

Operational questions in quantum gravity and quantum field theory

Based on Section 2 of arXiv: 2606.30853



Slides

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Model-first [Physics]

Given: an action/Hamiltonian/symmetry

Target: Correlators, dynamics, etc.

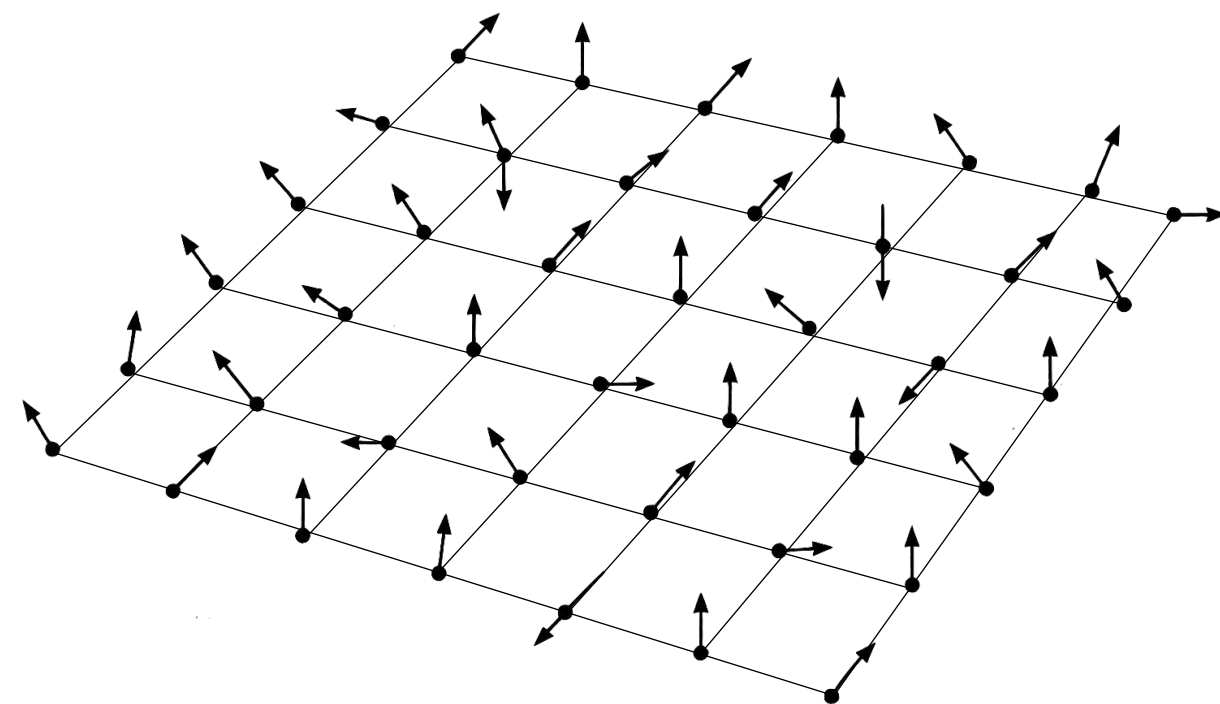
Question: Which phenomena does the model explain and how?

H

$U(N)$

$\langle S_i \cdot S_j \rangle$

$-\langle [W(t), V(0)]^2 \rangle$

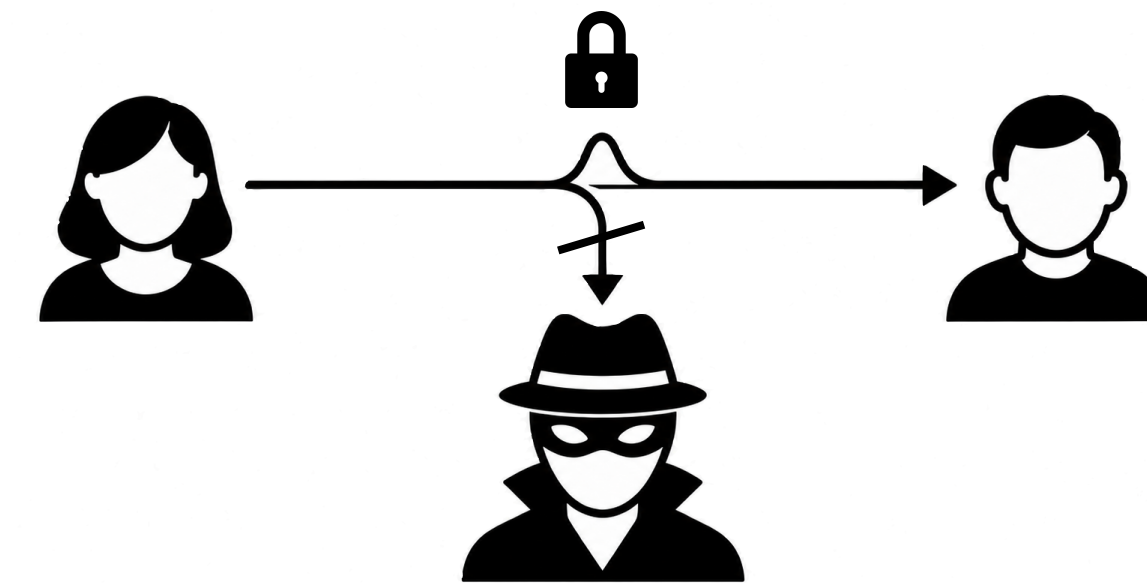


Task-first [Quantum information]

Given: an (information processing) task

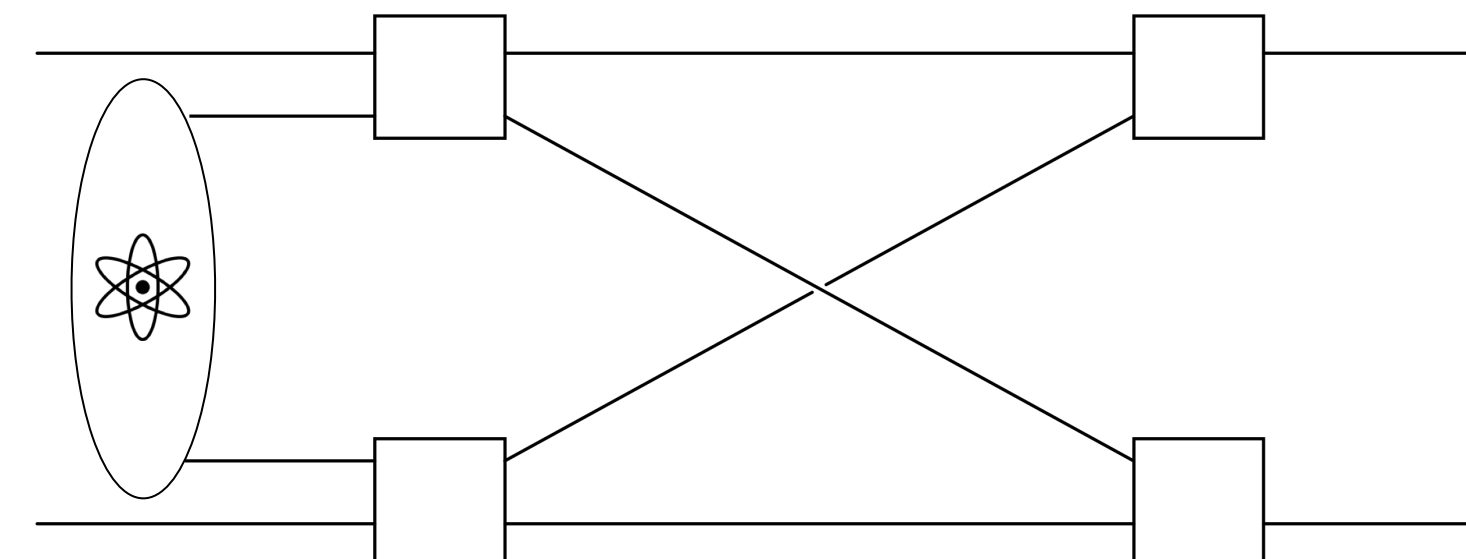
Target: achievability, rates, error of tasks

Question: What can be done at what cost?



