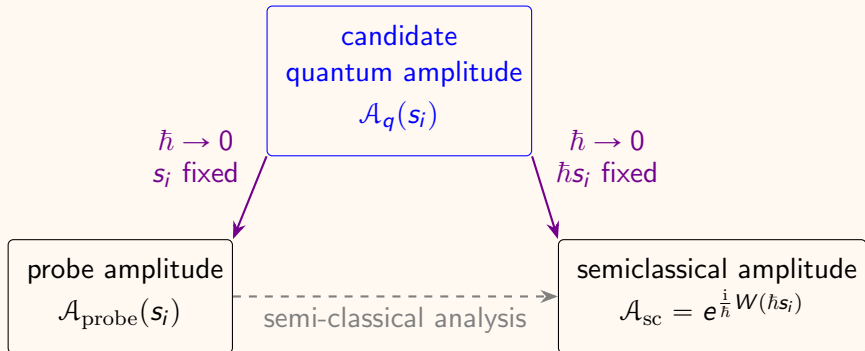


dS₃ OTOC =



$$A_{\text{OTOC}}^{\text{dS}} = \int_0^\infty \frac{dp_+ dp_-}{p_+ p_-} p_+^{iQ/2} p_-^{-iQ/2} \int_0^{2\pi\alpha} d\phi e^{ip_+ p_- h(\phi)}$$

$$q = e^{-\hbar} = e^{-8\pi G_N} = e^{-p^2/N}$$



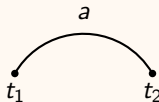
Diagrammatic rules for dS₃ TOCs and OTOCs

$$\begin{array}{c} a \\ \text{---} \text{---} \text{---} \\ t_1 \qquad t_2 \end{array} = e^{-iE(a)(t_2-t_1)}, \qquad \begin{array}{c} a \\ | \\ \text{---} c \text{---} \\ | \\ b \end{array} = \sqrt{\Gamma_8(a, b, c)}.$$

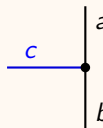
$$\Gamma_8(a, b, c) = \prod_{\sigma_1, \sigma_2, \sigma_3} \Gamma\left(\frac{1}{2} \pm \frac{i}{\hbar}(\sigma_1\theta_a + \sigma_2\theta_b + \sigma_3\theta_c)\right)$$

$$A_{\text{TOC}} = \begin{array}{c} d \\ \text{---} \text{---} \text{---} \\ e \\ \text{---} \text{---} \text{---} \\ f \\ \text{---} \text{---} \text{---} \\ a \end{array}$$

Diagrammatic rules for dS_3 TOCs and OTOCs

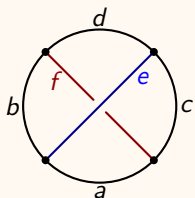


$$= e^{-iE(a)(t_2 - t_1)},$$



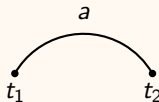
$$= \sqrt{\Gamma_8(a, b, c)}.$$

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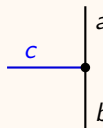


$$A_{\text{OTOC}} = A_{\text{TOC}} \times \begin{Bmatrix} a & c & e \\ b & d & f \end{Bmatrix}$$

Diagrammatic rules for dS₃ TOCs and OTOCs

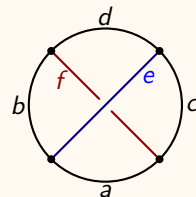


$$= e^{-iE(a)(t_2-t_1)},$$



$$= \sqrt{\Gamma_8(a, b, c)}.$$

$$\Gamma_8(a, b, c) = \prod_{\sigma_1, \sigma_2, \sigma_3} \Gamma_q\left(\frac{1}{2} \pm \frac{i}{\hbar}(\sigma_1\theta_a + \sigma_2\theta_b + \sigma_3\theta_c)\right)$$

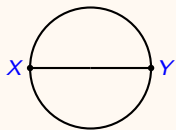


$$A_{\text{OTOC}} = A_{\text{TOC}} \times \left\{ \begin{matrix} a & c & e \\ b & d & f \end{matrix} \right\}_q$$

