

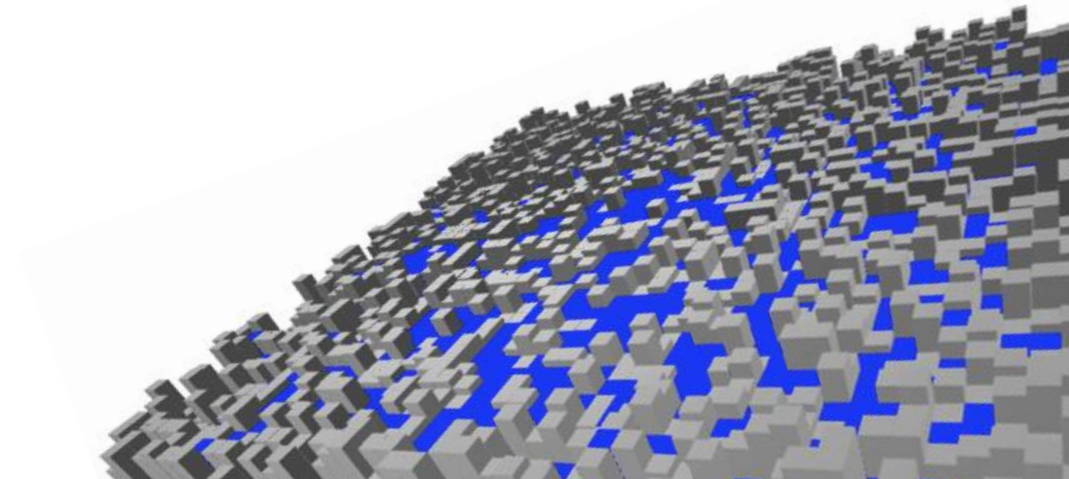
[Cotler, Ivo, Maldacena '26] 2609.38052

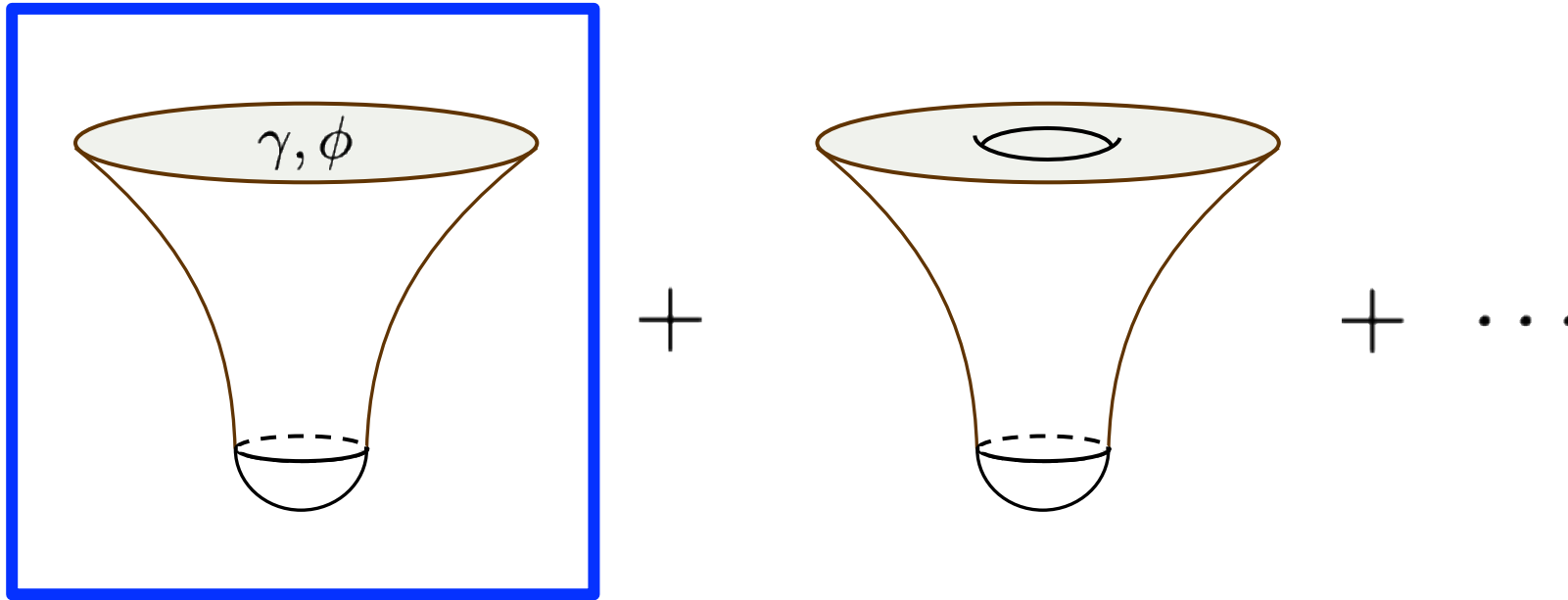
[Cotler, Ivo, Maldacena '26] 2608.26244

The “State” of Quantum Cosmology

JORDAN COTLER

HARVARD UNIVERSITY





e.g. in 3+1: $|\Psi|^2 \sim \exp\left(\frac{24\pi^2 M_{\text{pl}}^4}{V(\phi_\star)}\right)$



No-boundary state
favors as few e-folds of
inflation as possible!

(see e.g. [\[Maldacena '24\]](#))

Very slow roll limit of inflation

$$I = \frac{M_{\text{pl}}^{d-1}}{2} \int \sqrt{-g} R + \int \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \underbrace{V(\phi)}_{V(\phi) = V_0 + (\phi - \phi_r) V', \quad V_0 = \frac{d(d-1)}{2} M_{\text{pl}}^{d-1} H^2} \right]$$

Linear roll limit: $M_{\text{pl}} \rightarrow \infty$, H , $V'(\phi)$ constant (no tensor fluctuations)

Flat gauge: $ds^2 \simeq -dt^2 + e^{2Ht} \hat{\gamma}_{ij}(x) dx^i dx^j$

[Creminelli, Dubovsky, Nicolis,
Senatore, Zaldarriaga, '08]

[Chen, Penington, 24]

$$\phi(t, x) = \bar{\phi}(t) + \delta\phi(t, x)$$

Reheating surface: $\phi_r = \phi(t, x)$ complicated surface since the inflaton can reheat in different spatial regions for different t

Very slow roll limit of inflation

