

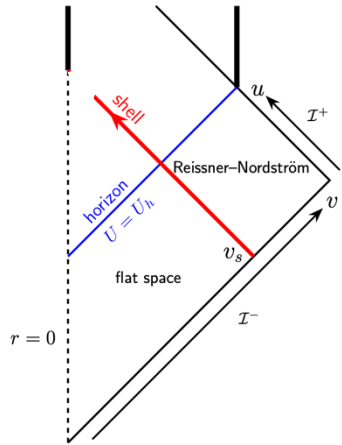
Enhanced Correlations in Hawking Radiation from Near-Extremal Collapse

Per Kraus

Mani L. Bhaumik Institute for Theoretical Physics, UCLA

- Based on 2607.12989 with S. Bintanja and S. Kim

Introduction



Collapse of spherical charged shell in asymptotically flat space to form a RN black hole (Vaidya solution)

Adding a massless scalar, we should be able to compute what comes out order-by-order in G (expect info loss!). Hawking gave answer in free field limit.

As is now well known, near extremality fluctuations of the geometry become important, and suitable observables are controlled by the effective coupling:

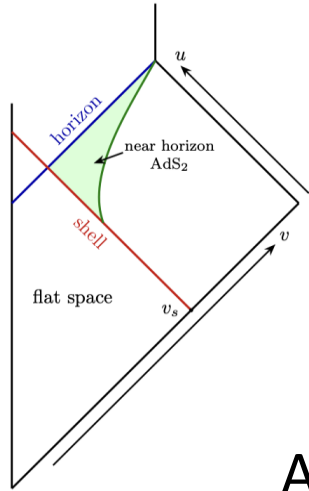
$$g \sim \frac{G}{r_0^3 T_H} \quad [\text{Preskill, Schwarz, Shapere, Trivedi, Wilczek 1991}]$$

Much activity over the past decade, based on AdS_2 , Schwarzian QM, etc. Not immediately applicable to full asymptotically flat collapse scenario.

Here I consider this problem using “old school” methods. Goal is to compute scalar field correlators on $\mathcal{I}^+$

Lightning review

Progress over past decade based on **near-horizon Schwarzan QM**



Strong coupling effects confined to AdS₂ region.
Solve theory in this region and then match to external world

$$ds_4^2 = ds_2^2 + \Phi^2(t, r) d\Omega^2, \quad \Phi = r_0(1 + \phi)$$

Action:

$$S = S_0 + \underbrace{\frac{r_0^2}{G} \int d^2x \sqrt{g_2} \phi (R_2 - \Lambda_2) + \frac{r_0^2}{G} \int_{bndy} dt \phi K_2 + O(\phi^2)}_{\text{JT gravity}}$$

Boundary curve: $\tau(u)$



JT gravity

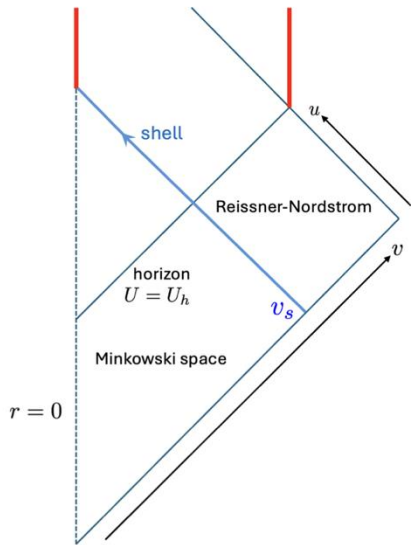
Euclidean action: $S = C \int du \text{Sch}[\tau(u)]$

[Maldacena, Stanford, Yang]

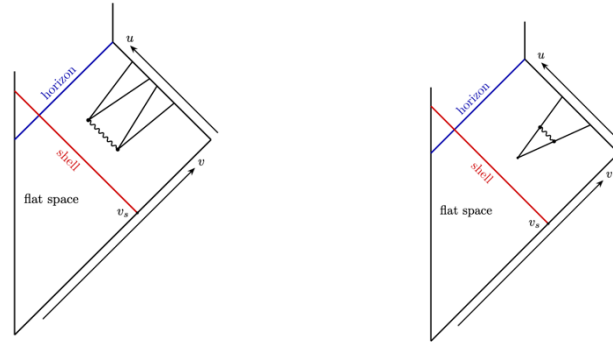
[Almheiri, Polchinski
Jensen
Engelsoy, Mertens, Verlinde;
Maldacena, Stanford, Yang
Iliesiu, Turiaci, ...]
[Brown, Iliesiu, Penington, Usatyuk
Biggs
Empanan
Betzios, Papadoulaki, Zhou
Luo, Pando-Zayas, ...]

