# *Black-hole scattering in N=8 supergravity to all loops*



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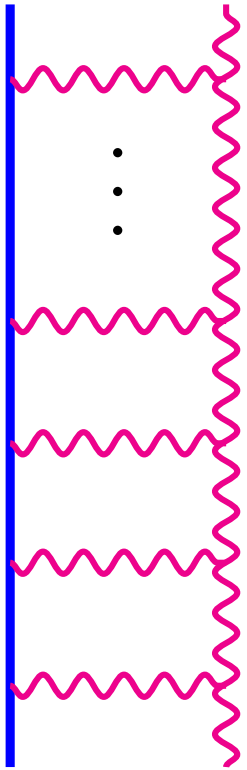




**Recent Progress in Amplitudes**

**ShanghaiTech University**

Work with: Maor Ben-Shahar, Lucile Cangemi, Chia-Hsien Shen

# This talk: BH Compton scattering

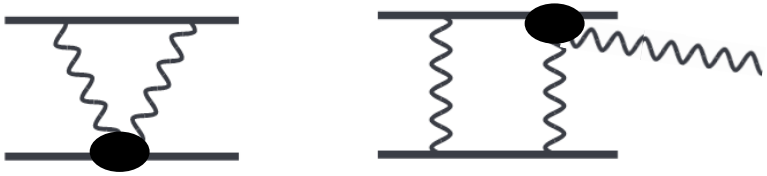
- **Compton scattering probes detailed structure of BH:**

$$\mathcal{A}_{\rm Compton} = G\left(\quad + \quad\right) + G^2\left(\quad + \quad\right) + G^3\left(\quad\right) + \cdots$$

- **Spin multipoles, Love/tidal, finite-size, dissip. effects (BHPT)**

Caron-Huot, Miguel Correia, Isabella, Solon; Bjerrum-Bohr, Chen, Eriksen, Shah; Akpinar, del Duca, Gonzo; Akpinar; Bautista, Driesse, Haddad, Jakobsen; Ivanov, Li, Parra-Martinez, Zhou; Bohnenblust, Eriksen, Hoogeveen, Jakobsen, Plefka; Brunello, Meo, Smith [....]
Aoude, Haddad, Helset; Bautista, Guevara, Kavanagh, Vines; Bjerrum-Bohr, Chen, Skowronek; Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov; Ben-Shahar, Cangemi, HJ [...]

- Needed for N$^k$LO calculations:



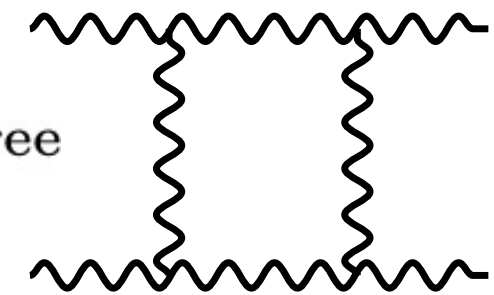
# A physical toy model: $N=8$

- $N=8$ supergravity is the "best" point-particle gravity model (not an unphysical toy model)
- Quantum 4D theory: UV finite, at least up to 6 loops
- Many BPS BH solutions, natural stringy extensions
- Will focus on ½-BPS BH's: no-triangle property → finite size effects absent in 4D Caron-Huot, Zahraee ('18)
- BH solutions valid in $D < 11$ dimensions, goes well with dim. reg.
- Lots of experience with $N=8$ amplitudes (up to 5 loops)

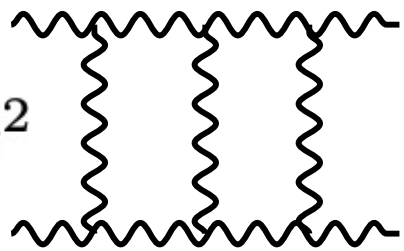
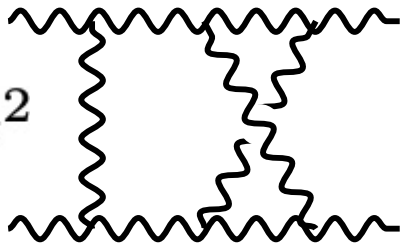
  Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng;
  Di Vecchia, Naculich, Russo, Veneziano, White; Heissenberg;
  Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng; [...]