$$I = \frac{M_{\text{pl}}^{d-1}}{2} \int \sqrt{-g} R + \int \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \underbrace{V(\phi)}_{V(\phi) = V_0 + (\phi - \phi_r) V', \quad V_0 = \frac{d(d-1)}{2} M_{\text{pl}}^{d-1} H^2} \right]$$

Linear roll limit: $M_{\text{pl}} \rightarrow \infty$, H , $V'(\phi)$ constant (no tensor fluctuations)

Flat gauge: $ds^2 \simeq -dt^2 + e^{2Ht} \hat{\gamma}_{ij}(x) dx^i dx^j$

$$\phi(t, x) = \bar{\phi}(t) + \delta\phi(t, x)$$

Reheating surface: $\phi_r = \phi(t_r + \delta t_r(x), x)$

[Creminelli, Dubovsky, Nicolis,
Senatore, Zaldarriaga, '08]

[Chen, Penington, 24]

Very slow roll limit of inflation

$$I = \frac{M_{\text{pl}}^{d-1}}{2} \int \sqrt{-g} R + \int \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \underbrace{V(\phi)}_{V(\phi) = V_0 + (\phi - \phi_r) V', \quad V_0 = \frac{d(d-1)}{2} M_{\text{pl}}^{d-1} H^2} \right]$$

Linear roll limit: $M_{\text{pl}} \rightarrow \infty$, H , $V'(\phi)$ constant (no tensor fluctuations)

Flat gauge: $ds^2 \simeq -dt^2 + e^{2Ht} \hat{\gamma}_{ij}(x) dx^i dx^j$

[Creminelli, Dubovsky, Nicolis, Senatore, Zaldarriaga, '08]

[Chen, Penington, 24]

$$\phi(t, x) = \bar{\phi}(t) + \delta\phi(t, x)$$

Reheating surface: $\phi_r = \bar{\phi}(t_r) + \underbrace{\delta\phi(t_r, x) + \dot{\bar{\phi}}(t_r) \delta t_r(x)}_{=0}$