Questions: role of dilaton as Lagrange multiplier; Euclidean versus Lorentzian; ...

Collapse problem



Collapse of spherical charged shell to form near-extremal BH.
Consider no incoming radiation (Unruh state). Want in-in expectation values of observables at future null infinity.



- $D = 3+1$ Einstein-Maxwell + charged shell + massless neutral scalar field
- Restrict to s-wave (captures strong coupling effects)
- Do not quantize shell: use it to set initial data for scalar field outside shell
- Treat scalar as free in flat region inside shell

Hamiltonian approach

Hamiltonian formulation is useful for isolating physical degrees of freedom in real time

ADM decomposition: $ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt)$

Action: $S = \int d^4x \left(\pi^{ij} \dot{h}_{ij} - N\mathcal{H} - N^i \mathcal{H}_i \right) - \int dt H_{\text{ADM}}$

Phase space variables are (π^{ij}, h_{ij}) . (N, N^i) are Lagrange multipliers enforcing constraints:

$$\mathcal{H} = \frac{1}{\sqrt{h}} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - \sqrt{h} R^{(3)} = 0$$

$$\mathcal{H}^i = -2D_j \pi^{ij} = 0$$

Poisson brackets: $\{\pi^{ij}, h_{kl}\} = \delta_k^i \delta_l^j + (i \leftrightarrow j)$

Canonical equations plus conservation of constraints are equivalent to Einstein equations

Hamiltonian approach

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt)$$
$$\mathcal{H} = \frac{1}{\sqrt{h}} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - \sqrt{h} R^{(3)} = 0 \quad \mathcal{H}^i = -2D_j \pi^{ij} = 0$$

Aside from UV issues, quantization involves various subtleties

Positivity of $M_{\text{ADM}}[h_{ij}]$ for AF initial data implies that not all h_{ij} allowed. [Ashtekar, Horowitz]

Consequence of quadratic dependence on momenta, unlike in gauge theory.

Quantizing as $\Psi = \Psi[h_{ij}]$, $\pi^{ij} = -i \frac{\delta}{\delta h_{ij}}$ lapse and shift do not appear; they are

derived quantities. Nothing obviously forbids, say, $N(x^i) = 0$ at some x^i . In fact needed to describe some physical reasonable phenomena, such as tunneling of false vacuum bubbles [Farhi, Guth, Guven] [Fischler, Morgan, Polchinski]

For Hamiltonian JT gravity, restriction to AdS2 metrics requires equations of motion, not just constraints, so doesn't hold inside phase space path integral.

S-wave Hamiltonian

Build on earlier work [Berger, Chitre, Moncrief, Nutku; Unruh]

s-wave reduction: $ds^2 = -N^2(t, r)dt^2 + L^2(t, r) (dr + N^r(t, r)dt)^2 + R^2(t, r)d\Omega^2$
 $A = A_t(t, r)dt$, $\phi = \phi(t, r)$

(N, N^r) enforce constraints: $\mathcal{H}_t^\phi + \mathcal{H}_t^G = 0$, $\mathcal{H}_r^\phi + \mathcal{H}_r^G = 0$.

$$\mathcal{H}_t^\phi = \frac{1}{2} \left(\frac{\pi_\phi^2}{LR^2} + \frac{R^2}{L} \phi'^2 \right), \quad \mathcal{H}_r^\phi = \pi_\phi \phi'$$

from Gauss law constraint

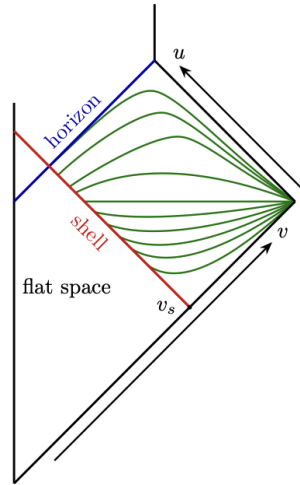
$$\mathcal{H}_t^G = \frac{GL\pi_L^2}{2R^2} - \frac{G\pi_L\pi_R}{R} + \frac{1}{G} \left[\left(\frac{RR'}{L} \right)' - \frac{R'^2}{2L} - \frac{L}{2} \right] + \frac{Q^2 L}{2R^2}, \quad \mathcal{H}_r^G = R'\pi_R - L\pi_L'$$

