Applications of Topological Holographic Principle : From holographic tensor network to discovering new CFTs (?)

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Recent Progress in Amplitudes,

10th Nov 2026

Based on works with : 2021-2025

Kaixin Ji, Ce Shen, Yidun Wan, Yu Zhao 2506.05324 (Accepted, Nature Physics)
Gong Cheng, Lin Chen, Zheng-Cheng Gu 2311.18005 Phys.Rev.X 15 (2025) 1, 011073
Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang
2210.12127 Physical Review X 14 (4), 041033
Hao Geng, Yikun Jiang 2505.20385 JHEP 262 (2026)
Yikun Jiang, Bingxin Lao, 2504.21660
Lin Chen, Yikun Jiang, Bingxin Lao
2404.00877, 2403.03179 *SciPost Phys.* 19 (2025) 6, 163



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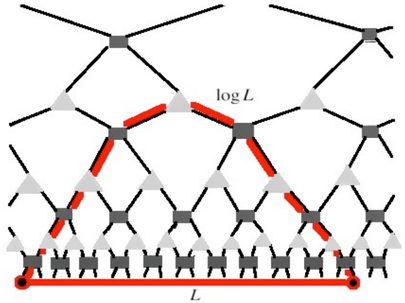
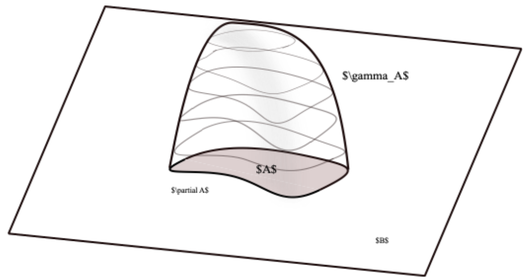
Yau Mathematical Sciences Center, Tsinghua University

Why did I care about holographic tensor network?

- Microscopic local map between CFT and AdS dofs - making the blackbox transparent!
- But continuous construction it was not possible to explicitly deal with a non-Gaussian path-integral in general.

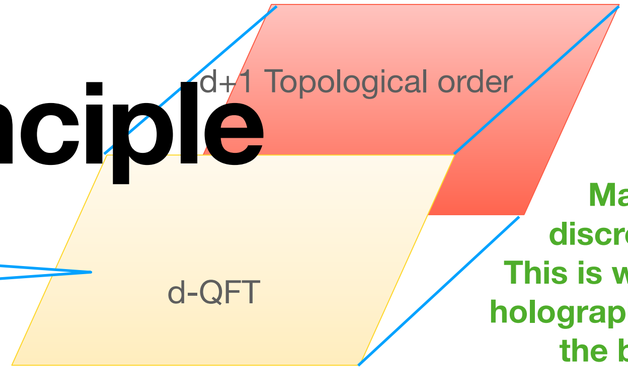
Picture courtesy Orus





To construct these tensor networks, entanglement is not enough — it is only a number , but CFT has way more structures!!!!

Topological Holographic Principle



But how to construct this boundary?!

Many TQFT has discrete formulation. This is why the topological holographic principle can be the bridge between continuity and discreteness

- Topological Holographic Principle states that a symmetric d-dimensional model can be expressed as a d+1 dimensional TQFT path-integral over a “sandwich”
- Bulk imposes the “generalised symmetries” of the boundary.
- “Generalised symmetries” refers to the collection of all the topological defects of different dimensions .

Sandwich and SymTFT:
Kong, Wen, Zheng 2015, 2017;
Kong, Zheng 2019;
Ji, Wen 2019;
Kong, Lan, Wen, Zhang, Zheng 2020;
Gaiotto, Kulp 2020;
Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki 2021;
Freed, Moore, Teleman 2022;

- The collection of these operators form a mathematical structure: a (higher) fusion category — we will need only 1-(spherical) fusion category in the following to discuss 1d topological defects of 2D CFT.
- Topological defects in the d-QFT corresponds to topological excitations (e.g. anyon lines) in the d+1 TQFT (symTFT/symTO).
- e.g. Verlinde lines in 2D CFT ⟶ Wilson lines/anyon lines in a corresponding 3D Chern-Simons theory. — this is known since the 80s.

Tying this to symmetry, making the correspondence a 2 way road, generalised to arbitrary D, is a new insight!

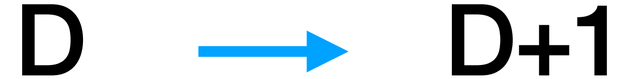
Overview -Applications of the topological holographic principle

- D ⟶ D+1
  Using CFT data to derive higher dimensional dual

- symmetry preserving RG algorithms - the bridge between continuity and discreteness

- D ⟵ D+1
  Predict new critical lattice models and discover new CFTs

- Is this an opportunity for more general strongly interacting field theories?



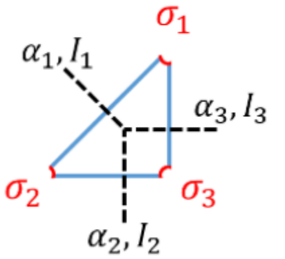
Discretise CFTs and Predict Higher dimensional dual

Cut 2D CFT path-integral into pieces

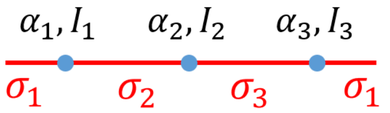
Gabriel Wong, LYH 2020
Brehm, Runkel 2022
Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2024
Gong Cheng, Lin Chen, Zhengcheng Gu, LYH 2025

- Cut 2D manifold into triangles.
- Each triangle = three-point function of boundary-changing operators.
- Glue triangles: summing over states on the edge (primaries + descendants).
- Leaving holes (radius R) at each vertex, labeled by conformal boundary conditions.

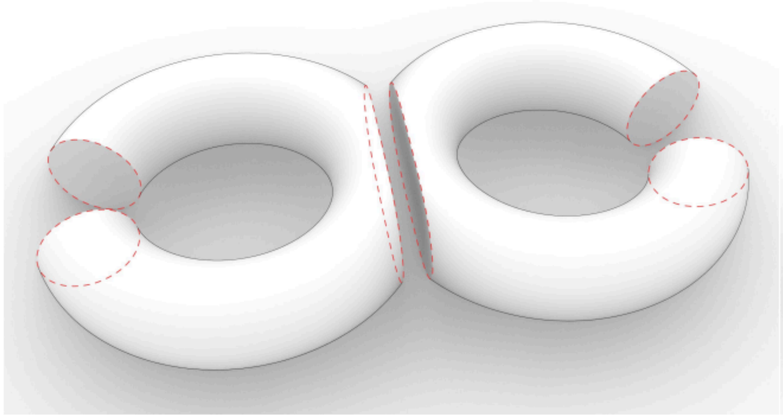
$$\mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle, R) =$$

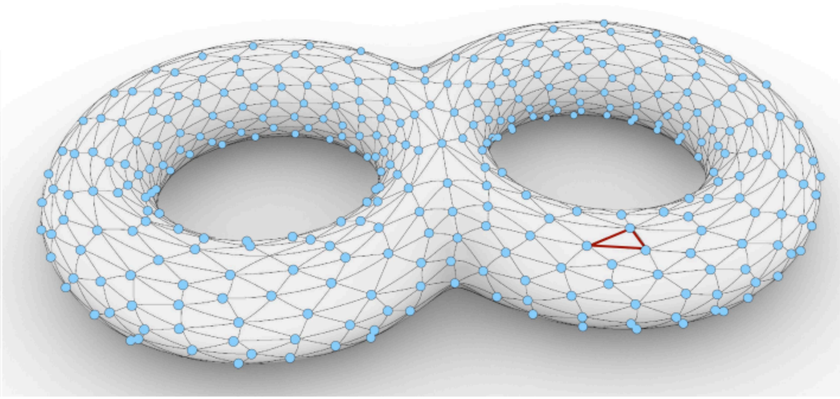


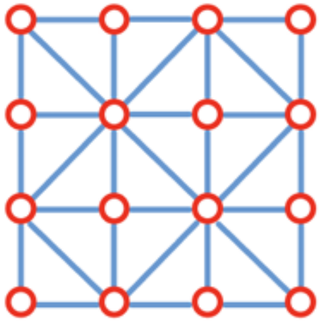
Conformal map



$$\mathcal{Z}(R, \sigma_a) = \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle, R)$$