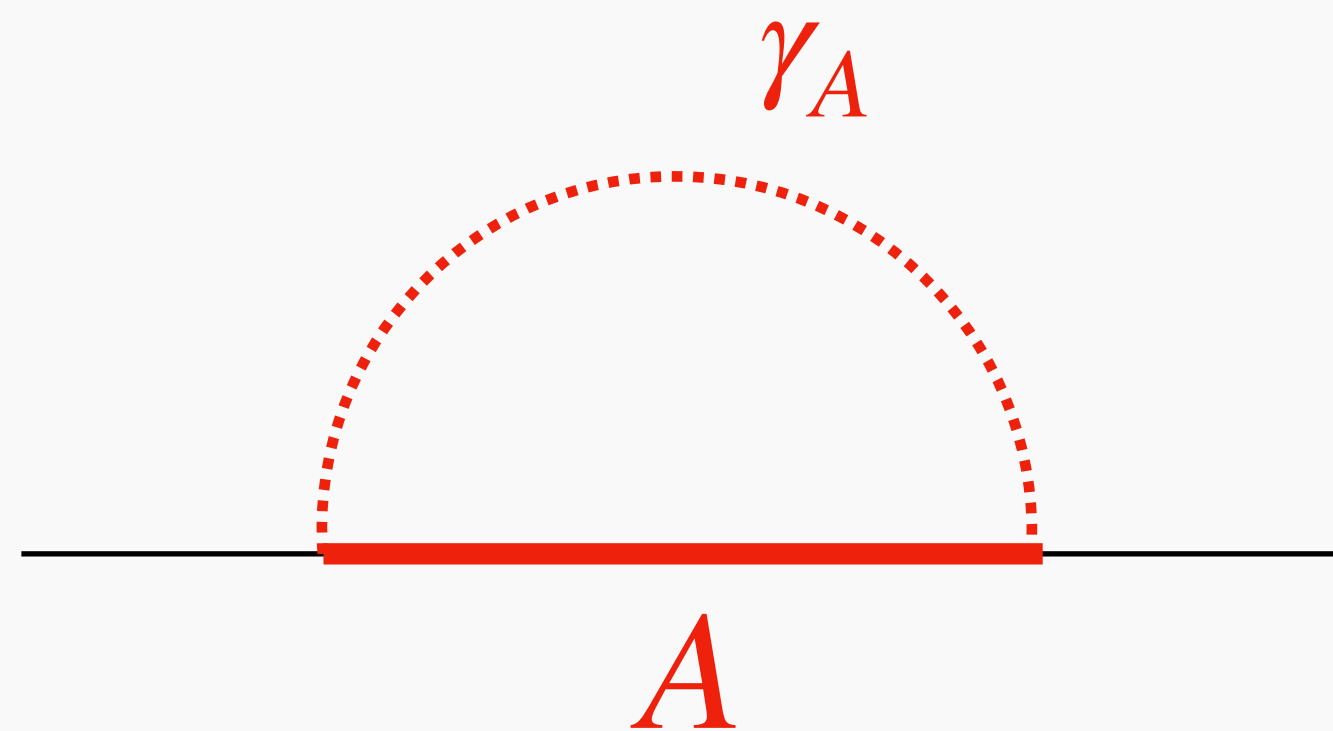
$$\beta_n \sim 2^{-nD(\rho\|\sigma)}$$

$$p_{\text{suc}} \geq 1 - \varepsilon$$



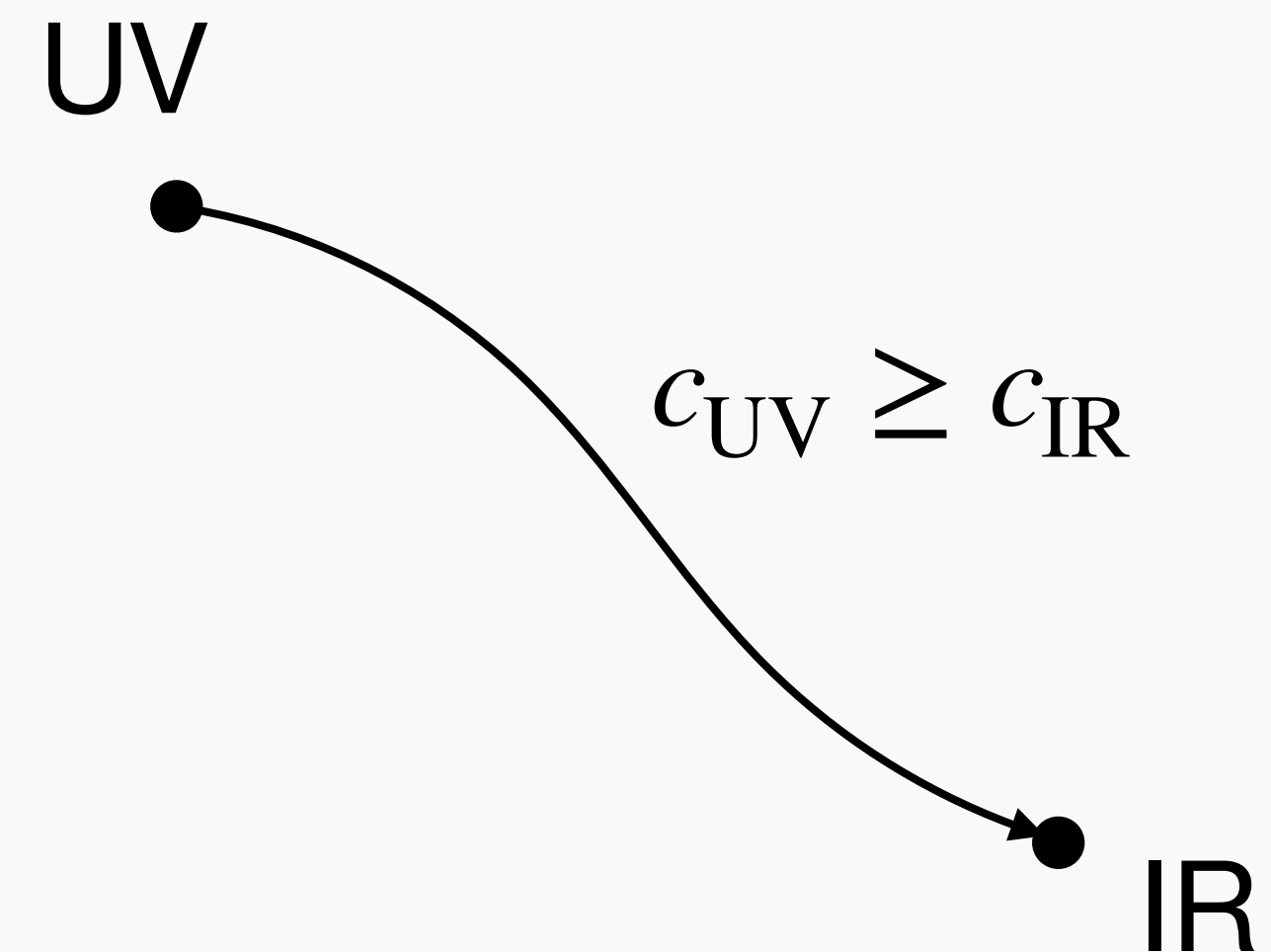
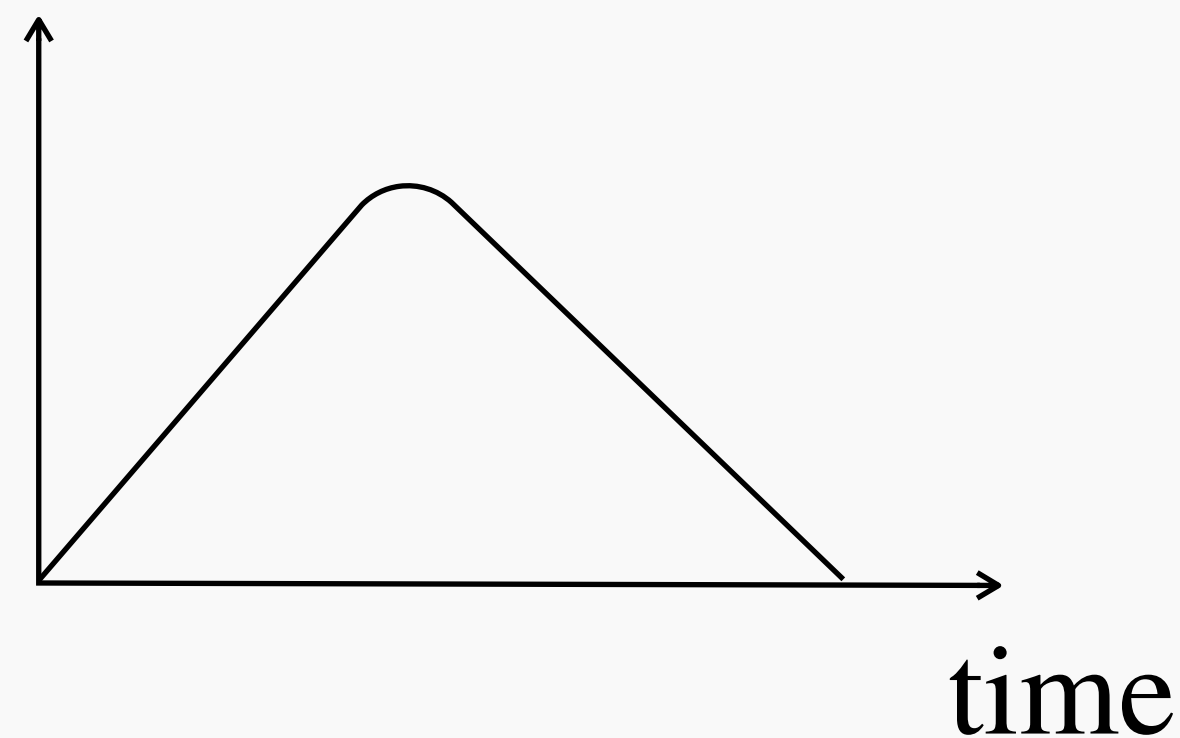
Where we stand (i) — the gravity/field-theory side

- Information-theoretic quantities are by now routine:
 - entanglement entropy, Rényi entropies, negativity,
 - reflected entropy, Markov gap, multi-entropy, discord, ...
- They are used as **diagnostics of a given model**: order parameters, c - and a -theorems, phase transitions, the Page curve.
- What has been imported is the **list of quantities**, not the operational viewpoint.



$$S(\rho_A) = \frac{c}{3} \log \frac{\ell}{\epsilon} = \min_{\gamma_A} \frac{|\gamma_A|}{4G_N}$$

$S(\text{radiation})$



Where we stand (ii) — the quantum information side

- Standard QI asymptotics: $n \rightarrow \infty$ copies at **fixed** local dimensions.
