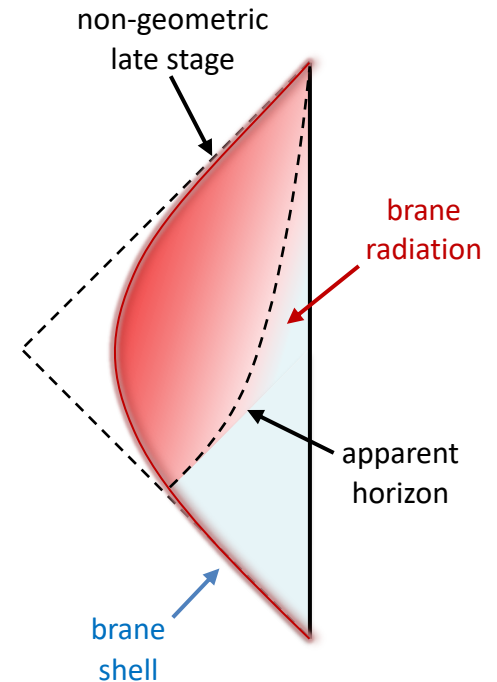


The black hole S-matrix in gauge/gravity duality

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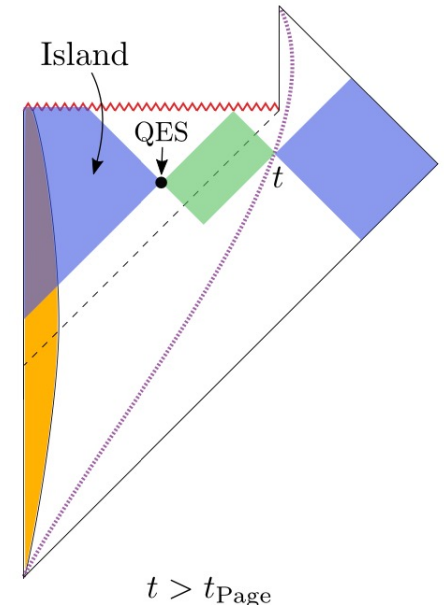
this work: [2607.18393](#)
brane Hawking rad: [2303.00234](#), [2303.17139](#)
brane + bulk EFT: [2411.16630](#), [2512.08729](#) (w/ Yoav Zigdon)

earlier work: [0904.3922](#) Horowitz-Lawrence-Silverstein
[1005.4439](#) Barbon-MartinezMagan



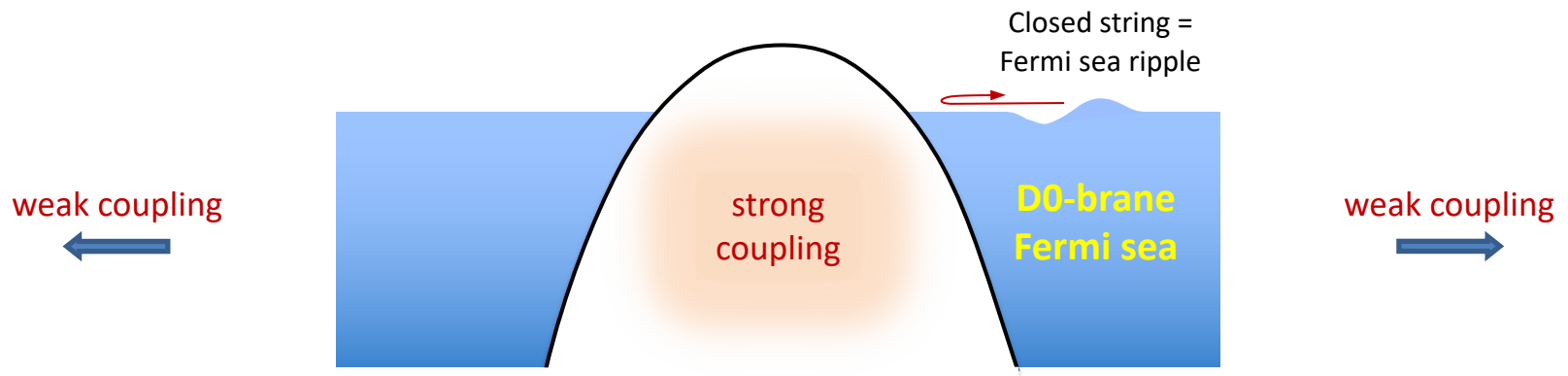
Story time in quantum gravity?