Euclidean 3D de Sitter Green functions satisfy a fusion algebra!

$$G_a(X, Y) G_b(X, Y) = \int_0^\infty d\rho(c) N_{abc} G_c(X, Y),$$

$$N_{abc} = \Gamma_8(a, b, c).$$



The fusion coefficients N_{abc} can be written as a Verlinde formula

$$N_{abc} = \int_0^1 d\tau \frac{S_a(\tau) S_b(\tau) S_c(\tau)}{S_1(\tau)}, \quad G_s(\tau) = \frac{S_s(\tau)}{S_1(\tau)}$$

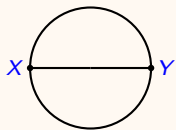
$$S_s(\tau) = \frac{\sin 2s\tau}{\sinh 2\pi s},$$

$$S_1(\tau) = \sinh \tau$$

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The q-deformation makes time discrete!

$$N_{abc} = \sum_{\tau_n = \frac{\hbar}{2} n} \frac{S_a(\tau_n) S_b(\tau_n) S_c(\tau_n)}{S_1(\tau_n)}.$$

⇔ Mass becomes an angle! ⇔

⇔ Matches CLS and doubled DSSYK! ⇔

The q-deformation makes time discrete!

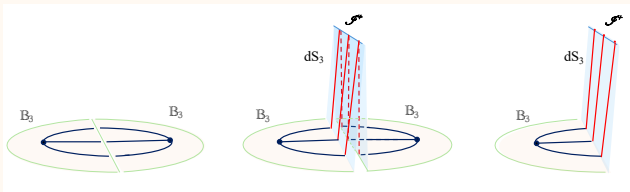
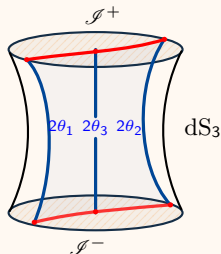
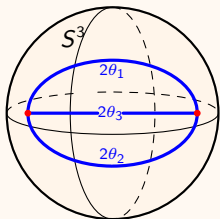
$$N_{abc} = \sum_{\tau_n = \frac{\hbar}{2} n} \frac{S_a(\tau_n) S_b(\tau_n) S_c(\tau_n)}{S_1(\tau_n)}.$$

$\Leftrightarrow$ Mass becomes an angle! $\Leftrightarrow$

$\Rightarrow$ Matches $\mathbb{CLS}$ and doubled scaled SYK^2 $\Leftarrow$

Semi-classical saddle point

$$S = \text{Li}_2(e^{2i\theta_1}) + \text{Li}_2(e^{2i\theta_2}) + \text{Li}_2(e^{2i\theta_3}) - \text{Li}_2(e^{i(\pi+\theta_3\pm\theta_1\pm\theta_2)}).$$

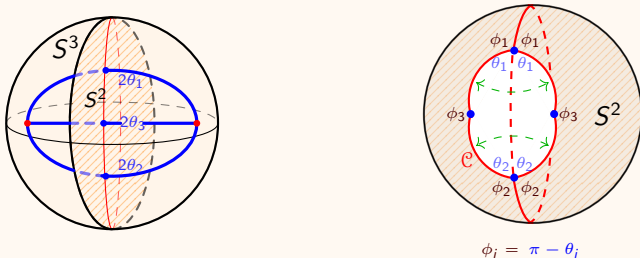


$$|\Psi(\theta_1, \theta_2, \theta_3)|^2$$

$$\Psi(\theta_1, \theta_2, \theta_3)$$

Semi-classical saddle point

$$S = \text{Li}_2(e^{2i\theta_1}) + \text{Li}_2(e^{2i\theta_2}) + \text{Li}_2(e^{2i\theta_3}) - \text{Li}_2(e^{i(\pi+\theta_3\pm\theta_1\pm\theta_2)}).$$

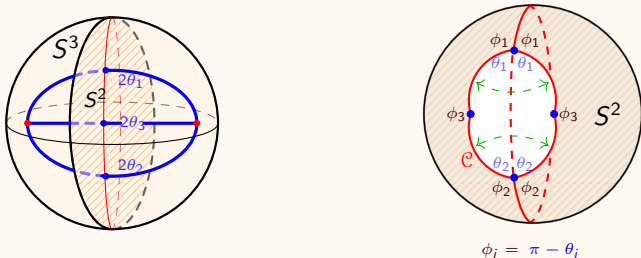


$$\text{Re}[S(\theta_1, \theta_2, \theta_3) = \text{Vol}(T)]$$

CLS and SYK² match with semi-classical dS³ gravity!