- Thermodynamic systems are infinitely large, often ending up with nontrivial von Neumann algebras.
- Quantum field theories (QFT) live at large (often infinite) local dimensions.
- Large-d techniques in QI exist but mostly as proof machinery and QI in QFT regimes are much less explored [but attempts exist; see Hayata's talk, L. van Luijk, et al.]

Why quantum gravity is a good place to try this

We do not have the model. We have constraints.

- causality, and an arrow of time
- diffeomorphism invariance, gauge redundancy, dressing
- holography and entropy bounds
- an observer with finite access, finite energy, finite complexity

So the well-posed questions are **already operational**:

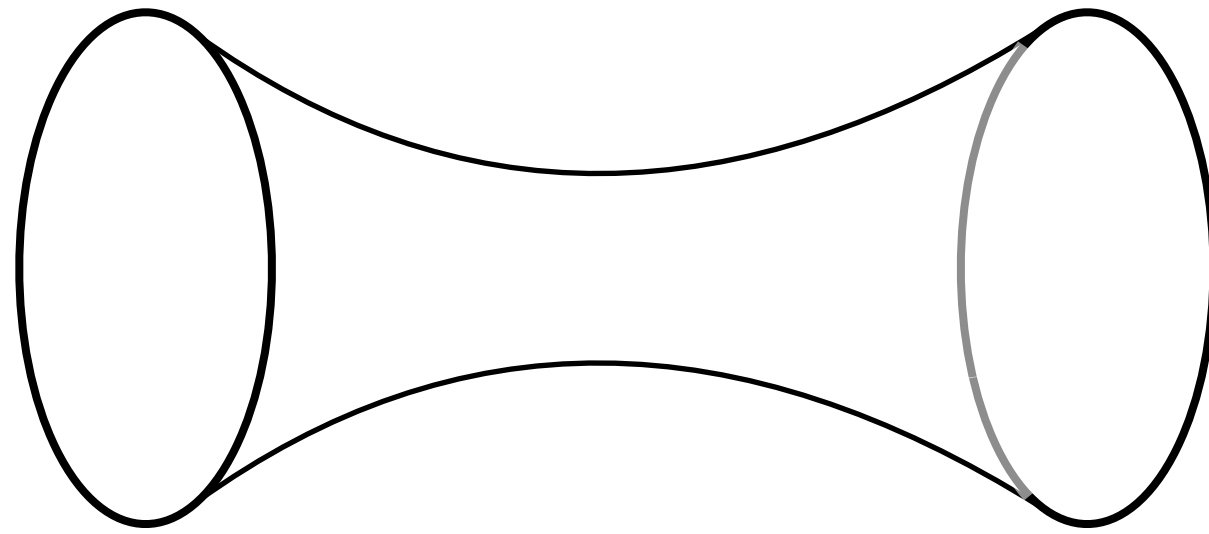
- What can a physical observer *prepare, transmit, reconstruct, verify*?
- Which statements survive without knowing H ?

