On string scattering

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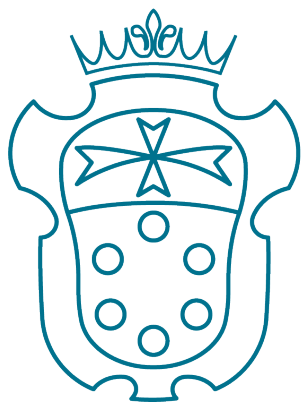
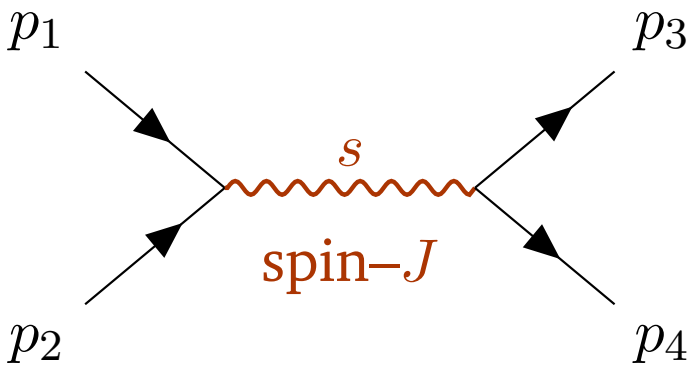
November 10, 2026
Workshop on Recent Progress in Amplitudes
ShanghaiTech University



SCUOLA
NORMALE
SUPERIORE

Strings were born from S–matrix theory!

- let's consider $2 \to 2$ scalar field scattering (s–channel):



diverges at high energies!

- Veneziano amplitude (s–channel):

$$\mathcal{A}_4 \sim \frac{\Gamma\big(-\alpha(s)\big)\Gamma\big(-\alpha(t)\big)}{\Gamma\big(-\alpha(s)-\alpha(t)\big)}\,, \quad \alpha(s) = 1 + \alpha' s$$

$\Rightarrow$ *cures* divergences via *infinitely many* simple poles

$$\text{e.g. @ } s = -\tfrac{1}{\alpha'}, 0, \tfrac{1}{\alpha'}, \tfrac{2}{\alpha'}, \ldots$$