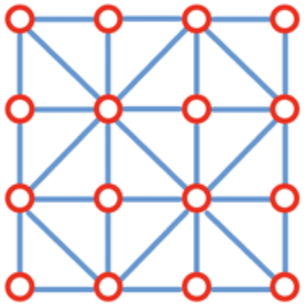






Cut a CFT into triangles

$$\mathcal{Z}(R,\sigma_a) = \sum_{\{\alpha_i\}}\sum_{\{I_i\}}\prod_{\triangle}\mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle,R)$$



- Shrink the holes.

Shrinkable Boundary Condition =
"Entanglement Brane Boundary Condition" in RCFT
Wong, LYH 2020,
Brehm, Runkel 2022

Vacuum Ishibashi state $|e\rangle\rangle = \sum_a \omega_{\sigma_a}|\sigma_a\rangle$

"Quantum dimensions"

Shrinkable boundary designed to preserve **all** the CFT generalised symmetries.

$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}}\prod_v \omega_{\sigma_a}\sum_{\{\alpha_i\}}\sum_{\{I_i\}}\prod_{\triangle}\mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle,R)\ .$$

$$Z_{\mathrm{CFT}} = \lim_{R\to 0}\prod_v e^{-c/6\ln R}\mathcal{Z}(R)$$

Fixed point tensors

Gong Cheng, Lin Chen, Zheng-Cheng Gu LYH 2023

Retain few (level lower than 4) descendants, and numerically recover precise closed CFT data

Ising model

States	Numerical dim	A
$\mathbb{1}$	0.0000	
σ	0.1250	
ϵ	0.9989	
$\partial\sigma, \bar{\partial}\sigma$	1.1253	
$\partial\epsilon, \bar{\partial}\epsilon$	2.0004	
$\mathbb{1}^{(-2)}, \bar{\mathbb{1}}^{(-2)}$	1.9986	
$\partial\bar{\partial}\sigma$	2.1099	2.1250
$\sigma^{(-2)}, \bar{\sigma}^{(-2)}$	2.1208	2.1250
$\partial\bar{\partial}\epsilon$	2.9517	3.0000
$\partial^2\epsilon, \bar{\partial}^2\epsilon$	3.0030	3.0000
$\mathbb{1}^{(-3)}, \bar{\mathbb{1}}^{(-3)}$	2.9496	3.0000
$\bar{\partial}\sigma^{(-2)}, \partial\bar{\sigma}^{(-2)}$	2.9900	3.1250
$\partial\sigma^{(-2)}, \bar{\partial}\bar{\sigma}^{(-2)}$	3.1167	3.1250
$\sigma^{(-3)}, \bar{\sigma}^{(-3)}$	3.1291	3.1250
$\mathbb{1}^{(-2,-\bar{2})}$	4.0323	4.0000

Tricritical Ising

States	Numerical dim	Accurate dim
$\partial\Lambda, \bar{\partial}\Lambda$	1.871	1.875
$\mathbb{1}^{(2)}, \bar{\mathbb{1}}^{(2)}$	2.013	2.000
$\partial\bar{\partial}\Omega$	2.064	2.075
$\Omega^{(2)}, \bar{\Omega}^{(2)}$	2.084	2.075
$\Omega^{(2)'}, \bar{\Omega}^{(2)'}$	2.087	2.075
$\partial\bar{\partial}\Phi$	2.190	2.200
$\Phi^{(2)}, \bar{\Phi}^{(2)}$	2.192	2.200
$\partial\Psi, \bar{\partial}\Psi$	2.210	2.200
$\partial\bar{\partial}\Lambda$	2.877	2.875
$\Lambda^{(2)}, \bar{\Lambda}^{(2)}$	2.878	2.875
$I^{(3)}, \bar{I}^{(3)}$	2.978	3.000
Ξ	3.004	3.000

Lee-Yang

States	Numerical dim	Accurate dim
	-0.403	-0.400
	0.000	0.000
	0.598	0.600
	1.591	1.600
	1.600	1.600
	1.996	2.000
	2.530	2.600
	2.594	2.600

- Precise recovery of CFT
- Graph independence



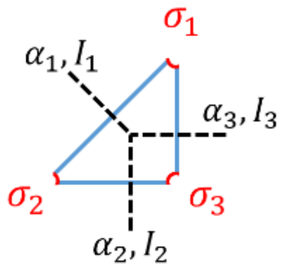
More than a decade old question resolved!

Levin,Nave 2006;
Gu, Levin, Wen 2008;
Evenbly, Vidal 2014, 2015;
Yang, Gu, Wen 2015;
.....

Emergence of 3D Bulk from BCFT OPE

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2024

$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}} \prod_v \omega_{\sigma_a} \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle, R) \ .$$



$$\equiv \tilde{C}^{\sigma_3\sigma_1\sigma_2}_{\alpha_1\alpha_2\alpha_3} \tilde{\gamma}^{\alpha_1\alpha_2\alpha_3}_{I_1I_2I_3}$$

related to “strange correlator” in lattice spin models
Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete 2018;
Aasen, Mong, Fendley 2016; 2020;

Rewrite Z(R) as an overlap of two 3D states

$$\mathcal{Z}(R) = \langle \Omega(R) | \Psi \rangle \quad = \sum_{\{\alpha_i\}} \Omega(\{\alpha_i\})(R) \Psi(\{\alpha_i\})$$

Collection of BCFT OPE (= 6j symbols= 3D tetrahedra)

Fuchs, Runkel, Schweigert 2002-2006

$$\Psi(\{\alpha_i\}) = \sum_{\{\sigma_a\}} \prod_v \omega_{\sigma_a} \prod_{\triangle} \tilde{C}^{\sigma_a\sigma_b\sigma_c}_{\alpha_i\alpha_j\alpha_k}$$

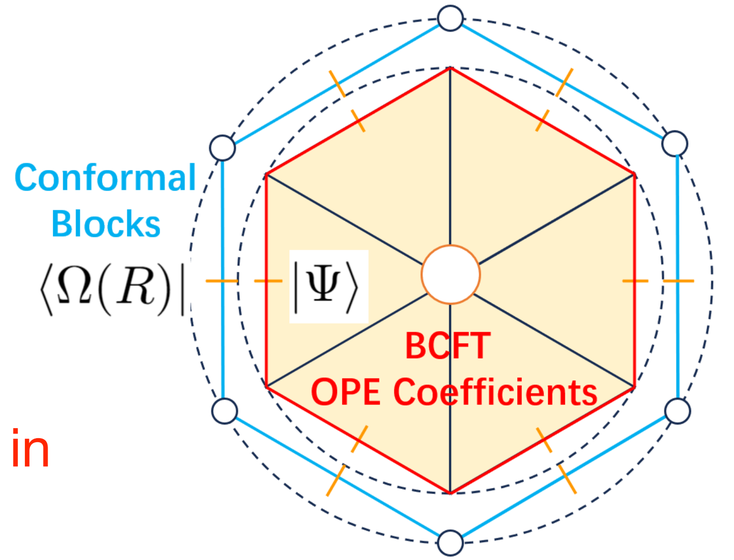
Ground state of a **3D** TQFT

Turaev, Viro 1992;
Levin, Wen 2004

Collection of conformal blocks

$$\Omega(\{\alpha_i\})(R) = \sum_{\{I_i\}} \prod_{\triangle} \tilde{\gamma}^{\alpha_i\alpha_j\alpha_k}_{I_iI_jI_k}(R)$$

