Some Recent Advances in dS Correlators

Chandramouli Chowdhury

ITP-CAS, Beijing

Shanghai Tech University
Recent Progress in Amplitudes

Based on:

2610.32956: All Order Self-Dual Correlators From Symmetries
w/ Silvia Nagy
+
2312.13803, 2501.10598, 2602.03841, 2604.26421: "Simplicity of Cosmological Correlators"
w/ Sadra Jazayeri, Arthur Lipstein, Joe Marshall, Jiajie Mei, Ivo Sachs, Pierre Vanhove; Song He, Dongyu Yang, Yong-Xiang Su
+
to appear w/ Paolo Benincasa, Maria Fernandez Lozano; Suvrat Raju

Observables

1. Scattering is one of the simplest process one can study in QFT and QG and allows one to quantify observables in flat space.

2. For example: Scattering amplitudes, which are directly related to physical observables (Cross Sections) that are measurable in experiments.

3. For this we usually compute a Feynman diagram for the amplitude and then square it and obtain the cross section

$$\sigma_{in-in} \sim \int d\psi_{\text{out}} \, \langle \psi_{\text{in}} | S | \psi_{\text{out}} \rangle \, \langle \psi_{\text{out}} | S | \psi_{\text{in}} \rangle$$

4. This computation often involves off-shell particles, spurious poles and can get very complicated very soon

1. For amplitudes this is stark in spinning theories. Eg: In Gravity, the 4-point function has 2000+ terms [DeWitt]

2. However many cancellations occur and the final expressions are simple. One of the first examples is DeWitt (1967) on the 2→2 graviton scattering computation:

quanta, eliminate many of the terms from these expressions. Nevertheless, a large amount of cancellation between terms still has to be dug out of the algebra, and this, combined with the fact that the final results are ridiculously simple, leads one to believe that there must be an easier way. The cross sections which one finds are

3. Similar quote in Parke-Taylor in 1986!

4. The "easier ways" were discovered in the early 2000's and eventually led to the "Amplitudes program"

5. Take Home Message: Many hidden patterns in final answers often not visible from the conventional methods
   – Sometimes require special choice of helicities, masses, etc.

6. Perhaps advanced AI can help us spot these patterns and simplicities much faster

Moving Forward

1. How hard is it to compute such in-in correlators in general spacetimes?
2. Can we directly obtain this without computing the "amplitude"?
3. One such example: Equal time in-in correlators,

$$\langle \Psi | O(t=0,\vec{x}_1) \cdots O(t=0,\vec{x}_n) | \Psi \rangle$$

   where O's are operators at a time slice. In our universe ($\approx$ FRW) these are cosmological correlators and related to measurements in the CMB.
4. Only good calculables in quantum gravity
5. Such correlators are also computed in other contexts with different physical applications and commonly computed via Schwinger-Keldysh path integrals
6. In this context the state $|\Psi\rangle$ (wave function of the universe) are related to AdS correlators via analytic continuation.

Why is it hard?

1. First, there is a technical difficulty. To see this, consider these correlators in flat space, where they are evaluated at finite time and hence lack LSZ prescription

2. This automatically introduces many complications even without a curvature

3. One example of such a complication are finite time integrals in perturbation theory, resulting in poles

$$\int_0^\infty dt e^{iEt} = \frac{1}{E}$$

rather δ-functions present in amplitudes

$$\int_{-\infty}^\infty dt e^{iEt} = \delta(E)$$

4. The situation only gets more complicated in curved space $\sqrt{g} = \frac{1}{t^4}$. For example, the same kind of integral can give $\int_0^\infty \frac{dt}{t} e^{ixt} \sim \log(x)$

What is new?

1. Such expressions are familiar from perturbation theory in Quantum Mechanics: Lippmann-Schwinger equation [eg: Matt Schwartz QFT book]
2. Which gives the perturbative corrections to ground state wavefunctions

$$\psi_n^{(1)} = \sum_m \frac{\langle \psi_m^{\text{free}} | H_{int} | \psi_n^{\text{free}} \rangle}{E_m - E_n} \psi_m^{\text{free}}$$

   where the energy eigenvalues are $E_n = |\vec{k}_n|$.

