N. Emil J. Bjerrum-Bohr

Shanghai Tech Nov 2026



Binary Black Hole Merger Dynamics from S-matrix Methodology

Talk about work done in collaboration with e.g.
Gang Chen, Poul Damgaard, Carl Eriksen, Konstantinos Papadimos,
Ludovic Plante, Nabha Shah, Marcos Skowronek, Pierre Vanhove, Yuexiang Zhang

MARIE CURIE ACTIONS

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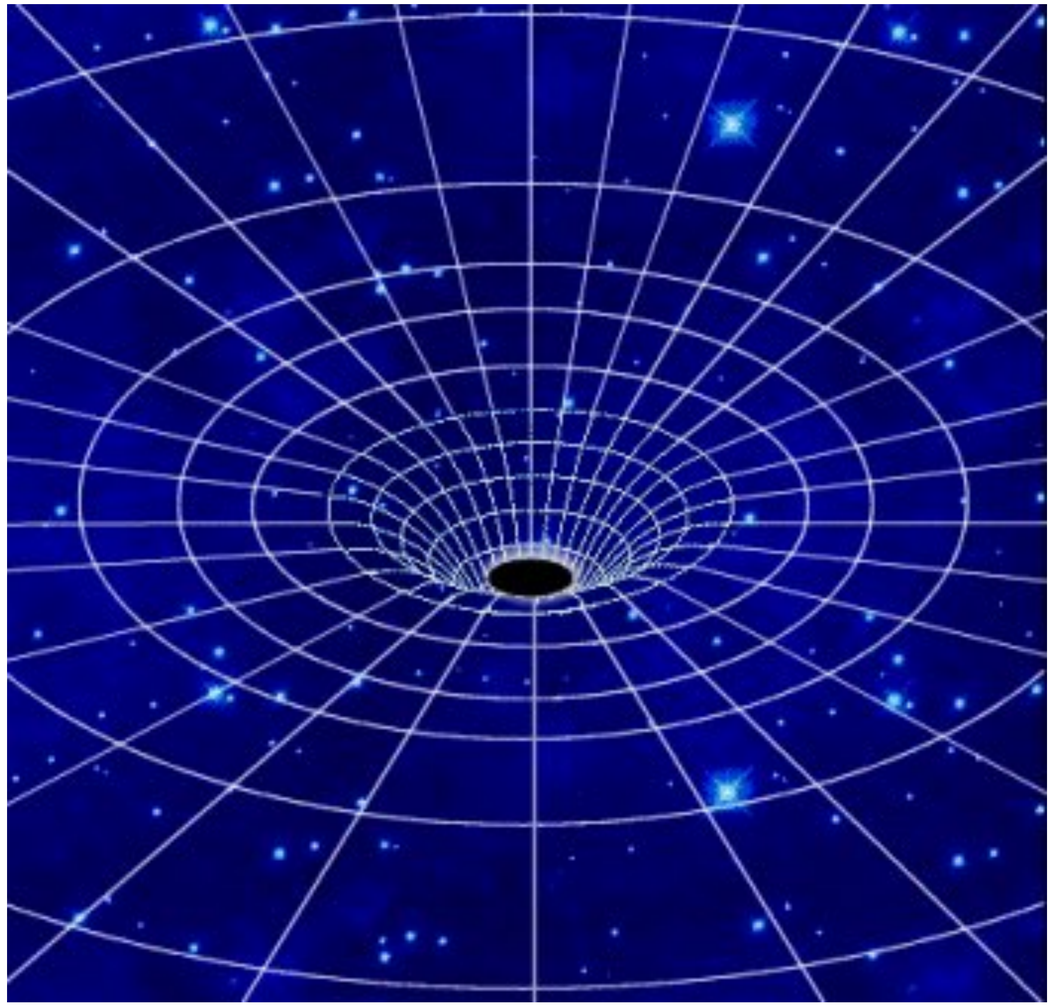
General Relativity

Einstein's theory presents us with a beautiful theory of gravity and several exciting questions, e.g.:

(See also Paolo's talk)

- Test of classical gravity principles and motion
- Geometrical description <-> EFT-QFT (flat space / curved space) formulations
- Strongly coupled gravity (resummation, etc)
- Quantum extensions
- Higher derivative operator bounds
- Graviton properties/mass, etc

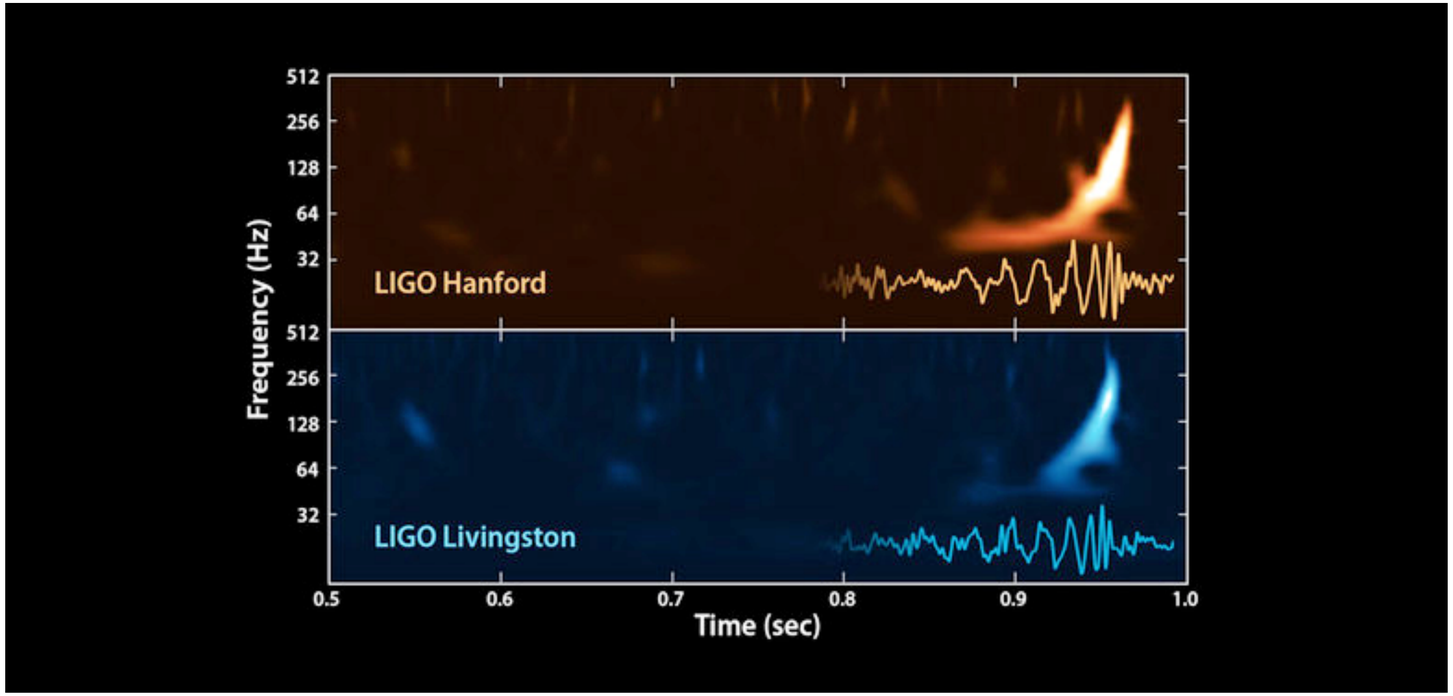


A new window for gravitational data

First direct observation of a binary black hole merger

Direct access to gravitational interactions in the most extreme regimes

Need for theory to match observational progress and precision.



Potential to address interesting questions

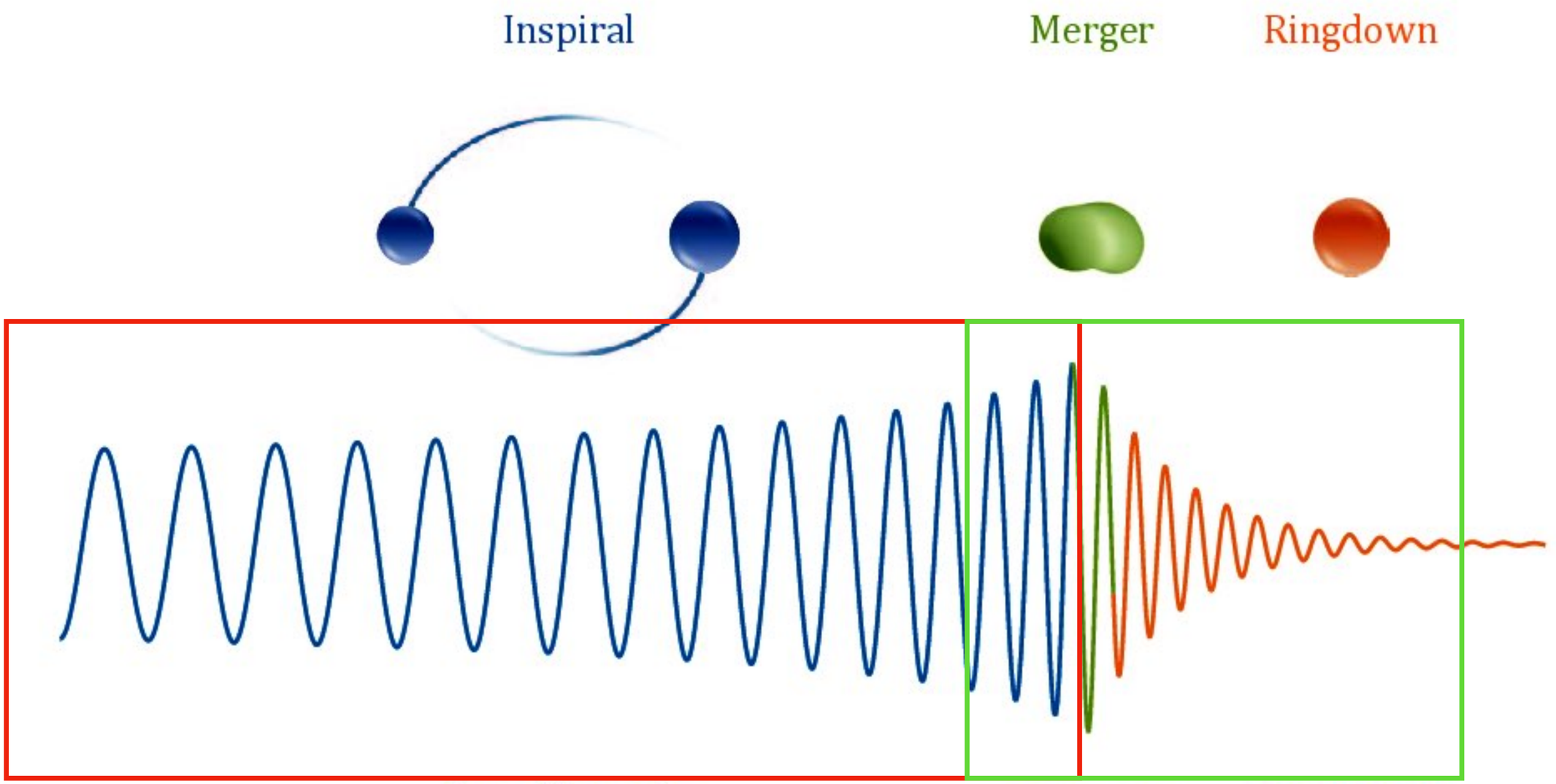
Environments

Tidal and spin effects, radiation

Amplitude methods enable refined computation and increased precision that complement conventional analysis.

A potential avenue for making new discoveries in gravity

Understanding the gravitational merger of Black Holes



Connecting weak and strong coupled physics?