3D graph independence in the bulk triangulation



Apply it to Liouville CFT - Closing Holes

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$\Phi_{\alpha}^{\sigma_1\sigma_2}(x)$$

$$\sigma \equiv Q/2 + iP_\sigma, \qquad \alpha \equiv Q/2 + iP_\alpha$$

A self-consistent collection of conformal boundary conditions

Fateev, Zamolodchikov, Zamolodchikov 2000; Teschner 2000;

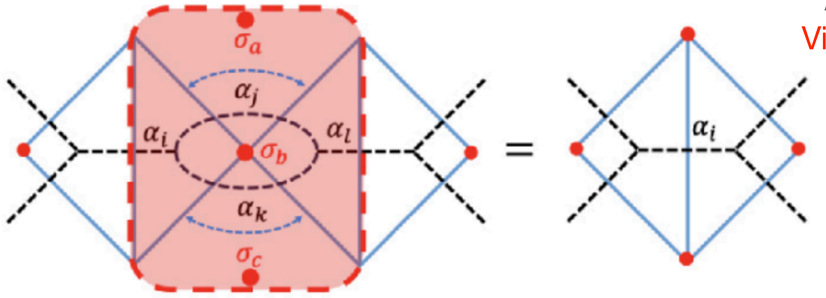
$$C_{\alpha_1,\alpha_2,\alpha_3}^{\sigma_3,\sigma_1,\sigma_2} = (\mu(P_{\alpha_1})\mu(P_{\alpha_2})\mu(P_{\alpha_3}))^{1/4} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix}_b .$$

Ponsot, Teschner 2001; Chen, LYH, Jiang, Lao 2024;

$$Q = b + b^{-1} \qquad c = 1 + 6Q^2$$

$$\mathcal{U}_q(SL(2,\mathbb{R})) \qquad q = e^{\pi i b^2}$$

Also Crossing Kernel of Virasoro conformal blocks



$$\mu(P) = 4\sqrt{2}\sinh(2\pi P b)\sinh\left(\frac{2\pi P}{b}\right).$$

Weight the hole with the Plancherel measure (Cardy density of states)

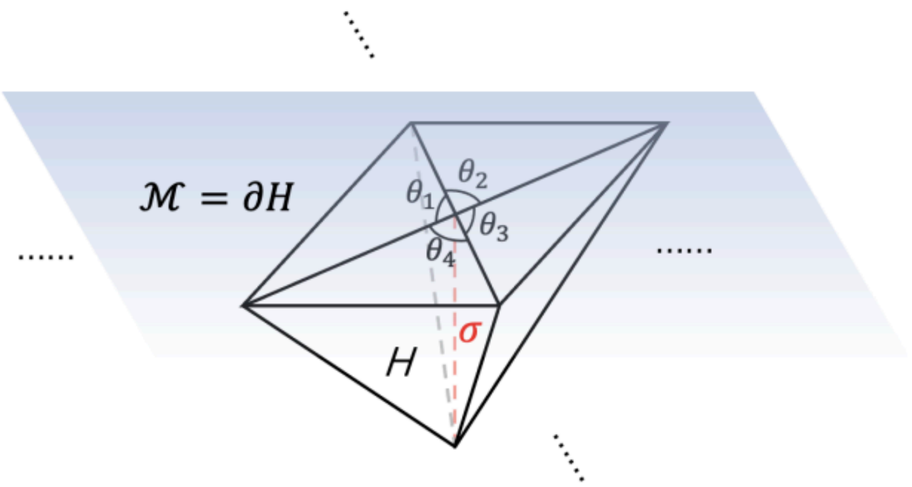
Reading off the bulk

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$Z_{\text{Liouville}}(R) = \langle \Omega(R) | \Psi \rangle$$

$$\Psi(\{\alpha_i\}) = \prod_v \int_0^\infty dP_{\sigma_a} \mu(P_{\sigma_a}) \prod_e \sqrt{\mu(P_{\alpha_i})} \prod_\triangle \begin{Bmatrix} \alpha_i & \alpha_j & \alpha_k \\ \sigma_c & \sigma_a & \sigma_b \end{Bmatrix}_b$$

$$\Omega(\{\alpha_i\})(R) = \prod_e \sum_{\{I_i\}} \prod_\triangle \left(\tilde{\gamma}^{\alpha_i \alpha_j \alpha_k}_{I_i I_j I_k}(R) \right)$$



Reading off the bulk in the large c limit

$$P = \frac{l}{2\pi b}$$

$$\left\{ \begin{matrix} \frac{Q}{2}+i\frac{l_4}{2\pi b} & \frac{Q}{2}+i\frac{l_5}{2\pi b} & \frac{Q}{2}+i\frac{l_6}{2\pi b} \\ \frac{Q}{2}+i\frac{l_1}{2\pi b} & \frac{Q}{2}+i\frac{l_2}{2\pi b} & \frac{Q}{2}+i\frac{l_3}{2\pi b} \end{matrix} \right\}_b \underset{b\to 0}{\to} \exp\left(-\frac{\mathrm{Vol}(T\{l_i\})+\sum_i l_i\theta_i/2}{\pi b^2}\right),$$

Kashaev 1997;
Teschner, Vartanov 2012;
Collier, Eberhardt, Zhang 2023;
Chen, LYH, Jiang, Lao 2024;
Hartman 2025;
Liu, Ming, Sun, Wu, Yang 2025;

Large c limit reduces to on-shell Einstein-Hilbert action on hyperbolic tetrahedron.

$$S_{EH} = -\frac{1}{16\pi G_N}\int_H d^3x\sqrt{g}(R-2\Lambda) = \frac{V(H)}{4\pi G_N} = \frac{V(H)}{\pi b^2},$$

$$\sum_i \frac{\theta_i l_i}{2\pi b^2} = \sum_i \frac{\theta_i l_i}{8\pi G_N} = \frac{1}{8\pi G_N}\sum_i \int_{\Gamma_i} \theta_i\sqrt{h},$$

Gibbons Hawking/ Hayward Term

BCFT primary labels ⟺ Bulk geodesic lengths

Low dimensional AdS/CFT and topological holographic principle have a large overlap here! 3D gravity imposes an infinite number of Virasoro Lines — details of the quantum measure of geometrical sum hides in the hole closing detail!