Strategy: choose gauge; solve constraints; substitute back into action after deriving expression for M_{ADM} by demanding good variational principle [Regge, Teitelboim]

No use of equations of motion! Metric variables determined by (π_ϕ, ϕ)

Choose gauge, solve constraints

Only need to cover region outside horizon, so we can choose Schwarzschild type gauge



$$ds^2 = -N^2(t, r) dt^2 + L^2(t, r) (dr + N^r(t, r) dt)^2 + R^2(t, r) d\Omega^2$$

Gauge choice: $\pi_L = 0$, $R = r$

Constraints can be solved explicitly: $\frac{1}{L^2(r)} = f(r) - \frac{2G}{r} \int_{r_s}^r dr' f(r') h(r') e^{-\int_{r'}^r dr'' \frac{2Gh(r'')}{r''}}$

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}, \quad h(r) = \frac{1}{2} \left(\frac{\pi_\phi^2}{r^2} + r^2 \phi'^2 \right)$$

ADM energy: $H_{\text{ADM}} = \lim_{r \rightarrow \infty} \mathcal{M}(r)$ $\mathcal{M} = \frac{G\pi_L^2}{2R} + \frac{R}{2G} \left[1 - \left(\frac{R'}{L} \right)^2 \right] + \frac{Q^2}{2R} \rightarrow \frac{r}{2G} \left[1 - \left(\frac{1}{L} \right)^2 \right] + \frac{Q^2}{2r}$

Constraint solution gives:

$$H_{\text{ADM}} = M + \int_{r_s}^{\infty} dr' f(r') h(r') e^{-\int_{r'}^{\infty} dr'' \frac{2Gh(r'')}{r''}}$$

Reduced action

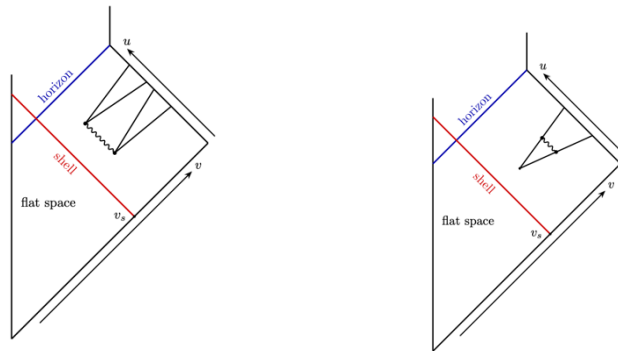
Reduced scalar action capturing s-wave gravity interactions

$$S = \int dt \int_{r_s}^{\infty} dr \left(\pi_{\phi} \dot{\phi} - f(r) h(r) e^{-\int_r^{\infty} dr' \frac{2Gh(r')}{r'}} \right) \quad f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}, \quad h(r) = \frac{1}{2} \left(\frac{\pi_{\phi}^2}{r^2} + r^2 \phi'^2 \right)$$

Integrate out π_{ϕ} :
$$S = \int dt \int_{r_s}^{\infty} dr \left[\frac{1}{2} r^2 \left(\frac{1}{f} \dot{\phi}^2 - f \phi'^2 \right) - 2G f(r) h(r) \int_r^{\infty} dr' \frac{h(r')}{r'} \right] + O(G^2)$$

Nonlocal in space, but local in time

Can use standard perturbative methods to compute correlations in outgoing Hawking radiation



Vertices can be anywhere, but enhanced effects occur in near-horizon region

Near-horizon scaling

Starting from $S = \int dt \int_{r_s}^{\infty} dr \left(\pi_{\phi} \dot{\phi} - f(r) h(r) e^{-\int_r^{\infty} dr' \frac{2Gh(r')}{r'}} \right)$ we write

$$r_{\pm} = r_0 \pm \lambda \rho_h$$

$$t = \frac{r_0^2}{\rho_h \lambda} \tilde{t}, \quad r = r_0 + \lambda \rho_h \coth z$$

$$\phi = \frac{1}{r_0} \tilde{\phi}, \quad \pi_{\phi} = \frac{r_0}{\rho_h \lambda} \sinh^2 z \tilde{\pi}_{\phi}$$

$$r_{\pm} = GM \pm \sqrt{G^2 M^2 - GQ^2}$$

and take $\lambda \rightarrow 0$

$$ds^2 \rightarrow r_0^2 \left(\frac{-d\tilde{t}^2 + dz^2}{\sinh^2 z} + d\Omega^2 \right) + \dots = \text{AdS}_2 \times S^2$$