3. More recently [Arkani-Hamed, Benincasa, Postnikov] revisited this in the language of "Feynman diagrams" and uncovered a geometric construction
4. Started a new area of exploration within the community and garnished interest in using amplitudes tools in such problems [reviews: Baumann, Joyce; Benincasa]
5. Due to the technical difficulties a lot of the progress in this direction has been made for correlators of fields that can be Weyl transformed from Flat space
6. Thus, while the problem itself is not new (though the motivations might be) there are new tools and ideas needed to make progress through the technical hurdles

What next?

1. In principle one could open their favourite statement about amplitudes and ask how it works in (A)dS via correlators
2. The rules for basic questions like the manifestation of unitarity, cluster decomposition, causality, holography, etc. are not always obvious
3. Bigger questions like formulating flat space holography, finite correlation functions in dS, double copy in curved spacetime, etc., can be good lampposts
4. Ideas from other areas of physics, eg. condensed matter, might be useful to gain some insights
   – Eg: cluster decomposition from the point of view of spin glasses [Anninos, Denef; Benincasa, CC, Lozano (upcoming)]
5. Pattern recognition and reformulating questions which lead to new structures

New Results

1. Simple structures emerging in dS correlators and wavefunctions – Dressing Rules
2. It provides a concrete bridge between flat space scattering amplitudes and dS correlators functions
3. Correlators in a sector of YM/GR have a nice structure in terms of determinants.

Conventions

1. Metric in Flat space

$$ds^2 = -dt^2 + d\vec{x}^2$$

2. Metric in dS_4,

$$ds^2 = \frac{-dt^2 + d\vec{x}^2}{t^2} \implies \sqrt{-g} = \frac{1}{t^4}$$

3. The correlation functions are evaluated at $t = 0$,

$$\langle \Psi | O(t = 0, \vec{x}_1) \cdots O(t = 0, \vec{x}_n) | \Psi \rangle$$

4. Since there is translational invariance along $\vec{x}$, momentum $\vec{k}$ is defined by

$$O(t = 0, \vec{x}) = \int d^3k \; e^{i\vec{k}\cdot\vec{x}} O(t = 0, \vec{k})$$

5. "Energy" $= |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$

Correlator $\sim |\Psi|^2$

1. Conventionally the in-in correlator is evaluated using the Born rule

$$\langle \Psi | \phi(\vec{k}_1) \cdots \phi(\vec{k}_n) | \Psi \rangle = \int D\phi \phi(\vec{k}_1) \cdots \phi(\vec{k}_n) |\Psi[\varphi]|^2$$

2. One can easily go from the correlator to the wavefunction and can be discussed interchangeably [Benincasa, Dian; Stefanyszyn, Tong, Zhu]

3. The state $\Psi[\varphi]$? is the Hartle-Hawking/Bunch-Davies state (ground state),

$$\Psi[\varphi(\vec{x})] = \int_{\phi(t=-\infty)=0}^{\phi(t=0,\vec{x})=\varphi(\vec{x})} D\phi \; e^{iS[\phi]}$$

   – By analytical continuation this is equivalent to the AdS partition function [Maldacena]

4. Alternatively, in some cases they can be solved by $\widehat{H}\Psi = 0$ eg. [DeWitt]

5. Diagrammatically these are expressed as Witten diagrams.

6. These correlators can also be computed via Schwinger-Keldysh formalism [Weinberg]

In-Out

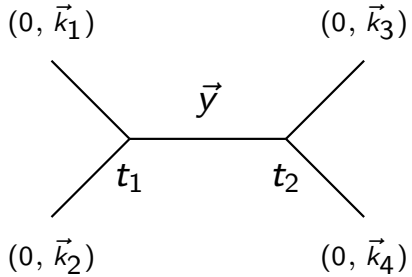
1. For equal time correlators in flat space we can use an alternate textbook formula

$$\langle\Psi|\phi(\vec{x}_1)\cdots\phi(\vec{x}_n)|\Psi\rangle = \langle\Psi|\mathcal{T}\Big[\phi(\vec{x}_1)\cdots\phi(\vec{x}_n)\Big]|\Psi\rangle$$

2. Hence they can be evaluated with standard Feynman Green function

$$G(t,t';\vec{k}) = \int_{-\infty}^{\infty}\frac{dp}{p^2-k^2-i0}e^{ip(t-t')} = \frac{1}{2k}e^{ik|t-t'|}$$

Example:



$$= \int_{-\infty}^{\infty} dt_1\,dt_2\,G(0,t_1;k_1)G(0,t_1;k_2)G(t_1,t_2;\vec{y})G(0,t_2;k_3)G(0,t_2;k_4)$$