Semi-classical saddle point

$$S = \text{Li}_2(e^{2i\theta_1}) + \text{Li}_2(e^{2i\theta_2}) + \text{Li}_2(e^{2i\theta_3}) - \text{Li}_2(e^{i(\pi+\theta_3\pm\theta_1\pm\theta_2)}).$$

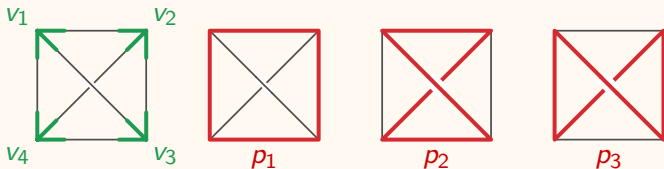


$$\text{Re}[S(\theta_1, \theta_2, \theta_3) = \text{Vol}(T)]$$

CLS and SYK² match with semi-classical dS³ gravity.

Murakami-Yano formula for tetrahedral volume

$$\text{Vol } \mathcal{T} = \text{Im } \mathcal{W}_{\text{MY}}(z_*) + \sum_v \Delta_v$$



$$\mathcal{W}_{\text{MY}}(z, \mathcal{T}) = \frac{1}{2} \left[\sum_i \text{Li}(p_i z) - \sum_j \text{Li}(-v_j z) \right].$$

$$\Delta_v = \sum_{\sigma_1 \sigma_2 \sigma_3 = -1} \text{Li}_2(e^{i\sigma_1 \theta_1 + i\sigma_2 \theta_2 + i\sigma_3 \theta_3})$$

Tetrahedral volumes are semi-classical limit of q-6j-symbol

$$\prod_i C_{v_i} \left\{ \begin{matrix} \theta_1 & \theta_2 & \theta \\ \theta_4 & \theta_3 & \theta \end{matrix} \right\} = \prod_i \Gamma_4(v_i) S(v, p)$$

$$S(p, v) \equiv \sum_{n \in \mathbb{Z}} \int d\nu \prod_i \frac{\Gamma_{\mathbb{C}}(\frac{1}{2} + v_i + z)}{\Gamma_{\mathbb{C}}(1 + p_i + z)}$$

$$\Gamma_{\mathbb{C}}(x+z) = \frac{\Gamma(x+z)}{\Gamma(1-x-\bar{z})}, \quad z = \frac{n}{2} + i\nu, \quad \bar{z} = \frac{n}{2} - i\nu$$

$$\Gamma_4(v) = \prod_{\sigma_1 \sigma_2 \sigma_3 = -1} \Gamma\left(\frac{1}{2} + \frac{i}{\hbar}(\sigma_1 \theta_1 + \sigma_2 \theta_2 + \sigma_3 \theta_3)\right)$$

Tetrahedral volumes are semi-classical limit of q-6j-symbol

$$\prod_{v_i} c_{v_i} \left\{ \begin{matrix} \theta_1 & \theta_2 & \theta \\ \theta_4 & \theta_3 & \theta \end{matrix} \right\}_q = \prod_i \Gamma_4(v_i) S(v, p)$$

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Relation between $SL(2, \mathbb{C})$ and SYK OTOC

$$A_{\text{OTOC}}^{\text{SL}(2, \mathbb{C})} = \left(\prod_v \gamma_q(v) \right) \langle \{6j\}_{\text{SYK}}, \{6j\}_{\text{SYK}}^v \rangle$$