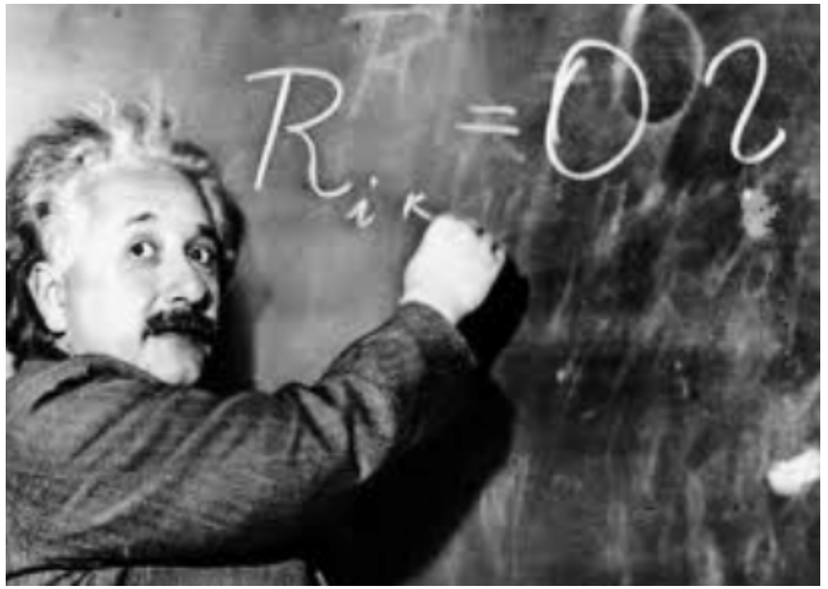
Quantization of Gravity

Known since the 1960s that a particle version of General Relativity can be derived from the Einstein-Hilbert Lagrangian (Feynman, DeWitt)

Expand the Einstein-Hilbert Lagrangian

$$\mathscr{L}_{\mathrm{EH}} = \sqrt{-g}\left[\frac{R}{16\pi G_N} + \mathscr{L}_{\mathrm{matter}}\right]$$

Develop the theory as a quantum field theory



Derive vertices as in a particle theory - compute amplitudes as Feynman diagrams! (GW Kovacs and Thorne 1977)

An effective field theory

A modern viewpoint (Weinberg) to view the quantization of general relativity from the viewpoint of effective field theory

$$\mathscr{L}_{\rm EH} = \sqrt{-g}\left[\frac{R}{16\pi G_N} + \mathscr{L}_{\rm matter}\right]$$

EFT -gravity EFT -matter

$$\mathscr{L}_{\rm eff\ GR} = \sqrt{-g}\left[\frac{2R}{16\pi G_N} + R^2 + R_{\mu\nu}^2 + \ldots + \mathscr{L}_{\rm matter} + \ldots\right]$$

Consistent quantum gravity at low energies long-distance (Donoghue; NEJBB, Donoghue, Holstein)

Advantages: Gravity as an EFT

Treating general relativity as an effective field theory:

- Natural generalization of Einstein's theory
- Ideal (low-energy) perturbative setup

Universal consequences of underlying fundamental quantum theory ~~ link to low-energy features, e.g., string and supergravity theories

Current key research directions

Key question: How can we extract classical physics precisely and efficiently from amplitudes to analyze gravitational dynamics in quantum field theory?

-> Discovery of new physics
Faster and more accurate theoretical breakdown of gravitational wave events!
New computational frameworks?

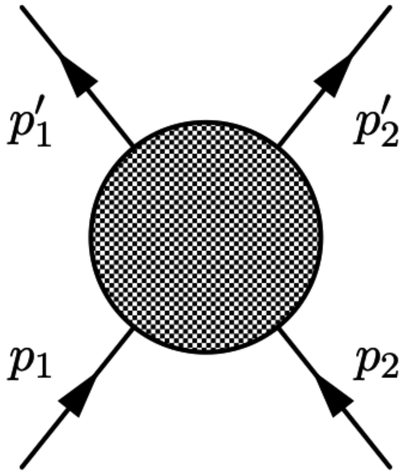
Post-Minkowskian framework and amplitudes

Post-Minkowskian expansion of Einstein's general theory of relativity has received much attention in the amplitude community (Damour)

Idea: use scattering amplitude to provide a self-contained framework for deriving the two-body scattering valid in all regimes of energy and employ the computational power of modern amplitude calculations in an expansion in G

On-shell Integrand construction
Use of KLT / spinor-helicity / numerator technology

$$= \sum_{L=0}^{\infty} \mathcal{M}_L(p_1, p_2, p_1', p_2') =$$



Spinless gravity from particle physics

We start with Einstein-Hilbert term

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + g^{\mu\nu}T_{\mu\nu}\right]$$

Other possibilities of course possible

where the minimal ‘energy-momentum’ tensor for spinless fields is

$$T_{\mu\nu} \equiv \partial_\mu\varphi\partial_n u\varphi - \frac{\eta_{\mu\nu}}{2}\left[\partial^\alpha\varphi\partial_\alpha\varphi - m^2\varphi^2\right]$$

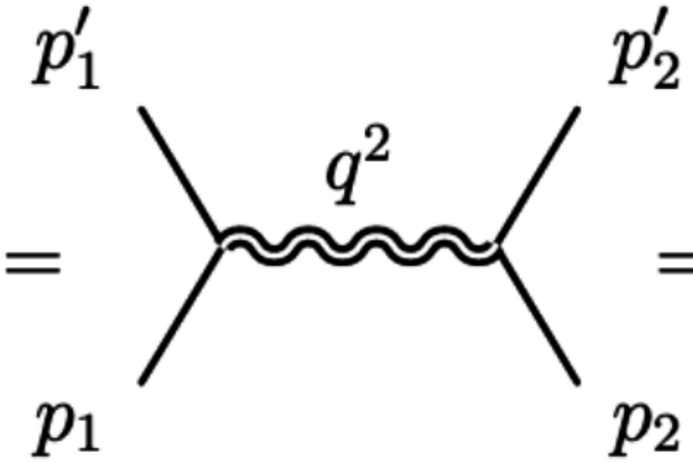
Consider the 2 -> 2 process from path integral

$$\varphi_1(p_1, m_1) + \varphi_2(p_2, m_2) \rightarrow \varphi_1(p_1', m_1) + \varphi_2(p_2', m_2)$$

Tree level — two ‘heavy’-masses

- γ^2 factor resum velocity and generate covariant expressions

$$\mathcal{M}_0(\gamma, \underline{q}^2, \hbar) = \quad = \hbar \frac{2\pi m_1^2 m_2^2 G_N (2\gamma^2 - 1)}{|\underline{q}|^2} + O(\hbar^0)$$



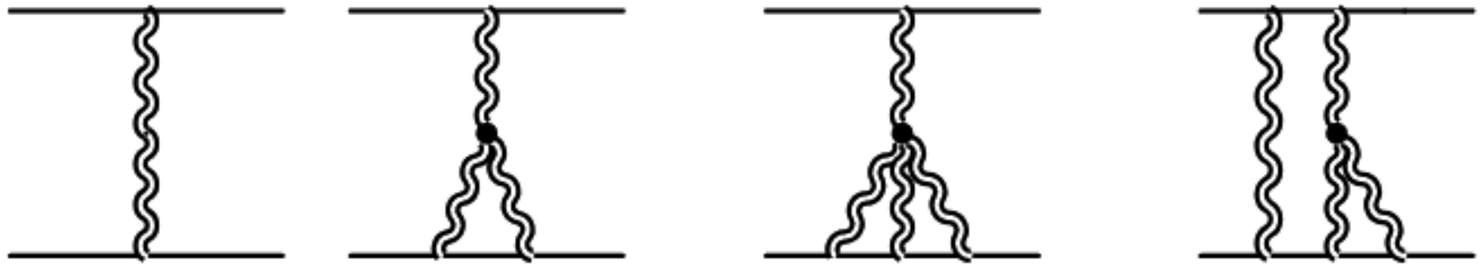
Newton’s law through Fourier transform

$$V(r) = -\frac{G m_1 m_2}{r}$$

- but does **not** give higher G correction effects

Classical gravity from particle scattering

Surprise: Non-linear (classical) corrections from loop diagrams!
Can consider the various exchanges

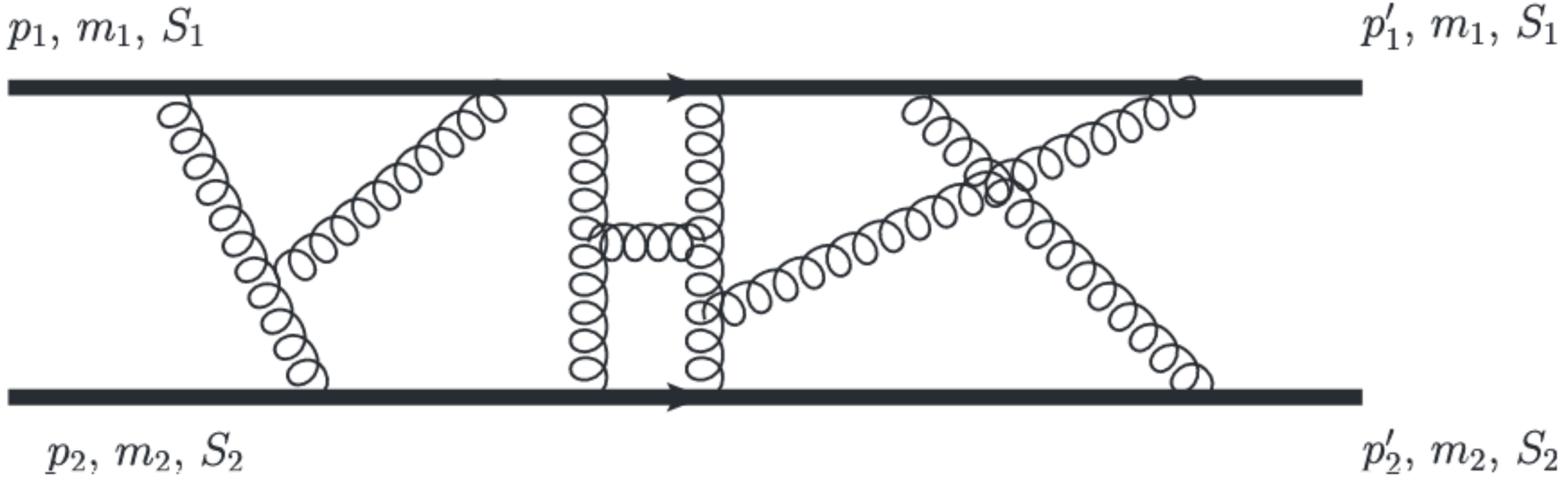


Normally we work with expressions where factors $\hbar$ are not displayed since ($\hbar$ = c =1)

$$q^2 \equiv (p_1 - p_1')^2 \qquad \gamma \equiv \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$\mathcal{E}_{CM}^2 \equiv (p_1 + p_2)^2 \equiv (p_1' + p_2')^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma$$

Generalization to any loop



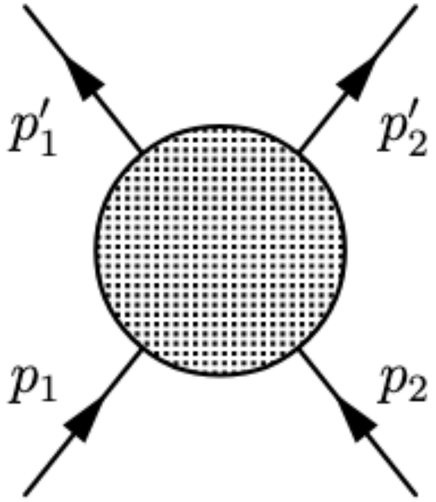
Reinstating $\hbar$ there is a difference between massive and massless propagation

Classical limit: we keep wave number fixed and take Planck's constant to zero, leads to the following Laurant expansion (quantum / classical / superclassical terms)

$$\mathcal{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(-L-1)}(\gamma, \underline{q}^2)}{\hbar^{L+1}|\underline{q}|^{\frac{L(4-D)}{2}+2}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma, \underline{q}^2)}{\hbar|\underline{q}|^{\frac{L(4-D)}{2}+2-L}} + O(\hbar^0) \qquad 1/\hbar$$