Symmetry preserving RG - The Bridge between continuity and Discreteness

Generalised Symmetry Preserving RG and Their Fixed Point

A lattice integrable model can also be written in this form:
R. Vanhove, M. Bal, D. J. Williamson, N. Bultinck, J. Haegeman, and F. Verstraete;
D. Aasen, P. Fendley, and R. S. K. Mong

$$\langle \Omega_N | \Psi_a^{LW} \rangle$$

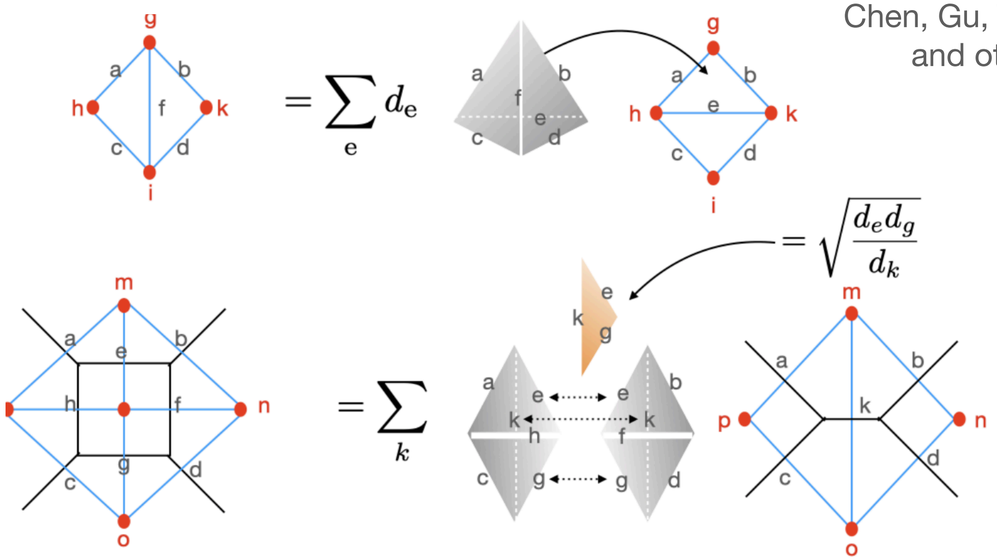
$$|\Psi\rangle_\Lambda = U|\Psi\rangle_{\sqrt{2}\Lambda}$$

$$\langle \Omega_N | FF | \Psi_{ka}^{LW} \rangle = \langle \Omega_{N-1} | \Psi_{ka}^{LW} \rangle$$

$$\sqrt{2}_\Lambda \langle \Omega | =_\Lambda \langle \Omega | U$$

CFT corresponds to a fixed point solution!

RG of topological order
Chen, Gu, Wen 2010;
and others..



- symmetric topological phase = topological eigenstate of RG operator from Frobenius algebra.
- symmetric CFT is an infinite bond dimension eigenstate of the RG operator from conformal blocks

Gong Cheng, Lin Chen, Zheng-Cheng Gu
2311.18005
Phys.Rev.X 15 (2025) 1, 011073

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2210.12127 Physical Review X 14 (4), 041033



Engineering seed boundary conditions to discover new critical points and CFTs

Discovering CFTs from Statistical Lattice models

Historically a treasure of CFTs

- Discretising CFT does not get you new CFTs. To predict new CFTs - we need to work backwards — Given a symmetry, construct a CFT.

- It's too hard to directly create fixed point bc — but it is possible to create bc corresponding to continuous phase transitions in a lattice — this is how historically statistical model helped discovered many CFTs!

- e.g. The Ising spin model solved by Onsagar — has a critical point which is related to the 2D Ising CFT

- The entire A-series minimal models were originally found in the A-series lattice integrable models.

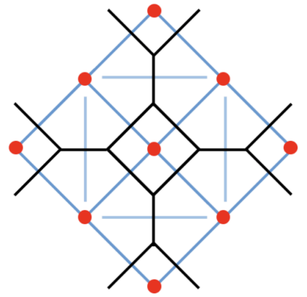
- This is no longer the main-stream of producing new CFT — writing down Lagrangian has become the main-stream. Why? Can we re-live this good old times?

Path-integral
of a 3-ball with a two dimensional surface

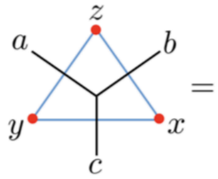


RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020;

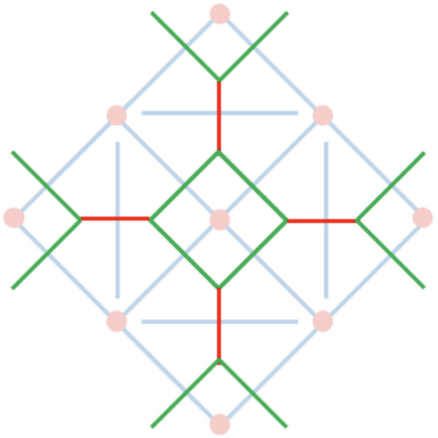


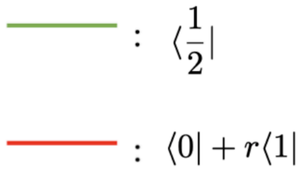
- PEPS representation of Levin-Wen models/Turaev-Viro ground state $|\Psi^{LW}\rangle$
  Gu, Levin, Swingle, Wen PRB 2009; Buershaper, Aguado, Vidal PRB 2009;



$$= \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})^*_{cz}$$

- Then pick some mysterious state $\langle \Omega_N |$ and take the overlap with $|\Psi^{LW}\rangle$ i.e. $\langle \Omega_N | \Psi_a^{LW} \rangle$. $\langle \Omega_N |$ is chosen such that the overlap matches exactly the partition function of well known families of integrable models — this is a realisation of the sandwich.





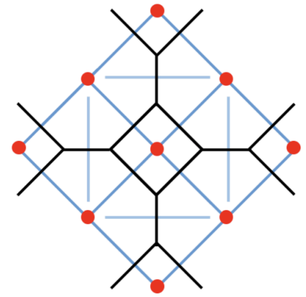
This observation is related to the anyon chain proposed in Feigen et al 2006

RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020;

Path-integral
of a 3-ball with a two dimensional surface

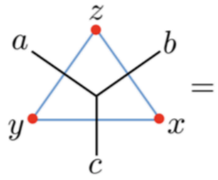




r is related to the temperature in the classical Ising spin model:

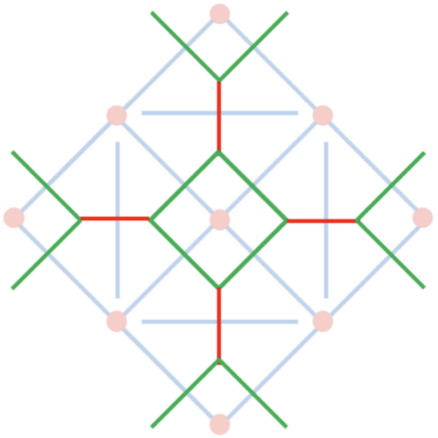
$$r = e^{-2\beta}$$

$$r_c = e^{-2\beta_c}, \qquad \beta_c = 1/2 \ln(1+\sqrt{2})$$



$$= \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})^*_{cz}$$

The critical temperature is known for many years. Now expressed as the boundary condition of the Levin-Wen model.