# $\mathcal{N}$=8 supergravity amplitudes

Massless 4-graviton amplitudes:

$$stu\mathcal{M}^{\text{tree}}$$  + perms

Green, Schwarz, Brink ('82)

$$stu\mathcal{M}^{\text{tree}}$$ $\left( s^2 \right.$  $+ s^2$  $\left. \right)$ + perms

Bern, Dixon, Dunbar, Perelstein, Rozowsky ('98)

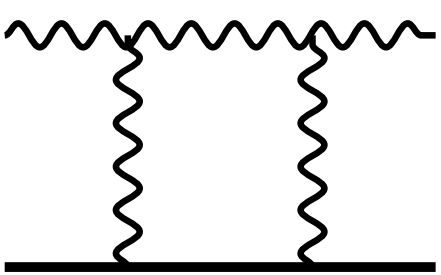
Valid in general dimension:

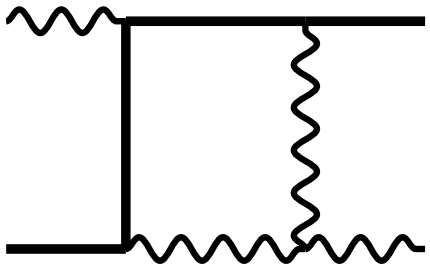
$$2 \leq D \leq 11$$

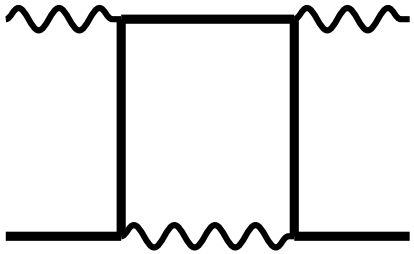
# Kaluza-Klein compactification

$$SO(1,10) \to SO(1,d) \times SO(10-d)$$

Massive multiplet $\mathcal{N}=8$ KK momentum: $p^\mu \to (p^\mu, M, 0, \ldots, 0)$

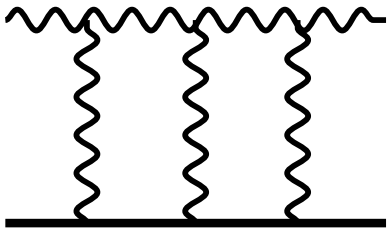






Classical Limit: $\mathcal{O}(\hbar^0)$ $\mathcal{O}(\hbar^1)$ $\mathcal{O}(\hbar^2)$

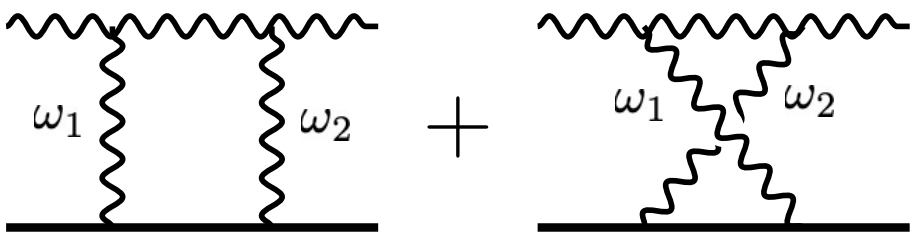
At 2 loops:



$\mathcal{O}(\hbar^0)$

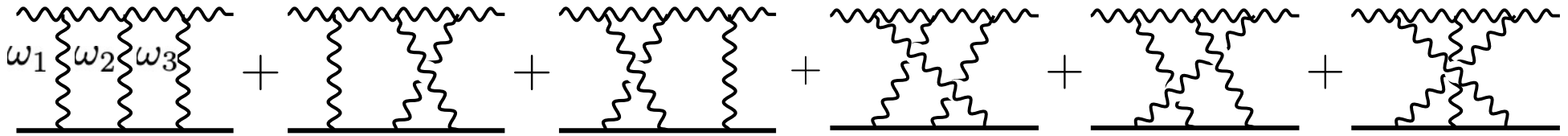
# Details of classical limit

Sum over classical ladder and crossed ladder:




BH background conserves graviton energy:

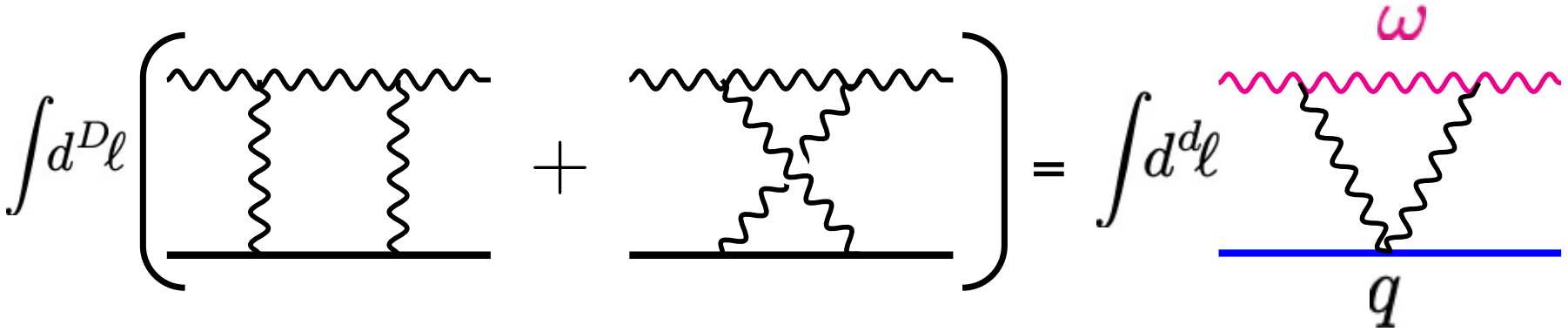
$$\hat{\delta}(\omega_1+\omega_2)\left[\frac{1}{\omega_1-i0}+\frac{1}{\omega_2-i0}\right]=i\hat{\delta}(\omega_1)\hat{\delta}(\omega_2)$$



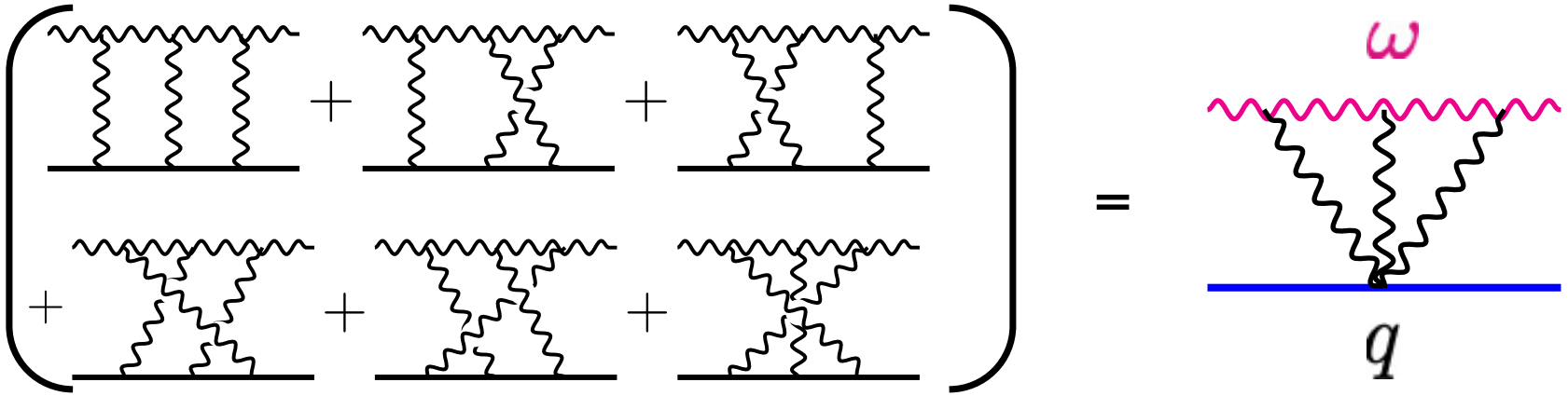

$$\hat{\delta}(\omega_1+\omega_2+\omega_3)\left[\frac{1}{\omega_1-i0}\frac{1}{\omega_1+\omega_2-i0}+\text{perm}\right]=-\hat{\delta}(\omega_1)\hat{\delta}(\omega_2)\hat{\delta}(\omega_3)$$

Saotome, Akhoury ('12)