3. The dressing rules are obtained by performing all t-integrals and working with p-integrands [CC, Lipstein, Marshall, Mei, Sachs, Vanhove] . Example:

$$\langle\phi(\vec{k}_1)\cdots\phi(\vec{k}_4)\rangle_{\text{tree}} = \frac{1}{k_1k_2k_3k_4}\int_{-\infty}^{\infty}dp\,\frac{k_{12}k_{34}}{(p^2-k_{12}^2)(p^2-k_{34}^2)}\,\frac{1}{p^2-y^2}$$

4. Using the in-out formula in dS [Donath Pajer] we can derive results for general masses [WIP] though IR divergent cases needs extra care [CC, He, Su, Yang]

Correlator: Fourier Space

1. Single Exchange:

$$\langle \phi_1 \cdots \phi_4 \rangle_{\text{tree}} = \int_{-\infty}^{\infty} dq \, \frac{k_{12} k_{34}}{(q^2 - k_{12}^2)(q^2 - k_{34}^2)} \, \frac{1}{q^2 - y^2}$$
$$= \frac{1}{(k_{12} + k_{34})(k_{12} + y)(k_{34} + y)} + \frac{1}{y(k_{12} + y)(k_{34} + y)}$$

   Tree level and loop integrands are rational functions in flat space

2. Each of these poles have a physical meaning [Maldacena, Pimentel; Raju; · · ·]

3. The Kernel is generally theory dependent (though for dS it has worked out explicitly for massless and CC cases) [work in progress for general mass]

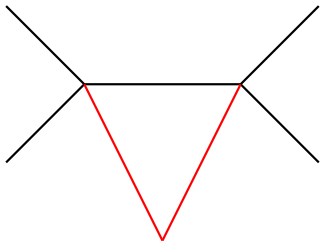
4. For CC theory in dS, tree level and loop integrands are MPLs and the Kernel $\sim \log$ (see [Ferro, et. al] for n-pt formulas)

5. In contrast to a Celestial Amplitude, this integral is on a single Feynman diagram (and not full amplitude)

Dressing Rules

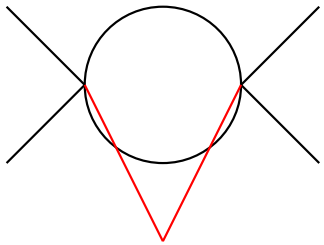
Rule: Take a Feynman graph, multiply with Kernels joining at a point (auxiliary propagators), integrate over energies

$$\langle \phi_1 \cdots \phi_4 \rangle^{\text{tree}} =$$



$$= \int_{-\infty}^{\infty} \frac{dp\ k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \frac{1}{p^2 + y^2}$$

$$\langle \phi_1 \cdots \phi_4 \rangle^{\text{loop}} =$$



$$= \int_{-\infty}^{\infty} \frac{dp\ k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \int \frac{d^4 L}{L^2 (L+P)^2}$$

where $P^\mu = (p, \vec{y})$ & the red lines denote the 1-D propagators, carrying energy

Loop Integrals: Correlators

1. Not a lot of loop integrals have been performed explicitly
2. In dS space, for general mass, the bubble spectral integrand can be written in terms of ${}_7F_6$ [Marolf, Morrison]
3. Even for CC (conf. coupled, $\Delta = 1$), position space integrals already give elliptic polylogs [Heckelbacher, Sachs, Skvortsov, Vanhove]
4. In flat space, the massless tadpole, bubble (chains) and triangle can be done explicitly in terms of MPL
5. Massive bubbles in flat space, even in low dimensions, give non-MPL functions [work in progress]
6. Integrals are different from the standard amplitude kind. Eg: n gons in $n+1$ dimensions admit a dlog representation but they are non-MPL [upcoming work w/ Yang, Su; Brown, Duhr]

(CC dS) Flat Bubble

1. From the dressing rule we get an integral of kind,

$$B(k_{12}, k_{34}) = \int_{-\infty}^{\infty} \frac{dp k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \int \frac{d^4 L}{L^2 (L+P)^2} \sim \int_{-\infty}^{\infty} \frac{dp}{p^2 + a^2} \log(p^2 + k^2)$$