- A variety of ideas have surfaced in recent years regarding the resolution of the black hole information problem, and in general how holography works – entanglement wedges, islands, replica wormholes, baby universes ...
- How much of it is true? Do we have the right picture about how holography works, what happens when you couple a holographic field theory to an external system, *etc*?
- For instance, there are claims that you can nonlocally manipulate the state of black hole interiors by manipulating the state of Hawking radiation far away
- Rather than filtering the dynamics through some poorly understood holographic map, an alternative approach is to set up an initial condition and let the system evolve. In other words, we study the S-matrix. Can we engineer that in the context of holography?



2d string theory and matrix QM

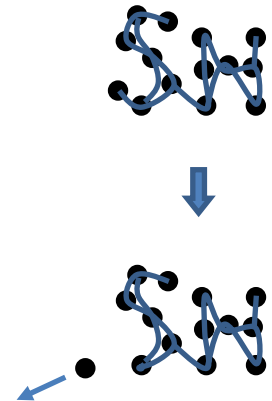
- In hindsight, the first example of holography was matrix QM, which describes 1+1d noncritical string theory. Around the same time the black hole solution of 1+1d dilaton gravity was discovered.
(Gross-Klebanov '91, Witten '91)
- But because there are no non-abelian excitations, there is no deconfined phase of the gauge theory and correspondingly **no black hole states** in the dual string theory.



- We'll consider a similar situation in which branes live in such an inverted oscillator potential, but **do** possess the non-abelian structure of black hole μ states. The branes coalesce & form a black hole near the top of the hill, which then decays.

Building a better matrix S-matrix

- Many examples of gauge/gravity duality (*e.g.* the BFSS matrix model of M-theory) admit a **Coulomb branch** for the brane dynamics, allowing the possibility to study scattering processes in which initial and final states consist of collapsing or separating branes
- A shortcoming of BFSS is that the D-brane wavefunctions spread over a large fraction of the region where it describes M-theory, and there is no known semi-classical approximation
- But the region over which a gravitational description holds extends far beyond the M-theory regime – for instance, while M-theory on $\mathbb{T}^3$ is dual to SYM on $\mathbb{T}^3$ at very low ($O(1/N)$) energies, at higher energies the gauge theory on $\mathbb{T}^3$ is dual to supergravity on the periodically identified Poincaré patch of $AdS_5 \times S^5$
- The duality then holds at arbitrarily high energies and out to spatial infinity. (The wavefunction spread still makes it difficult to localize events on sub-AdS scales)
- We can again use the Coulomb branch dynamics to construct an S-matrix ...



Refurbishing old constructions

- Our S-matrix in-state will be a collapsing shell of branes, heading in from infinity along the Coulomb branch; the intermediate state is a black hole, and the out-state consists of Hawking-radiated branes heading back out to spatial infinity.
- An abelianized effective field theory of explicit D3-brane sources coupled to supergravity describes bulk dynamics in the absence of a horizon, for instance during the collapse phase; it also describes the outgoing radiated branes far from the black hole, as well as the entire spacetime below the black hole threshold
- This EFT should emerge from the exact gauge theory dual upon integrating out non-abelian degrees of freedom that are frozen in their ground state
- When a horizon forms, low-energy non-abelian modes that are no longer frozen in their ground state ***must be integrated back in***. Not all the non-abelian modes – only those at the scale of the temperature and below. Their dynamics
 - (i) is ***local to the near-horizon region***,
 - (ii) accounts for black hole thermodynamics
 - (iii) can maintain unitarity throughout the evolution