Problem 2.1 — can gravitational observables be justified, not merely interpreted, operationally?

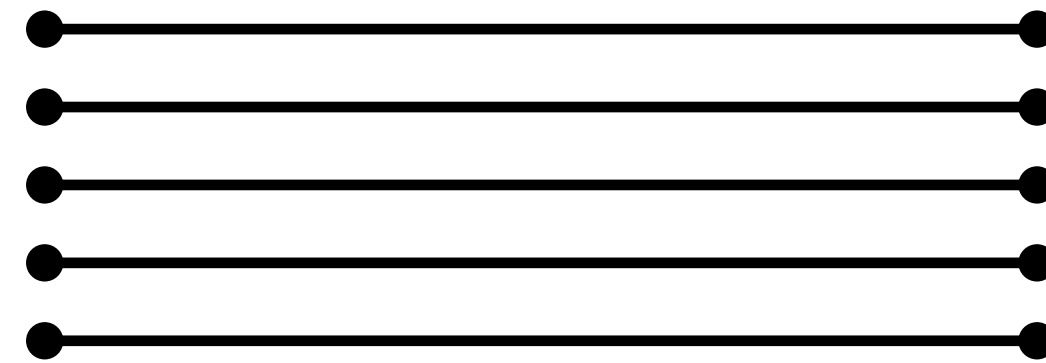
Some examples

Example 1: Operational aspects of spatial connectivity

- 2001, 2013 ER=EPR [Maldacena; Maldacena-Susskind]
- 2016-2018 Bit thread [Freedman-Headrick; Cui-Hayden-He-Headrick-Stoica-Walter]



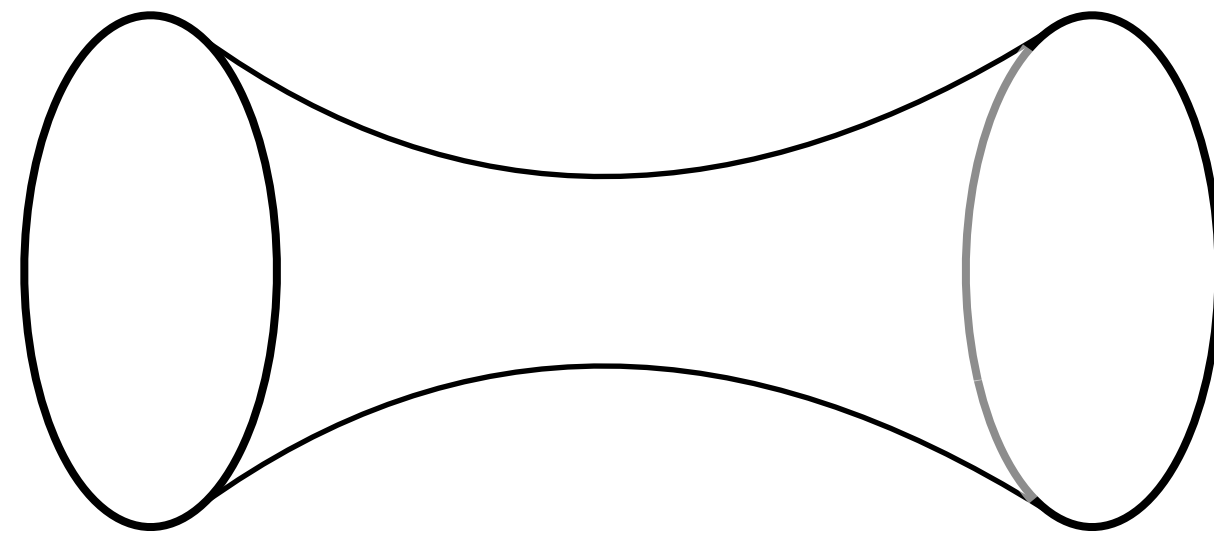
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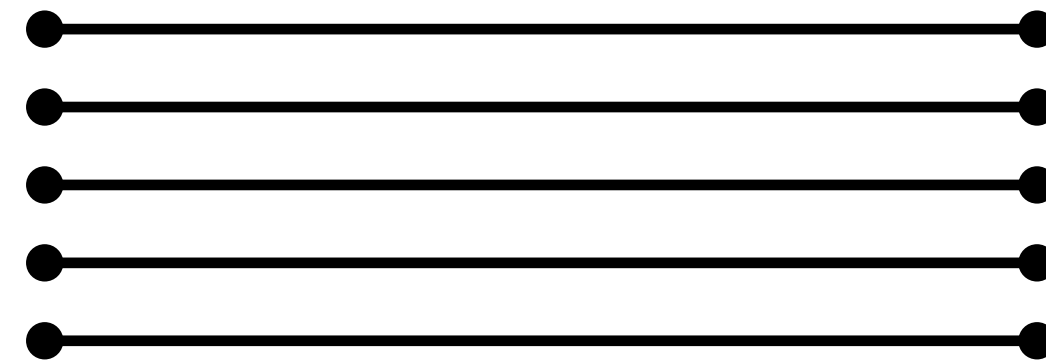
Spatial connectivity = EPR?