# Ladders become fan diagrams



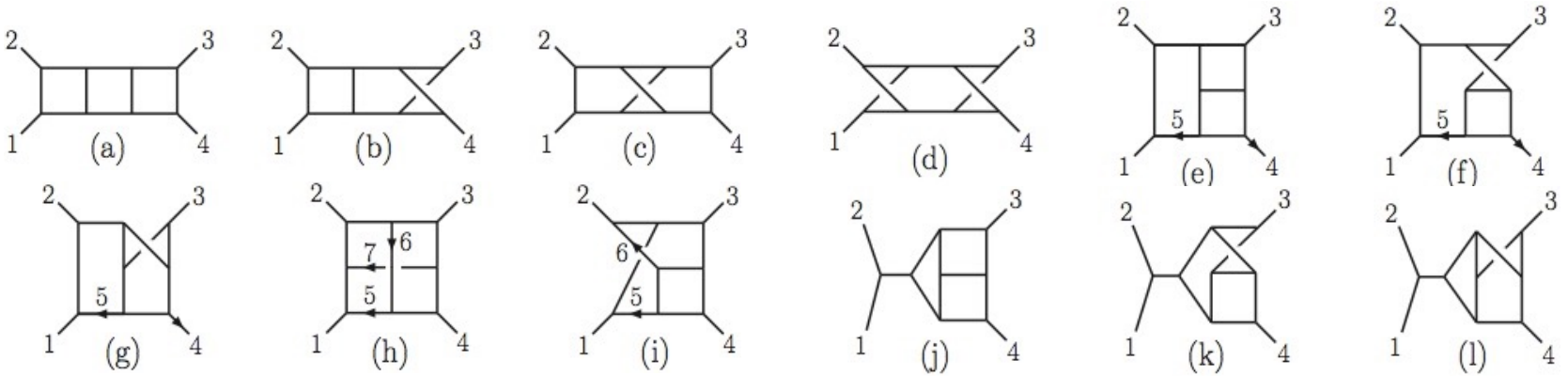

Only spatial integral: $d = D - 1$




Graviton energy $\omega$ → effective mass (tachyonic)

# What about Numerators?

$N$=8 supergravity has complicated numerators:




| $\mathcal{N}=8$ Supergravity non-BCJ numerators: |
|---|
| $[s^2]^2$ |
| $[s\,(l_1+k_4)^2]^2$ |
| $(sl_{1,2}^2+tl_{3,4}^2-st)^2-s^2(2(l_{1,2}^2-t)+l_5^2)l_5^2-t^2(2(l_{3,4}^2-s)+l_6^2)l_6^2$ $-s^2(2l_1^2l_8^2+2l_2^2l_7^2+l_1^2l_7^2+l_2^2l_8^2)-t^2(2l_3^2l_{10}^2+2l_4^2l_9^2+l_3^2l_9^2+l_4^2l_{10}^2)+2stl_5^2l_6^2$ |
| $(sl_{1,2}^2-tl_{3,4}^2)^2-(s^2l_{1,2}^2+t^2l_{3,4}^2+\frac{1}{3}stu)l_5^2$ |

Bern, Carrasco, Dixon, HJ, Kosower, Roiban ('08)

# Numerators simplify!

In the classical limit:

→ observe that all $\mathcal{N}=8$ supergravity ladder numerators become ''rung-rule'' numerators:

$$\sim stu\mathcal{M}^{\text{tree}}(s^2)^{L-1} \quad \rightarrow \quad q^2\mathcal{M}^{\text{tree}}(2M\omega)^{2L}$$

→ and all non-ladder diagrams vanish (quantum)

→ However, standard 3-4 loop BCJ numerators not well behaved (super-classical and quantum terms mix). [Open problem!]

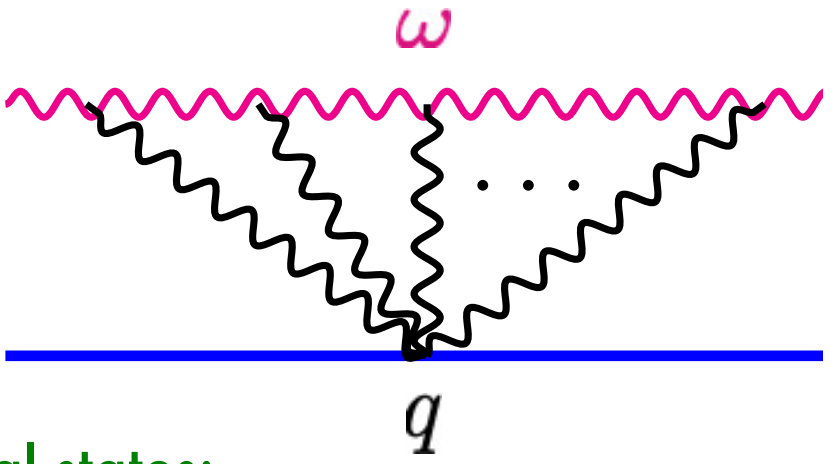
→ No proof that ladder ''rung-rule'' numerators always emerge, but one can make heuristic arguments e.g. Carrasco, Edison, HJ