D3-brane + supergravity EFT

- Let's consider the the joint effective action of supergravity coupled to the DBI actions for N sufficiently separated D3-branes, in the decoupling limit

$$\mathcal{S}_{sugra} = -\frac{1}{2\kappa^2} \int d^{10}x e^{-2\Phi} \sqrt{G} \left[\mathcal{R} + 4(\nabla\Phi)^2 - \frac{1}{2}H_3^2 - \frac{1}{2}F_1^2 - \frac{1}{2}F_3^2 - \frac{1}{4}F_5^2 \right] \\ - \frac{1}{4\kappa^2} \int d^{10}x C_4 \wedge H_3 \wedge F_3$$

$$\mathcal{S}_{DBI} = -\tau_3 \int d^4\xi e^{-\Phi^*} \sqrt{-\det [G^* + B^* + 2\pi\mathcal{F}]} + \tau_3 \int d^4\xi C^* \wedge e^{B^* + 2\pi\alpha' \mathcal{F}}$$

- This effective action is valid if acceleration effects are small, and the open strings stretching between the D3's are sufficiently heavy that they can be consistently integrated out – basically, when the branes are well out on their Coulomb branch.
- Let's consider the collapse of a shell of branes, and the process of horizon formation. We will also be interested in the subsequent decay via Hawking radiation of the D3's

Shell game $\mathcal{I}$

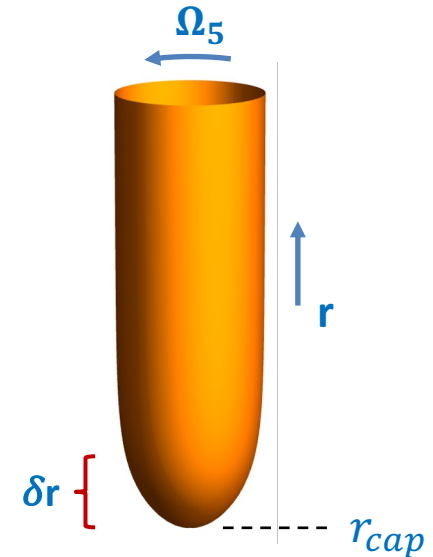
- Reference state: Consider BPS D3-branes wrapped on $\mathbb{T}^3$ and uniformly sprinkled with density $\rho(\mathbf{x})$ over the transverse $\mathbb{S}^5$ in a radial shell of thickness δr :

$$ds^2 = H^{-1/2} \left(-dt^2 + ds_{\mathbb{T}^3}^2 \right) + H^{1/2} \left(dr^2 + r^2 d\Omega_5^2 \right) , \quad e^\Phi = g_s$$

$$C_{01234} = g_s^{-1} \frac{1}{2} (H^{-1} - 1) ,$$

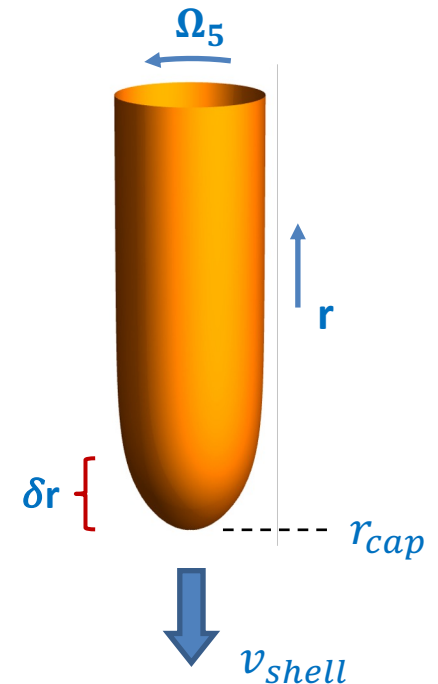
$$H(\mathbf{x}) = \int d^6 \mathbf{x}' \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^4}$$