Very slow roll limit of inflation

$$I = \frac{M_{\text{pl}}^{d-1}}{2} \int \sqrt{-g} R + \int \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \underbrace{V(\phi)}_{V(\phi) = V_0 + (\phi - \phi_r) V', \quad V_0 = \frac{d(d-1)}{2} M_{\text{pl}}^{d-1} H^2} \right]$$

Linear roll limit: $M_{\text{pl}} \rightarrow \infty$, H , $V'(\phi)$ constant (no tensor fluctuations)

Reheating gauge: $ds^2 \simeq -dt^2 + e^{2H(t + \delta t_r(x))} \hat{\gamma}_{ij}(x) dx^i dx^j$

[Creminelli, Dubovsky, Nicolis, Senatore, Zaldarriaga, '08]

[Chen, Penington, 24]

$$\phi(t, x) = \bar{\phi}(t)$$

Reheating surface: $\phi_r = \bar{\phi}(t_r) + \underbrace{\delta\phi(t_r, x) + \dot{\bar{\phi}}(t_r) \delta t_r(x)}_{=0}$

Very slow roll limit of inflation

$$I = \frac{M_{\text{pl}}^{d-1}}{2} \int \sqrt{-g} R + \int \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \underbrace{V(\phi)}_{V(\phi) = V_0 + (\phi - \phi_r) V', \quad V_0 = \frac{d(d-1)}{2} M_{\text{pl}}^{d-1} H^2} \right]$$

Linear roll limit: $M_{\text{pl}} \rightarrow \infty$, H , $V'(\phi)$ constant (no tensor fluctuations)

Reheating gauge: $ds^2 \simeq -dt^2 + e^{2Ht} e^{2\zeta(x)} \hat{\gamma}_{ij}(x) dx^i dx^j$ Reheating surface is at constant t but has non-trivial scale factor

$$\phi(t, x) = \bar{\phi}(t)$$

Reheating surface: $\phi_r = \bar{\phi}(t_r) + \underbrace{\delta\phi(t_r, x) + \dot{\bar{\phi}}(t_r) \delta t_r(x)}_{=0}$

Wavefunctional for reheating geometry

Global slicing:

$$|\Psi[\zeta, \hat{\gamma}]|^2 \propto \exp \left[-\frac{d Q^2}{2\Gamma(d) \text{Vol}(S^d)} \int d^d x \sqrt{\hat{\gamma}} (\zeta \mathcal{P}_{\hat{\gamma}} \zeta + 2 \mathcal{Q}_{\hat{\gamma}} \zeta) \right]$$

$$Q^2 = \frac{\dot{\phi}^2}{H^{d+1}} \text{Vol}(S^{d+1}) = \frac{d-1}{d} \epsilon S_{\text{dS}}$$

Holds for any ζ (!)

Invariant under CKVs:

$$\delta_{\xi} \zeta = \xi^i \partial_i \zeta + \frac{1}{d} \hat{\nabla}_i \xi^i$$

$$\begin{cases} \mathcal{P}_{e^{2\omega} \gamma} = e^{-d\omega} \mathcal{P}_{\gamma} \\ \mathcal{Q}_{e^{2\omega} \gamma} = e^{-d\omega} (\mathcal{Q}_{\gamma} + \mathcal{P}_{\gamma} \omega) \end{cases} \quad \begin{array}{l} \text{conformally covariant} \\ \text{kinetic operator and} \\ \text{generalized curvature} \end{array}$$

$$\mathcal{P}_{\gamma} \sim (-\nabla^2)^{d/2} \text{ at short distances} \quad \longrightarrow \quad \langle \zeta(x) \zeta(y) \rangle = \frac{2}{d Q^2} \log \frac{1}{|x - y|} + \dots$$

$$\text{2+1 dimensions: } \frac{Q^2}{4\pi} \int d^2 x \sqrt{\hat{\gamma}} \left[(\hat{\nabla} \zeta)^2 + \hat{R} \zeta \right]$$

Wavefunctional for reheating geometry

Global slicing:

$$|\Psi[\zeta, \hat{\gamma}]|^2 \propto \exp \left[-\frac{d Q^2}{2\Gamma(d) \text{Vol}(S^d)} \int d^d x \sqrt{\hat{\gamma}} (\zeta \mathcal{P}_{\hat{\gamma}} \zeta + 2\mathcal{Q}_{\hat{\gamma}} \zeta) \right]$$

$$Q^2 = \frac{\dot{\phi}^2}{H^{d+1}} \text{Vol}(S^{d+1}) = \frac{d-1}{d} \epsilon S_{\text{dS}}$$

Holds for any ζ (!)