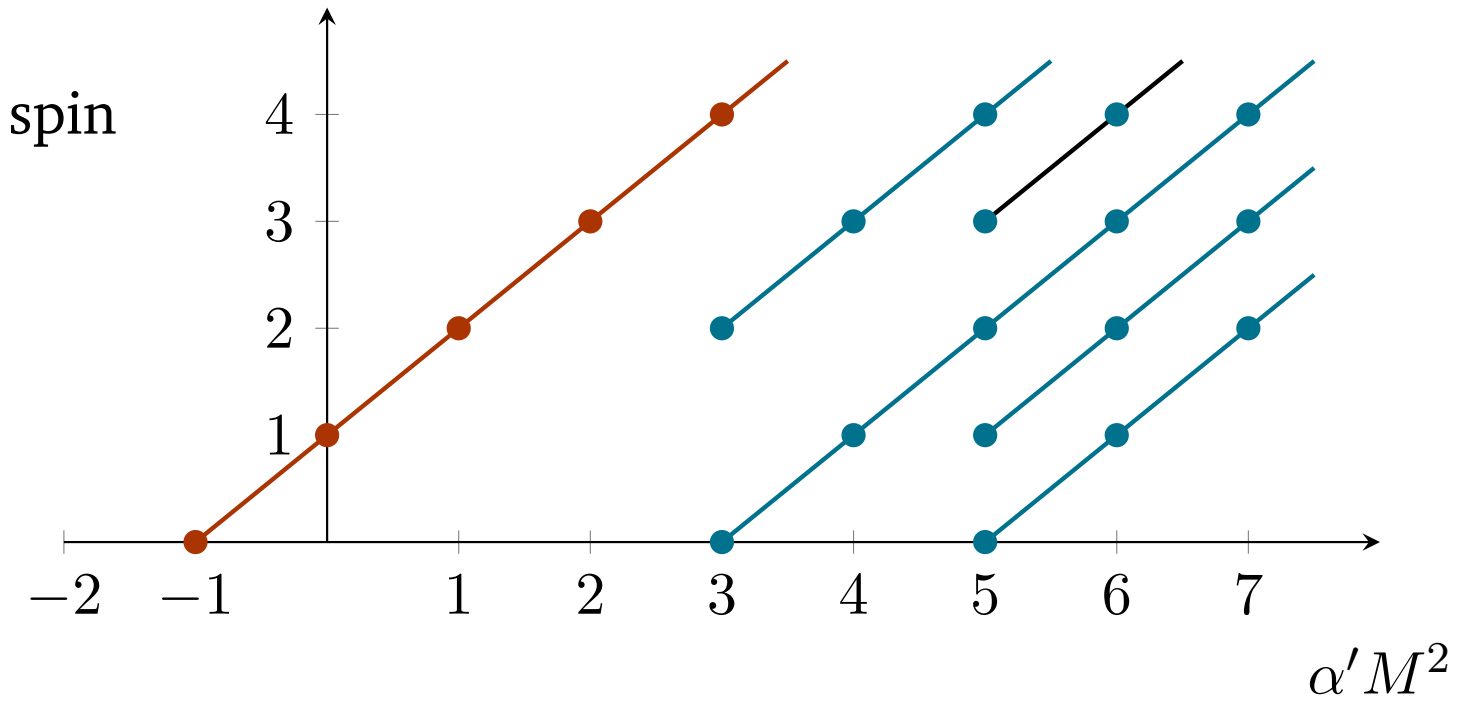
Veneziano '68

A theory of ∞–many states & trajectories

- result: *string* theory with *infinitely many physical states*

$$M^2 = \text{integer} \times \frac{1}{\alpha'}$$

- Regge behaviour $s^{\alpha(t)}$ iff parent has infinite daughters!
  (for external states of *unequal* mass)



A theory of ∞–many states

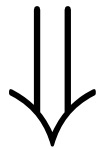
(one box per spacetime index, symmetric rows, $\mathfrak{so}(D-1)$ or $\mathfrak{so}(D-2)$)

N	decomposition in physical states (open bosonic string)
0	$\bullet$
1	$\square_{\mathfrak{so}(D-2)}$
2	$\square\square$
3	$\square\square\square \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}$
4	$\square\square\square\square \oplus \square\!\square\!\square \oplus \square\square \oplus \bullet$
5	$\square\square\square\square\square \oplus \square\!\square\!\square\square \oplus \square\square\square \oplus \square\!\square\!\square \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \square$
6	$\square\square\square\square\square\square \oplus \square\!\square\!\square\!\square \oplus \square\square\square\square \oplus \square\!\square\!\square\!\square \oplus \square\!\square\!\square \oplus \square\square\square \oplus \square\!\square \oplus 2\,\square\square \oplus \ldots$

see e.g. Weinberg 1985, Mañes, Vozmediano 1989

beyond the leading Regge, the spectrum is *repetitive*

∞–many massive higher spins



crucial to string UV softness

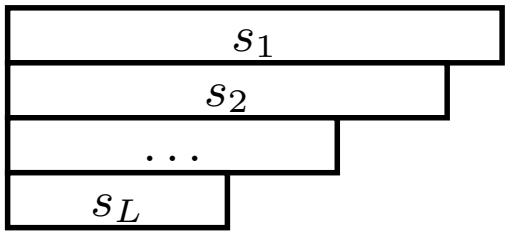
it is thus important to understand string higher spins

Level by level

- general vertex operator:

$$V(z,p) = F(\partial X^\mu, \partial^2 X^\nu, \dots) e^{ip\cdot X(z)}$$

where F contains the polarization



$$\Leftrightarrow \quad \varepsilon^{\mu(s_1),\,\lambda(s_2),\dots,\,\nu(s_L)}(p)$$

$\Rightarrow$ ∞–many ways to contract with $\partial X^\mu, \partial^2 X^\nu, \dots$

- *leading* Regge trajectory: highest spins $\forall$ level

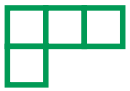
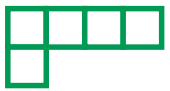
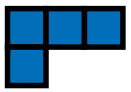
$$V_{\text{leading}} = \boldsymbol{\varepsilon_{\mu_1 \dots \mu_s}\, \partial X^{\mu_1} \dots \partial X^{\mu_s}}\, e^{ip\cdot X(z)}$$

- each traditional method (old covariant, light cone, DDF, partition function) has its own restriction

Is the spectrum concealing a *bigger organizing symmetry*?