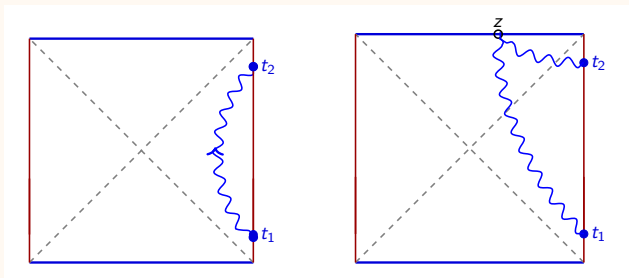
Virasoro CFT $c \in \mathbb{R}$ $(6j)_b = \exp\left(\frac{i}{\hbar} W\right)$

DSSYK/CLS $c \in 13 \pm i\mathbb{R}$ $(6j)_q = \exp\left(\frac{1}{\hbar} W\right)$

Geometry	q-parameter	Pairing	Exponent
Euclidean AdS_3	$q_b = e^{i\hbar}$	$6j_b \overline{6j_b}$	$4\text{Im}W/\hbar$
Lorentzian AdS_3	$q_b = e^{i\hbar}$	$6j_{b,L} 6j_{b,R}$	$2i(W_L + W_R)/\hbar$
Euclidean dS_3	$q = e^{-\hbar}$	$\langle 6j_q, \overline{6j_q} \rangle$	$4\text{Re}W/\hbar$
Lorentzian dS_3	$q = e^{-\hbar}$	$\langle 6j_q, 6j_{qv} \rangle$	$4i\text{Im}W/\hbar$

Physical operators in SYK²

$$O(t) = \int ds O_L(s) O_R(t-s) \quad \Rightarrow \quad \langle O(t) O(0) \rangle = G_{\text{dS}}(t+i\pi)$$



$$\mathcal{A}_{\text{SYK}^2}(t) = \langle V(t) W(0) V(t) W(0) \rangle$$

de Sitter₃ OTOC in recoil limit

$$\mathcal{A}_{\text{dS}}(Q) = \frac{\Gamma(iQ)}{\Gamma(1-iQ)} = \sinh(\pi Q) \Gamma(iQ)^2$$

$$\mathcal{A}_{\text{SYK}}(Q) = \Gamma(iQ) \qquad \mathcal{A}_{\text{SYK}^2}(Q) = \Gamma(iQ)^2$$

$$\mathcal{A}_{\text{dS}}(t) = \mathcal{A}_{\text{SYK}^2}(t + i\pi) - \mathcal{A}_{\text{SYK}^2}(t - i\pi)$$

Q = energy dual to OTOC time variable t

Deconstructing the de Sitter TOC and OTOC

$$\hat{V} = \begin{pmatrix} 0 & \zeta V_+ \\ \zeta^{-1} V_- & 0 \end{pmatrix}, \quad V_{\pm}(t) = V(t \pm \frac{i\pi}{2})$$

$$|\Omega\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}, \quad \zeta^8 = -1.$$

OTOC

$$\mathcal{A}_{\text{dS}}(t) = \langle \hat{V}(t) \hat{W}(0) \hat{V}(t) \hat{W}(0) \rangle$$

TOCs

$$\Omega | \hat{V}(t) \hat{V}(0) | \Omega \rangle = \frac{1}{2} (G_A(t + i\pi) + G_A(t - i\pi)).$$

$$\begin{aligned} & \langle \Omega | \hat{V}(x_1) \hat{W}(y_1) \hat{W}(y_2) \hat{V}(x_2) | \Omega \rangle = \\ & \frac{1}{2} \left[G_A(x_{12} + i\pi) G_A(y_{12} - i\pi) + G_A(x_{12} - i\pi) G_A(y_{12} + i\pi) \right]. \end{aligned}$$