Invariant under CKVs:

$$\delta_{\xi} \zeta = \xi^i \partial_i \zeta + \frac{1}{d} \hat{\nabla}_i \xi^i$$

$$\left\{ \begin{array}{l} \mathcal{P}_{e^{2\omega} \gamma} = e^{-d\omega} \mathcal{P}_{\gamma} \\ \mathcal{Q}_{e^{2\omega} \gamma} = e^{-d\omega} (\mathcal{Q}_{\gamma} + \mathcal{P}_{\gamma} \omega) \end{array} \right. \quad \begin{array}{l} \text{conformally covariant} \\ \text{kinetic operator and} \\ \text{generalized curvature} \end{array}$$

$$\mathcal{P}_{\gamma} \sim (-\nabla^2)^{d/2} \text{ at short distances} \quad \longrightarrow \quad \langle \zeta(x) \zeta(y) \rangle = \frac{2}{d Q^2} \log \frac{1}{|x-y|} + \dots$$

$$\text{2+1 dimensions: } \frac{1}{4\pi} \int d^2 x \sqrt{\hat{\gamma}} \left[(\hat{\nabla} \varphi)^2 + Q \hat{R} \varphi \right] \quad \text{d = 2 Liouville with no CC (!)}$$

Wavefunctional for reheating geometry

Global slicing:

$$|\Psi[\zeta, \hat{\gamma}]|^2 \propto \exp \left[-\frac{d Q^2}{2\Gamma(d) \text{Vol}(S^d)} \int d^d x \sqrt{\hat{\gamma}} (\zeta \mathcal{P}_{\hat{\gamma}} \zeta + 2\mathcal{Q}_{\hat{\gamma}} \zeta) \right]$$

$$Q^2 = \frac{\dot{\phi}^2}{H^{d+1}} \text{Vol}(S^{d+1}) = \frac{d-1}{d} \epsilon S_{\text{dS}}$$

Holds for any ζ (!)

Invariant under CKVs:

$$\delta_{\xi} \zeta = \xi^i \partial_i \zeta + \frac{1}{d} \hat{\nabla}_i \xi^i$$

$$\left\{ \begin{array}{l} \mathcal{P}_{e^{2\omega} \gamma} = e^{-d\omega} \mathcal{P}_{\gamma} \\ \mathcal{Q}_{e^{2\omega} \gamma} = e^{-d\omega} (\mathcal{Q}_{\gamma} + \mathcal{P}_{\gamma} \omega) \end{array} \right. \quad \begin{array}{l} \text{conformally covariant} \\ \text{kinetic operator and} \\ \text{generalized curvature} \end{array}$$

$$\mathcal{P}_{\gamma} \sim (-\nabla^2)^{d/2} \text{ at short distances} \quad \longrightarrow \quad \langle \zeta(x) \zeta(y) \rangle = \frac{2}{d Q^2} \log \frac{1}{|x-y|} + \dots$$

$$\text{3+1 dimensions: } \frac{3}{8\pi^2} \int_{S^3} d^3 x \sqrt{\hat{\gamma}} \left[\varphi (-\hat{\nabla}^2) \sqrt{1 - \hat{\nabla}^2} \varphi + 4Q\varphi \right] \quad \text{d = 3 Liouville with no CC (!)}$$

1-loop effects

No additional ζ -dependence in the probability measure is generated at one loop

1. The inflaton potential is linear, so its quadratic fluctuation operator is independent of the classical inflaton solution

Gravity also decouples

➡ The one-loop factors are **independent of the boundary value of the inflaton**

2. Asymptotic time-reparameterization invariance relates changes in the boundary inflaton to Weyl rescalings of the boundary metric

➡ Since one-loop factors are independent of the inflaton, the symmetry implies that one-loop factors should also be **independent of the conformal factor**

Subtlety: regularization of one-loop determinants can acquire an apparent Weyl anomaly, but boundary measure should cancel it with opposite anomaly