Re–organizing the spectrum

the spectrum consists of $w = 0$ trajectories and their ∞–many clones!

N	decomposition in physical states
0	•
1	$\square_{so(D-2)}$
2	$\square\square$
3	$\square\square\square \oplus$ 
4	$\square\square\square\square \oplus$  $\oplus$  $\oplus$ •
5	$\square\square\square\square\square \oplus$  $\oplus$  $\oplus$  $\oplus$  $\oplus$ 
6	$\square\square\square\square\square\square \oplus$  $\oplus$  $\oplus$  $\oplus$  $\oplus$  $\oplus 2$  $\oplus$  $\oplus \ldots$

let's call
$w = N - N_{\text{min}}$
the "*depth*"
CM, Skvortsov '23

$\Rightarrow$ polynomial *complexity*: measured by w (and number of rows)

A bigger organizing symmetry

- can *rewrite* (transverse) Virasoro constraints as

$$L_{n>0}F = \left[\sum_{m=1}^{n-1} T_{m,n-m} + 2\sum_{m=1}^{\infty} m\, T^{m}{}_{n+m}\right] F = 0$$

$\mathfrak{sp}$ lowering operators

- $\mathfrak{sp}(2K)$ algebra generated by $(k,\ell = 1,2,\ldots,K)$

$$T^{k}{}_{\ell} := \tfrac{1}{k}\,\alpha_{-k}\cdot\alpha_{\ell}\,, \quad T_{k\ell} := \alpha_{k}\cdot\alpha_{\ell}\,, \quad T^{k\ell} = \tfrac{1}{k\ell}\,\alpha_{-k}\cdot\alpha_{-\ell}$$

entire spectrum splits in

$w = 0$ → $\mathfrak{sp}$ lowest weight states (contract i–th row with $\partial^{i}X^{\mu} \leftrightarrow \alpha^{\mu}_{-i}$ see e.g. Weinberg 1985)