$: \langle \frac{1}{2}|$

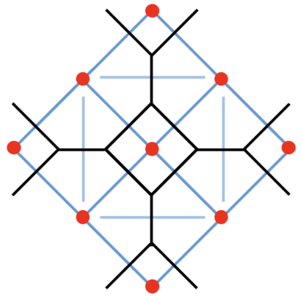
$: \langle 0| + r\langle 1|$

RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020; Chen et al 2022;

Path-integral
of a 3-ball with a two dimensional surface

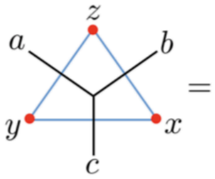




More general there is the A series integrable models with the same ansatz :

Expressed in this form, the well known critical coupling is located at :

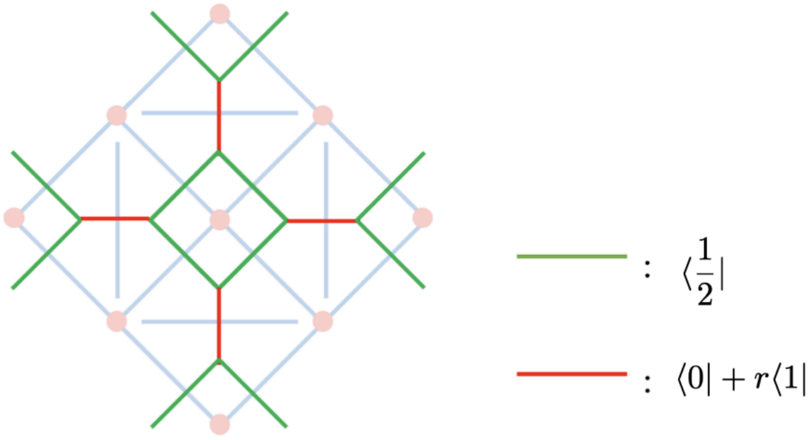
$$r_c = \sqrt{2\cos(2\pi/(k+2))+1}/(2\cos(\pi/(k+2)+1)$$



$$= \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})^*_{cz}$$

What is this? Does the Levin Wen/TV model knew about these numbers?

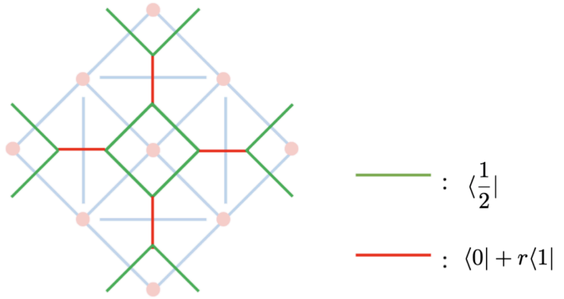
i.e. How exactly is this ansatz chosen in principle????? In particular, which $\langle \Omega_N |$ can produce critical points = CFTs? Is there anything special about these critical couplings???



A 2D CFT Factory : an ansatz of unit cell, and competing anyon condensation

Ansatz of Unit cell

Module and Frobenius algebra



Example: In the Ising example, we notice that at small beta and large beta, the red edges are coloured by one of the two Frobenius algebra of the Ising model respectively. One can check that in this case they are closely related to topological boundary condition of the 3D TV model.

There are 2 Frobenius algebra:

$A_0 = 0$ Electric condensate: equivalent to Lagrangian condensate = $1 \oplus e$

Low temperature Z2 spontaneously sym breaking

How to connect the two things? There are many ways . See Wan et al. But bottom line is to use the Ribbon operator for the Levin Wen model.

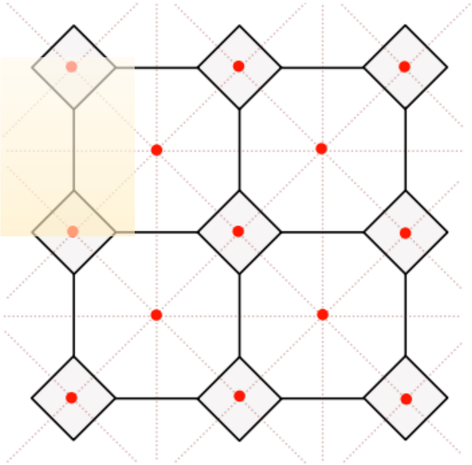
$A_1 = 0 \oplus 1$ Magnetic condensate: equivalent to Lagrangian condensate = $1 \oplus m$

High temperature dual Z2 spontaneously sym breaking

Two anyons with non-trivial braiding cannot condense together. CFT is an equilibrium point in this competition!

Ansatz of Unit cell

Module and Frobenius algebra

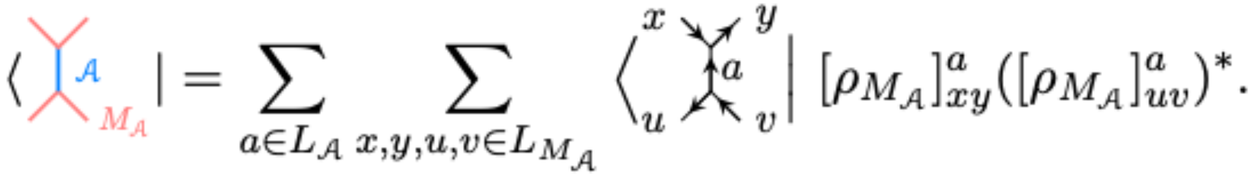


Consider a octagon-square lattice. (Inspired by the Ising model)

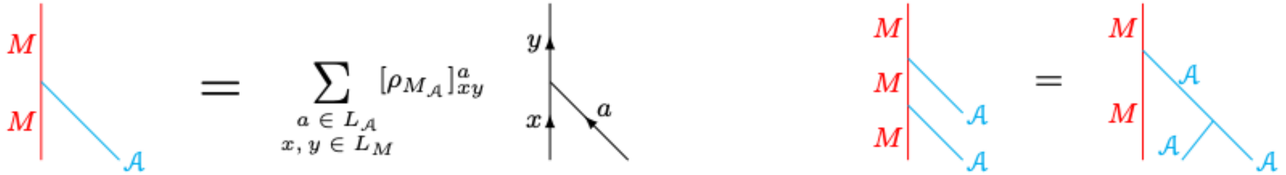
We consider a unit cell in this lattice as highlighted above. We want to define a state corresponding to pouring the condensate into this cell, while it is well “isolated” from the other cells.

This is given by:

This state is a topological boundary condition of the TQFT!



Here, M is a module of A. It satisfies:



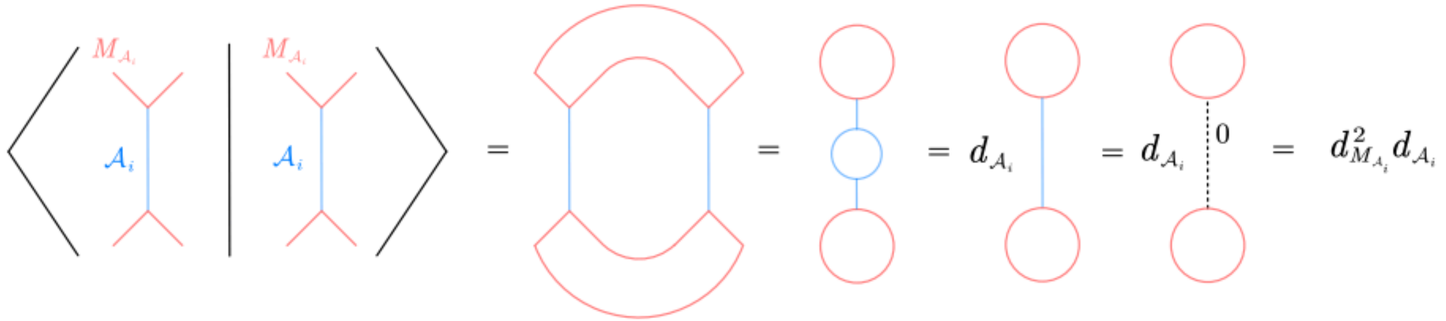
(a) (b)