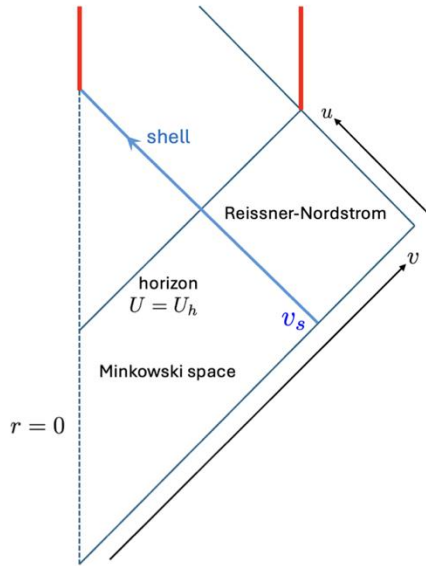
$$T_H = \frac{r_+ - r_-}{4\pi r_+^2} \approx \frac{\rho_h}{2\pi r_0^2} \lambda.$$

Near-horizon action: $S = \int dt \int_0^{\infty} dz \left[\tilde{\pi}_{\phi} \dot{\tilde{\phi}} - \frac{1}{2} \left(\tilde{\pi}_{\phi}^2 + \tilde{\phi}'^2 \right) e^{-\frac{1}{2} g \int_0^z dz' \sinh^2 z' (\tilde{\pi}_{\phi}^2 + \tilde{\phi}'^2)} \right]$

Dimensionless coupling: $g = \frac{2G}{\rho_h r_0 \lambda} \approx \frac{G}{\pi r_0^3 T_H}.$

Large fluxes in (ϕ, π_{ϕ}) translate via constraints to large fluxes in $L \rightarrow$ throat length

Hawking radiation from collapse



inside shell: (U, V)

outside shell: (u, v)

Matching at shell:
$$u = U - \frac{2r_+^2}{r_+ - r_-} \ln \left(\frac{v_s - U}{2} - r_+ \right) + \frac{2r_-^2}{r_+ - r_-} \ln \left(\frac{v_s - U}{2} - r_- \right)$$

$$v = V$$

State emerging from shell is (U, v) vacuum (Unruh vacuum)

In free field theory, ignoring greybody factors:
$$\langle \partial_U \tilde{\phi}(U_1, v_1) \partial_U \tilde{\phi}(U_2, v_2) \rangle = -\frac{1}{4\pi} \frac{1}{(U_1 - U_2 - i\varepsilon)^2}$$

$$\langle \partial_v \tilde{\phi}(U_1, v_1) \partial_v \tilde{\phi}(U_2, v_2) \rangle = -\frac{1}{4\pi} \frac{1}{(v_1 - v_2 - i\varepsilon)^2}$$

$$\rightarrow \langle T_{uu} \rangle = -\frac{1}{2} \lim_{U_2 \rightarrow U_1} \left[\frac{\left(\frac{\partial u_2}{\partial U_2} \frac{\partial u_1}{\partial U_1} \right)^{-1}}{(U_2 - U_1 - i\varepsilon)^2} - \frac{1}{(u_2 - u_1 - i\varepsilon)^2} \right] = \frac{1}{12} \left(\frac{\partial u_1}{\partial U_1} \right)^{-2} \text{Sch}(u_1, U_1) \approx \frac{(r_+ - r_-)^2}{96r_+^4} = \frac{\pi^2 T_H^2}{6} \text{ as } u \rightarrow \infty$$

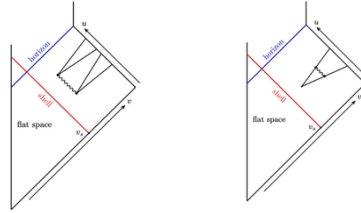
To incorporate gravitational interactions, apply perturbation theory to

$$S = \int dt \int_0^\infty dz \left[\tilde{\pi}_\phi \dot{\tilde{\phi}} - \frac{1}{2} \left(\tilde{\pi}_\phi^2 + \tilde{\phi}'^2 \right) e^{-\frac{1}{2}g \int_0^z dz' \sinh^2 z' (\tilde{\pi}_\phi^2 + \tilde{\phi}'^2)} \right]$$

Perturbation theory

Perturbative action: $S = \int dt \int_{r_s}^{\infty} dr \left[\frac{1}{2} r^2 \left(\frac{1}{f} \dot{\phi}^2 - f \phi'^2 \right) - 2G f(r) h(r) \int_r^{\infty} dr' \frac{h(r')}{r'} \right] + O(G^2)$

Sufficient for computing:



Full computation “straightforward” but messy due to non-universal early time behavior, grey-body factors.