1-loop effects

2+1 dimensions: Let $S_L[\omega; \gamma] = \frac{1}{4\pi} \int d^2x \sqrt{\gamma} [(\nabla\omega)^2 + R_\gamma\omega]$

Object of Weyl-anomaly coefficient c transforms as $Z[e^{2\omega}\gamma] = e^{\frac{c}{6}S_L[\omega;\gamma]} Z[\gamma]$

Inner product: $\langle \Psi | \Psi \rangle \sim \int \underbrace{D_{\gamma_b} \zeta}_{c=1} \underbrace{Z_{\text{FP}}^{\text{bdy}}}_{c=-26} \underbrace{(A_{\text{grav},+} A_{\text{grav},-})}_{c=26} \underbrace{(A_{\phi,+} A_{\phi,-})}_{c=-1} \underbrace{e^{-\mathcal{I}_{\text{grav}} - \mathcal{I}_{\phi}}}_{\text{invariant}}$

Change evaluation surface: $\zeta \longrightarrow \zeta + \omega, \quad \phi_b \longrightarrow \phi_b - \frac{V'}{d} \omega$

 $c_{\text{tot}} = 0$

Transition to eternal inflation

Global slicing:

$$|\Psi[\zeta, \hat{\gamma}]|^2 \propto \exp \left[-\frac{d Q^2}{2\Gamma(d) \text{Vol}(S^d)} \int d^d x \sqrt{\hat{\gamma}} (\zeta \mathcal{P}_{\hat{\gamma}} \zeta + 2\mathcal{Q}_{\hat{\gamma}} \zeta) \right]$$

Transition to eternal inflation

See Victor Ivo's talk

Global slicing:

$$|\Psi[\varphi, \hat{\gamma}]|^2 \propto \exp \left[-\frac{d}{2\Gamma(d) \text{Vol}(S^d)} \int d^d x \sqrt{\hat{\gamma}} (\varphi \mathcal{P}_{\hat{\gamma}} \varphi + 2Q \mathcal{Q}_{\hat{\gamma}} \varphi) \right]$$

$$Q^2 = \frac{\dot{\phi}^2}{H^{d+1}} \text{Vol}(S^{d+1})$$

Eternal inflation phase transition at $Q = 2$ (!) $\longrightarrow$ infinite reheating volume

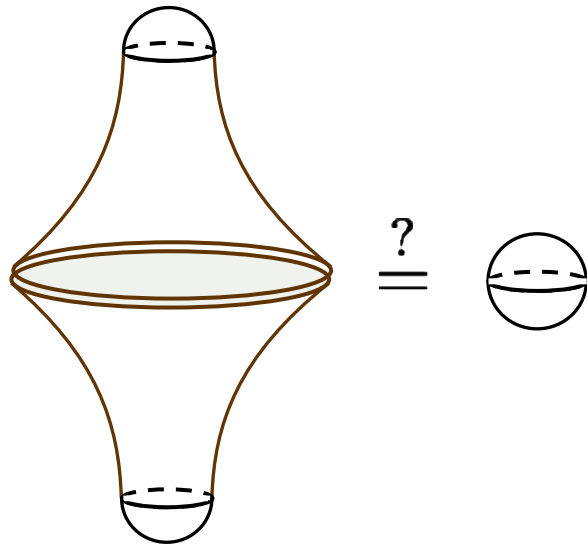
c.f. [Creminelli, Dubovsky, Nicolis, Senatore, Zaldarriaga, '08]

In two dimensions this coincides with the $c_m = 1$ barrier in Liouville

$$\left(Q = b + \frac{1}{b}, \quad c_L = 1 + 6Q^2, \quad c_m = 26 - c_L = 25 - 6Q^2 \xrightarrow{Q=2, b=1} c_L = 25, \quad c_m = 1 \right)$$

Norm of the no-boundary state

Pure Einstein gravity:



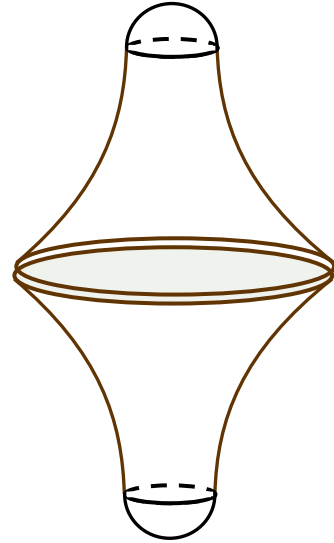
Norm of the no-boundary state

Note: in *neither* case does the norm equal the sphere partition function

[Cotler, Jensen '26]