- Any harmonic function singularities are an artifact of treating the branes in the classical supergravity approximation (*e.g.* their locations live in a wavefunction on moduli space)
- A Gaussian sphere measures the number of enclosed D3's and thus determines the proper size of $\mathbb{S}^5$; as one dives into the shell, the $\mathbb{S}^5$ shrinks until at the inner edge of the shell it smoothly pinches off, capping the throat
- The **depth** of the cap is a measure of how far out on the Coulomb branch the branes are in their configuration space (large Higgs vevs $\leftrightarrow$ shallow cap)



Shell game II

- Now consider *infalling* D3-branes wrapped on toroidal or compact hyperbolic Σ_3 . The geometry outside the brane shell is approximately that of the black brane
- Set the initial condition so that the shell moves coherently down the throat at asymptotically constant proper velocity
- Treating the infalling shell as as a domain wall, in the thin-wall limit the bulk field equations gain discontinuities governed by the *Israel junction conditions*, which determine the motion of the shell
- A probe approximation **agrees** with the back-reacted thin-shell approximation for $E_{probe} = 0$ (i.e. so that the probe's back-reaction yields the same ADM mass)
- A more precise treatment uses consistent truncations of type II supergravity (a special case involving NS5-branes is exactly solvable (Ithaki-Kutasov-Seiberg '05))



Black D3-branes

- The resulting black D3-brane geometry outside the shell is given by

$$ds^2 = \ell^2 \left(\underbrace{-f(r)dt^2 + r^2 d\Sigma_3^2}_{\parallel \text{ to branes}} + \underbrace{\frac{dr^2}{f(r)} + d\Omega_5^2}_{\perp \text{ to branes}} \right) \quad \underbrace{C_{0123}^+ = r^4}_{\text{AST gauge flux}}$$

$$f(r) = \underbrace{r^2}_{\text{AdS}} + k - \underbrace{\frac{\mu}{r^2}}_{\text{ADM}} \quad k = \begin{cases} +1 & \text{sphere} \\ 0 & \text{torus} \\ -1 & \text{hyperboloid} \end{cases} \quad \leftarrow \text{our main focus}$$

- There are horizons at $r_{\pm}^2 = \frac{1}{2} \left(-k \pm \sqrt{k^2 + 4\mu} \right)$
- For $\mu > 0$ $\exists$ one horizon ($r_-^2 < 0$), and a spacelike singularity. } any k
 - For $-1 < 4\mu < 0$ $\exists$ two horizons, and a timelike singularity. } $k = -1$ only
 - For $\mu < -1/4$ has no horizon – there is a naked singularity at $r = 0$

Brane effective action

- The DBI effective action for a probe brane with embedding $T(\xi), R(\xi)$ takes the form

$$\begin{aligned}\mathcal{S}_{D_p} &= -\tau_p \int d^{p+1}\xi \, e^{-\Phi^*} \sqrt{-\det[G^*]} + \tau_p \int C_{p+1}^* \\ &= -\tau_3 \int d\xi_0 \left(R^3 \sqrt{f(R) \dot{T}^2 - \dot{R}^2 / f(R)} - R^4 \dot{T} \right)\end{aligned}$$