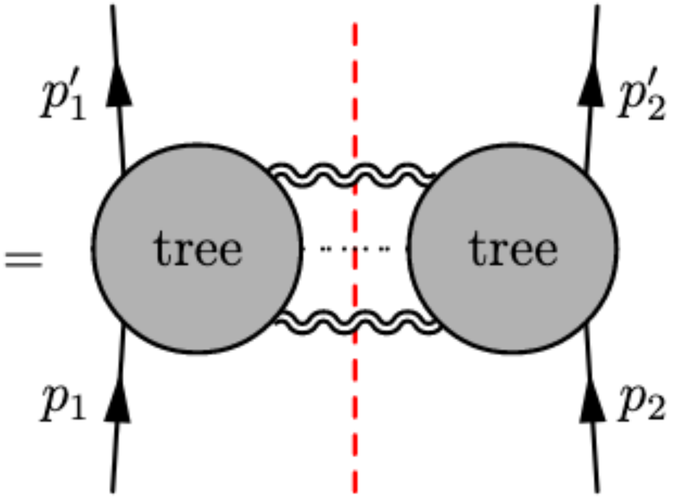
Classical gravitational scattering: Generic loop level

Extraction of integrand similar to QCD
Spinor-helicity and D-dimension
covariant tree
amplitudes can be used in cuts



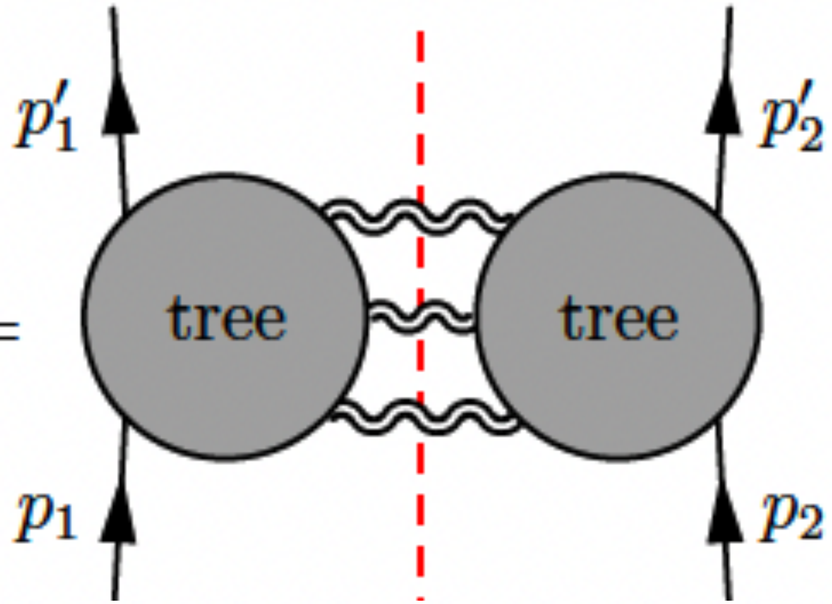
$$= \mathcal{M}(\gamma, q^2) = \sum_{L=0}^{\infty} \mathcal{M}_L(\gamma, q^2)\,.$$

$$\mathcal{M}^{\text{cut}}_{L+1}(\gamma, q^2) =$$



Example: Einstein gravity at two-loop order

$$\mathcal{M}^2_{3-\text{cut}}(\sigma, q^2) =$$



$$\mathcal{M}_2^{3-\text{cut}}(\sigma, q^2) = \int \frac{d^D l_1 d^D l_2 d^D l_3}{(2\pi)^{3D}} (2\pi)^D \delta^{(D)}(l_1 + l_2 + l_3 + q) \frac{i^3}{l_1^2 l_2^2 l_3^2}$$

$$\times \frac{1}{3!} \sum_{\substack{\text{Perm}(l_1, l_2, l_3) \\ \lambda_1 = \pm, \lambda_2 = \pm, \lambda_3 = \pm}} \mathcal{M}_0(p_1, p_1', l_1^{\lambda_1}, l_2^{\lambda_2}, l_3^{\lambda_3}) (\mathcal{M}_0(p_2, p_2', -l_1^{\lambda_1}, -l_2^{\lambda_2}, -l_3^{\lambda_3}))^*$$

Einstein gravity at two-loop order

Integrals

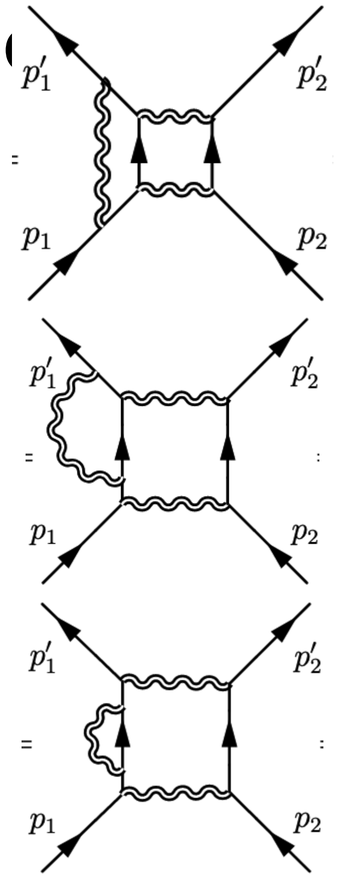
$$\mathcal{M}_2^{3-\text{cut}}(\sigma, q^2) = \mathcal{M}_2^{\square\square} + \mathcal{M}_2^{\triangleleft\square} + \mathcal{M}_2^{\square\triangleright} + \mathcal{M}_2^{\triangleleft\triangleleft} + \mathcal{M}_2^{\triangleright\triangleright} + \mathcal{M}_2^{H} + \mathcal{M}_2^{\square\circ}$$

We use the unitarity cut to fix coefficients in front of master integrals. The full result can be written

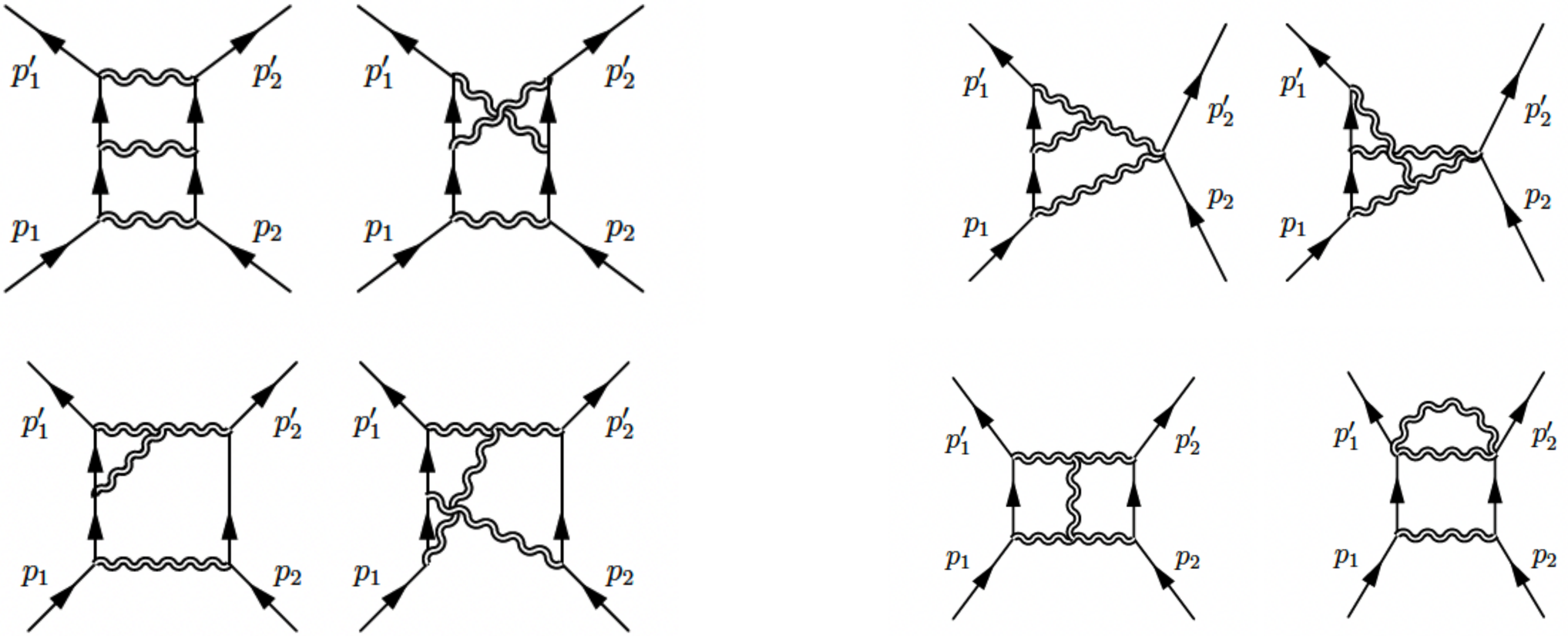
$$\mathcal{M}_2(\gamma, q^2) = \mathcal{M}_2^{3-\text{cut}}(\gamma, q^2) + \mathcal{M}_2^{\text{SE}}(\gamma, q^2)$$

Where the SE contribution is

$$\mathcal{M}_2^{\text{self}-\text{energy}}(\gamma, \underline{q}^2) = -4(16\pi G_N)^3 \sum_{i=I}^{IV} (J_{SE}^{i,s} + J_{SE}^{i,u}) + (m_1 \leftrightarrow m_2)$$



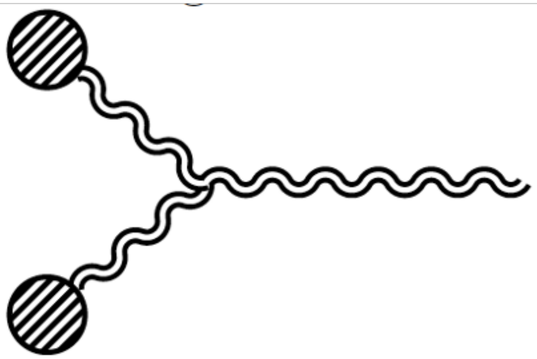
Einstein gravity at two-loop order



Needed master integrals at two-loops for the conservative part of the amplitude - determined by LiteRed2.

Interpretation

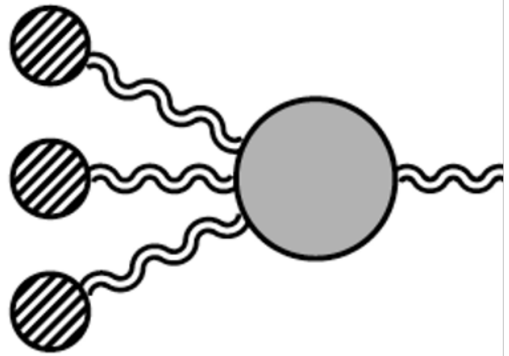
$$\int \frac{d^3\vec{\ell}}{(2\pi)^3}\frac{1}{\vec{\ell}^2}\frac{1}{(\vec{\ell}+q)^2} \longleftrightarrow$$



Integration of classical sources on tree graphs – no loops!