[Cotler, Jensen, Shi, Turiaci WIP]

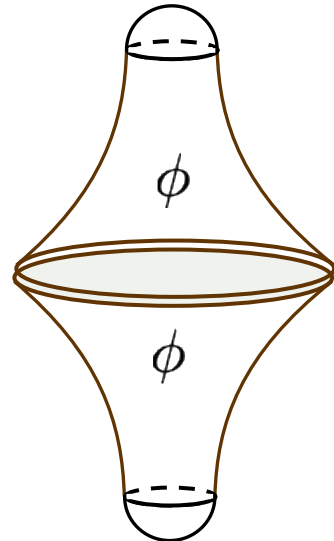
Pure Einstein gravity:



$$\propto \frac{e^{S_{\text{dS}}}}{\text{Vol}(SO(d+1, 1))} = \mathbf{0}$$

[Cotler, Jensen '25]

**Einstein gravity plus
very slow roll inflaton:**



$$\sim \frac{1}{\text{Vol}(SO(d+1))} \int d\zeta_0 Z'_{1\text{-loop}} e^{-\mathcal{I}_* - dQ^2 \zeta_0} > \mathbf{0}$$

[Cotler, Ivo, Maldacena '26]

The e-fold problem, reprised

2+1 dimensions, genus g : $|\Psi|^2 \sim \underbrace{e^{(1-g)S_{\text{dS}}}}_{\text{subdominant}} \underbrace{e^{-2Q^2(1-g)\zeta_0}}_{\text{pressure for large } \zeta_0 \text{ when } g > 1}$

But: such spacetimes violate KSW for $g > 0$

More general story: $|\Psi|^2 \sim \exp\left(-\frac{d\dot{\phi}^2}{H^{d+1}} V_E \zeta_0\right)$ [Cotler, Ivo, Maldacena '26]

$$V_E = \frac{\text{Vol}(S^{d+1})}{\Gamma(d) \text{Vol}(S^d)} \int_{\Sigma_d} d^d x \sqrt{\gamma} \mathcal{Q}_\gamma$$

$$\text{KSW} \longrightarrow \text{Re } V_E > 0$$



KSW implies that higher-topology contributions to the no-boundary state prefer **as few e-folds of inflation as possible**

[Kontsevich, Segal '21]
[Witten '21]

[Hertog, Janssen,
Karlsson '23]

Volume enhancements from observers

[Hartle, Hawking,
Hertog '07, '08]

Suppose the probability that an observer like us arises in a particular Hubble-sized region is $p_E \ll 1$

If a reheating universe contains N_H approximately independent Hubble-sized regions, the probability that at least one observer arises is

$$P(\text{at least one observer} \mid N_H) = 1 - (1 - p_E)^{N_H} \simeq p_E N_H \propto \text{Vol}(\Sigma_d)$$

$$\begin{array}{c} \text{blue arrow} \quad P_{\text{obs}}[\zeta] \propto \text{Vol}[\zeta] \quad P_{\text{HH}}[\zeta] \sim e^{-dQ\zeta_0/b} \\ \qquad \qquad \qquad \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \\ \qquad \qquad \qquad e^{dQb\zeta_0} \quad e^{-dQ^2\zeta_0} \end{array}$$

[Cotler, Ivo, Maldacena '26]

Pressure for small number of e-folds
everywhere outside of eternal regime

Volume becomes infinite in eternal
regime, so not clear if there is a
controlled calculation in that regime

Discussion

The very slow roll limit of inflation gives rise to a probability functional for the reheating surface described by (higher-dimensional) **Liouville without a CC**

The eternal inflation phase transition occurs at **$Q = 2$**

The norm of the no-boundary state with an inflaton is **positive (and thus non-zero)**

KSW implies that higher-topology contributions to the no-boundary state prefer **as few e-folds of inflation as possible**

The volume-enhancement of probabilities due to condition in the presence of an observer **does not alleviate the e-fold problem in the non-eternal regime**