# Recap: $\mathcal{N}$=8 simplifications

Ben-Shahar, Cangemi, HJ, Shen

The one diagram that rules them all:

$$q^2\mathcal{M}^{\text{tree}} \times (4GM\omega^2)^L$$




Many options for selecting external states:

- e.g. gravitons & spinning BH:

$$\mathcal{M}^{\text{tree}}(h^+ \to h^\pm) = \frac{2GM(u\cdot\chi_\pm)^4}{q^2\omega^2} e^{a\cdot q_\pm} + \mathcal{O}(\hbar)$$

- e.g. scalars & spherically-symmetric BH:

$$\mathcal{M}^{\text{tree}}(\phi \to \phi) = \frac{2GM\omega^2}{q^2} + \mathcal{O}(\hbar)$$

all unified by the same diagram!

$$q_\pm = k \pm k'$$
$$\chi^\mu_\pm = \{[k'|u\sigma^\mu|k], \langle k'|\sigma^\mu|k]\}$$

# Integrate by solving the wave equation!

$$\Box\phi = 0$$

What is the metric?

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = 0$$

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Consider a perturbative solution to: $\Delta\phi = 0$

$$\Delta = \Delta_0 + r_S\Delta_1 + r_S^2\Delta_2 + \dots$$

$$\phi = \phi_0 + \phi_1 + \phi_2 + \dots \qquad \phi_0(x) = e^{-ip\cdot x}$$

$$\phi_1 = -r_S\Delta_0^{-1}\Delta_1\phi_0$$

$$\phi_2 = -r_S^2\Delta_0^{-1}\Delta_2\phi_0 + r_S^2\Delta_0^{-1}\Delta_1\Delta_0^{-1}\Delta_1\phi_0$$

ladder/fan diagram

# Integrate by solving the wave equation!

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ladder/fan diagram

Effective metric linear in coupling!

$$\Delta_1 = \frac{1}{4\pi r}(-\partial_t^2) \xrightarrow{F.T.} \frac{1}{\vec{\ell}^2}\omega^2 \quad \Delta_2 = \Delta_3 = 0$$

# Finding the metric!

Ben-Shahar, Cangemi, HJ, Shen

Wave equation must take the form:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\left(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\right)\phi = 0\,,$$

Gives inverse metric: $$g^{\mu\nu} = \frac{1}{\sqrt{-g}}\left(\eta^{\mu\nu} + 4GM\frac{u^\mu u^\nu}{r}\right)$$

and metric: $$g_{\mu\nu} = \sqrt{|g|}\left(\eta_{\mu\nu} - \frac{4GM}{4GM+r}u_\mu u_\nu\right)$$

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line element form: $$ds^2 = \frac{1}{\sqrt{1+\frac{4GM}{r}}}dt^2 - \sqrt{1+\frac{4GM}{r}}\,(d\vec{x}^2)$$

volume element/warp factor: $$\sqrt{-g} = \sqrt{1+4GM/r}$$

# The $D$-dimensional metric

Since the BH amplitude exists up to $D = 10$, let's generalize:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\left(\Box_0 + \frac{\lambda}{r^{D-3}}(u\cdot\partial)^2\right)\phi = 0$$

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Mass parameter: $$\lambda = 4GM\frac{\Gamma(\frac{1}{2}(D-3))}{\pi^{\frac{1}{2}(D-3)}}$$

volume element/warp factor: $$\sqrt{|g|} = \left(1 + \lambda/r^{D-3}\right)^{\frac{1}{D-2}}$$

This is the known (KK) metric of a D0 brane in maximal supergravity!