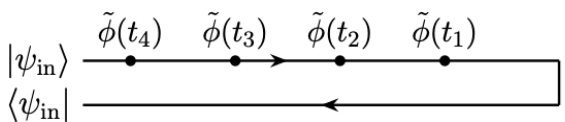
Focus on late-time correlators: restrict vertices to near-horizon region, and include grey-body factors at the end.

Near-horizon action (dropping tildes) $S[\phi] = \frac{1}{2} \int dt \int_0^{\infty} dz \left(\dot{\phi}^2 - \phi'^2 \right) + g \int dt \int_0^{\infty} dz \int_0^z dz' \sinh^2 z' T_{tt}(z) T_{tt}(z') + O(g^2)$

$$T_{tt} = \frac{1}{2}(\dot{\phi}^2 + \phi'^2)$$

Compute correlators at boundary of near-horizon (AdS2) region, then use free field equations to propagate them to null infinity

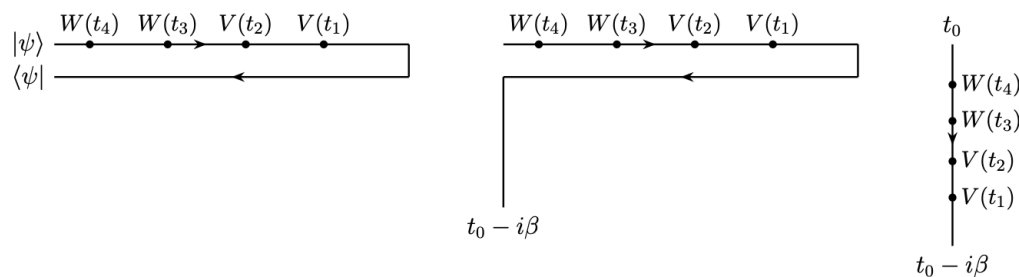
Correlator computations

Need to compute in-in expectation value:  $|\psi_{in}\rangle = \text{Unruh vacuum}$

Insertions are at boundary of AdS2 region.

Outgoing modes in AdS2 have reflection amplitude near -1 (standard AdS low energy decoupling).

So in AdS2 region Unruh vacuum correlators become equivalent to thermal correlators, which can be computed in imaginary t and then continued to Lorentzian:



Thermal correlators

For Euclidean AdS2 in coordinates $ds^2 = \frac{d\tau^2 + dz^2}{\sinh^2 z}$, $\tau \cong \tau + 2\pi$

reduced scalar action is: $I[\phi] = \frac{1}{2} \int_0^{2\pi} d\tau \int_0^\infty dz \left(\dot{\phi}^2 + \phi'^2 \right) - g \int_0^{2\pi} d\tau \int_0^\infty dz \int_0^z dz' \sinh^2 z' T_{\tau\tau}(z) T_{\tau\tau}(z') + O(g^2)$

$$T_{\tau\tau} = \frac{1}{2} \left(\dot{\phi}^2 - \phi'^2 \right)$$

Compute boundary correlators from Witten diagrams as usual

Bulk-boundary propagator: $K(z, \tau; \tau_i) = \frac{1}{2\pi} \frac{\sinh z}{\cosh z - \cos(\tau - \tau_i)} = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{-|n|z - in(\tau - \tau_i)}$

$$\lim_{z \rightarrow 0} K(z, \tau; \tau_i) = \delta(\tau - \tau_i)$$

Bulk-bulk propagator: $G(\tau, z; \tau', z') = \frac{1}{4\pi} \log \left[\frac{\cosh(z + z') - \cos(\tau - \tau')}{\cosh(z - z') - \cos(\tau - \tau')} \right] = \frac{z_{<}}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sinh(nz_{<}) e^{-nz_{>}} \cos(n(\tau - \tau'))$

$$\nabla_x^2 G = -\delta^{(2)}(x - x')$$

Quartic vertex from $T_{\tau\tau}(z) T_{\tau\tau}(z')$

Four-point function

$$G_4(\tau_i) = \langle V(\tau_1) V(\tau_2) W(\tau_3) W(\tau_4) \rangle =$$

$$+ (z \leftrightarrow z')$$

$$= g \int d\tau \int_0^\infty dz \int_0^z dz' \sinh^2 z' (T_\tau(\tau, z; \tau_1, \tau_2) T_{\tau\tau}(\tau, z'; \tau_3, \tau_4) + T_{\tau\tau}(\tau, z; \tau_3, \tau_4) T_\tau(\tau, z'; \tau_1, \tau_2))$$