Can be very easily evaluated using contour integrals

$$B(k_{12}, k_{34}) = \frac{\pi}{k_{12} + k_{34}} \frac{1}{\epsilon} + \frac{\pi}{k_{12} + k_{34}} \left[\log \left(\frac{(k_{12} + y)(k_{34} + y)}{(k_{12} + k_{34})^2} \right) + \frac{k_{12} + k_{34}}{k_{12} - k_{34}} \log \left(\frac{k_{34} + y}{k_{12} + y} \right) \right]$$

where there is some care needed with the regularization [CC, Lipstein, Mei, Sachs, Vanhove]

2. Singular structure as $k_{12} + k_{34} \to 0$ crucial for the correct flat space limit [Raju]

3. Originally motivated the "Correlators are simpler than wavefunctions" ($\Psi \sim \mathrm{Li}_2$ [Albayrak, CC, Kharel]) also see [Arkani-Hamed, Glew, Vazao]

4. Things start getting more complicated when we encounter $\sqrt{p^2 + c^2}$,

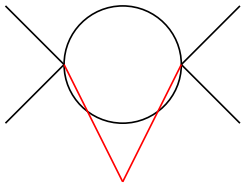
$$\text{Triangle/Box} \sim \int \frac{dp}{p^2 + a^2} \frac{\mathrm{Li}_2(\cdots)}{\sqrt{\lambda(p_1, p_2, p_3)}}$$

Surprisingly the triangle is Li_2 [Pimentel, Westerdijk; Henn, Mei, Yang; Sachs] , but Box is non-MPL ($\lambda \sim p^4$).

Cuts and Discs.

1. Correlator = Flat space diagram + Extra 1D propagators,

$$B(k_{12}, k_{34}) = \quad = \int_{-\infty}^{\infty} \frac{dp\ k_{12} k_{34}}{(p^2 - k_{12}^2)(p^2 - k_{34}^2)} \int \frac{d^4 L}{L^2 (L+P)^2}$$

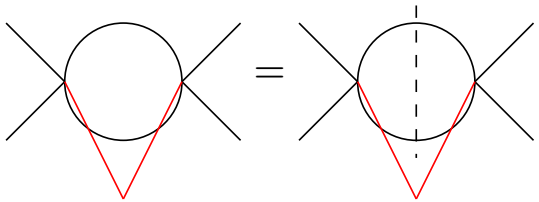


Similar to scattering amplitudes cut propagators to set things on-shell

[CC, S. Jazayeri, A. Lipstein, J. Marshall, J. Mei, I. Sachs]

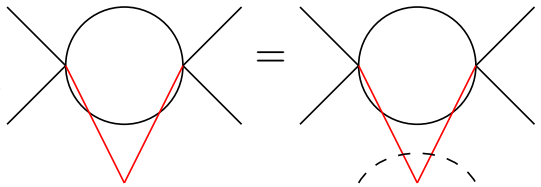
2. Can relate to disc. via Cutkosky Rules and sequential disc. Cuts of Feynman Propagators are very similar to amplitudes

disc_{y^2} =



3. Cuts of auxiliary propagators are similar to mass cuts in amplitudes

$\text{disc}_{k_{12}^2}\,\text{disc}_{k_{34}^2}$ = $= \delta(k_{12} - k_{34}) \int \frac{d^4 L}{L^2 (L+P)^2}$



Leads to consistency conditions sum rules, $\sum_{\pm} B(\pm k_{12}, \pm k_{34}) = 0$.

4. Can be used to bootstrap the function ! [work in progress]

Self-Dual Theories

1. Until the last slide we looked at some nice structures emerging for scalar theories
2. I will now summarize some results on the nice structures emerging in a sector of spinning theories (YM/GR) known as the self-dual theory
3. At tree level this describes the single-minus sector of the full theory
4. This theory has an infinite tower of symmetries [Plebanski, · · · , Campiglia, Nagy, · · ·]
5. Has been used to study loop amplitudes, double copy, holography, etc.
6. I will only describe a tiny tiny result for correlators (or wavefunctions) which we stumbled upon (lots of recent interesting works by [Miller; Skvortsov, Van Dongen; Heuveline, Ruzziconi, Sheta, Strominger; Gomez, Jusinskas, Lipstein, Lopez-Arcos, · · ·])

Ground State Result

1. The infinite tower of symmetries δ_{Λ_n} manifest themselves as infinite number of conserved charges Q_{Λ_n}

2. These give rise of infinite number of Ward identities $Q_{\Lambda_n}\Psi = 0$

3. Tower by Tower (n by n) these give the soft limits for wavefunctions (historically soft limits from ward identities were first discovered in cosmology [Creminelli; Maldacena; Joyce; Khoury; Hinterbichler; $\cdots$])

$$\lim_{\vec{k}_i \to 0} \Psi_m = \mathcal{S}_n \Psi_{m-1}$$

In particular the first rung of the tower gives an Adler Zero

4. More interestingly, the full tower (resumming the symmetries) results in an unique solution for the ground state

$$Q_\Lambda \Psi = 0$$

can be shown that Ψ agrees with the perturbiner solution.