# Solving the wave 4D equation to all loops

The BH wave equation $\left(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\right)\phi = 0$

is now equivalent to the Coloumb wave eqn in flat space

$$\left(\nabla^2 + \omega^2 + \frac{4GM\omega^2}{r}\right)\phi = 0$$

The radial equation $v(r) = rR(r)$

$$\frac{d^2v}{dr^2} + \left(\omega^2 + \frac{4GM\omega^2}{r} - \frac{\ell(\ell+1)}{r^2}\right)v = 0$$

have well-known solutions (hypergeometric/Whittaker)

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The all loop amplitude is then

$$\mathcal{M}(\theta) = \frac{1}{2i|\omega|}\sum_{\ell=0}^{\infty}(2\ell+1)(e^{2i(\delta_\ell+\delta_{\rm IR})}-1)P_\ell(\cos\theta)$$

with the partial-wave phase shift $e^{2i\delta_\ell} = \frac{\Gamma(\ell+1+i2GM\omega)}{\Gamma(\ell+1-i2GM\omega)}$

# Resumming the 4D $\mathcal{N}$=8 Compton amplitude

Ben-Shahar, Cangemi, HJ, Shen

Shift-enter directly gives:

$$\mathcal{M}(\theta) = -\frac{GM}{2\sin^2(\theta/2)} \exp\left[-2iGM\omega \ln\sin\frac{\theta}{2} + 2i\delta_0 + 2i\delta_{\rm IR}\right]$$

Using $\sin^2\frac{\theta}{2} = -q^2/(4\omega^2)$ one can massage it into

$$\mathcal{M}(\theta) = \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln\frac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\rm IR}\right]$$

$$= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega\left(\frac{1}{\epsilon} - \ln\frac{-q^2}{\mu^2}\right) + 2i\delta_0\right]$$

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---

$$\delta_0 = -GM\omega\gamma_E + (GM\omega)^3\frac{\zeta_3}{3} - (GM\omega)^5\frac{\zeta_5}{5} + (GM\omega)^7\frac{\zeta_7}{7} + \ldots$$

# IR divergence -- sleight of hand

$$\mathcal{M}(\theta) = \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln \tfrac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\rm IR}\right]$$
$$= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega\left(\frac{1}{\epsilon} - \ln \tfrac{-q^2}{\mu^2}\right) + 2i\delta_0\right]$$

The 4D IR divergent phase is matched to dim. reg. by sleight of hand

$$\delta_{\rm IR} = -GM\omega \ln(4\omega^2 R^2) \stackrel{!}{=} -\frac{GM\omega}{\epsilon}\left(\frac{\mu^2}{4\omega^2}\right)^{-\epsilon} = -GM\omega\left(\frac{1}{\epsilon} + \ln\frac{4\omega^2}{\mu^2} + \mathcal{O}(\epsilon)\right)$$

# Solving the wave 5D equation to all loops

The radial equation in 5D

$$\left(\frac{d^2}{dr^2}+\omega^2-\frac{(\ell+1/2)(\ell+3/2)}{r^2}+\frac{4GM\omega^2}{\pi r^2}\right)v(r)=0$$

→ free eqn after shifting angular momentum:

$$\nu_\ell=-1+\sqrt{(\ell+1)^2-(4GM\omega^2/\pi)^2}$$

solution in terms of Bessel J: → phase shift: $\delta_\ell=\frac{\pi}{2}(\ell-\nu_\ell)$

The all loop amplitude is then

$$\mathcal{M}(\theta)=\frac{i}{\omega^2\sin\theta}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(2iGM\omega^2)^{n+1}}{(2i\pi)^{k-2}}\frac{(n+k)!}{k!(n+1)!(n-k)!}\left(\mathrm{Li}_{n+k}(e^{i\theta})-\mathrm{Li}_{n+k}(e^{-i\theta})\right)$$

Related to all-loop ladder resum. in 4D SYM [Usyukina, Davydychev]

# Solving the wave 7D equation to all loops

The radial equation in 7D

$$\left(\frac{d^2}{dr^2}+\omega^2-\frac{(\ell+3/2)(\ell+5/2)}{r^2}+\frac{4GM\omega^4}{\pi^2 r^4}\right)v(r)=0$$

→ Mathieu equation after change of variables:

$$\left(\frac{d^2}{dz^2}-a+2q\cosh(2z)\right)y(z)=0$$

$$a=(\ell+2)^2\,,\quad q^2=\frac{4GM\omega^4}{\pi^2}\,,\quad r=\frac{q}{\omega}e^z$$

Phase shift → Mathieu characteristic exponent: $\delta_\ell=\frac{\pi}{2}(\ell-\nu_\ell)$

Tree: $$\delta_\ell\Big|_{q^2}=\frac{\pi}{2^3}\frac{1}{(\ell+1)(\ell+2)(\ell+3)}$$

1-loop $$\delta_\ell\Big|_{q^4}=\frac{\pi}{2^7}\frac{108+340\ell+325\ell^2+120\ell^3+15\ell^4}{\ell(\ell+1)^3(\ell+2)^3(\ell+3)^3(\ell+4)}$$

recursive formula is known!