Integrals are all trivial exponentials, giving

$$G_4(\tau_i) = \frac{g}{(2\pi)^3} \sum_{n_1 \dots n_4} \left(\frac{|n_1 n_2| + n_1 n_2}{|n_1| + |n_2|} \right) \frac{2}{(|n_1| + |n_2| + |n_3| + |n_4|)^2 - 4} \left(\frac{|n_3 n_4| + n_3 n_4}{|n_3| + |n_4|} \right) e^{in_1 \tau_1 + in_2 \tau_2 + in_3 \tau_3 + in_4 \tau_4}$$

reindex:

$$G_4(\tau_i) = \frac{2g}{(2\pi)^3} \sum_{|k|>1} \frac{J_k^V(\tau_1, \tau_2) J_{-k}^W(\tau_3, \tau_4)}{k^2 (k^2 - 1)}$$

$$J_k^V(\tau_1, \tau_2) = \sum'_{n_1, n_2} n_1 n_2 e^{in_1 \tau_1 + in_2 \tau_2} \delta_{n_1 + n_2, k}, \quad \leftarrow n_1 n_2 > 0$$

$$J_{-k}^W(\tau_3, \tau_4) = \sum'_{n_3, n_4} n_3 n_4 e^{in_3 \tau_3 + in_4 \tau_4} \delta_{n_3 + n_4, -k}$$

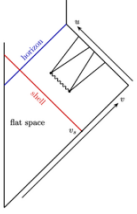
Propagator comes from (quadratic part of) Schwarzian action: $I_{\text{Sch}} = \frac{2\pi}{g} \sum_{|k|>1} k^2 (k^2 - 1) \epsilon_k \epsilon_{-k}$

$$G_4(\tau_i) = \int [D\epsilon] e^{-I_{\text{Sch}}} C^V(\tau_1, \tau_2) C^W(\tau_3, \tau_4) \quad C(\tau_i, \tau_j) = -\frac{i}{\pi} \sum_{|k|>1} \text{sgn}(k) J_k(\tau_i, \tau_j) \epsilon_{-k}$$

Reproduces dressed bilocals coupled to Schwarzian

[Maldacena, Stanford, Yang]

Four-point function on $\mathcal{I}^+$



$$\langle \partial_u V(u_1) \partial_u V(u_2) \partial_u W(u_3) \partial_u W(u_4) \rangle_c = \frac{(2\pi T_H)^4}{16} \frac{g}{4\pi^3} \frac{(-2 + 2\pi T_H u_{12} \coth(\pi T_H u_{12}))(-2 + 2\pi T_H u_{34} \coth(\pi T_H u_{34}))}{(2 \sinh(\pi T_H u_{12}))^2 (2 \sinh(\pi T_H u_{34}))^2}.$$

Compare to free (disconnected) 4pt function $\frac{\pi T_H^2}{4 \sinh^2(\pi T_H u_{12})} \frac{\pi T_H^2}{4 \sinh^2(\pi T_H u_{34})}$

Comments

Connected part has relative factor of $g = \frac{G}{\pi r_0^3 T_H}$, becomes large as $T_H \rightarrow 0$ at fixed $T_H u_{ij}$

→ infrared effect

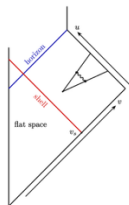
Fluctuations in canonical ensemble controlled by heat capacity $C_Q = \left. \frac{dM}{dT_H} \right|_Q \approx 4\pi^2 \sqrt{G} Q^3 T_H$

Indeed: $\langle \partial_u V(u_1) \partial_u V(u_2) \partial_u W(u_3) \partial_u W(u_4) \rangle_c = \frac{1}{C_Q} \left(T_H \frac{\partial}{\partial T_H} \langle \partial_u V(u_1) \partial_u V(u_2) \rangle \right) \left(T_H \frac{\partial}{\partial T_H} \langle \partial_u W(u_3) \partial_u W(u_4) \rangle \right)$

[Maldacena, Stanford, Yang]