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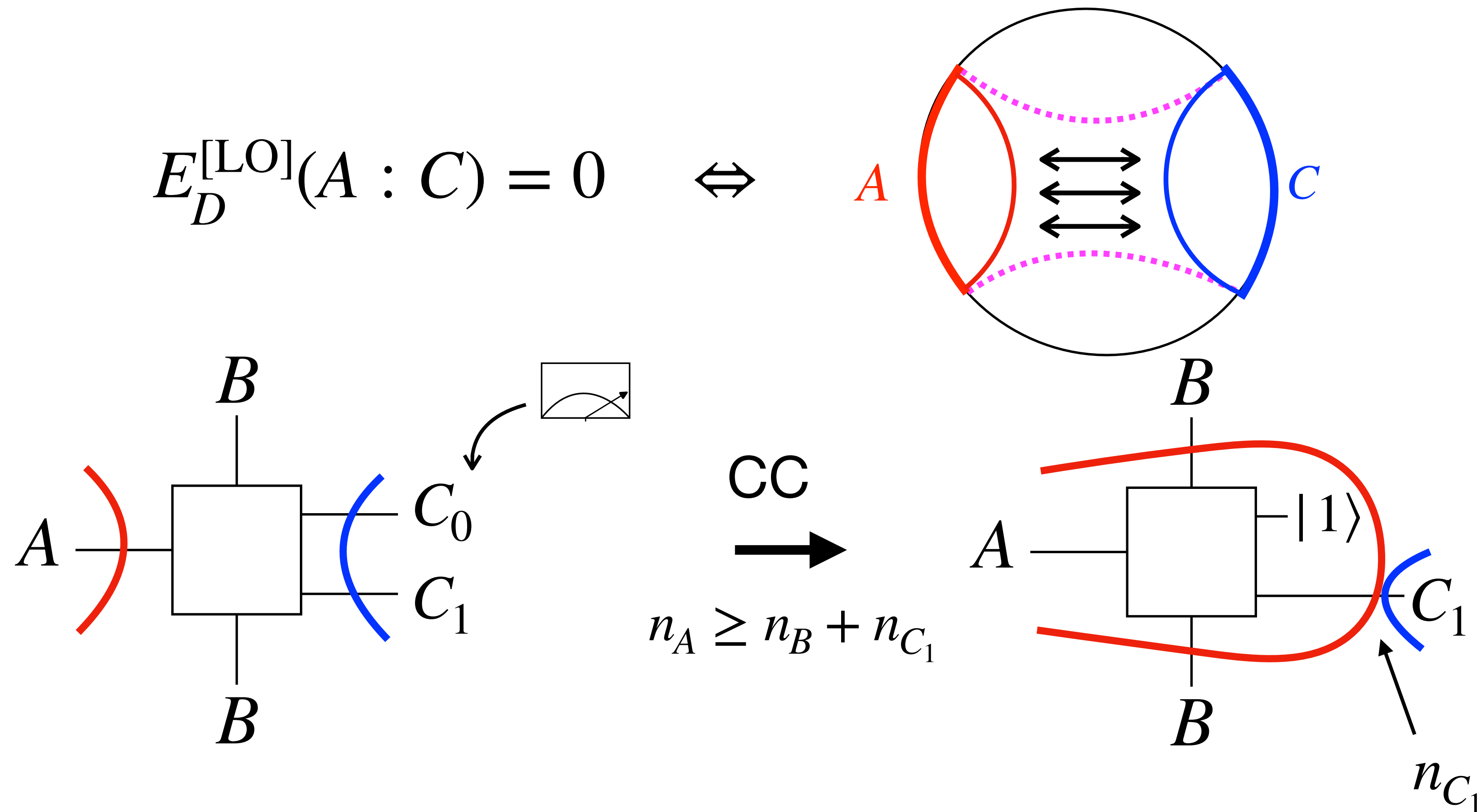


Spatial connectivity = EPR?

- 2019, 2021 **Not mostly bipartite** [Akers-Rath; Hayden-Parrikar-Sorce]

Example 1: Operational aspects of spatial connectivity

- 2025 **Mostly non-bipartite** [Mori-Yoshida 2411.03426] [Li-Mori-Yoshida 2502.04437]

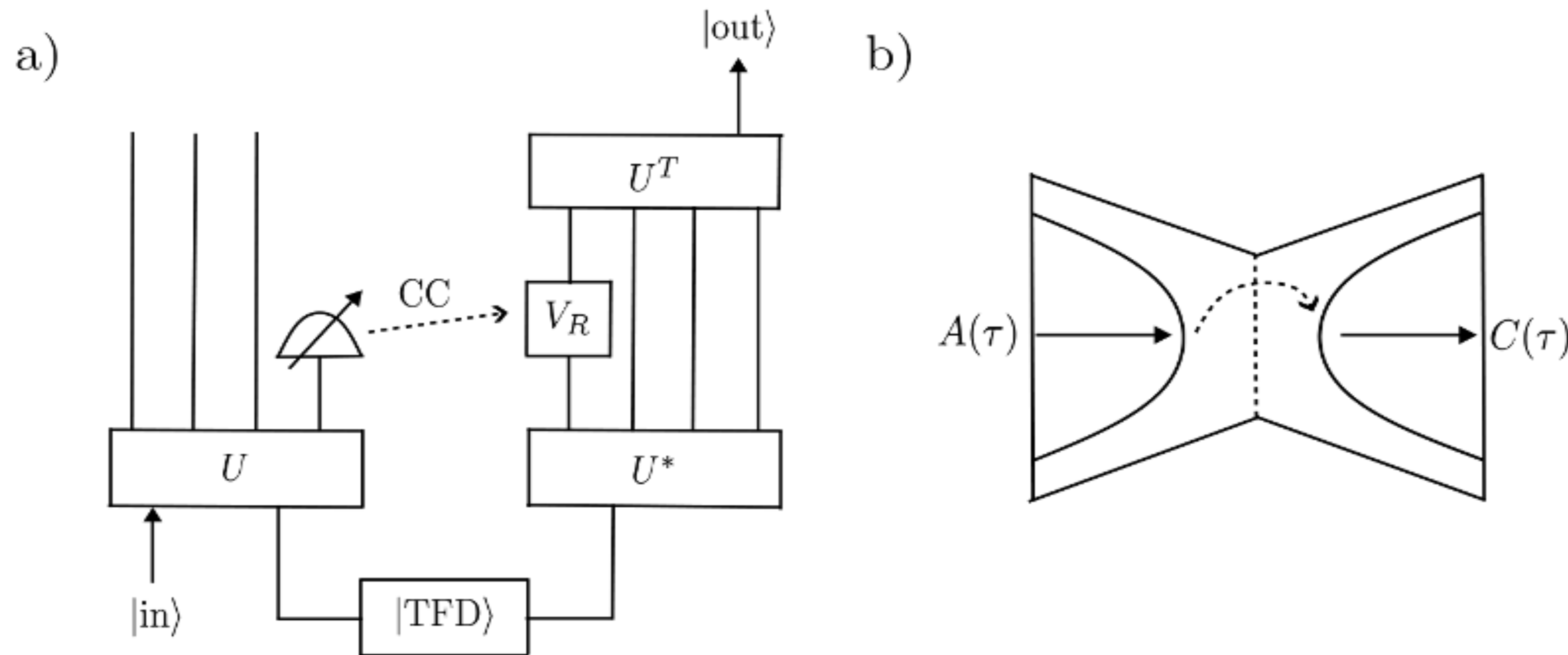


Entanglement distillation task $\Rightarrow$ more information about geometry

Example 1: Operational aspects of spatial connectivity

- 2025 **Mostly non-bipartite** [Mori-Yoshida 2411.03426] [Li-Mori-Yoshida 2502.04437]