# UV divergences in 7D:

The one-loop triangle is UV divergent → phase shift has pole $\ell = 0$

$$\delta_\ell\Big|_{q^4} = \frac{\pi}{2^7}\frac{108+340\ell+325\ell^2+120\ell^3+15\ell^4}{\ell(\ell+1)^3(\ell+2)^3(\ell+3)^3(\ell+4)}$$

and at similar poles at *L* loops: $\ell < L$ modes are divergent

The theory of Mathieu functions can be used to renormalize:

Remarkable fact: the function $\cos(\pi\nu_\ell)$ is Entire !

$$\delta_\ell = \frac{\pi}{2}(\ell - \nu_\ell)$$

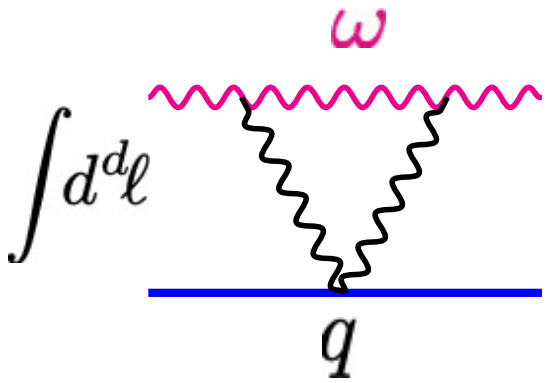
thus it has no UV poles!

Is general dimension possible to solve ?

# D-dimensional warm-up at 1-loop

Ben-Shahar, Cangemi, HJ, Shen

Consider the 1-loop classical Compton amplitude in *d*-dimensions:




Observations:

- IR divergent: $d \leq 3$
- UV divergent: $d \geq 6$
- divide by $\Gamma(3 - d/2)$ renders it UV finite!
- evaluates to log() in integer dimensions $d \geq 5$

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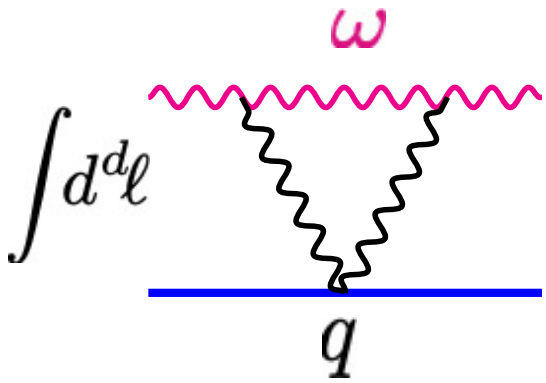


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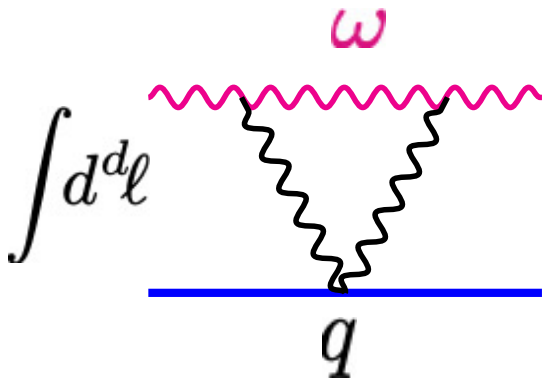
Ben-Shahar, Cangemi, HJ, Shen

Consider the 1-loop classical Compton amplitude in *d*-dimensions:




Observations:

- IR divergent: $d \leq 3$
- UV divergent: $d \geq 6$
- divide by $\Gamma(3-d/2)$ renders it UV finite!
- evaluates to log() in integer dimensions $d \geq 5$