Two-point function on $\mathcal{I}^+$

2-point function



Other diagrams either vanish identically or by renorm. prescription setting scale free integrals to zero as in dim reg

$$\langle \partial_u \phi(u_1) \partial_u \phi(u_2) \rangle_{1\text{-loop}} = \frac{T_H^2 g}{8 \sinh^2 x} \left[\frac{-2x^2 - 2\pi i x + 1 - \cosh(2x) + (i\pi + 2x) \sinh(2x)}{4 \sinh^2 x} - \left(-1 + \frac{x}{\tanh x} \right) \left(-1 + \frac{x + i\pi}{\tanh x} \right) \right]$$

$x = \pi T_H u_{12}.$

Correction to energy flux: $T_{uu}^{1\text{-loop}} = 2\pi \lim_{u_1 \rightarrow u_2} \langle \partial_u \phi(u_1) \partial_u \phi(u_2) \rangle_{1\text{-loop}} = \frac{\pi g}{8} T_H^2 \sim g T_{uu}^{\text{tree}}$

Can be interpreted as correction to mass-energy relation

$$T_{uu}^{\text{tree}} = \frac{GE}{12r_0^3} \quad E = M - \frac{Q}{\sqrt{G}} = \frac{2\pi}{g} T_H + O(g^0) \quad \leftarrow \text{1-loop correction}$$

Write $T_{uu}^{1\text{-loop}} = \frac{G\delta E}{12r_0^3}$ 1-loop computation gives $\delta E = \frac{3}{2} \frac{\pi r_0^3}{G} g T_H^2 = \frac{3}{2} T_H$

From standard thermo relations $\ln Z(T_H) = \ln Z_{\text{cl}}(T_H) + \frac{3}{2} \ln T_H$

Reproduces 1-loop correction to Schwarzian path integral [Stanford, Witten]

More general correlators

Can ask for agreement with Schwarzian for general correlators at all orders in g

$$\int [D\pi_\phi D\phi] e^{-I} \mathcal{O}(\tau_1) \mathcal{O}(\tau_2) \dots \mathcal{O}(\tau_{2n}) \quad I = \int dt \int_0^\infty dz \left[\pi_\phi \dot{\phi} - \frac{1}{2} (\pi_\phi^2 + \phi'^2) e^{-\frac{1}{2}g \int_0^z dz' \sinh^2 z' (\pi_\phi^2 + \phi'^2)} \right]$$

Versus: $\int [DT] e^{-I_{\text{Sch}}(T)} B(\tau_1, \tau_2) \dots B(\tau_{2n-1}) B(\tau_{2n})$

$$I_{\text{Sch}}(T) = \frac{2}{g} \int d\tau \text{Sch}(T(\tau), \tau)$$

$$B(\tau_1, \tau_2) = \frac{T'(\tau_1) T'(\tau_2)}{\left(2 \sin \frac{T(\tau_1) - T(\tau_2)}{2} \right)^2}$$

Some work shows that these actions agree on-shell, i.e. for tree-level correlators. Proof involves showing how to reconstruct an AdS2 metric given any solution of nonlocal scalar field action

At loop level need a prescription for renormalizing UV divergences in reduced scalar field description

Time scale

As measured on null infinity, when do large effects kick in?

Shell needs to be sufficiently close to horizon on fixed u slice to get long AdS throat

Diagrams involve integrals of type $\int_0^{z_s} dz e^{-|k|z} \int_0^z dz' \sinh^2 z' e^{-|k|z'}$

Dominant contributions from $z \sim O(1)$ so need $z_s \gtrsim 1$ which is $r_s - r_+ \lesssim r_0^2 T_H$

From the shell trajectory, and taking u_0 to be the characteristic time at which $r_s \sim r_+$ find that large effects turn on at

$$u - u_0 \sim \frac{1}{T_H}$$

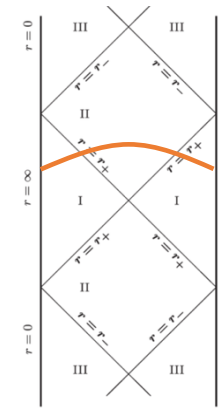
Extensions

- Replacing neutral scalar by charged scalar gives correlators matching Schwarzian + phase mode theory
- OTOCs can be computed and usual Lyapunov growth obtained
- A different gauge choice allows for eternal two-sided black hole
- Bulk-boundary and bulk-bulk correlators can be computed
- In AdS3 case s-wave assumption can be dropped, and one gets a theory of bulk matter coupled to boundary gravitons
- Scattering