Distillation-based traversable wormhole by LOCC

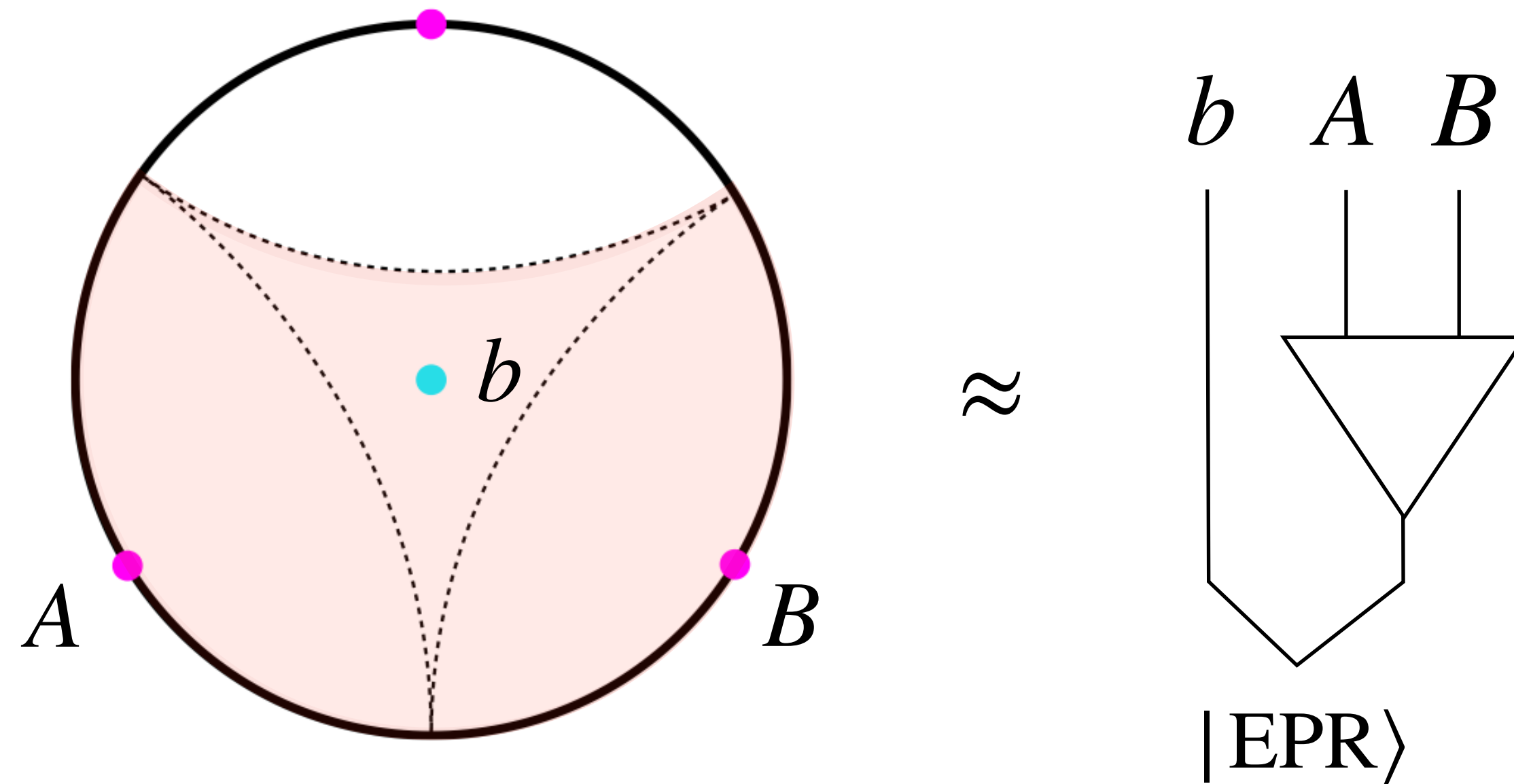


Entanglement distillation task $\Rightarrow$ more information about geometry

Example 1: Operational aspects of spatial connectivity

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Entanglement wedge reconstruction as **entanglement distillation**



Alternative interpretation:
state merging

[Akers-Penington 2008.03319]

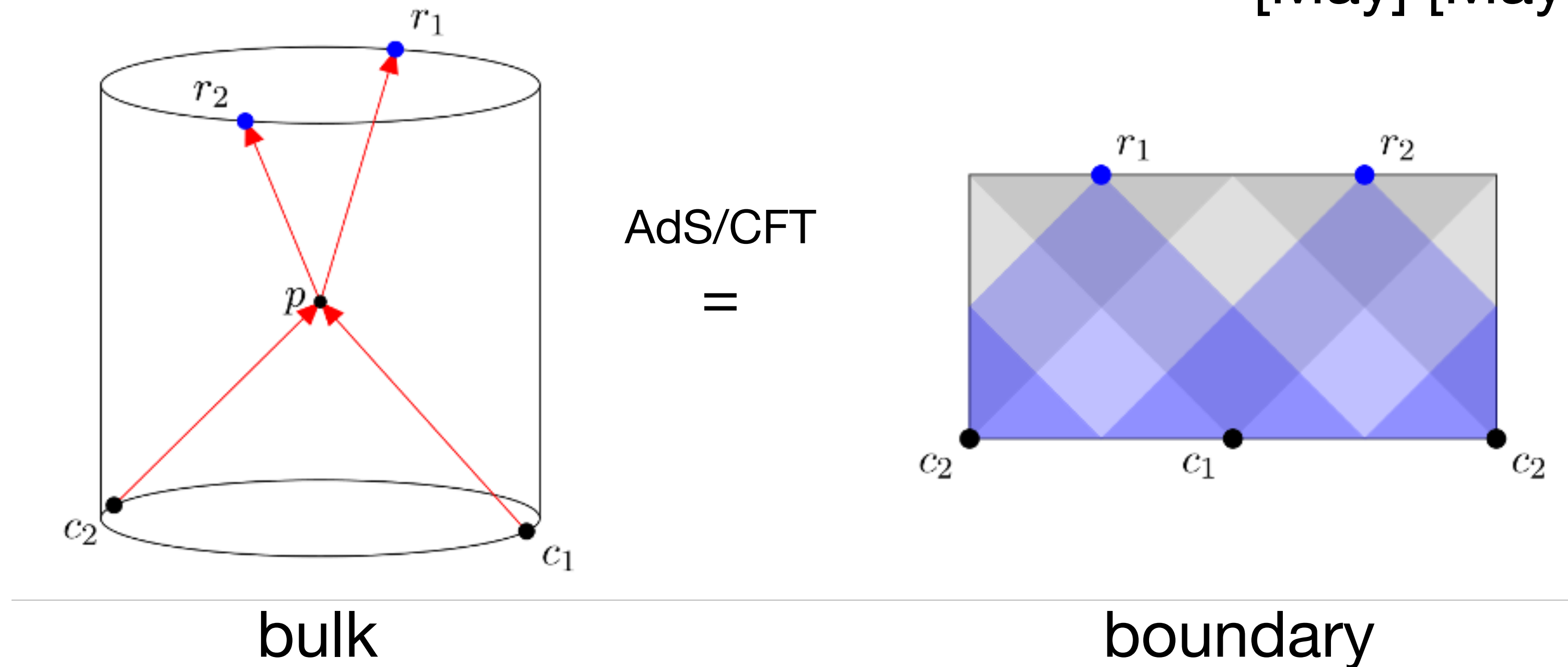
[Akers, et al. 2307.13032]

[Yeh-Wang-Hayden 2510.07418]

Entanglement distillation task $\Rightarrow$ more information about geometry

Example 2: Causality/correlation constraints from quantum tasks

[May] [May-Sorce-Penington]



The inputs can meet, interact, and reach both outputs.

No common meeting point is available before the deadlines.