Picture extends to higher loops

$$I_{\triangleright\triangleright(1)}(p_1,q), I_{\triangleright\triangleright(2)}(p_1,q) \leftrightarrow$$

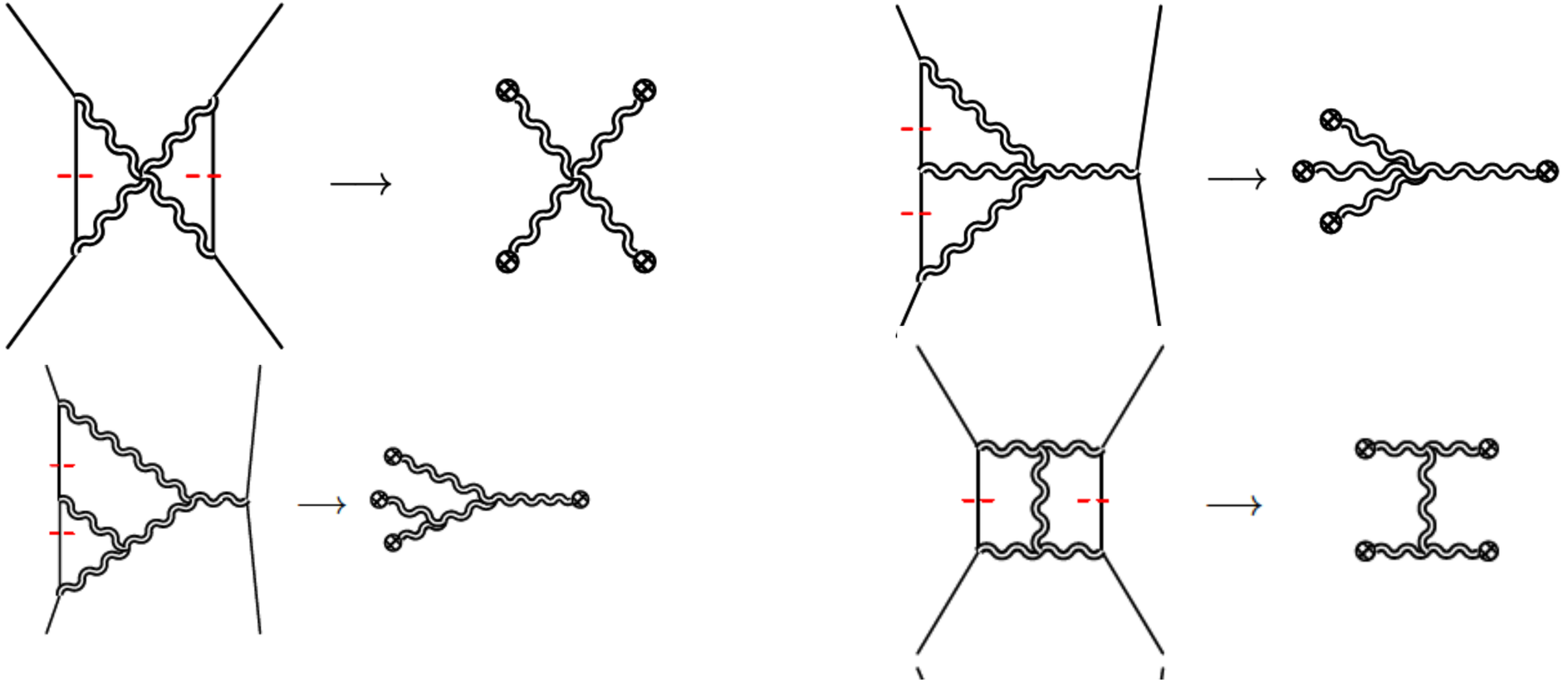


Explains the metric computation by (Duff)

(NEJB, Damgaard, Festuccia, Plante, Vanhove)

Velocity cuts and relation to world lines

Can open up massive propagators - direct connection to world-line formulation - direct computation of probe amplitude to four-loop order



(NEJBB, Damgaard, Plante, Vanhove; NEJBB, Plante, Vanhove)

Gravity at multi-loop order

Needed master integrals - determined by programs such as LiteRed2/FIRE6/KIRA, etc. (See also talk by Lee)

Each loop order poses a new challenge

Multi-loop integration
- IBP relation reduction (various programs)
- Modern integration techniques
- Integration regions: PDE approaches

Many interesting amplitude-related research problems can be considered in this framework

Even simpler organisation of results — velocity cuts, exponentiation and soft expansion

Velocity cuts is a clever to organise the integrand for simpler computations. The basic observation is that the combination of linear propagators

$$\left(\frac{1}{(p_A\cdot\ell_A+i\varepsilon)(p_A\cdot\ell_B-i\varepsilon)}-\frac{1}{(p_A\cdot\ell_B+i\varepsilon)(p_A\cdot\ell_A-i\varepsilon)}\right)\times \left(\frac{1}{(p_B\cdot\ell_A-i\varepsilon)(p_B\cdot\ell_C+i\varepsilon)}-\frac{1}{(p_B\cdot\ell_C-i\varepsilon)(p_B\cdot\ell_A+i\varepsilon)}\right)$$

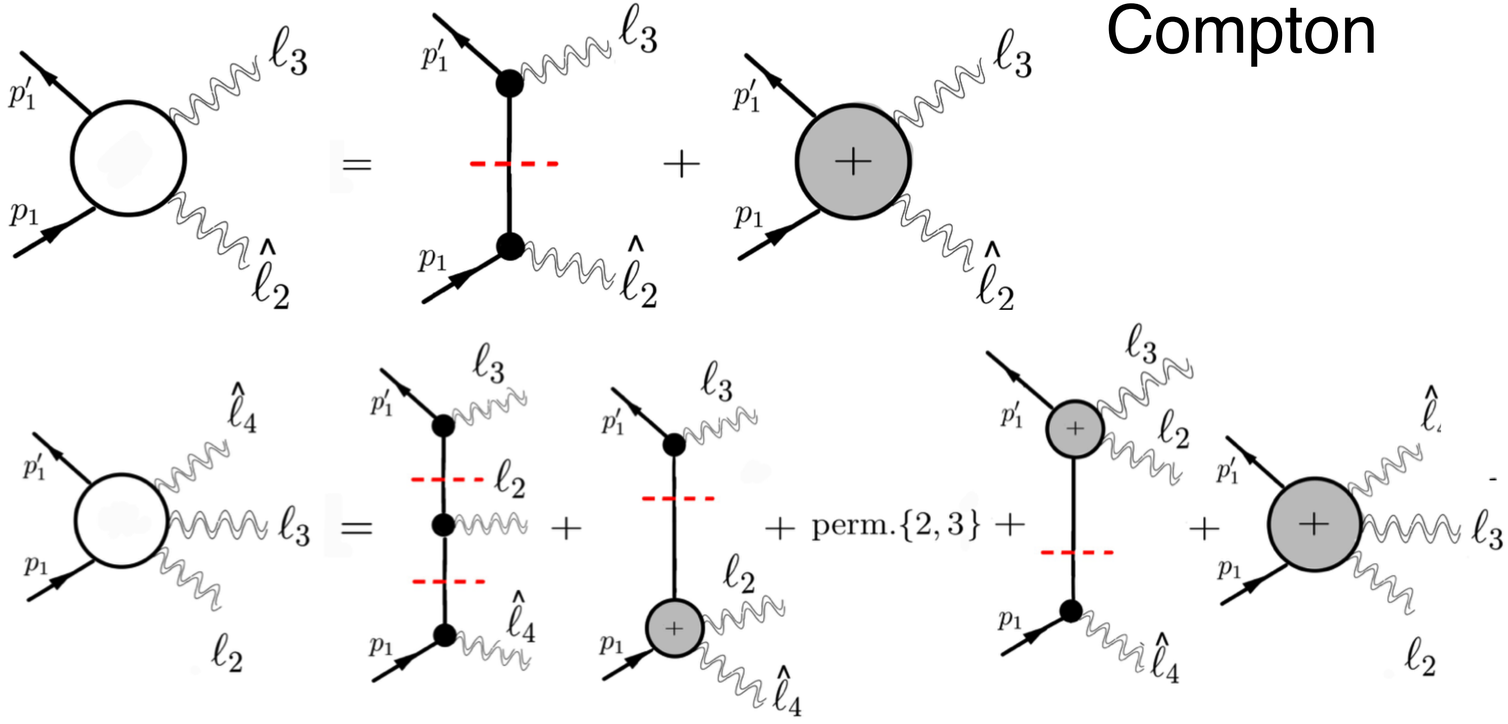
can be expressed as

$$\left(\frac{\delta(p_A\cdot\ell_A)}{p_A\cdot\ell_B+i\varepsilon}-\frac{\delta(p_A\cdot\ell_B)}{p_B\cdot\ell_A+i\varepsilon}\right)\times\left(\frac{\delta(p_B\cdot\ell_C)}{p_B\cdot\ell_A+i\varepsilon}-\frac{\delta(p_B\cdot\ell_A)}{p_B\cdot\ell_C+i\varepsilon}\right)$$

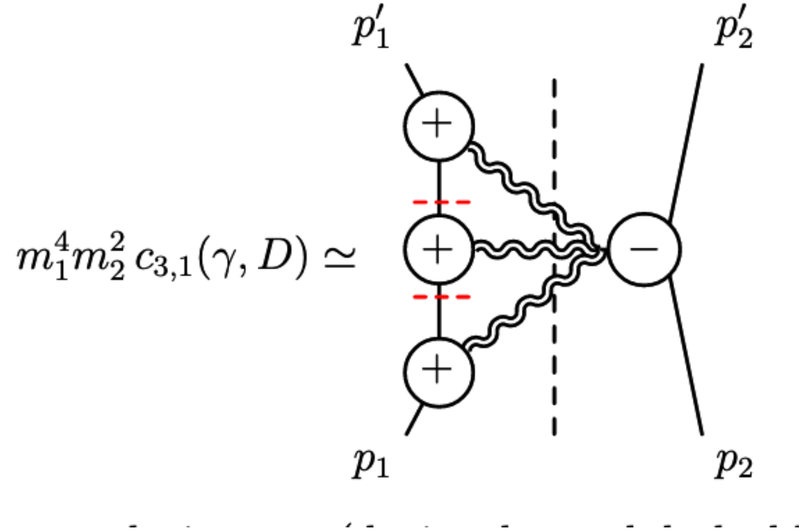
using

$$\frac{1}{x+i\varepsilon}-\frac{1}{x-i\varepsilon}=-2i\pi\delta(x)$$

Velocity cuts and tree amplitudes / soft expansion



Compton



Using the N-operator

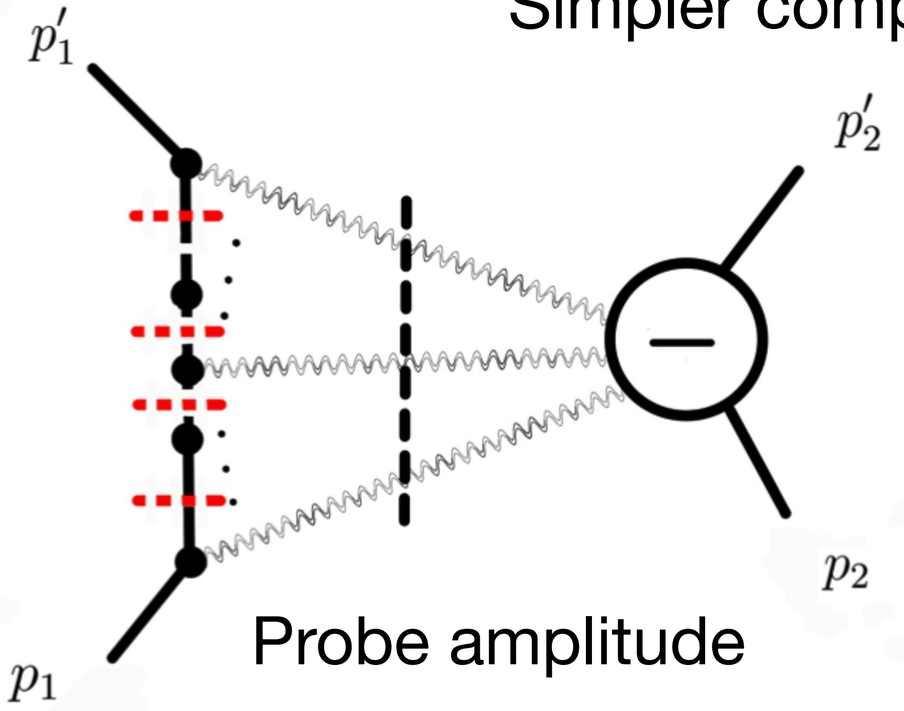
$$\mathcal{M}^{\text{tree}}_{L+1} \sim (\mathcal{M}^{\text{tree}(+)}_1)^{L+1} \prod_i^L \delta_i(\ldots) + (\mathcal{M}^{\text{tree}(+)}_1)^{L-1} (\mathcal{M}^{\text{tree}(+)}_2) \prod_i^{L-1} \delta_i(\ldots) + \cdots$$

$$+ \mathcal{M}^{\text{tree}(+)}_1 \mathcal{M}^{\text{tree}(+)}_L \delta(\ldots) + \mathcal{M}^{\text{tree}(+)}_{L+1}$$

(NEJBB, Plante, Vanhove)

Simpler integrand - velocity cuts tree topologies!