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OTOC

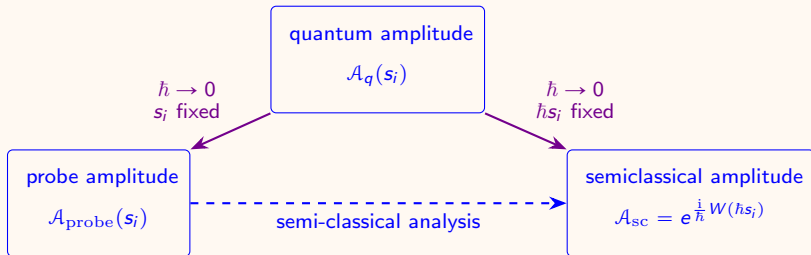
$$\mathcal{A}_{\text{dS}}(t) = \langle \hat{V}(t) \hat{W}(0) \hat{V}(t) \hat{W}(0) \rangle$$

$$\langle \hat{V}(t) \hat{V}(0) \rangle = \frac{1}{2} (G_A(t + i\pi) + G_A(t - i\pi)),$$

$$\begin{aligned} \langle \hat{V}(x_1) \hat{W}(y_1) \hat{W}(y_2) \hat{V}(x_2) \rangle = \\ \frac{1}{2} \left[G_A(x_{12} + i\pi) G_A(y_{12} - i\pi) + G_A(x_{12} - i\pi) G_A(y_{12} + i\pi) \right]. \end{aligned}$$

Concluding remarks

- Study of TOCs and OTOCs in dS_3 using "q-Virasoro TQFT".
- Proposed explicit expressions via q-deformation of $SL(2, \mathbb{C})$.
- Reproduces the correct probe and semi-classical amplitudes.
- Quantitative matches with $\mathbb{C}LS$ and SYK^2 model.
- Open problems: shockwave with defect angle and G_N corrections.



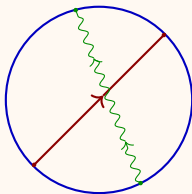
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$$A_{\text{OTOC}}^{\text{dS}} = \int_0^\infty \frac{dp_+ dp_-}{p_+ p_-} p_+^{iQ/2} p_-^{-iQ/2} \int_0^{2\pi\alpha} d\phi e^{ip_+ p_- h(\phi)}$$

OTOC = twisted 2-point function

$$G_\ell(\tau_1, \tau_2; \tau_3, \tau_4) = \left\langle \psi(\tau_1) \psi(\tau_2) \exp \left[\frac{\ell}{2} \sum_i \psi_i(\tau_3) \psi_i(\tau_4) \right] \right\rangle_\beta$$



$$\mathcal{G} = \left[\frac{\cos\left(\frac{\pi\nu}{2}\right)}{e^{-\ell} \sin \phi_{13} \sin \phi_{23} + e^{\ell} \cos\left(\phi_{13} + \frac{\pi\nu}{2}\right) \cos\left(\phi_{23} + \frac{\pi\nu}{2}\right)} \right]^{2\Delta}$$

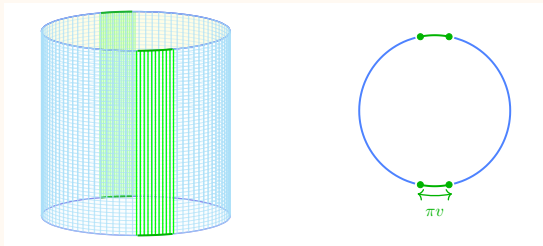
Thermal SYK 2-point function

$$G(\tau_1, \tau_2) \equiv \langle O_{\Delta}(\tau_1) O_{\Delta}(\tau_2) \rangle_{\beta} = \left(\frac{\cos\left(\frac{\pi\nu}{2}\right)}{\cos\left(\frac{\phi_1 - \phi_2}{2}\right)} \right)^{2\Delta}$$

$$\phi_1 = \nu \left(\frac{2\pi}{\beta} \tau_1 + \frac{\pi}{2} \right)$$

$$\phi_2 = \nu \left(\frac{2\pi}{\beta} \tau_2 - \frac{\pi}{2} \right)$$

$$\beta = \frac{\pi\nu}{\cos\left(\frac{\pi\nu}{2}\right)}$$



$$\tau \rightarrow \tau + \beta_{\text{fake}},$$

$$\beta_{\text{fake}} = \frac{\beta}{\nu}$$

KMS property implies that the fake
thermal cylinder splits into a two disjoint cylinders