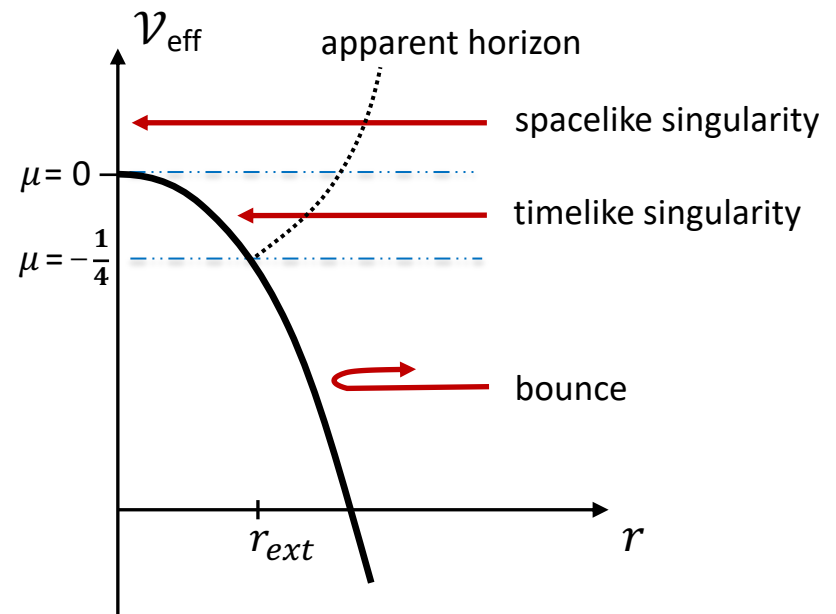
where $f(R) = R^2 + k - \mu/R^2$ is the D3-brane's emblackening factor

- The leading R^4 terms in the large R effective potential $\mathcal{V}_{eff}$ cancel, and the next term is proportional to kR^2 (the strong coupling equivalent of the conformal coupling of SYM scalars)
 - For $k = +1$ (global AdS_5 , $\mathbb{S}^3$ spatial geometry), D3's feel a confining effective potential
 - For $k = 0$ (Poincare AdS_5 , $\mathbb{T}^3$ spatial geometry) the branes have a moduli space
 - For $k = -1$ (compact hyperbolic spatial geometry $\mathbb{H}^3/\Gamma$), the effective potential is quadratically unstable, much like the matrix QM dual of 2d noncritical string theory. The difference is that now there are non-abelian excitations...

The hyperbolic effective potential

- Depending on the mass of the shell:

- For $4\mu < -1$, it bounces
- For $-1 < 4\mu < 0$, it climbs just below the top of the hill but still gets trapped and forms a black hole
- For $\mu > 0$, it crashes into itself and forms a black hole



$\mathcal{W}$ -strings and thermodynamics

- With the D3-branes uniformly spread over the angular S^5 , the energy it costs to excite W-strings is large until the branes fall to sufficiently large redshift.
- If/when the shell gets close to its apparent horizon, the redshift compensates the distance in string units between the branes, and the free energy cost of stretched strings is small and compensated by the growth in the entropy from $O(N^0)$ to $O(N^2)$
- A deconfinement transition ensues, and an apparent horizon forms.
- The thermodynamics is that of a deconfined SYM plasma:

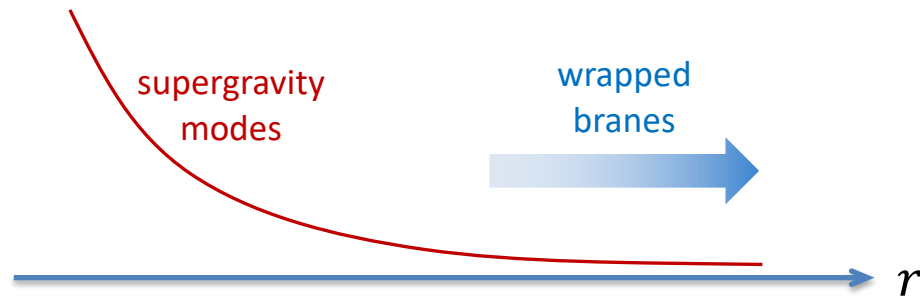
$$\ell M = \frac{3\hat{V}_\Sigma N^2}{8\pi^2} \left(\mu + \frac{1}{4} \right) = \frac{3\hat{V}_\Sigma N^2}{8\pi^2} \left(r_+^2 - \frac{1}{2} \right)^2$$

$$S = \frac{\hat{V}_\Sigma N^2}{2\pi} r_+^3$$

- Supergravity knows nothing about W-strings, yet they are the carriers of the entropy