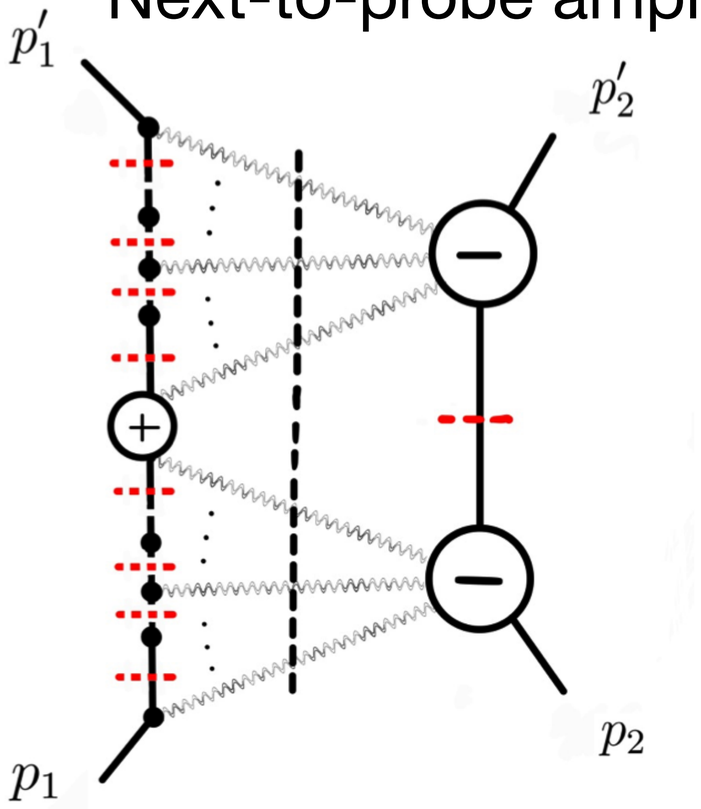
Simpler computations



Probe amplitude

(NEJBB, Plante, Vanhove)

Next-to-probe amplitude



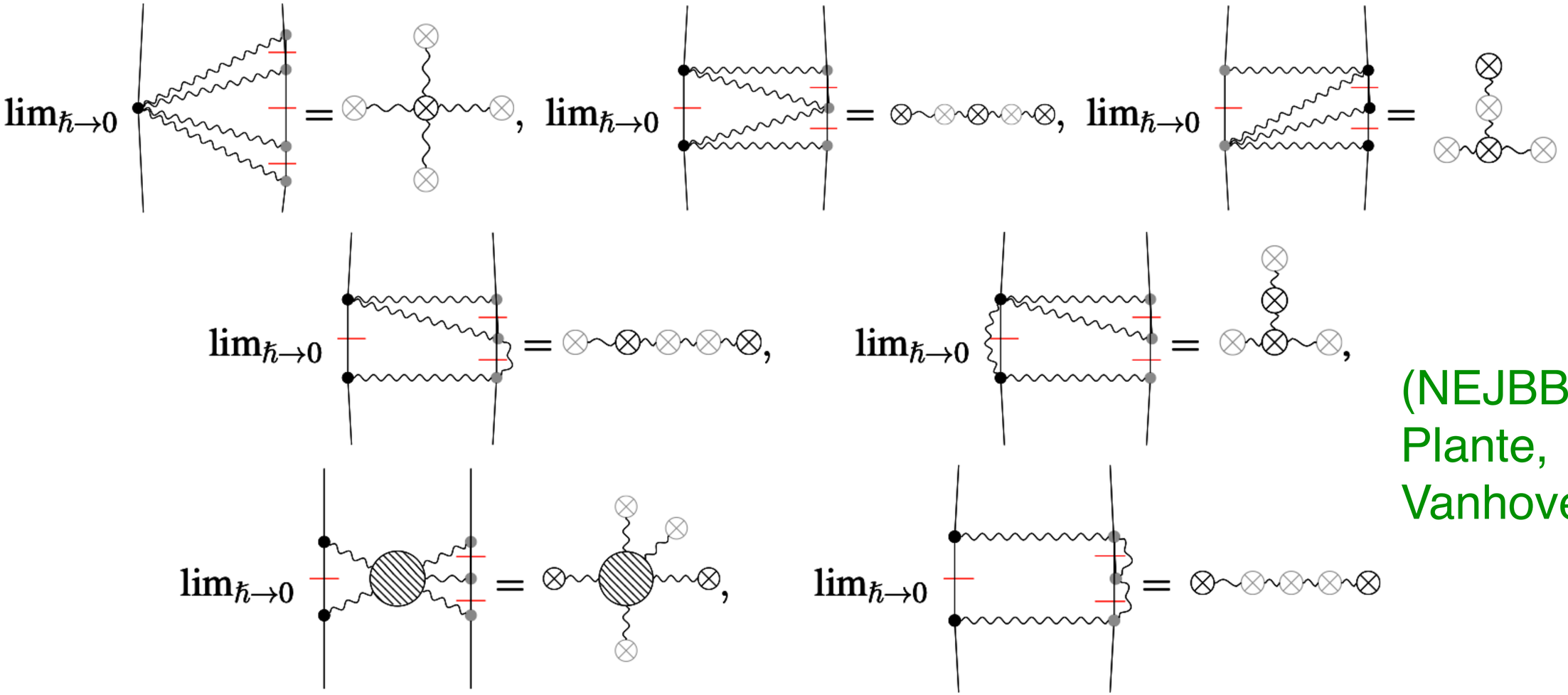
Heavy-quark—EFT inspiration:

(Damgaard, Haddad, Helset)

(Brandhuber, Chen, Travaglini, Wen)

- heavy mass vs small |q| expansion?
- some similarities / some differences

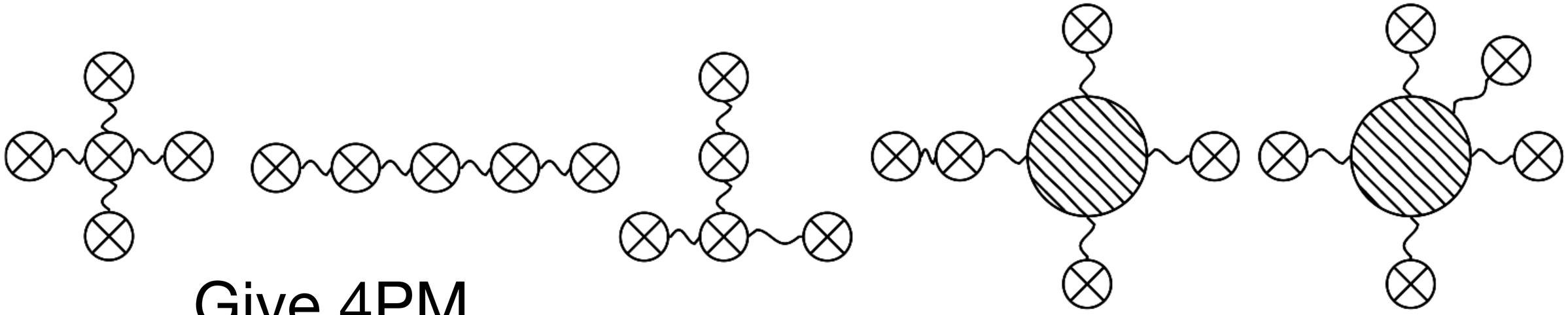
Enables a simple extension to fourth order in Newton's constant (conservative pieces)



(NEJBB, Plante, Vanhove)

Extension to fourth order in Newton's constant (conservative pieces)

Only five integrand topologies have to be considered



Give 4PM conservation amplitude

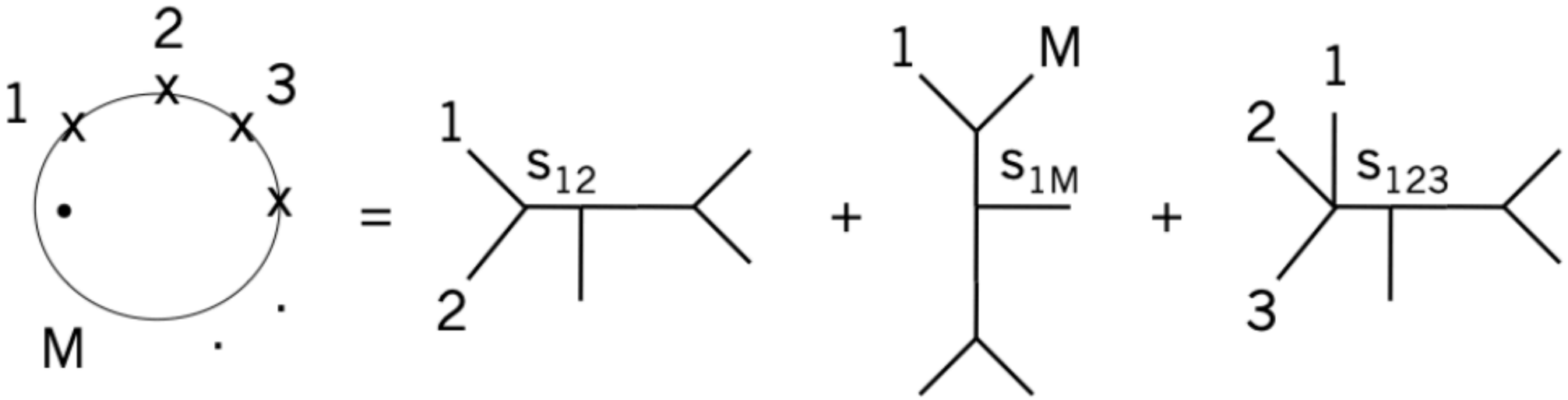
(NEJBB, Plante, Vanhove)

Efficient trees from 'Stringy' inspiration?

String theory
add channels up..

<->

Feynman diagrams
sums separate
kinematic poles



Example : Compact massive trees

Find ‘stringy’ structure in the scattering equation prescription (CHY)

(NEJB, Damgaard, Tourkine, Vanhove)

$$A_{n-2}(1,\{2,\ldots,n-1\},n) = \int \frac{\prod_{i=1}^{n} dz_i}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C}))} \prod_{i=1}^{n} \delta'\left(\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{k_i\cdot k_j}{z_{ij}}\right) \frac{1}{z_{12}\cdots z_{n-1\,n}}$$
$$\times \sum_{\beta\in\mathfrak{S}_{n-2}} \frac{N_{n-2}(1,\beta(2,\ldots,n-1),n)}{z_{1\beta(2)}z_{\beta(2)\beta(3)}\cdots z_{\beta(n-1)\,n}},$$

Scattering equation can be solved analytically
leading to very compact results

Example : Compact massive trees

We can generate gravity amplitudes in the following way

$$M_{n-2}^{\text{tree}}(1,2,\ldots,n) = i \sum_{\beta \in \mathfrak{S}_{n-2}} N_{n-2}(1,\beta(2,\cdots,n-1),n) A_{n-2}(1,\beta(2,\ldots,n-1),n)$$

Advantage that all poles are simple — no spurious poles!