Comments on eternal black holes

Two-sided black hole

Consider eternal BTZ $S = -\frac{1}{2\pi} \int d^3x \sqrt{-g} \left[\frac{1}{8G} \mathcal{R} - \frac{1}{4G\ell^2} + \frac{1}{2} (\partial\phi)^2 \right]$



on fixed t surfaces: $ds_{\Sigma}^2 = L^2 dy^2 + R^2 (d\theta + A dy)^2$

action: $S = \int d^3x \left[\pi_{\phi} \dot{\phi} + \pi_L \dot{L} + \pi_R \dot{R} + \pi_A \dot{A} - N\mathcal{H} - N^y \mathcal{H}_y - N^{\theta} \mathcal{H}_{\theta} \right] - \int dt (H_L + H_R)$

Reduced action describes interacting theory of bulk scalar and bndy gravitons.

Can be worked out perturbatively.

All order results in special cases: s-wave, near-horizon, etc

All functions θ – independent

$$ds_\Sigma^2 = L^2 dy^2 + R^2 (d\theta + A dy)^2$$

constraints:

$$\mathcal{H} = -4G\pi_L\pi_R + \frac{4GL}{R^3}\pi_A^2 + \frac{1}{4G}\left(\frac{R'}{L}\right)' - \frac{LR}{4G\ell^2} + \frac{1}{2L}\left(\frac{\pi_\phi^2}{R} + R\phi'^2\right) = 0,$$

$$\mathcal{H}_y = -L\pi'_L + R'\pi_R - A\pi'_A + \pi_\phi\phi' = 0,$$

$$\mathcal{H}_\theta = -\pi'_A = 0.$$

mass function:

$$\mathcal{M} = 2G\pi_L^2 + \frac{1}{8G}\left[\frac{R^2}{\ell^2} + \frac{16G^2\pi_A^2}{R^2} - \left(\frac{R'}{L}\right)^2\right]$$

gauge choice:

$$\pi_R = 0, \quad L = \frac{R'}{\sinh(y/\ell)}$$

Setting $\phi = \pi_\phi = 0$ constraints yields:

$$R^2 + \frac{(4G\ell J)^2}{R^2} = \ell^2 \sinh^2(y/\ell) + C(t)$$

$$\pi_L = P(t) \Rightarrow \int d^2x \pi_L \dot{L} = \int dt P \dot{D} \quad D = \lim_{y_c \rightarrow \infty} \left(\int_{-y_c}^{y_c} L dy - 2y_c \right)$$

$$\pi_A = -J(t) \Rightarrow \int d^2x \pi_A \dot{A} = \int dt J \dot{\Theta} \quad \Theta = - \int_{-\infty}^{\infty} A dy$$

reduced action: $S = \int dt (P\dot{D} + J\dot{\Theta} - H)$

$$H = 4GP^2 + \frac{1}{4G} \sqrt{e^{-2D/\ell} + \left(\frac{8GJ}{\ell}\right)^2}$$

Setting $J=0$ gives Liouville QM as in [Harlow, Jafferis]

s-wave with matter

To solve constraints with matter we set $J=0$ and take near horizon limit: $\left(\frac{\pi_\phi^2}{R} + R\phi'^2\right) \rightarrow \left(\frac{\pi_\phi^2}{r_h} + r_h\phi'^2\right)$

Proceeding as before, reduced action works out to be:

$$S = \int dt \left[P\dot{D} + \int dx \Pi \dot{\Phi} - 4GP^2 - \frac{1}{4G} e^{-D/\ell} - G \left(\int_{-\infty}^{\infty} dx \Pi \partial_x \Phi \right)^2 - \frac{1}{2} e^{-D/2\ell} \int_{-\infty}^{\infty} dx \cosh(x/\ell) \left(\Pi^2 + (\partial_x \Phi)^2 \right) \right]$$

$$x(y) = \int_0^y dy' L(y') \quad , \quad \Phi(x) = \phi(y), \quad \Pi(x) = \frac{\pi_\phi(y)}{L(y)}$$

Reproduces [Penington,Witten][Kolchmeyer]

Theory is essentially solvable because matter dependence appear through $SL(2,R)$ charges:

$$L_0 = \int dx \Pi \partial_x \Phi \quad , \quad L_{\pm 1} = \int dx e^{\pm x/\ell} \left(\Pi^2 + (\partial_x \Phi)^2 \right)$$

Holographic theory [Penington,Witten]

