

Quantum black hole cohomologies

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Leinweber Network Workshop “*Quantum Black Holes*”

LITP, University of Michigan

Oct 11, 2026

Talk based on: Some review on classical cohomology

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“Quantum black hole cohomologies” arXiv:2606.27955.

Introduction

BPS black hole states in $AdS_5 \times S^5$ from heavy BPS operators in maximal SYM

- Studies at strong coupling $\rightarrow$ hard.
- Study at **weak-coupling**: Hope non-renormalization certain properties (spectrum)
& qualitative lessons on these quantum states.
- BPS BH entropy counted from protected index $\rightarrow$ Ensures certain weak-coupling studies

Various weak-coupling studies:

- Free limit $\rightarrow$ Too many BPS states than supergravitons, even at low $E \ll N$.
(Non-BPS oscillating strings become accidentally BPS at $\lambda = Ng_{YM}^2 \rightarrow 0$.)
- 1-loop BPS states: Correct low E spectrum (accidentally BPS string modes lifted)
 $\rightarrow$ Question: BPS BH states at $E \sim N^2$?
- Some (many?) 1-loop BPS states may still lift by higher order corrections, but qualitatively better describes the BPS spectrum & provides a good starting point for quantum studies.

Plan: classical supercharge cohomologies ($\sim$ 1-loop BPS) & their quantum corrections

Classical cohomology

1/16-BPS states in 4d maximal SYM

3 chiral multiplets: $\phi_m, \bar{\phi}^m$ and $\psi_{m\alpha}, \bar{\psi}^m_{\dot{\alpha}}$ ($m = 1, 2, 3$)

vector multiplet: $A_\mu \sim A_{\alpha\dot{\beta}}$ and $\lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}$ ($\mu = 1, \dots, 4$) ($\alpha = \pm, \dot{\alpha} = \pm$)

- Among $16Q + 16S$, pick a pair Q & $S = Q^\dagger$ that the BPS states preserve.

$$2\{Q, Q^\dagger\} = E - E_{BPS} = E - (R_1 + R_2 + R_3 + J_1 + J_2)$$

- Nilpotency $Q^2 = 0$: BPS states $\sim$ harmonic forms $\xleftrightarrow{1 \text{ to } 1}$ Q-cohomology classes.

(Q-closed operators w/ $O \sim O + [Q, \Lambda]$)

Gauge-invariant local BPS operators: (at $x^\mu = 0$ on R^4)

- Free ($g_{YM} \rightarrow 0$): All gauge invariants of the **invariant fields** under Q & $Q^\dagger$:

$$\bar{\phi}^m, \psi_{m+}, \bar{\lambda}_{\dot{\alpha}}, f_{++} \equiv F_{1+i2, 3+i4} \text{ \& holomorphic derivatives } \partial_1 + i\partial_2 \equiv \partial_{++}, \partial_3 + i\partial_4 \equiv \partial_{+ \dot{+}}$$

- Many of them are non-BPS at $g_{YM} \neq 0$: At small g_{YM} ,

$$Q \bar{\phi}^m = 0, Q \psi_{m+} \sim g_{YM} \epsilon_{mnp} [\bar{\phi}^n, \bar{\phi}^p], Q f_{++} \sim g_{YM} [\psi_{m+}, \bar{\phi}^m], Q \bar{\lambda}_{\dot{\alpha}} = 0, [Q, D_{+\dot{\alpha}}] \sim g_{YM} [\bar{\lambda}_{\dot{\alpha}}, \cdot]$$

- Classical Q w/ interaction $\rightarrow$ anomalous dimension $QQ^\dagger + Q^\dagger Q \sim E - E_{BPS}$ at **1-loop**, $O(g_{YM}^2)$.