(NEJB, Brown, Gomez)

Straightforward to compute any tree order needed with manifest color-kinematic numerators no double poles (from KLT) Spin-0, spin-1/2 .. easy to derive

Compact massive tree amplitudes

CHY formalism leads to the following very compact amplitudes

$$M_1^{\text{tree}}(p,\ell_2,-p') = i\,N_1(p,\ell_2,-p')A_1(p,\ell_2,-p') = i\,N_1(p,\ell_2,-p')^2,$$

$$M_2^{\text{tree}}(p,\ell_2,\ell_3,-p') = i\,N_2(p,2,3,-p')A_2(p,2,3,-p') + \text{perm.}\{2,3\}$$

$$= \frac{iN_2(p,2,3,-p')^2}{(\ell_2+p)^2-m^2+i\varepsilon} + \frac{iN_2(p,3,2,-p')^2}{(\ell_3+p)^2-m^2+i\varepsilon} + \frac{i(N_2{}^{[2,3]})^2}{(\ell_2+\ell_3)^2+i\varepsilon}$$

$$N_1(p,\ell_2,-p') = i\sqrt{2}\,\zeta_2\cdot p, \quad A_1(p,\ell_2,-p') = N_1(p,\ell_2,-p'),$$

$$N_2(p,\ell_2,\ell_3,-p') = \frac{i}{2}\Big(s_{2p}(\zeta_2\cdot\zeta_3) - 4(\zeta_2\cdot p)\zeta_3\cdot(p+\ell_2)\Big)$$

Nested commutators from massless factorizations

HEFT tree formalism

Direct numerator construction approach connected to quasi-shuffle Hopf algebras - numerators always manifest gauge invariant - and compact - but numerators feature spurious poles

(Brandhuber, Chen, Travaglini, Wen; Brandhuber, Chen, Travaglini, Wen; Johansson)

$$\mathcal{N}_f([12],\overline{3},\overline{4}) = -\frac{1}{p_{41}^2-m^2}\left(\bar{v}\bullet(p_4\cdot F_{12})\bullet u+\frac{1}{4}\bar{v}\bullet F_1\bullet(p_1\cdot F_2)\bullet u\right)$$

E.g. massive fermion-fermion numerators for gluon emission

$$\begin{aligned}\mathcal{N}_f([123],\overline{4},\overline{5}) = \bar{v}\bullet\Bigg[&\frac{(p_5\cdot F_{123})+\frac{1}{4}F_1\bullet(p_1\cdot F_{23})}{p_{51}^2-m^2}\\ &-\frac{(p_5\cdot F_{12})+\frac{1}{4}F_1\bullet(p_1\cdot F_2)}{p_{51}^2-m^2}\bullet\frac{2(p_{12}\cdot F_3\cdot p_5)+(p_{12}\cdot F_3)\bullet p_3}{p_{512}^2-m^2}\\ &-\frac{(p_5\cdot F_{13})+\frac{1}{4}F_1\bullet(p_1\cdot F_3)}{p_{51}^2-m^2}\bullet\frac{2(p_{13}\cdot F_2\cdot p_5)+(p_{13}\cdot F_2)\bullet p_2}{p_{513}^2-m^2}\Bigg]\bullet u\end{aligned}$$

(NEJBB, Chen, Miao, Skowronek)

HEFT tree formalism

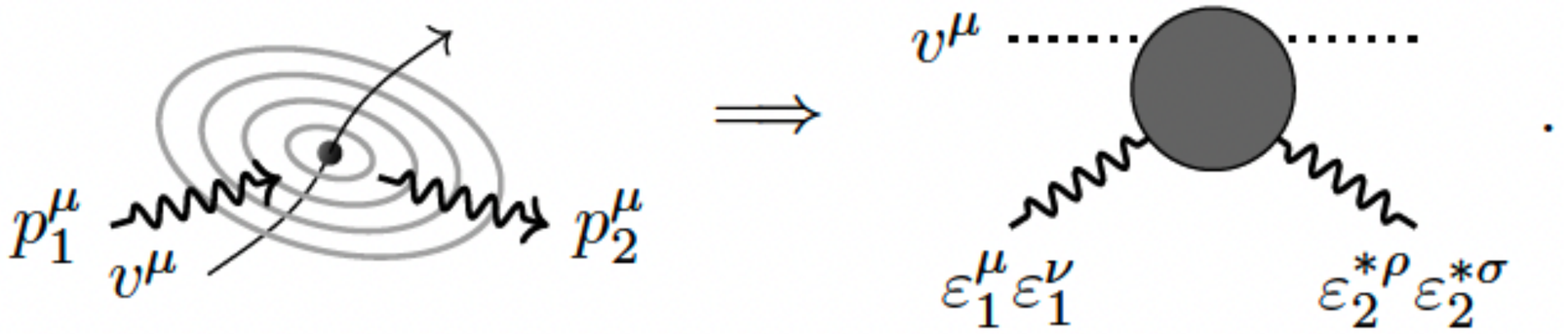
Very compact n-point expressions

$$\mathcal{N}_f(12\ldots n{-}2,\overline{n{-}1},\overline{n}) = (-1)^{n-3}\ \bar{v}\bullet\left[\frac{H_{1\ldots n-2}}{p_{n1}^2-m^2}\right]\bullet u$$

$$+\sum_{\tau_R\subset\{2,\ldots,n-2\}}(-1)^{|\tau_R|}\ \bar{v}\bullet\left[\frac{\mathcal{X}(1\rho(\tau_R))\bullet H_{\tau_R}(p_{\Theta_L(\tau_R)},p_{\Theta_R(\tau_R)})}{p_{n1\tau_L}^2-m^2}\right]\bullet u$$

(NEJBB, Chen, Miao, Skowronek; Fu, Vanhove, Wang)

Resummed integrand construction



The Compton amplitude offers insight into the two-body problem that links directly to first-order self-force theory through resummation.

(See also Johansson's talk)

Equally generalized Compton amplitudes connect to higher self-force orders in a similar way.

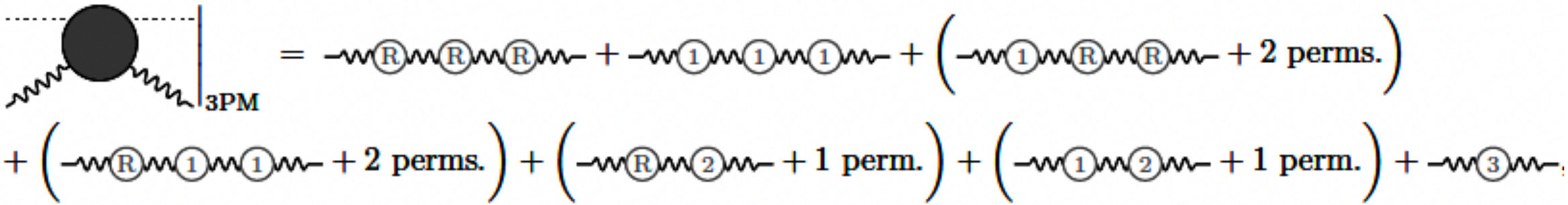
Compton amplitude

Compton amplitude provide a pathway for determining the classical dynamics of gravitational perturbations around stable, curved background and open avenue for further insights through the development of curved space perturbative S-matrix methods.

Using a world-line formalism that can incorporate curved backgrounds

(Kosmopoulos, Solon)

We studied the one and two-loop amplitudes



using a flat-space and curved-space worldline effective field theory

(NEJBB,Chen, Eriksen,Shah)

Compton amplitude and BHPT

Amplitude takes the form

Dim reg ϵ, infrared reg. j < 0

$$\mathcal{M} = \mathcal{M}_0^{\rm 1PM} + \sum_{n=2}^{\infty} \sum_{j=1-n}^{\infty} \epsilon^j \mathcal{M}_j^{n\rm PM}$$

$$\bar{\epsilon} \;=\; 2GM\omega$$

(NEJBB,Chen, Eriksen,Shah)

The Weinberg theorem allows us to identify the finite amplitude

$$\begin{aligned}\mathcal{M}_{\rm finite}(\mu, \varepsilon_1, \varepsilon_2^*) &= e^{{\rm i}\bar{\epsilon}/\epsilon}\, \mathcal{M}\big|_{\epsilon\to 0} \\ &= \mathcal{M}_0^{\rm 1PM} + \mathcal{M}_0^{\rm 2PM} + (\mathcal{M}_0^{\rm 3PM} + {\rm i}\bar{\epsilon}\mathcal{M}_1^{\rm 2PM}) + \cdots\end{aligned}$$

And we can write the cross section in terms of (

$$\frac{{\rm d}\sigma}{{\rm d}\Omega} = \frac{|\mathcal{M}_{\rm finite}(\mu, \varepsilon_1, \varepsilon_2^*)|^2}{(8\pi M)^2} \qquad = |f_{\rm amp}(\theta)|^2 + |g_{\rm amp}(\theta)|^2$$

Compton amplitude

$$\mathsf{F}_1 = \frac{(v\cdot f_1\cdot f_2^*\cdot v)}{\omega^2},\quad \mathsf{F}_2 = \frac{(v\cdot f_1\cdot p_2)(v\cdot f_2^*\cdot p_1)}{\omega^4}$$

$$\omega = p_1\cdot v = p_2\cdot v$$

$$\mathcal{M}_0^{\rm 2PM} \sim \Bigg[\mathsf{F}_1^2\Bigg(\frac{i\log\left(\frac{4\omega^2}{\mu^2}\right)}{64\pi x^2} - \frac{(x(15x+28)+15)(x-1)^2}{1024x(x+1)^2(2x^2-1)} - \frac{i(4x^2-7)}{192\pi(2x^4-3x^2+1)} - \frac{i(2x^4-2x^2+1)\log(x)}{32\pi x^2(x^2-1)^2(2x^2-1)}\Bigg)$$

$$+\frac{\mathsf{F}_2^2}{2x^2-1}\left(\frac{-i(14x^4-19x^2+11)}{1536\pi x^2(x^2-1)^3} + \frac{15x^2+28x+15}{4096(x+1)^4x} + \frac{i(2x^4-3x^2+2)\log(x)}{128\pi(x^2-1)^4}\right)\Bigg]$$

$$x = \frac{|q|}{2\omega} = \sin\left(\frac{\theta}{2}\right)$$

Trying to compare to black hole perturbation theory - leads to divergences when transforming to spherical harmonic functions

Compton amplitude

We can see this from looking at the amplitude contributions

$$\mathcal{M}_0^{1\mathrm{PM}} \sim \frac{F_1^2}{x^2} \qquad \mathcal{M}_0^{2\mathrm{PM}} : \frac{1}{x^2}\log\left(\frac{4\omega^2}{\mu^2}x\right) \qquad \mathcal{M}_{\mathrm{finite}}^{3\mathrm{PM}} \quad \frac{1}{x^2}\log^2\left(\frac{4\omega^2}{\mu^2}x\right)$$

$$\mathcal{M}_{\mathrm{finite}}^{3\mathrm{PM}} = \mathcal{M}_0^{3\mathrm{PM}} + i\bar{\epsilon}\mathcal{M}_1^{2\mathrm{PM}} = \frac{\pi M(2GM\omega)^3}{\omega}$$

$$\times\Bigg[d_1\left(G_+(x) + \mathrm{i}\pi\log(1+x)\right) + d_2\left(G_-(x) + \mathrm{i}\pi\log\frac{4x}{1+x}\right)$$

$$+ d_3\log^2\frac{4\omega^2x^2}{\mu^2} + d_4\left(4\zeta(2) - \log^2\frac{4\omega^2}{\mu^2}\right) + d_5\log x^2$$

$$+ (d_6 + d_7\mathrm{i}\pi)\log\frac{4\omega^2}{\mu^2} + d_{\mathrm{Re}} + d_{\mathrm{Im}}\Bigg]$$

$$G_\pm(x) = \mathrm{Li}_2(x) \pm \mathrm{Li}_2(-x) + (\log(1-x) \pm \log(1+x))\log x.$$

(Also Bautista, Driesse, Haddad, Jakobsen; Ivanov, Li, Parra-Martinez, Zhou)

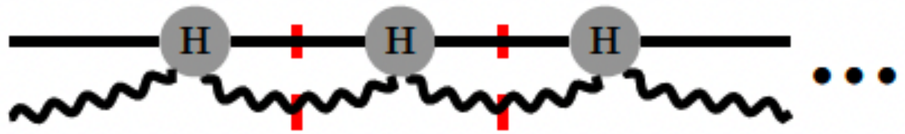
Compton amplitude

Setting $\mu = 2e^{-\gamma}\omega$ allow us to write

$$\mathcal{M}_{\text{finite}}(2e^{-\gamma}\omega, \varepsilon_1, \varepsilon_2^*) = 32\pi G M^2 \frac{\Gamma(1-i\bar{\epsilon})}{\Gamma(1+i\bar{\epsilon})} \frac{\mathrm{F}_1^2}{4x^{2(1-i\epsilon)}}$$

(Dolan)

$$+\sum_{n=2}^{\infty} \mathcal{M}^{\text{nPM}}_{\text{finite,rm}}(2e^{-\gamma}\omega, \varepsilon_1, \varepsilon_2^*)\,,$$



Interesting non-perturbative relation (Isospectrality)

This split-up allows us to project into the same basis of spherical harmonic functions (resummation of Newtonian terms regulates divergences). We can thus make a direct comparison with Black Hole perturbative theory

$$g_{\text{B}}^{(l)} = \frac{6i\bar{\epsilon}}{(-1)^l l(l^2-1)(l+2)} f_{\text{B}}^{(l)}$$

$$f_{\text{amp}}^{\text{Newtonian}(l)} + f_{\text{amp}}^{\text{other}(l)} = f_{\text{B}}^{(l)} \qquad g_{\text{amp}}^{\text{Newtonian}(l)} + g_{\text{amp}}^{\text{other}(l)} = g_{\text{B}}^{(l)}$$

(NEJBB,Chen,Eriksen,Shah)