Ansatz of Unit cell

Module and Frobenius algebra

- Normalising a Frobenius algebra

$$N^2_{(\mathcal{A}_i, M_{\mathcal{A}_i})} =$$

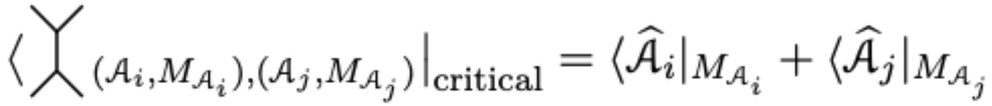


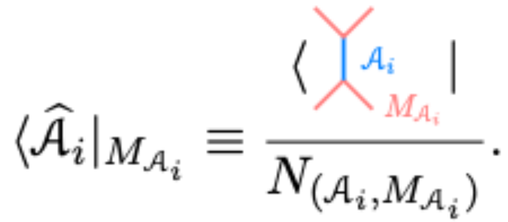
$$d_{M_{\mathcal{A}_i}} \equiv \sum_{c \in M_{\mathcal{A}_i}} m_c \, d_c, \qquad d_{\mathcal{A}_i} \equiv \sum_{a \in \mathcal{A}_i} n_a \, d_a.$$

Ansatz of Unit cell

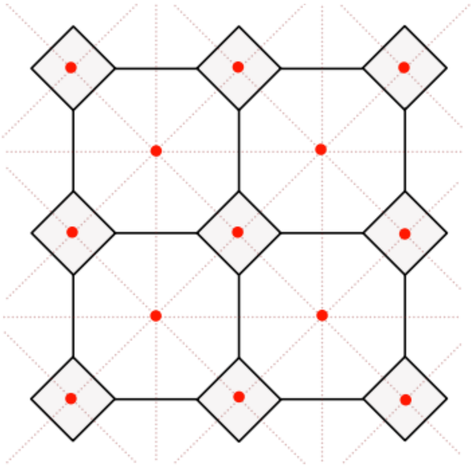
Module and Frobenius algebra

Interpolating between condensates:





Here, we require that Ai and Aj share the module objects in M, even if the module function may not be the same.
This is crucial to preserving the isolation of the condensate between unit cells in the full lattice.



Ansatz of Unit cell

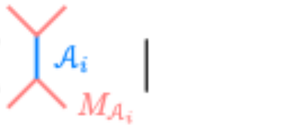
Module and Frobenius algebra

Example:
Going back to
the A-series

Interpolating between condensates:

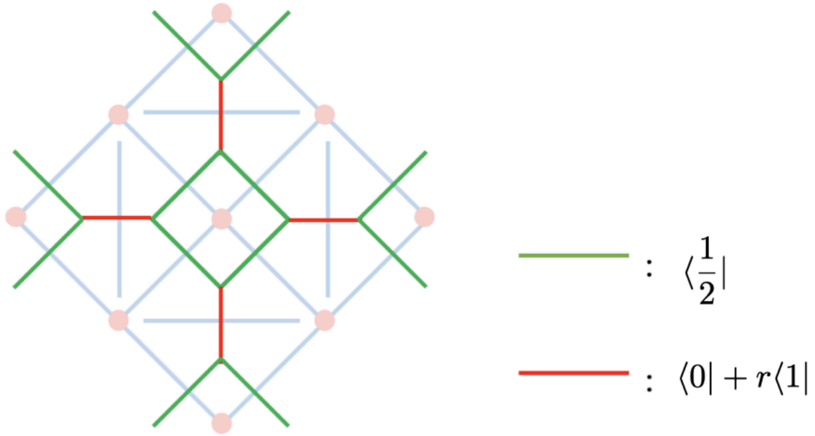


$$\langle \mathcal{Y}_{(\mathcal{A}_i, M_{\mathcal{A}_i}),(\mathcal{A}_j, M_{\mathcal{A}_j})}\big|_{\text{critical}} = \langle \widehat{\mathcal{A}}_i|_{M_{\mathcal{A}_i}} + \langle \widehat{\mathcal{A}}_j|_{M_{\mathcal{A}_j}}$$



$$\langle \widehat{\mathcal{A}}_i|_{M_{\mathcal{A}_i}} \equiv \frac{\langle \mathcal{Y}_{\mathcal{A}_i}|}{N_{(\mathcal{A}_i, M_{\mathcal{A}_i})}}.$$

$$r_c = \sqrt{2\cos(2\pi/(k+2)) + 1}/(2\cos(\pi/(k+2) + 1)$$



For all k we have the following Frobenius algebra:

$$A_0 = 0$$

$$A_1 = 0 \oplus 1$$

$$\mathcal{M} = \frac{1}{2}$$

Phase Diagrams and Phase Boundaries

3 phase competition: Ashkin-Teller model from A5

phase boundaries and tri-critical point

- The module: $M = 1$ $A_0 = 0 \subset A_1 = 0 \oplus 4 \subset A_3 = 0 \oplus 4 \oplus 2$

Note that this choice of module kills $A_2 = 0 \oplus 2$

nM = 3, Dc = 2 => All phase transitions are second order.

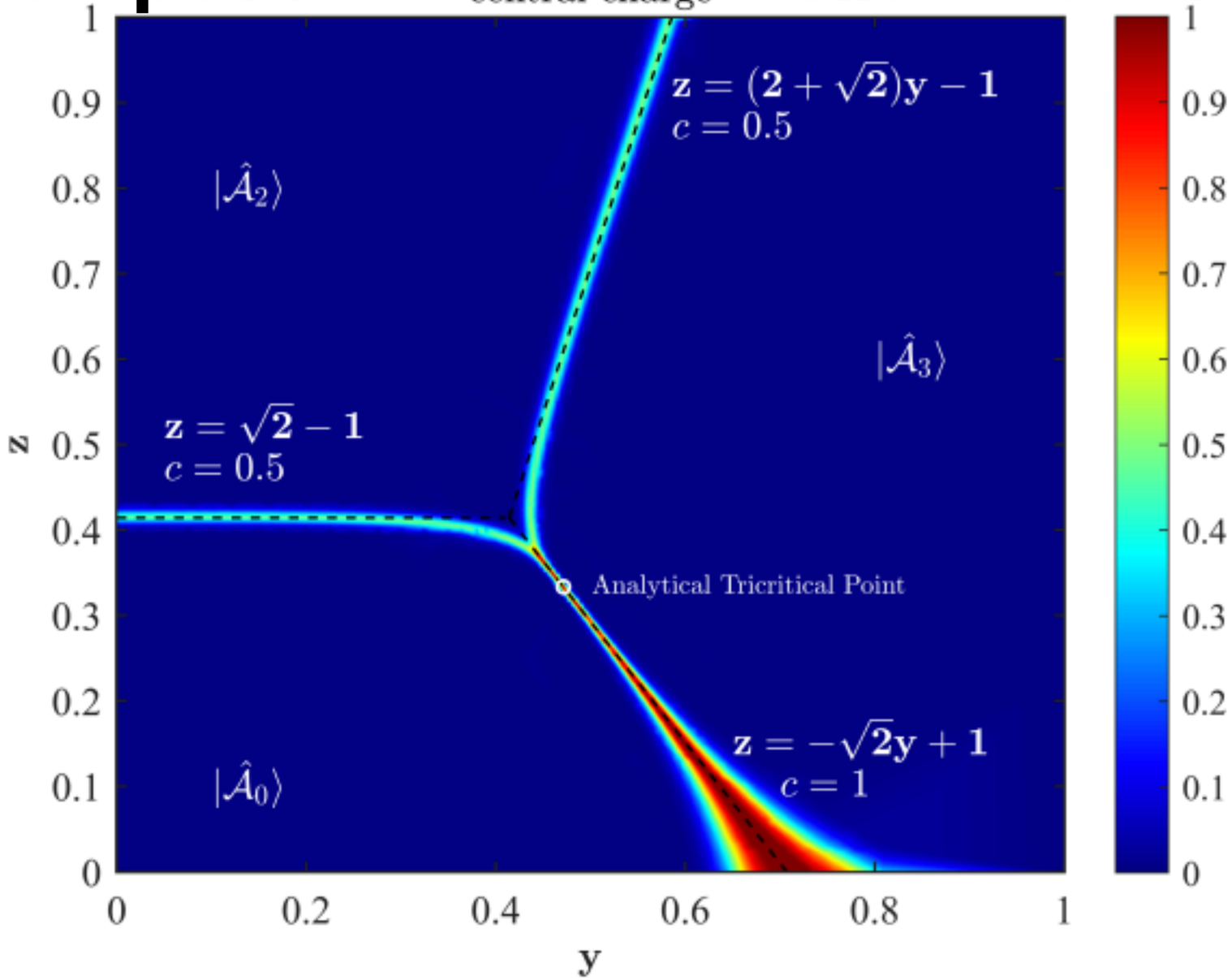
3 phase competition: Ashkin-Teller model from A5