Hawking radiation of branes

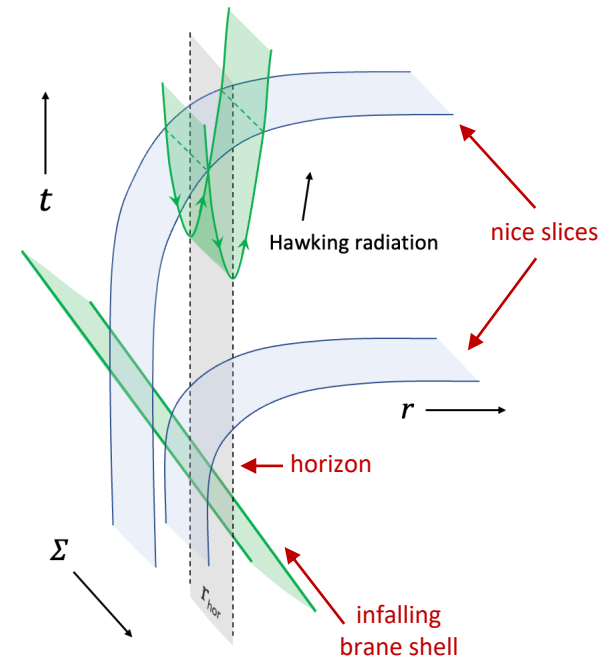
- Once the brane star collapses into a black hole, it sets up a thermal atmosphere of Hawking quanta. In the AdS decoupling limit, supergravity modes are trapped in AdS; the **only** available decay channel is to radiate the branes that source the background flux



- However, the branes are produced very near threshold, and so when the effective potential is flat, they hang around the thermal atmosphere for a very long time. We want instead a clean separation between the black hole and the radiated branes.
- We can engineer this separation by making the Coulomb branch unstable; and this option favors wrapping the branes on $\mathbb{H}^3/\Gamma$ rather than $\mathbb{T}^3$. Then the radiated branes are swept away to the asymptotic region after they are emitted.

Hawking's theory of nothing

- In a purely gravitational effective theory, ordinary matter can't be supported at the horizon; energy bounds force it to focus inward and it collapses, leaving behind a vacuum state at the horizon and beyond
- The Hawking process is described as vacuum pair creation of branes near the horizon, with the brane member of the pair tunneling out and escaping
- Vacuum structure near the horizon places the collapsing initial shell deep in the interior. Tunneling these branes out is much more highly suppressed than the Hawking process
- There is still a Page time & information paradox
- The *Small Corrections Theorem* (Mathur '09, Guo-Hughes-Mathur-Mehta '21) then forces each emitted brane to be entangled predominantly with antibranes in negative energy states just inside the horizon of the remaining black hole, and not with previously emitted branes, or those that initially formed the black hole



The Hawking process in brief

- One can use the first-quantized formalism to calculate the probability Γ for a black hole or black brane to emit quanta carrying the various conserved charges $E, Q=n_3, J, etc.$
- Particles/strings/branes travel a non-classical trajectory near the horizon; they “tunnel across” and escape
(Kraus-Wilczek '94, Parikh-Wilczek '98)
- The escaping quantum arises from either a vacuum fluctuation (Hawking) or is present in the microstate (fuzzball) wavefunction
- The WKB tunneling amplitude is $\exp[-Im \mathcal{S}]$ in terms of the classical action for the brane center of mass dynamics

$$\mathcal{S} = \tau_3 \int \left[r^3 \left(\frac{1}{\gamma} G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \gamma \right) - A_\mu \dot{x}^\mu \right] \int_{\Sigma_3} C$$