Black Hole Perturbation Toolkit; Markovic, Ivanov

Compton amplitude

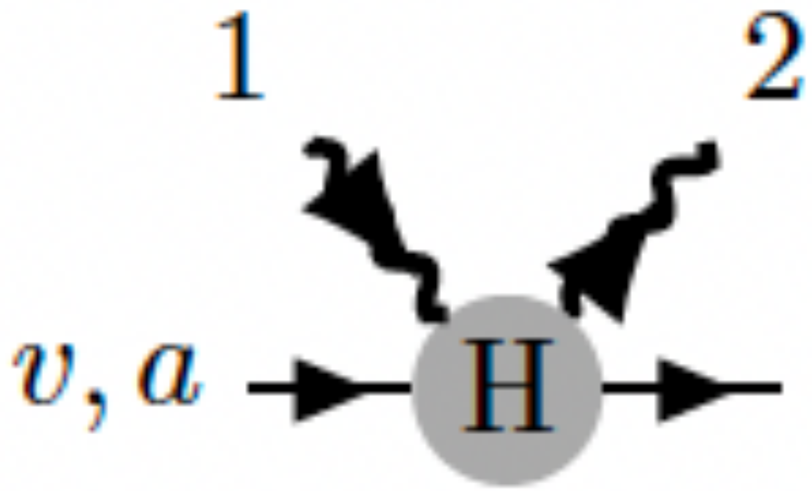
Formalism that includes recoil effects in a non-spinning background can be extended to the third post-Minkowskian order

The established bridge suggests that black hole perturbation theory could constrain tree-level Compton amplitudes, including spin and tidal responses, thereby improving the understanding of near- and far-zone dynamics

Generalizing the framework to higher-multiplicity amplitudes is of considerable interest, as it may provide access to nonlinear features other mass-ratio expansion results - and link to higher order self-force results

Application: Effective Compton for spinning systems

Result of a covariant Compton spinning particle amplitude with classical ring radius a = s/m interacting with gravitons from quantum field theory bootstrap (NEJBB, Chen,Skowronek)



Covariant formalism that use a basis of entire functions

Match with the result by (Cangemi, Chiodaroli, Johansson, Ochiov, Pichini, Skvortsov)

Spin Compton from bootstrap

Bootstrap of spinning Compton amplitude (NEJBB, Chen,Skowronek)

$$\mathcal{M}^{\text{tree}}_{\text{Kerr}} = 32\pi G M^2 \left(\frac{\mathcal{N}_0 \mathcal{N}_a}{2p_1\cdot p_2} - \frac{\mathcal{N}_{\text{r}}}{4\, p_1\cdot v\, p_2\cdot v} + \mathcal{N}_{\text{c}} \right)$$

First term : double-copy contribution
Second term : reminder contribution
Third term : contact contribution

$$\mathcal{N}_0 = \frac{v\cdot F_1\cdot F_2^*\cdot v}{p_1\cdot v}$$

Spin Compton from bootstrap

Covariant formalism that uses a basis of entire functions. Start with

$$\mathcal{N}_a(1,v) = m\,(v\cdot\varepsilon_1)\,\exp\left(\,i\,\frac{p_1\cdot S\cdot\varepsilon_1}{m\,v\cdot\varepsilon_1}\right)$$

Arkani-Hamed, Huang, and Huang;
Guevara, Ochirov, Vines
Chung, Huang, Kim, Lee

Rewrite

$$m\,v^\mu\exp\left(\,i\,\frac{p_1\cdot S\cdot\varepsilon_1}{m\,v\cdot\varepsilon_1}\right) \doteq m\,\cosh(p_1\cdot a)v^\mu - i\,\left(\frac{\sinh(p_1\cdot a)}{p_1\cdot a}\right)(p_1\cdot S)^\mu$$

$$G_1(x) \;= \frac{\sinh x}{x}. \qquad G_2(x_1,x_2)\equiv\frac{1}{x_2}\left(\frac{\sinh(x_{12})}{x_{12}}-\cosh(x_2)\frac{\sinh(x_1)}{x_1}\right)$$

$$G_j(x_1,x_2,\cdots,x_j)\equiv\frac{1}{x_j}\Big(G_{j-1}(x_1{+}x_j;x_2,\cdots,x_{j-1}) - G_{j-1}(x_1,x_2,\cdots,x_{j-1})\cosh(x_j)\Big)$$

We can cancel spurious poles because of basis of entire fuctions.

Spin Compton from bootstrap

And we have from physical factorization

$$\mathcal{N}_a = \frac{\mathsf{w}(p_1)\cdot F_1\cdot F_2^*\cdot \mathsf{w}(-p_2)}{M^2(p_2\cdot v)} \qquad F_i^{\mu\nu} = p_i^\mu \varepsilon_i^\nu - \varepsilon_i^\mu p_i^\nu \qquad \tilde{F}^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$\mathsf{w}^\mu(p_i) = Mv^\mu \cosh(a\cdot p_i) \qquad (p_i\cdot S)^\mu\, G_1(a\cdot p_i)$$

$$+\frac{i}{M}\Big(G_2\left(a\cdot p_1, -a\cdot p_2\right)\left(a\cdot F_1\cdot F_2^*\cdot S\cdot p_2 - a\cdot F_2^*\cdot F_1\cdot S\cdot p_1\right) - G_1\left(a\cdot p_1 - a\cdot p_2\right)\mathrm{tr}\left(F_1\cdot S\cdot F_2^*\right)\Big)$$

$$-G_1(a\cdot p_1)G_1(-a\cdot p_2)\Big((a\cdot F_1\cdot v)(a\cdot F_2^*\cdot p_1)+(a\cdot F_1\cdot p_2)(a\cdot F_2^*\cdot v)+\frac{p_1\cdot v+p_2\cdot v}{2}(a\cdot F_1\cdot F_2^*\cdot a)\Big)$$

As well as

(NEJBB, Chen,Skowronek)

$$\mathcal{N}_{\mathrm{r}} = (\partial_{a\cdot p_1}+\partial_{a\cdot p_2})\Big(G_1(a\cdot p_1)G_1(-a\cdot p_2)\Big)$$

$$\times\Big(p_2\cdot v\big((a\cdot F_1\cdot F_2^*\cdot a)(a\cdot F_2^*\cdot F_1\cdot v)+a\cdot a(a\cdot F_1\cdot F_2^*\cdot v)(v\cdot F_1\cdot F_2^*\cdot v)\big)+(1\leftrightarrow 2)\Big)$$

$$+i(\partial_{a\cdot p_1}+\partial_{a\cdot p_2})G_2(a\cdot p_1,-a\cdot p_2)$$

$$\times\Big(p_1\cdot v(a\cdot F_1\cdot F_2^*\cdot v)\big((a\cdot F_1\cdot v)(a\cdot \tilde{F}_2^*\cdot v)-(a\cdot F_2^*\cdot v)(a\cdot \tilde{F}_1\cdot v)\big)-(1\leftrightarrow 2)\Big)\,.$$

Spin Compton from bootstrap

In the spinning case, the finite amplitude takes the form (using the bootstrap ansatz) and after regulating (as in the scalar case):

$$\begin{aligned}\mathcal{M}_{\text{fin.}}(2e^{-\gamma_{\text{E}}}\omega,\varepsilon_1,\varepsilon_2^*) &= 32\pi GM^2\frac{\Gamma(1-i\bar{\epsilon})}{\Gamma(1+i\bar{\epsilon})}\Big(\frac{\mathcal{N}_0\mathcal{N}_0}{4\omega^2x^{2(1-i\bar{\epsilon})}}\Big)\\ &\quad+32\pi GM^2\Big(\frac{\mathcal{N}_0\mathcal{N}_a-\mathcal{N}_0\mathcal{N}_0}{4\omega^2x^2}-\frac{1}{4M^2\omega^2}\mathcal{N}_r+\mathcal{N}_{\text{c}}\Big)\\ &\quad+\mathcal{M}^{\text{Loop}}_{\text{fin.,rm}}(2e^{-\gamma_{\text{E}}}\omega,\varepsilon_1,\varepsilon_2^*)\,.\end{aligned}$$

where the finite amplitude does not have a finite divergence and thus can be easily transformed into a partial wave basis expression (the subscript “rm” denotes the remaining loop contributions not captured by the resummed leading forward-divergent term).

Spin Compton from bootstrap

Using the coordinate choice:

$$v = (1,0,0,0)\,, \quad p_1 = \omega(1,0,0,1)\,, \quad \varepsilon_1^+ = \frac{1}{\sqrt{2}}(0,1,i,0)\,, \quad \varepsilon_1^- = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$a = |a|(0,-\sin(\gamma),0,\cos(\gamma))\,, \qquad p_2 = \omega(1,\sin(\theta)\cos(\phi),\sin(\theta)\sin(\phi),\cos(\theta)),$$

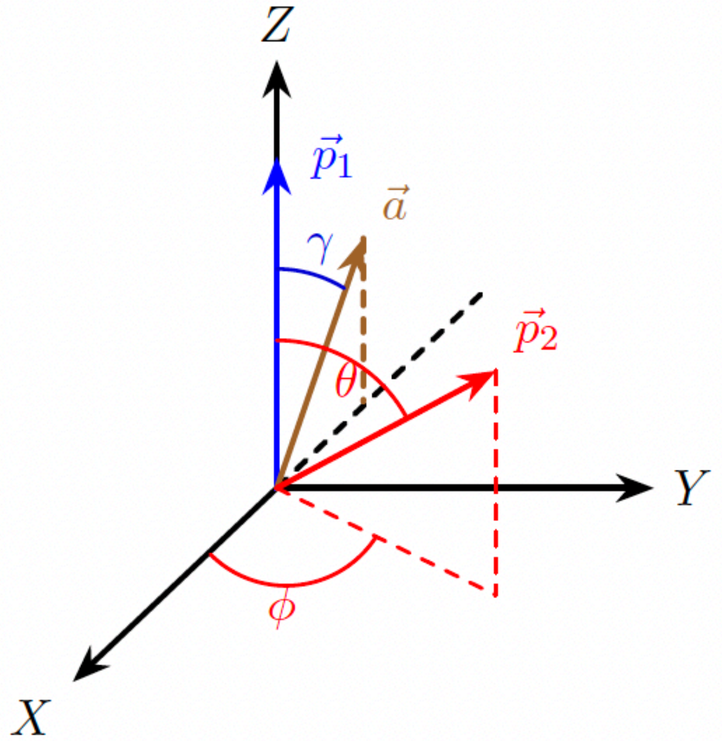
$$\varepsilon_2^+ = \frac{1}{\sqrt{2}}(0,\cos(\theta)\cos(\phi) - i\sin(\phi),\cos(\theta)\sin(\phi) + i\cos(\phi),-\sin(\theta))\,,$$

$$\varepsilon_2^- = \frac{1}{\sqrt{2}}(0,\cos(\theta)\cos(\phi) + i\sin(\phi),\cos(\theta)\sin(\phi) - i\cos(\phi),-\sin(\theta))\,,$$

We can write

$$f_{\text{amp.}}(\theta,\phi) = \frac{1}{(8\pi M)}\mathcal{M}_{\text{fin.}}(2e^{-\gamma_{\text{E}}}\omega, \varepsilon_1^+, \varepsilon_2^{+*})$$

$$g_{\text{amp.}}(\theta,\phi) = \frac{1}{(8\pi M)}\mathcal{M}_{\text{fin.}}(2e^{-\gamma_{\text{E}}}\omega, \varepsilon_1^+, \varepsilon_2^{-*})$$



Spin Compton from bootstrap

A partial wave expansion (so we can can compare a cross formalisms) can be generated from

$$\mathbb{P}_f^{(l,m)}(\mathcal{X}) \equiv \int_0^{\pi} \sin(\theta) d\theta \int_0^{2\pi} d\phi \, {}_{-2}Y^*_{lm}(\theta,\phi)\, \mathcal{X}(\theta,\phi)\,,$$

$$\mathbb{P}_g^{(l,m)}(\mathcal{X}) \equiv \int_0^{\pi} \sin(\theta) d\theta \int_0^{2\pi} d\phi \, {}_{-2}Y^*_{lm}(\pi-\theta,\phi)\, \mathcal{X}(\theta,\phi)\,,$$

$$f_{\rm B}^{(l,m)} = \mathbb{P}_f^{(l,m)}(f_{\rm B}), \qquad g_{\rm B}^{(l,m)} = \mathbb{P}_g^{(l,m)}(g_{\rm B}),$$

$$f_{\rm amp.}^{(l,m)} = \mathbb{P}_f^{(l,m)}(f_{\rm amp.}), \qquad g_{\rm amp.}^{(l,m)} = \mathbb{P}_g^{(l,m)}(g_{\rm amp.})\,.$$