phase bou

- The module

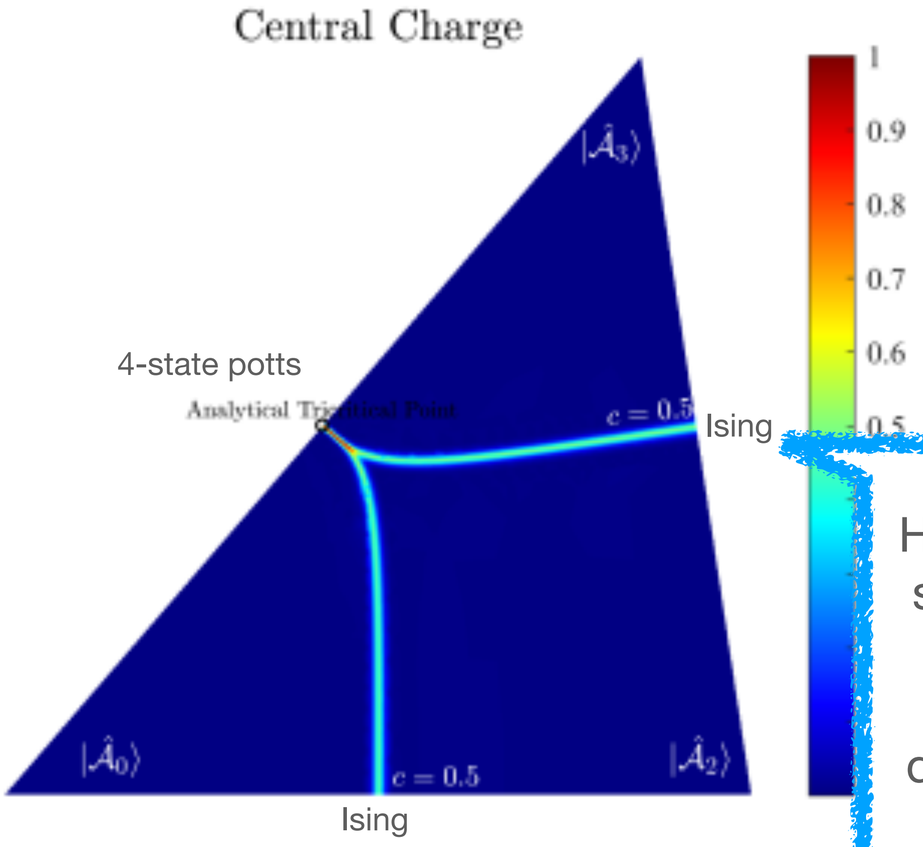
Note that this

nM = 3, Dc =
second order.



3 phase competition: Ashkin-Teller model from A5

phase boundaries and tri-critical point



4-state potts

Analytical Tricritical Point

Ising

Ising

Phase boundaries:

$$\langle C_{ij}| = \frac{1}{2}(\langle \hat{\mathcal{A}}_i| + \langle \hat{\mathcal{A}}_j|)$$

$$\langle C_{ij}(p)| = \langle C_{ij}| + p\left(\langle \hat{\mathcal{A}}_k| - \frac{\langle \hat{\mathcal{A}}_k|B_{ij}\rangle}{\langle B_{ij}|B_{ij}\rangle}\langle B_{ij}|\right)$$

How do we understand which CFT would show up here? Particularly they seem to preserve different amount of (non)-invertible symmetries. How do we determine what is the *minimal* amount of symmetries preserved?

Novel Examples - Haagerup TQFT

Haagerup Model

$$1, \quad \alpha, \quad \alpha^2, \quad \rho, \quad \alpha\rho, \quad \alpha^2\rho,$$

The fusion rules are

1	α	α^2	ρ	$\alpha\rho$	$\alpha^2\rho$
α	α^2	1	$\alpha\rho$	$\alpha^2\rho$	ρ
α^2	1	α	$\alpha^2\rho$	ρ	$\alpha\rho$
ρ	$\alpha^2\rho$	$\alpha\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$
$\alpha\rho$	ρ	$\alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$
$\alpha^2\rho$	$\alpha\rho$	ρ	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$

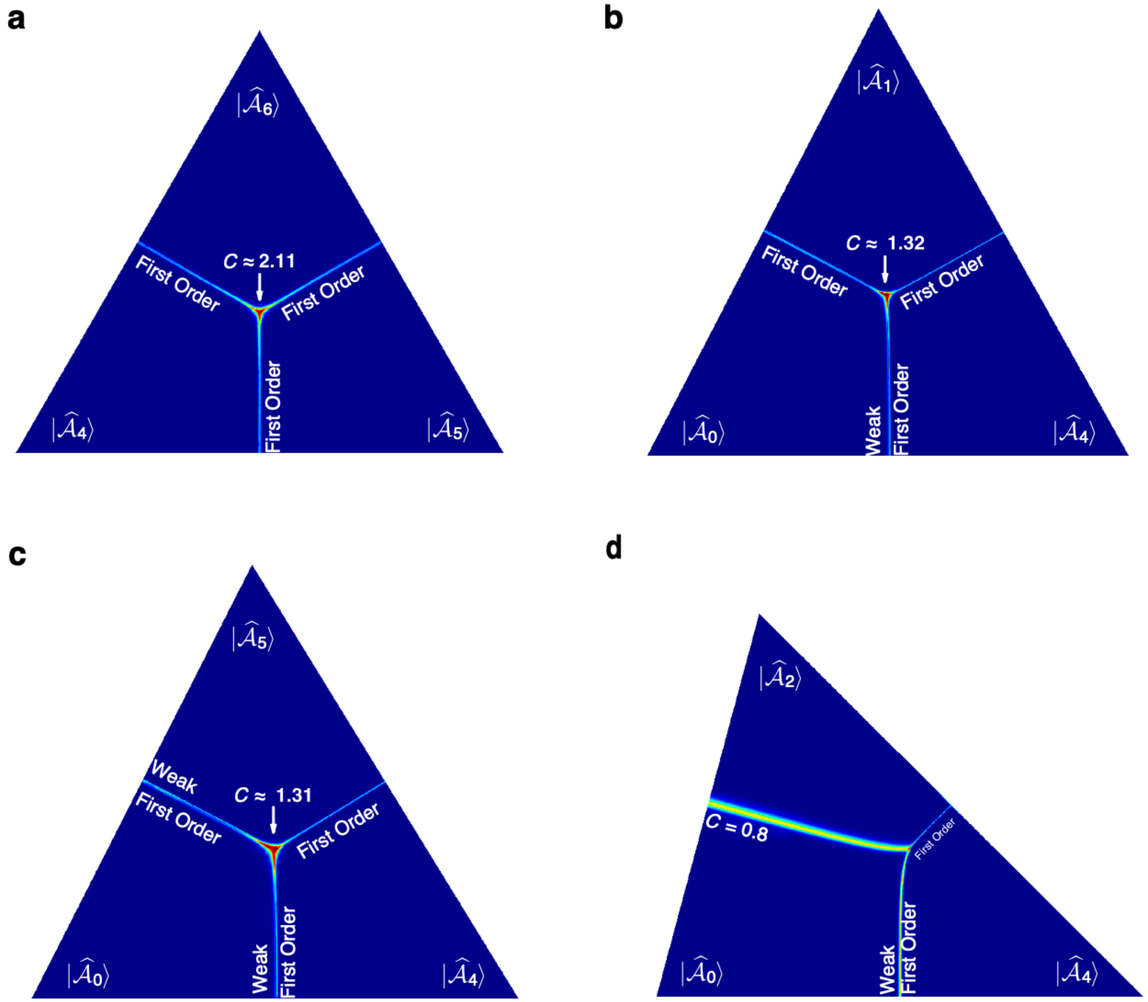
$$d_1 = d_\alpha = d_{\alpha^2} = 1, \qquad d_\rho = d_{\alpha\rho} = d_{\alpha^2\rho} = \frac{3+\sqrt{13}}{2}.$$