$\rightarrow$ “**Classical cohomology $\sim$ 1-loop BPS states**”

Gravitons vs. black holes

Constructing the representatives of supergraviton cohomologies:

1. (Partly) anti-symmetrized scalars in trace are Q-exact. E.g.

$$\text{tr}(\bar{\phi} \cdots [\bar{\phi}^m, \bar{\phi}^n] \cdots \bar{\phi}) \sim Q \text{tr}(\bar{\phi} \cdots \epsilon^{mnp} \psi_{p+} \cdots \bar{\phi})$$

Symmetrized scalars $\rightarrow$ chiral primaries of single trace ($\sim$ single particle) cohomologies

$$\text{tr}[\bar{\phi}^{(m_1} \cdots \bar{\phi}^{m_n)}]$$

2. $PSU(1,2|3) \subset PSU(2,2|4)$ descendants: All single-trace cohomologies

E.g. $\text{tr}(\bar{\phi}^{(m} \bar{\phi}^{n)})$, $\text{tr}(\bar{\phi}^m \bar{\lambda}_{\dot{\alpha}})$, $\text{tr}(\bar{\lambda}_{\dot{\alpha}} \bar{\lambda}_{\dot{\beta}})$,

$$\text{tr}(\bar{\phi}^m \psi_{n+}) - \frac{1}{3} \delta_n^m \text{tr}(\bar{\phi}^l \psi_{l+})$$
 , $\text{tr}(\bar{\lambda}_{\dot{\alpha}} \psi_{m+} - \epsilon_{mnp} \bar{\phi}^n D_{+\dot{\alpha}} \bar{\phi}^p)$,
$$\text{tr}(\bar{\phi}^m f_{++} - \frac{1}{4} \epsilon^{mnp} \psi_{n+} \psi_{p+})$$
 , $\text{tr}(\bar{\lambda}_{\dot{\alpha}} f_{++} - \frac{2}{3} \psi_{m+} D_{+\dot{\alpha}} \bar{\phi}^m + \frac{1}{3} \bar{\phi}^m D_{+\dot{\alpha}} \psi_{m+})$

& the derivatives $\partial_{+\dot{\alpha}}$ acting on them (conformal descendants)

3. Products of all these single-traces: multi-trace ($\sim$ multi-particle) cohomologies

- Large N & low E: $\#(\text{trace}) = \#(\text{particle})$. This construction provides BPS multi-gravitons.
- We regard **all the rest** as (at least containing) “**black hole cohomologies.**”
- To be more precise, we have to (re)view **all these cohomologies at finite N.**

Monotonous vs. fortuitous

Even at finite N ($E > N$), “gravitons” (previous slide) still represent cohomologies:

- Continue to be Q-closed at any matrix size: didn’t use specific value of N .
- But these operators may be redundant (i.e. overcomplete) due to trace relations:
E.g. for $SU(2)$, $2 \operatorname{tr}(Z^4) - [\operatorname{tr}(Z^2)]^2 = 0$
- They are “giant gravitons”: heavy particles polarized to D3-branes

A characterization of the two types of finite N cohomologies: [Chang, Lin] (2024)

- gravitons → “monotone”
Since I didn’t specify N on the previous slide, Q-closed at all N → sequence of operators
- black hole states → “fortuitous”: all the remainders (those not cohomologous to gravitons)
A necessary condition for this type → Q-closed by trace relations at specific N .
- This algebraic characterization distinguishes two types even at small rank, e.g. $SU(2)$.
- The latter type depends more erratically on discrete N . (grav. coupling $G_N \sim 1/N^2$)
- Many examples constructed/studied in finite N (often ‘small N ’) models
(maximal SYM, the conifold SCFT, 3d ABJM, 2d D1-D5 SCFT, SUSY SYK, other toy models)

Quantum corrections

Quantum correction → They may survive or get lifted to non-BPS states.

- Convention/notation change: $Q\Phi_{can} \sim g_{YM}[\Phi_{can}, \Phi_{can}] \rightarrow \Phi_{can} = \Phi/g_{YM} \rightarrow Q\Phi \sim [\Phi, \Phi]$.
- From now on, call this tree-level supercharge Q_0 , and exact supercharge Q .