The boundary must implement the same task using its distributed resources (nonlocal quantum computation). ← QI

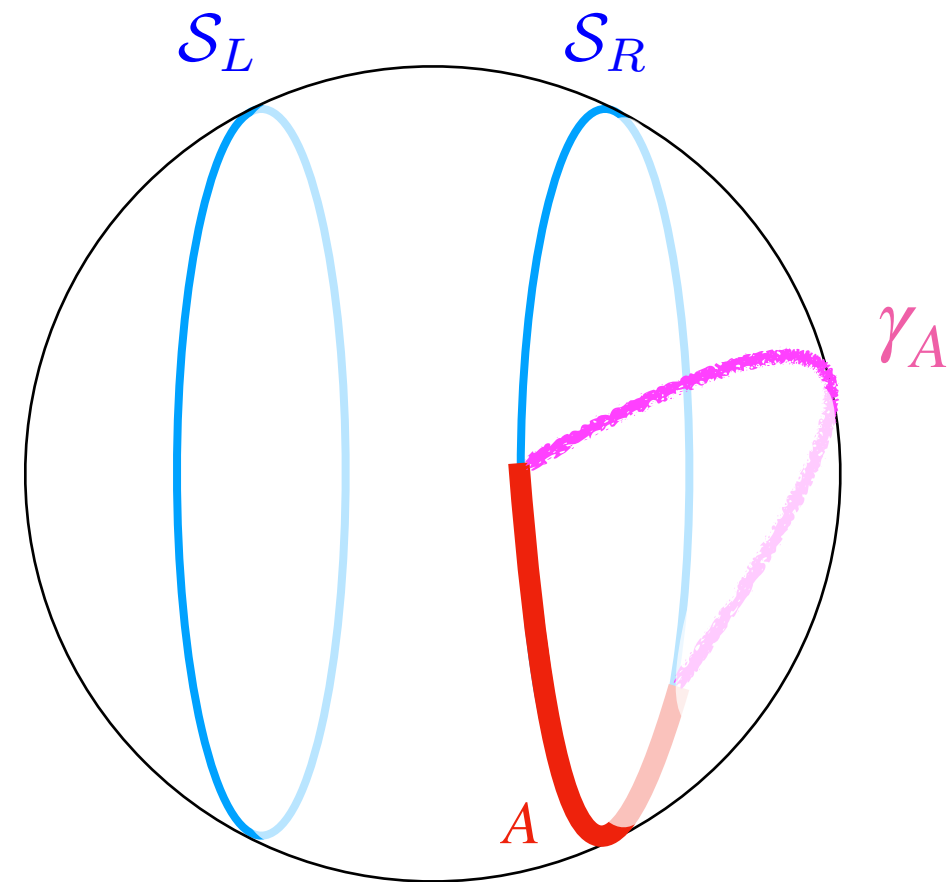
Connected wedge theorem (CWT): Bulk scattering requires a connected wedge.

Example 2: Causality/correlation constraints from quantum tasks

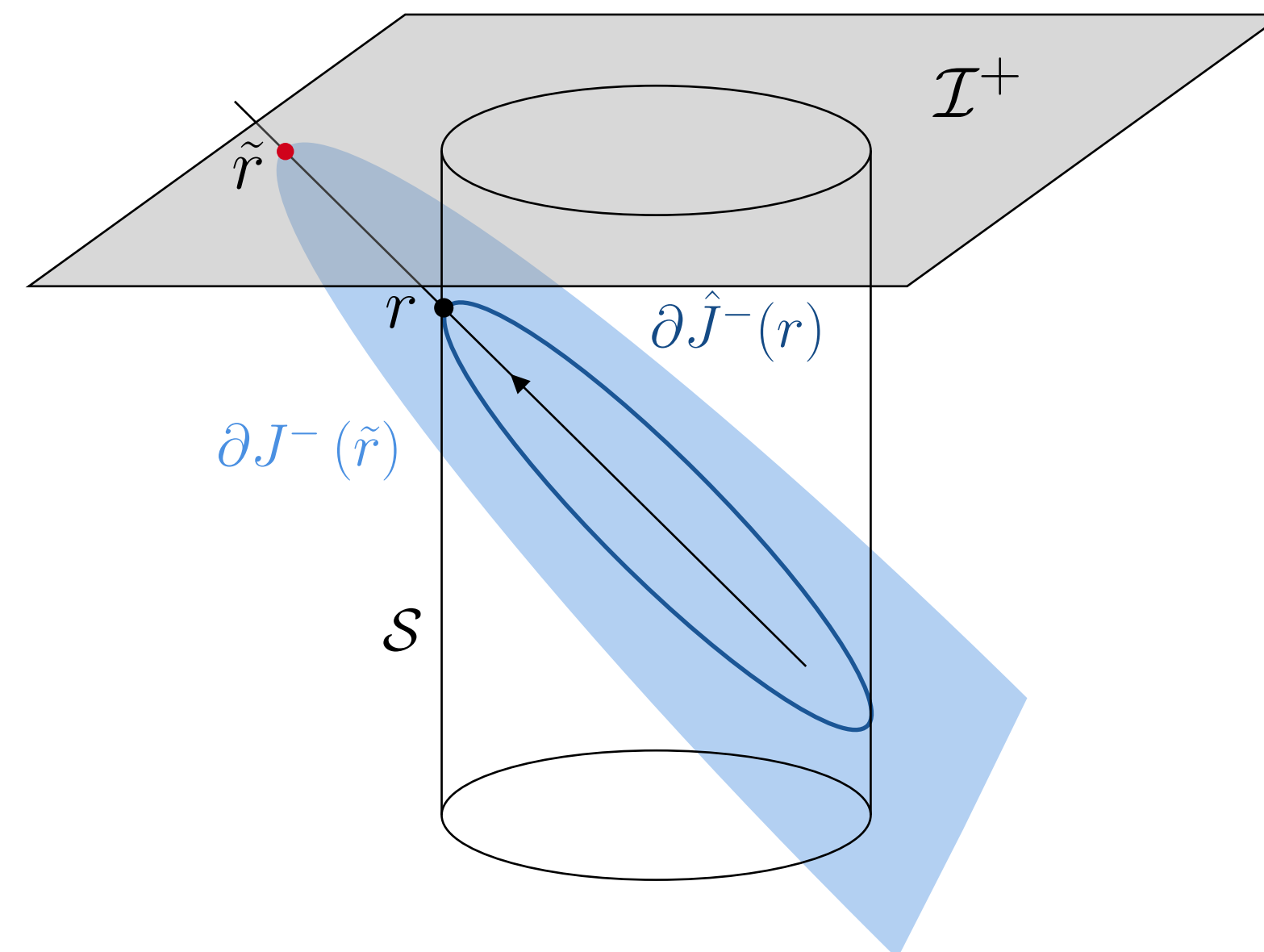
[Franken-Mori 2410.09050]

- Use CWT to **infer the causal structure of unknown holographic boundary systems**.
- de Sitter static patch holography ~ dS/CFT?