Work out the integer dimensional results → closed *d*-dimensional formula

$$f^{(d)} = \frac{(1+y^2)^{(3-d)/2}}{d-4}\Bigg[(y^2)^{d/2-2}\Big(\ln(-y^2) + \sum_{k=1}^{(d-5)/2}\frac{(1+y^2)^k}{k}\Big)$$

$$- i^{1-d}2^{(d-3)/2}\sqrt{\pi}\Gamma(d/2-1)\Big(\frac{(-y^2)^{d/2-2}}{(d-4)!!} - \sum_{k=0}^{(d-5)/2}\frac{(-1)^k(1+y^2)^k}{2^k k!(d-4-2k)!!}\Big)\Bigg]$$

$$\propto \left(\tfrac{4\omega^2}{q^2}\right)^{d/2-2}\Phi\left(1+\tfrac{4\omega^2}{q^2},1,\tfrac{d-3}{2}\right) - B\left(\tfrac{1}{2},\tfrac{d}{2}-1\right){}_2F_1\left(\tfrac{1}{2};1;\tfrac{d-1}{2},1+\tfrac{4\omega^2}{q^2}\right)$$

# Partial-wave amplitude at 1-loop

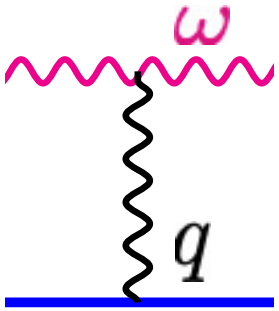
In general $d$ Gegenbauer polynomials describe amplitude

$$\mathcal{M}(\theta) = \frac{1}{2i|\omega|} \sum_{\ell=0}^{\infty} \frac{2\ell + d - 2}{d-2} (e^{2i(\delta_\ell + \delta_{\rm IR})} - 1)\, C_\ell^{(d/2-1)}(\cos\theta)$$

For fixed $\ell, d$ it is easy to transform to partial waves:

First, at tree-level we get that the $q$ propagator transform to

$$e^{2i(\delta_\ell + \delta_{\rm IR})} = 1 - \frac{2iGM|\omega|\ell!}{(d-3)(d-1)_{\ell-1}} + \mathcal{O}(G^2)$$




IR divergent in $d \leq 3$

# Partial-wave amplitude at 1-loop

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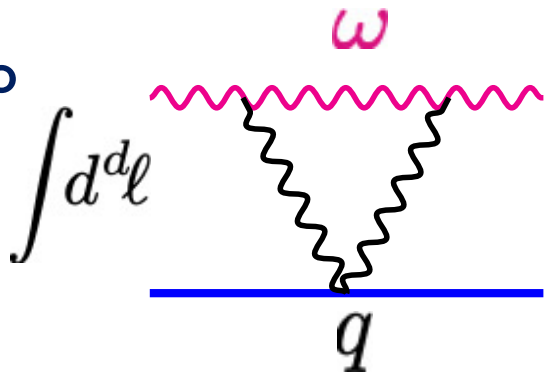
For fixed $\ell, d$ it is easy to transform to partial waves:

At 1-loop level we get that triangle transforms to




$$\frac{d-2}{2\ell+d-2}\frac{\mathcal{M}_\ell^{(1)}}{G^2M^2\omega^2} = -\frac{(-i)^d 2^{2d-11}}{(d-4)(d-2)}\left[\frac{\ell!}{(d-3)(d-1)_{\ell-1}}\right]^2$$
$$+(-1)^\ell 2^{d-\ell-8}B\left(\tfrac{1}{2},\tfrac{d}{2}-1\right)\frac{\Gamma(d-4)\Gamma(d/2)\Gamma(2d-5)}{\Gamma(d+\ell-2)^2\Gamma(3d/2+\ell-4)}\mathcal{P}^{(\ell)}(d)$$

Recursion formula for polynomial:

$$\mathcal{P}^{(\ell+2)} = (2\ell+d)\big(d^2-2\ell^2-2d(\ell+5)-16\big)\mathcal{P}^{(\ell+1)} - (\ell+1)^2(\ell+d-2)^2(2\ell-d+4)(2\ell+3d+8)\mathcal{P}^{(\ell)}\,,$$

# Conclusion

- $N = 8$ supergravity is a testbed for advanced classical calc.
- Compton ampl. is dramatically simper than: 2→2, waveform
- Showed that all-loop resummation is possible for ½ BPS BH
- Identified the metric with D0 brane (11D KK mode)
- BH has no horizon, no finite-size corrections. Exact PP.
- Calculation captures infinitely many ladder diagrams of GR
- Further work required for higher-loop $D$-dimensional solution
- Outlook: Analyzing spin multipoles; UV divergences/finite-size corrections in higher dimensions, connection to ¼-BPS BH.