Perturbative corrections to Q and cohomologies:

$$\begin{aligned} Q &= Q_0 + g_{YM}^2 Q_1 + O(g_{YM}^4) & Q\mathcal{O} &= (Q_0 + g_{YM}^2 Q_1)(\mathcal{O}_0 + g_{YM}^2 \mathcal{O}_1) + O(g_{YM}^4) \\ \mathcal{O} &= \mathcal{O}_0 + g_{YM}^2 \mathcal{O}_1 + O(g_{YM}^4) & &= Q_0\mathcal{O}_0 + g_{YM}^2 (Q_1\mathcal{O}_0 + Q_0\mathcal{O}_1) + O(g_{YM}^4) \end{aligned}$$

- Start from a classical cohomology $\mathcal{O}_0$ with respect to Q_0 .
- Promoted to 1-loop cohomology if Q -closed & not Q -exact at 1-loop level:

Q -closed : $Q_1\mathcal{O}_0$ is Q_0 -exact.

Not Q -exact: Shouldn't be $\mathcal{O}_0 = Q_1\Lambda_0 + Q_0\Lambda_1$ w/ a classical cohomology Λ_0

At 1-loop, some classical cohomologies can pair & “lift” to be non-BPS.

- Such 1-loop lifts were first found in the $SO(7)$ maximal SYM, related to the S-duality of the $SO(7)/Sp(3)$ pairs. [Gadde, E. Lee, Raj, Tomar] [Chang, Lin] [J. Choi, E. Lee] (2025)
- We shall discuss the 1-loop effects in the $SU(2)$ & related theories.

$SU(2)$: classical

Many $SU(2)$ classical cohomologies known. E.g.

1. An ∞ tower of “core” fortuitous cohomologies ($n = 0, 1, 2, \dots$; $E_n = 19/2 + 4n$)

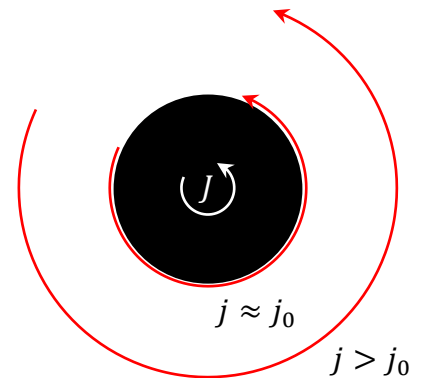
$$\begin{aligned} O_n = & \text{tr}(ff)^n \epsilon^{c_1 c_2 c_3} \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\phi^b \psi_{c_2}) \text{tr}(\psi_a \psi_b \psi_{c_3}) \\ & - \frac{n}{2} \text{tr}(ff)^{n-1} \epsilon^{b_1 b_2 b_3} \epsilon^{c_1 c_2 c_3} \text{tr}(f \psi_{b_1}) \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\psi_{b_2} \psi_{c_2}) \text{tr}(\psi_a \psi_{b_3} \psi_{c_3}) \\ & + \frac{2n^2 + n}{432} \text{tr}(ff)^{n-1} \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} \epsilon^{c_1 c_2 c_3} \text{tr}(\psi_{a_1} \psi_{b_1} \psi_{c_1}) \text{tr}(\psi_{a_2} \psi_{b_2} \psi_{c_2}) \text{tr}(\psi_{a_3} \psi_{b_3} \psi_{c_3}) \end{aligned}$$

- May look a bit odd, in that the would-be BH sector exhibits very regular/organized pattern of states in E . Would they remain in the full quantum BPS spectrum?

2. Product cohomologies: (O_n or its descendants) $\times g_j$

g_j : graviton cohomology with spin j .