Spin Compton from BHPT

The coefficients in BHPT can be derived from the Mano–Suzuki–Takasugi solution of the Teukolsky equation

$$e^{2i\delta^{\pm}_{lm}} = (-1)^{l+1}\frac{C_{\rm TS}}{16\omega^4}\frac{B^{(\rm ref)}_{lm}}{B^{(\rm inc)}_{lm}}$$

$$f_B(\theta,\phi) = \frac{\pi e^{-i\Phi}}{i\omega}\sum_{l=2}^{\infty}\sum_{m=-l}^{l} {}_{-2}\widehat{S}_{lm}(\theta,\phi;|a|\omega)\,{}_{-2}S_{lm}(\gamma,0;|a|\omega)\Delta^{+}_{lm},$$

$$g_B(\theta,\phi) = \frac{\pi e^{-i\Phi}}{i\omega}\sum_{l=2}^{\infty}\sum_{m=-l}^{l} {}_{-2}\breve{S}_{lm}(\pi-\theta,\phi;|a|\omega)\,{}_{-2}S_{lm}(\gamma,0;|a|\omega)\,(-1)^{l+m}\Delta^{-}_{lm}$$

$$\Delta^{+}_{lm} \equiv \left(e^{2i\delta^{+}_{lm}} + e^{2i\delta^{-}_{lm}}\right), \qquad \Delta^{-}_{lm} \equiv \left(e^{2i\delta^{+}_{lm}} - e^{2i\delta^{-}_{lm}}\right)$$

Spin Compton from bootstrap

Again, we have an overlap region for the cross section so that BHPT and weak-field amplitudes is linked

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi M)^2} \sum_{h=\pm} |\mathcal{M}_{\rm fin.}(2e^{-\gamma_{\rm E}}\omega, \varepsilon_1^+, \varepsilon_2^{h*})|^2 = |f_{\rm amp.}(\theta,\phi)|^2 + |g_{\rm amp.}(\theta,\phi)|^2,$$

we get

$$\frac{d\sigma}{d\Omega} = |f_{\rm B}|^2 + |g_{\rm B}|^2,$$

$$f_{\rm amp.}^{(l,m)} = f_{\rm B}^{(l,m)}, \qquad g_{\rm amp.}^{(l,m)} = g_{\rm B}^{(l,m)}$$

Effective Spin Compton from bootstrap and BHPT

Only contribution that has to be determined is the contact term

$$\mathcal{N}_{\rm c} = \mathcal{N}_{>,{\rm c}} + \mathcal{N}_{<,{\rm c}}$$

We fixed the first term previously (to all orders) using spin-dependent far-zone data at fourth and fifth orders in spin, obtained from the Teukolsky equation.

By matching to the weak-field result to the BHPT sector, we see that

	$S^{j\leqslant 4}$	S^5	S^6	S^7
$\mathcal{N}_{>,{\rm c}}$ match	all	$l \geqslant 3$	$l \geqslant 4$	$l \geqslant 5$
$\mathcal{N}_{<,{\rm c}}$ needed	none	$l = 2$	$l = 2, 3$	$l = 2, 3, 4$

Effective Spin Compton

We can now write the ansatz for the terms in $\mathcal{N}_{<,\mathrm{c}}$ and compute the difference compared to BHPT (helicity-conserving and helicity-reversing components)

$$\mathcal{M}_{\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1,\varepsilon_2^*)=\mathcal{M}_{>,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1,\varepsilon_2^*)+\mathcal{M}_{<,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1,\varepsilon_2^*)$$

$$\mathcal{M}_{<,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1,\varepsilon_2^*)=\mathcal{M}_{<,\mathrm{c}}(\varepsilon_1,\varepsilon_2^*)+\mathcal{M}^{\mathrm{Loop}}_{<,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1,\varepsilon_2^*)$$

$$\mathcal{M}_{<,\mathrm{c}}(\varepsilon_1,\varepsilon_2^*)\equiv 32\pi G M^2\mathcal{N}_{<,\mathrm{c}}$$

$$f^{(l,m)}_{\mathrm{diff.}}\equiv f^{(l,m)}_{\mathrm{B}}-\frac{1}{8\pi M}\mathbb{P}^{(l,m)}_f\left[\mathcal{M}_{>,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1^+,\varepsilon_2^{+*})\right]$$

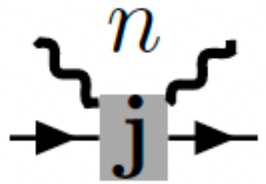
$$g^{(l,m)}_{\mathrm{diff.}}\equiv g^{(l,m)}_{\mathrm{B}}-\frac{1}{8\pi M}\mathbb{P}^{(l,m)}_g\left[\mathcal{M}_{>,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1^+,\varepsilon_2^{-*})\right]$$

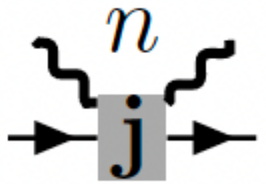
Leading to (compact object scattering involving finite-size effects - loop terms)

$$\mathbb{P}^{(l,m)}_f\left[\mathcal{M}_{<,\mathrm{c}}(\varepsilon_1^+,\varepsilon_2^{+*})\right]+\mathbb{P}^{(l,m)}_f\left[\mathcal{M}^{\mathrm{Loop}}_{<,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1^+,\varepsilon_2^{+*})\right]=8\pi M f^{(l,m)}_{\mathrm{diff.}}$$

$$\mathbb{P}^{(l,m)}_g\left[\mathcal{M}_{<,\mathrm{c}}(\varepsilon_1^+,\varepsilon_2^{-*})\right]+\mathbb{P}^{(l,m)}_g\left[\mathcal{M}^{\mathrm{Loop}}_{<,\mathrm{fin.}}(2e^{-\gamma_{\mathrm{E}}}\omega,\varepsilon_1^+,\varepsilon_2^{-*})\right]=8\pi M g^{(l,m)}_{\mathrm{diff.}}$$

Effective Spin Compton

$$\mathcal{M}_{<,\mathrm{c}}(\varepsilon_1,\varepsilon_2^*) = \sum_{n=0}^{3}\sum_{j=5-n}^{\infty}$$ 

$$+\sum_{n=4}^{\infty}\sum_{j=6-n}^{\infty}$$ 

	S^0	S^1	S^2	S^3	S^4	S^5	S^6	S^7	$\cdots$
1PM	×								
2PM	×	×							
3PM	×	×	×						
4PM	×	×	×	×					
5PM	×	**4**	**3**	**2**	×				
6PM	**6**	**5**	**4**	**3**	**2**	**1**	**0**	**-1**	low $\bar{\epsilon}$-order
7PM	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**	
8PM	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	
⋮	tidal								

Effective Spin Compton

$$\mathbb{P}_f^{(l,m)}\Big[\;\Big]=8\pi M f^{(l,m)}_{\text{diff.}}\Big|_{\bar\epsilon^j|a|^n}-\mathbb{P}_f^{(l,m)}\Big[\;\Big]$$

$$\mathbb{P}_g^{(l,m)}\Big[\;\Big]=8\pi M g^{(l,m)}_{\text{diff.}}\Big|_{\bar\epsilon^j|a|^n}-\mathbb{P}_g^{(l,m)}\Big[\;\Big]$$

$$\mathcal{M}^{\text{Loop}}_{<,\text{fin.}}\Big|_{\bar\epsilon^j|a|^n}\equiv\;\;,\qquad \mathcal{M}_{>,\text{fin.}}\Big|_{\bar\epsilon^j|a|^n}\equiv\;\;$$

Graphically we can use the convenient notation (where the fat blob is a Loop term)

This leads to equations to fulfill which can be solved iteratively

$$\mathbb{P}_f^{(l,m)}\Big[\;\Big]=8\pi M f^{(l,m)}_{\text{diff.}}\Big|_{\bar\epsilon^2|a|^4},\qquad \mathbb{P}_f^{(l,m)}\Big[\;\Big]=8\pi M f^{(l,m)}_{\text{diff.}}\Big|_{\bar\epsilon^6|a|^0},$$

$$\mathbb{P}_f^{(l,m)}\Big[\;\Big]=8\pi M f^{(l,m)}_{\text{diff.}}\Big|_{\bar\epsilon^7|a|^0}-\mathbb{P}_f^{(l,m)}\Big[\;+\;\Big].$$

Effective Spin Compton

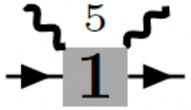
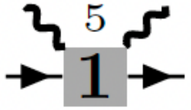
In general, determining for contact terms associated with spin independent tidal effects or the blue high GM spin sector requires challenging loop calculations.

	S^0	S^1	S^2	S^3	S^4	S^5	S^6	S^7	$\cdots$
1PM	×								
2PM	×	×							
3PM	×	×	×						
4PM	×	×	×	×					
5PM	×	4	3	2	×				
6PM	6	5	4	3	2	1	0	-1	low $\bar{\epsilon}$-order
7PM	7	6	5	4	3	2	1	0	
8PM	8	7	6	5	4	3	2	1	
⋮	tidal								

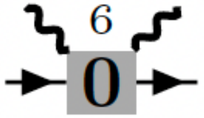
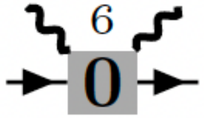
Effective Spin Compton

In the (red) spin sector, we do not need to perform challenging loop calculations.

For the fifth order we e.g. find

$\mathbb{P}_f^{(l,m)}\Big[$  $\Big] = \begin{cases} 8\pi M f^{(2,m)}_{\text{diff.}}\Big|_{\bar{\epsilon}|a|^5}, & l=2 \\ 0, & l\neq 2 \end{cases}$, $\quad \mathbb{P}_g^{(l,m)}\Big[$  $\Big] = 0.$

thus fixing a unique solution for $\mathcal{M}^{(1,5)}_{<,\mathrm{c}}$ for the next order we have

$\mathbb{P}_f^{(l,m)}\Big[$  $\Big] = \begin{cases} 8\pi M f^{(2,m)}_{\text{diff.}}\Big|_{\bar{\epsilon}^0|a|^6}, & l=2 \\ 0, & l\neq 2 \end{cases}$, $\quad \mathbb{P}_g^{(l,m)}\Big[$  $\Big] = 0$

here the unique solution is an imaginary amplitude which encodes dissipative finite-size effects, $\mathcal{M}^{(0,6)}_{<,\mathrm{c}}$

We have verified that the absorption coefficient defined in effective field theory is consistent with the absorption cross section obtained from classical gravitational scattering.

Outline

Our results for the Kerr-Compton system give a better understanding of massive classical high-spin interactions in gravity reorganised very suitable for a self-force expansion.

Use BHPT and amplitude factorization techniques to fix an effective covariant form for the generalized spinning Compton amplitude

(see also Vanhove's recent paper)

Use the tree-level result as input for investigating the post-Minkowskian dynamics in a binary spinning black hole system

Bending angle (Chen, Wang)

Waveforms (Brandhuber, Brown, Chen, Gowdy, Travaglini)

Resummed spin effects [2603.04214]

(NEJBB,Chen,Papadimos, Yuexiang)

Outlook

The Amplitude toolbox for computations already provides many new efficient methods for computation

Amplitude tools very useful

- Double-copy/KLT
- Unitarity
- Spinor-helicity
- Numerator formalism
- Low-energy limits of string theory

- Identifying IBP-relations solving DE equations/integral
- Recycling tools from QCD computations
- Numerical programs for amplitude computation

Outline

The effective field theory approach enables the use of efficient particle physics methods to study quantum gravitational effects using on-shell S-matrix techniques

Analyzing gravity with general relativity and quantum field theory reveals how classical concepts apply to quantum mechanics, including spin, tidal effects, and strongly coupled regimes via resummation.. much more to investigatate