- Pass to the Hamiltonian form of the action

$$\mathcal{S} = \int (p_\mu dx^\mu - \gamma \mathcal{H})$$

The tunneling amplitude

- Solving the constraint $\mathcal{H}=0$ for p_r , the vanishing of g_{tt} near the horizon implies that p_r has a pole there (Keski-Vakkuri/Kraus '96)

$$p_r \sim \frac{E - \Omega j - \Phi q}{\kappa(r-r_0)} \quad (E=p_t, j=p_\phi \text{ etc.})$$

whose residue involves only conserved brane charges, and geometrical properties of the horizon: the angular velocity Ω , surface gravity κ , chemical potential Φ

- Deform the integration contour into the complex plane to avoid the pole, and use the principal value prescription $\lim_{\varepsilon \rightarrow 0} \frac{1}{x+i\varepsilon} = P \frac{1}{x} + \underline{i\pi \delta(x)}$.
- The integral of the trajectory across the horizon yields (using $T=\kappa/2\pi$)

$$Im \mathcal{S} = \frac{E - \Omega j - \Phi q}{2T} = - \frac{dM - \Omega dJ - \Phi dQ}{2T} = - \frac{1}{2} dS_{BH}$$

where in the middle equality we used the conservation laws
and in the last equality we have used the first law of BH thermodynamics

Emission probability

- The tunneling probability is a universal result

$$\Gamma \sim e^{-2 \Im S_{cl}} = e^{\Delta S_{BH}} = e^{S_f - S_i}$$

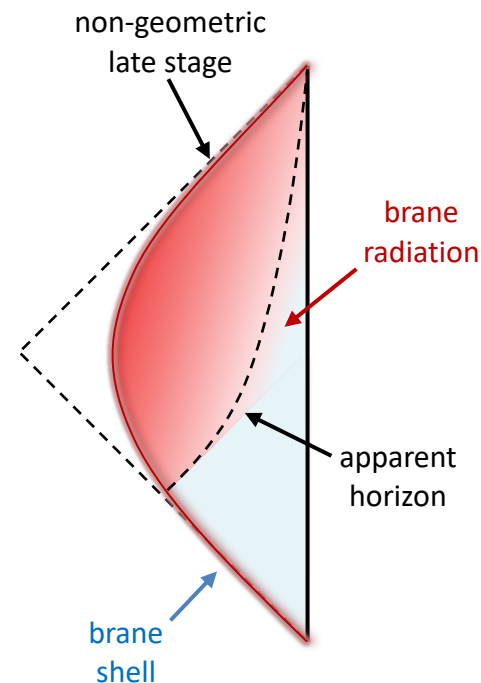
where S_{BH} is the black hole entropy as a function of the various conserved charges E, J, Q , etc. The e^{S_f} factor is the density of final states, while the e^{-S_i} factor results from ETH-like transition matrix elements

(c.f. EJM 2303.00234, 2303.17139)

- D3-brane radiation is exponentially rare: $S_{BH} \propto N^2 \Rightarrow e^{\Delta S_{BH}} \sim \exp[-\gamma N]$
- After tunneling out, the emitted brane still needs to traverse the near-horizon graybody effective potential (graybody factors affect γ)
- As the brane leaves the thermal atmosphere of the black hole, it decouples and is once again governed by the DBI effective action of the Coulomb branch
- In particular, there is no non-local interaction in the effective theory between the escaping D3-branes and those left behind, once they are far away.**

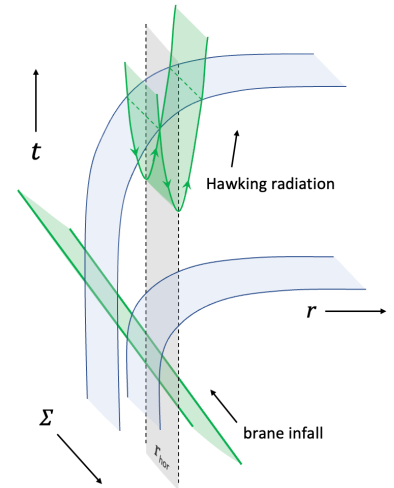
End times for black holes ...