- Many of them are nontrivial if j of graviton is large enough.
- Interpreted as $N = 2$ version of microstates for “grey galaxy” type dressed BPS BH’s [SK, Kundu, E. Lee, J. Lee, Minwalla, Patel] (2023)



“ $SU(2)$ related” $SU(3)$: classical

3. ‘dual giant dressed’ BH & microstates:

- D3 domain wall carries electric charges

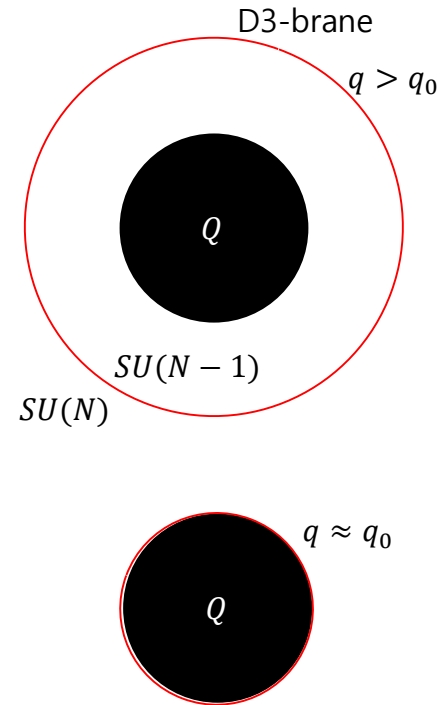
[S. Choi, Jain, SK, Krishna, E. Lee, Minwalla, Patel] (2024)

- Morally, core BH is an $SU(N - 1)$ phenomenon:

So should be the microstate cohomologies.

- Promoting a BH cohomology O_{N-1} of $SU(N-1)$ to $SU(N)$ operator fails to be Q-closed due to fortuity.

$$Q[(O_{N-1} \text{ promoted to } SU(N))(\text{dual giant operator})] \neq 0$$



Want to realize a brane “wrapping” BH: Considered “ $N = 3$ ” version

- suitable “fusion product” (gauge orientations entangled)

$$O_0 = \epsilon^{c_1 c_2 c_3} \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\phi^b \psi_{c_2}) \text{tr}(\psi_a \psi_b \psi_{c_3}) \text{ of } SU(2) + \text{try to 'wrap it' w/ a dual giant } \sim \phi\phi\phi$$

$$\rightarrow \text{tr}(\phi\phi\psi)\text{tr}(\phi\phi\psi)\text{tr}(\phi\psi\psi\psi) + \text{tr}(\phi\psi)\text{tr}(\phi\phi\psi)\text{tr}(\phi\phi\psi\psi\psi) + \dots$$

- Constructed ∞ tower of such brane-fused cohomologies of the form: $O_0 \times \phi^n$

for $n = 3, 4, 5, \dots$ [de Mello Koch, M. Kim, SK, J. Lee, S. Lee] (2024)

$SU(2)$ or “ $SU(2)$ related”: 1-loop

Core ‘BH states’ for $SU(2)$: [SK, S. Kim, S. Lee, J. Park] (2026)

- Lightest one is **unlifted**: $O_0 = \epsilon^{c_1 c_2 c_3} \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\phi^b \psi_{c_2}) \text{tr}(\psi_a \psi_b \psi_{c_3})$
- Heavier ones are **lifted** to non-BPS sector: checked for $n = 1, 2, 3$

$$\begin{aligned} O_n = & \text{tr}(ff)^n \epsilon^{c_1 c_2 c_3} \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\phi^b \psi_{c_2}) \text{tr}(\psi_a \psi_b \psi_{c_3}) \\ & - \frac{n}{2} \text{tr}(ff)^{n-1} \epsilon^{b_1 b_2 b_3} \epsilon^{c_1 c_2 c_3} \text{tr}(f \psi_{b_1}) \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\psi_{b_2} \psi_{c_2}) \text{tr}(\psi_a \psi_{b_3} \psi_{c_3}) \\ & + \frac{2n^2 + n}{432} \text{tr}(ff)^{n-1} \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} \epsilon^{c_1 c_2 c_3} \text{tr}(\psi_{a_1} \psi_{b_1} \psi_{c_1}) \text{tr}(\psi_{a_2} \psi_{b_2} \psi_{c_2}) \text{tr}(\psi_{a_3} \psi_{b_3} \psi_{c_3}) \end{aligned}$$