$$L_0 = 1 \oplus \pi_1 \oplus 2\pi_2$$
$$L_1 = 1 \oplus \pi_1 \oplus 2\sigma_1$$
$$L_2 = 1 \oplus \pi_1 \oplus \pi_2 \oplus \sigma_1.$$

$$\mathcal{A}_0 = 1$$
$$\mathcal{A}_1 = \rho \otimes \mathcal{A}_0 \otimes \rho = 1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$$
$$\mathcal{A}_2 = 1 \oplus \alpha \oplus \alpha^2$$
$$\mathcal{A}_3 = \rho \otimes \mathcal{A}_2 \otimes \rho = (1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho) \otimes (1 \oplus \alpha \oplus \alpha^2).$$
$$\mathcal{A}_4 = 1 \oplus \rho \oplus \alpha\rho,$$
$$\mathcal{A}_5 = \alpha \otimes \mathcal{A}_4 \otimes \alpha^2 = 1 \oplus \alpha\rho \oplus \alpha^2\rho,$$
$$\mathcal{A}_6 = \alpha^2 \otimes \mathcal{A}_4 \otimes \alpha = 1 \oplus \rho \oplus \alpha^2\rho.$$

$$L_0 : \ \mathcal{A}_0, \mathcal{A}_1$$
$$L_1 : \ \mathcal{A}_2, \mathcal{A}_3$$
$$L_2 : \ \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6.$$

Haagerup Model



Haagerup Model - summary of results

H_3	$\mathcal{A}_0,\ \mathcal{A}_1$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	3	First	—	—	—
H_3	$\mathcal{A}_0,\ \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	2	Cont.	$\mathcal{Z}(S_3)$	0.801	0.013
H_3	$\mathcal{A}_0,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	2	Weak	—	1.3	—
H_3	$\mathcal{A}_1,\ \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	First	—	—	
H_3	$\mathcal{A}_1,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	First	—	—	
H_3	$\mathcal{A}_2,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	First	—	—	
H_3	$\mathcal{A}_{(4,5,6)},\ \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	First	—	—	
H_3	$\mathcal{A}_0,\ \mathcal{A}_1,\ \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	Cont.	$\mathcal{Z}(H_3)$	0.793	0.014
H_3	$\mathcal{A}_0,\ \mathcal{A}_2,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	Cont.	$\mathcal{Z}(H_3)$	0.8047	0.0077
H_3	$\mathcal{A}_0,\ \mathcal{A}_1,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	Cont.	$\mathcal{Z}(H_3)$	1.3154	0.0065
H_3	$\mathcal{A}_0,\ \mathcal{A}_{(4,5,6)},\ \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	Cont.	$\mathcal{Z}(H_3)$	1.3141	0.0042
H_3	$\mathcal{A}_1,\ \mathcal{A}_{(4,5,6)},\ \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	7	First	—	—	—
H_3	$\mathcal{A}_2,\ \mathcal{A}_{(4,5,6)},\ \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	6	First	—	—	—
H_3	$\mathcal{A}_1,\ \mathcal{A}_2,\ \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	7	First	—	—	—
H_3	$\mathcal{A}_4,\ \mathcal{A}_5,\ \mathcal{A}_6$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	6	Cont.	$\mathcal{Z}(H_3)$	2.106	0.022

Summary and Outlook

- AdS/CFT seems to coincide with the topological holographic principle, at least in low dimensions!
- Topological holographic principle — connects the discrete and the continuous world
- We produce a systematic way of constructing *critical* lattice models based on the strange correlator
- This gives the Landau paradigm for generalised symmetries a more precise handle to construct effective field theories — here, the order parameter is precisely the anyon creation operator defined on some choice of unit cells

To appear :

1) Generalisation to Fermionic CFT in 2D - recovering global spin structures from local pieces. 2026.10xxx

   (Recently there seems to be a lot of activity in rigorous lattice formulation of chiral fermions .. our construction is an orthogonal framework that may also works for chiral theories)

2) More generalised 2D CFT factory —

   - More general lattice. 2026.10xxx
   - Some new competitions between generalised topological boundaries 2026.11xxx

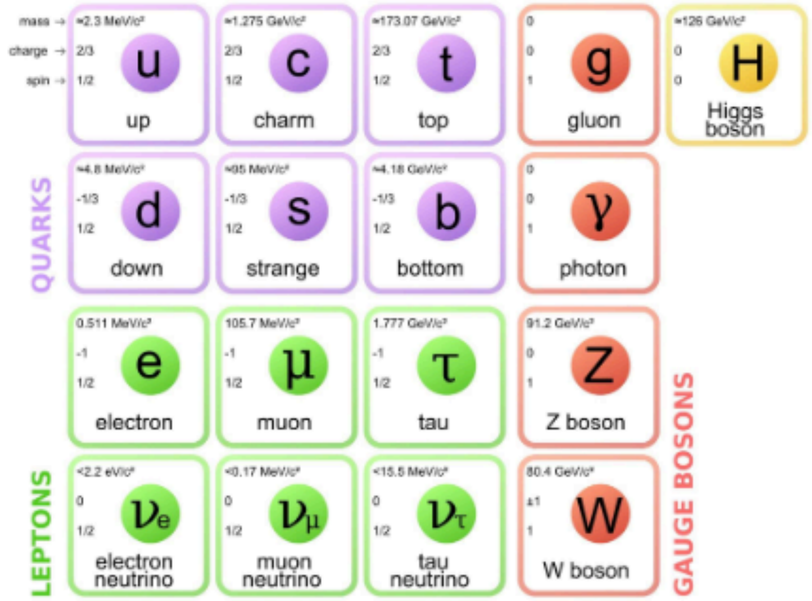
3) Higher dimensional generalisations

Future: Strongly Interacting Field Theory ???

- Higher dimensions

- Include fermions and introduce chirality (lots of recent works ….)

Quantum Field Theory and the Electroweak Standard Model - Arbuzov, A.B. - arXiv:1801.05670



Particle content of the Standard Model. Courtesy to Wikipedia: 'Standard Model of Elementary Particles' by MissMJ --- Own work by uploader, PBS NOVA, Fermilab, Office of Science, United States Department of Energy, Particle Data Group.

But the success is mostly
in the high energy limit, when the theory approaches free theory— where perturbation based on Feynman diagrams have been extremely successful.

pictures courtesy Cern

Thank you!