- Each radiated brane reduces the radius of curvature of the remaining $\text{AdS}_5 \times \mathbb{S}^5$ near-horizon geometry; the D3-brane gauge theory of the remaining black hole is getting less and less strongly coupled as the gauge coupling $\lambda = g_s N$ is getting smaller
- The very late stages of evaporation take place in a stringy non-geometric near-horizon regime, even if the initial formation and intermediate black hole phase are described by semi-classical gravity
- Eventually the non-abelian excitations are ineffective at trapping the remaining D3-branes, and brane radiation is no longer exponentially suppressed. The bound state falls apart and runs away
- The asymptotic geometry at spatial infinity is *always weakly curved* $\text{AdS}_5 \times \mathbb{S}^5$ of size N , but this is simply a description of the gauge theory vacuum at scales $\gg$ Coulomb vevs



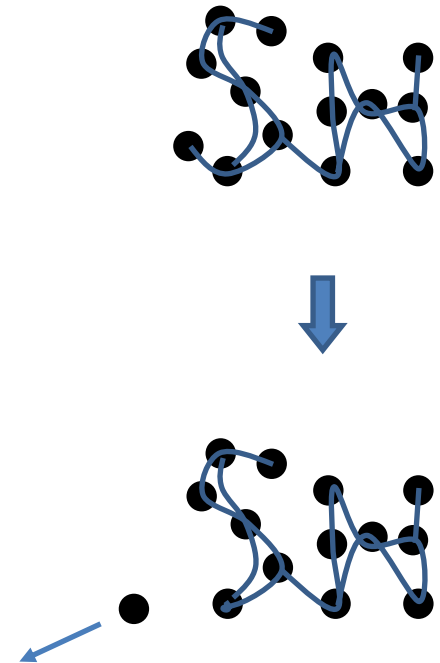
Fuzz from flux

- Keeping track of charge during the evaporation process reveals a clear dichotomy between Hawking's description of the evaporation *vs.* that of string theory:
- For Hawking,
 - The radiation stems from a brane pair creation process, a large fluctuation in the near-horizon vacuum state
 - The radiated branes are never the original branes
 - The antibrane partners remain near the horizon and screen the flux of the original infalling shell
 - The gauge group rank *increases* by two with each emission
- For string theory,
 - The radiation is a rare process of de-excitation of the thermal gas of W-strings
 - The rank of the gauge group is conserved; the part describing the hole decreases
 - The radiated branes are the original (albeit scrambled) branes
 - The branes must be available at the horizon to escape
 - The Gauss law tells us that the near-horizon region is **not** the vacuum



String theory soapbox

- Hawking radiation of charge from charged black holes is a **rare** event, but can be the dominant decay channel, leading to a black hole S-matrix **within** the context of gauge/gravity duality. Hyperbolic $\text{AdS}_5 \times \mathbb{S}^5$, dual to gauge theory on a compact hyperbolic space, provides an example
- A radiated brane leaves behind a changed near-horizon microstate, and there is no issue with causality, locality or unitarity of the evolution. Having charged objects as the dominant decay channel provides an extra handle
- *Gravity does not resolve the microstructure; the Hawking process is a thermodynamic approximation that coarse grains over the microscopics, and treats emission as a tunneling event near the horizon of the analytically continued vacuum geometry, whereas the actual microscopic process is the simultaneous de-excitation of all the W-strings that bind a given brane, allowing its escape.*



A modest proposal

- The naïve EFT of vacuum supergravity w/o explicit branes is **not the correct effective theory** in the presence of a black hole – it doesn't account for the non-abelian brane degrees of freedom that were liberated at black hole formation and are deconfined in its interior
- These dof's need to be “integrated in” and incorporated into the effective action. Not *all* the non-abelian dof's – just those on the scale of the black hole temperature and below, which are responsible for the entropy and can't be integrated out
- This deconfined phase **only** applies to the black hole interior, which is perhaps not dual to geometry.
- The exterior is described as usual by the singlet sector *i.e.* supergravity