Dressed BH states of O_0 (or its SC descendants):

- Grey galaxy type (product): Checked a few of them → **unlifted** [SK, S. Kim, S. Lee, J. Park] (2026)

$$w_2 O_0 = \text{tr} \left(\phi^1 f - \frac{1}{2} \psi^2 \psi^3 \right) \cdot \epsilon^{c_1 c_2 c_3} \text{tr}(\phi^a \psi_{c_1}) \text{tr}(\phi^b \psi_{c_2}) \text{tr}(\psi_a \psi_b \psi_{c_3})$$

$$\frac{1}{8} \partial^\alpha \text{tr} \left(f \lambda_\alpha - \frac{2}{3} D_\alpha \phi^n \psi_n + \frac{1}{3} \phi^n D_\alpha \psi_n \right) \cdot O_0$$

$$\epsilon_{mnp} (Q_+^m Q_+^n O_0) (w_2)^p$$

- dual giant dressed (fusion product): The whole ∞ tower shown to be **unlifted**.
[Seunggyu Kim, Jehyun Lee, Siyul Lee, Jaewon Song] (to appear)
- So the remaining E -towers seem to come only from gravitons/branes, not core BH states.

Estimation of the over-estimate

Over-estimation of ‘large charge **entropy**’ by classical cohomologies?

- E.g., even if 50% of the large charge classical cohomologies are quantum-lifted,

$$e^{S_{cl}} = 2 e^{S_{true}} \rightarrow S_{cl} = S_{true} + \log 2 \approx S_{true}$$

- We can’t directly address this, but did a closely related estimate based on indices.

Index vs. ‘refined index’ [Chang, Yin] (2013) for classical cohomologies.

- The index: Counts protected states w/ $(-1)^F$ [Romelsberger] [Kinney, Maldacena, Minwalla, Raju] (2005)

Strongly believed that “indicial entropy $\approx$ true 1/16-BPS entropy” asymptotically.

- ‘Refined’ index: $\exists U(1)$ ‘bonus’ symmetry of classical cohomologies.

“ $b \equiv \#$ of letters” refines the index as
$$e^{S(j_i)_{\text{protected}}} \sim e^{S_{\text{BH}}(j_i)} = \sum_b (\pm) e^{S(j_i; b)}$$

$\rightarrow$ maximal term of RHS in b : a lower bound on S_{cl} .

Large charge “Cardy” limit (charge $\gg N^2$):

- We obtain $S_{cl}/S_{BH} \geq 1.012$: $\sim$ **1.2% over-estimate** [SK, S. Kim, S. Lee, J. Park] (2026)
- So most classical cohomologies are lifted at large charges: $e^{S_{BH}}/e^{S_{cl}} \leq e^{-0.012 S_{BH}} \ll 1$

Concluding remarks

Today, discussed:

- Classical cohomology program for heavy 1/16-BPS operators
- Their possible quantum lifts at 1-loop
- Irregular behaviors of BH states in N (classical) or in E (needs quantum)
- Large fraction of classical cohomologies may be lifted (e.g. Cardy limit).

“Free vs. 1-loop vs. exact” BPS entropies in the Cardy limit (charges $\gg N^2$),

$$S_{cl}/S_{BH} \geq 1.012$$

$$S_{free\ BPS}/S_{BH} \approx 1.413 \quad [\text{Kinney, Maldacena, Minwalla, Raju}] (2005)$$

- Compared to the free QFT, the “detected” over-estimate at 1-loop seems much smaller.
- Related to the amount of accidental symmetries? ∞ higher spin (free) vs. just $U(1)$ (1-loop)
- Implications to the higher loop cohomologies?