Determinant, Double Copy?

1. The solution for Ψ_n can be expressed as a single determinant [CC, Nagy] For eg.

$$\Psi_4^{SDYM} = \det \begin{pmatrix} 1 & 0 & \frac{1}{\zeta_3-\zeta_2} \\ 0 & 1 & \frac{1}{\zeta_2-\zeta_1} \\ \frac{\zeta_{23}-\zeta_{123}}{\zeta_{23}-\zeta_1} & \frac{\zeta_3-\zeta_{123}}{\zeta_3-\zeta_{12}} & \frac{\zeta_3-\zeta_{123}}{(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)} \end{pmatrix}, \qquad \zeta_Q = \frac{k_{Qw}}{k_{Qu}}$$

2. Has the flavour of Hodge's formula but still not as simple. Perhaps the first step?

3. One motivation for studying these was to explore a possible double copy structure in curved space [Lipstein, Nagy]

4. For low points (upto 5) there was an attempt at exploring a structure of the kind [CC, Doran, Lipstein, Monteiro, Nagy, Singh]

$$\Psi_n^{SDYM} = X\frac{1}{S} \implies \Psi_n^{SDG} = X\widehat{\mathcal{D}}\frac{1}{S}$$

where $\widehat{\mathcal{D}}$ is a single-order differential operator depending on external kinematics. More recently this was shown to hold at all points [Gomez, Jusinskas, Lipstein, Lopez-Arcos] .

5. Unclear yet how to see this from the determinant structure.

Conclusion

1. For scalar in-in correlators there is a simple expression as an integral over flat space amplitudes

$$\langle \Psi | O_1(\vec{k}_1) \cdots O_n(\vec{k}_n) | \Psi \rangle = \int dE \underbrace{K(E,k)}_{\text{Kernel}} \underbrace{\mathcal{F}(E,\vec{k})}_{\text{Flat Feynman Graphs}}$$

Originally found in conjunction with "correlators are simpler than wavefunctions" and shed light on transcendentality of the integrals at loop level [CC, Lipstein, Marshall, Mei, Sachs, Vanhove; CC, He, Su Yang; Arkani-Hamed, Glew, Vazao]

2. More recently a flat space based cutting rules study was initiated and led to non-trivial consistency relations [CC, Jazayeri, Lipstein, Marshall, Mei, Sachs]

3. In a separate study we stumbled across a determinant formula for correlators of self-dual sector of YM/GR [CC, Nagy] ,

$$\Psi_n = \det A_{\text{upper triangular}}$$

further, this can be shown to admit Adler zeros, soft limits, and completely determined from Ward identities.

Future Directions

1. Sum of channels via BCFW, similar to AdS [Raju] ?
2. Generalization to FRW, connection with kinematic flow? [Baumann, Goodhew, Joyce, Lee, Pimentel, Westerdijk] [Glew, Pokraka] [Capuano, Ferro, Lukowski, Palazio]
3. What can we learn about UV properites of correlators and wave functions/BPHZ in dS? [Creminelli, Renaux-Petel, Tambalo, Yingcharoenrat] [WIP w/ Suvrat Raju]
4. Does this structure sustain after Renormalization?
5. How to handle higher masses [Raman, Yang] [Liu, Qin, Xianyu] ?
6. Lessons from previous cutting rules/tree theorems [Benincasa; Melville, Pajer; Salcedo, Melville] ?
7. Do similar dressing formulas exist for OTOC like asymptotic observables in flat space [Caron-Huot, Giroux, Hannesdottir, Mizera] ?
8. Other correlators $\langle\psi|\phi\pi\cdots|\psi\rangle$ and unequal time correlators [Kitching, Heavens] ?
9. Resumming graphs [Starobinsky; Senatore, Gorbenko; Cespedes, Davis, Wang] ?
10. IR finite correlators? Can we use flat space ideas? [Capatti, Hirschi, Pelloni, Ruijl]