$$S(\rho_A) = \min_{\gamma_A} \frac{|\gamma_A|}{4G_N}$$



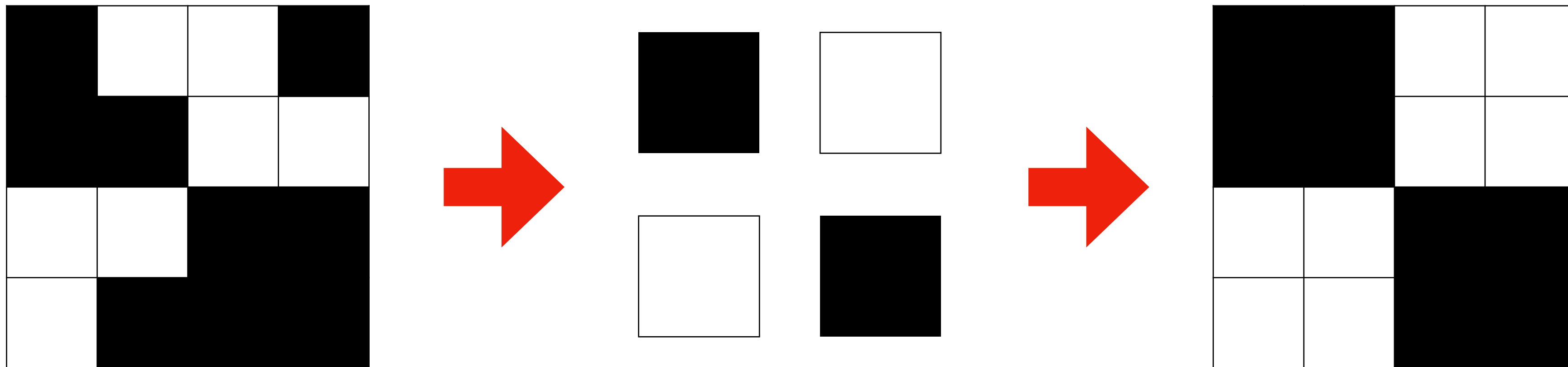
$\Rightarrow$



Cryptographic task $\Rightarrow$ prediction on unknown holographic proposals

Example 3: Resource theoretic interpretation of renormalization group

Renormalization group (RG): **coarse graining** and **rescaling**



can be summarized as a quantum channel [Mori-Nagasawa, wip]:

$$\mathcal{E}_\sigma(\rho) = \text{Tr}_>(\rho) \otimes \sigma_>, \quad \mathcal{G} = \mathcal{V} \circ \mathcal{E}_\sigma$$

Example 3: Resource theoretic interpretation of renormalization group

Resource theory clarifies *which* operations can make use of *what* resources of states.

- What is the **resource monotone** under RG? (partial answer from physics: c -function)
 - Resource theory allows us to compare two different theories in terms of resources, not just along a single RG trajectory.
- Can we determine whether a theory can be realized as another theory after RG map? (complete monotone characterizing convertibility)
- What has no resource (**free states**) and what does not produce any resources (**free operations**)?

QI input:

They are characterized completely once we have the resource-destroying map.

Example 3: Resource theoretic interpretation of renormalization group

[Mori-Nagasawa, wip]

Let the RG fixed points as the free states. We have a resource-destroying map

$$\mathcal{G}_\infty = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathcal{G}^n \quad (\text{Cesàro mean; long time average}) \quad \mathcal{G} : \text{RG map}$$

This exists even when the IR limit does not exist (e.g. limit cycles).

With this, one can information theoretically identify the resource monotone of RG:

$$\mathcal{R}(\rho) := D(\rho \| \mathcal{G}_\infty(\rho)) \quad \text{relative entropy from the IR limit}$$

Around the IR fixed point, information geometry relates to physical metric and dof:

$$D(\rho(g) \| \rho_*) = \frac{C_*}{2} g_{ij}^{\text{Zam}}(g_*) \delta g^i \delta g^j + O(\delta g^3) = \frac{C_*}{\kappa \omega} (c(s) - c_{\text{IR}}) + O(\delta g^3).$$

Resource theory $\Rightarrow$ more unified, broadly applicable physics formulation

Example 4: Holography-inspired QI theorems/conjectures

For random tripartite states,

- [Li-Mori-Yoshida 2502.04437]: Nothing is **LU distillable** and no global symmetry.
- [Mori-Yoshida 2608.05274]: **Relative entropy of entanglement** equals entanglement of formation or mutual information.

These are geometrically supported and later proven rigorously.

Holography-assisted tasks $\Rightarrow$ conjectures about random/generic states

<https://arxiv.org/abs/2606.30853>

Prob.	Topic (paraphrased)
2.1	Operational definitions of gravitational/geometrical quantities
2.2	Causality and distributed tasks
2.3	Correlators and task performance
2.4	Task sensitivity to quantum gravity
2.5	Physically admissible free states/operations
2.6	Monotones and their operational meaning in physics
2.7	Resource theory of RG
2.8	Holographic resource theory



Slides

Appendix

Operational aspects of black holes

Almheiri-Marolf-
Polchinski-Sully
(AMPS)

Late radiation cannot have two independent purifying partners.
The contradiction constrains semiclassical subsystem assumptions.

monogamy of entanglement → firewall? island?

Harlow–Hayden

A decoder may exist while requiring too much computation.
Accessibility depends on the observer's time and computational resources.

observer's accessible algebra does matter physics?

Resource theory

Resource theory clarifies *which* operations can make use of *what* resources of states.

Given an operation Δ , which we call the resource-destroying map (RDM),

- **Free states** $\forall \rho \in \mathcal{F}(\Delta)$ leave invariant under Δ :

$$\Delta(\rho) = \rho \quad (\text{free states have no resource to be destroyed})$$

- (A broader class of) **free operations** are those commuting with Δ :

$$\mathcal{E} \circ \Delta = \Delta \circ \mathcal{E} \quad (\text{free operations do not generate any resource that can be destroyed})$$

If ρ is a free state, $\Delta(\mathcal{E}(\rho)) = \mathcal{E}(\Delta(\rho)) = \mathcal{E}(\rho)$ so free operations does not generate any resources for free states.

- **Resource** of free states should be zero $\mathcal{R}(\mathcal{F}) = 0$ and **resource** under free operations

$$0 \leq \mathcal{R}(\mathcal{E}(\rho)) \leq \mathcal{R}(\rho), \quad \forall \rho$$

Quantify resource $\mathcal{R}$
given Δ ?

Example: Entanglement as resource. Local Operations and Classical Communication (LOCC) is free operations. Separable states are free.

RG resource monotone = RG monotone

- This RG monotone is rigorously non-increasing under RG flow.
- It can be related to Zamolodchikov metric and central charge near the IR fixed point:

Let $\rho \propto e^{-\beta H}$ then near the IR f.p. ρ_* ,

$$D(\rho(g) || \rho_*) = \frac{C_*}{2} g_{ij}^{\text{Zam}}(g_*) \delta g^i \delta g^j + O(\delta g^3)$$

↑
Bogoliubov-Kubo-Mori (BKM) metric

For a single linearized RG Eigen-direction $\beta = -\omega \delta g$,

$$D(\rho(g(s)) || \rho_*) = \frac{C_*}{\kappa \omega} (c(s) - c_{\text{IR}}) + O(\delta g^3).$$

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{g_{ij}^{\text{Zam}}}{|x - y|^{2\Delta}}$$

Zamolodchikov c-theorem:

$$\frac{dc}{ds} = -\kappa g_{ij}^{\text{Zam}} \beta^i \beta^j, \quad \kappa > 0$$