$w > 0$ → ∞–many clones *polynomials?*

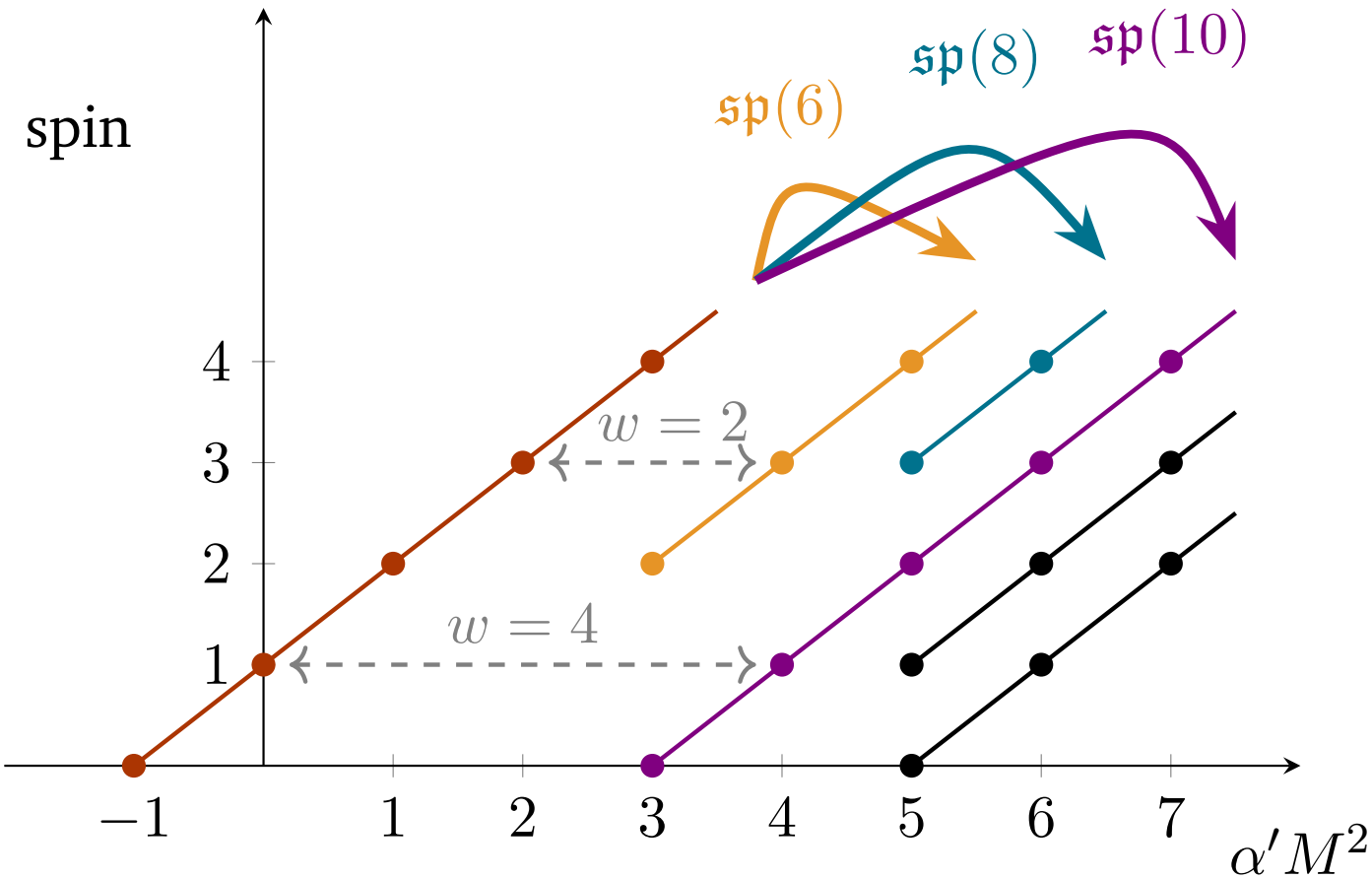
CM, Skvortsov '23

A new technology

- *cloning* possible thanks to *Howe duality* (Howe 1989):

$$F^{f}_{w>0} = f_{w>0}(T^{mn}, T^{k>\ell}{}_{\ell})\, \boldsymbol{F_{w=0}} \quad \Rightarrow \quad \text{solve } L_1 \,\&\, L_2$$

$\Rightarrow$ excavate *entire* trajectories! (spin is a free parameter)



CM, Skvortsov '23

A new technology

- example: reach the first clone, at $w = 2$, of the leading Regge

$$\boldsymbol{F^{f}_{w=2}} = \left[\beta_1 T^{11} + \beta_2 T^{3}{}_{1} + \beta_3 (T^{2}{}_{1})^2\right] \boldsymbol{F_{w=0}}$$

$\Rightarrow$ solve the Virasoro constraints for $F^{f}_{w=2}$:

$$\Rightarrow \quad \beta_2 = -\beta_1 \frac{D + 2\boldsymbol{s} - 1}{\boldsymbol{s}}\,, \quad \beta_3 = \beta_1 \frac{D + 2\boldsymbol{s} - 1}{\boldsymbol{s}(\boldsymbol{s} - 1)}$$

spin is not fixed $\Rightarrow$ entire trajectory is known!

- e.g.: the lightest member–state $\square\!\square$

$$\begin{aligned} \boldsymbol{F_B} &= B_{\mu_1\mu_2}\, \alpha^{\mu_1}_{-1} \alpha^{\mu_2}_{-1} \\ \boldsymbol{F_{\widetilde{B}}} &= \widetilde{B}_{\mu_1\mu_2} \left(12\, \alpha_{-1} \cdot \alpha_{-1} \alpha^{\mu_1}_{-1} \alpha^{\mu_2}_{-1} - 116\, \alpha^{\mu_1}_{-3} \alpha^{\mu_2}_{-1} + 87\, \alpha^{\mu_1}_{-2} \alpha^{\mu_2}_{-2}\right) \end{aligned}$$

CM, Skvortsov '23

The superstring, super quickly

- add *fermionic* modes b^{μ}_{r}, with $\{b^{\mu}_{m}, b^{\nu}_{n}\} = \delta_{m+n}\,\eta^{\mu\nu}$

  $\Rightarrow$ favour columns, but how do we *glue rows to columns*?

1. Neveu–Schwarz sector: $\mathfrak{osp}(2M|2K)$
   - $\exists$ mixed bilinears: $(n = 1, \ldots, N\,,\ r = 1, \ldots, M)$

$$Q^{nr} := \tfrac{1}{n}\,\alpha_{-n}\cdot b_{-r+\frac{1}{2}}\,, \quad Q^{n}{}_{r} := \tfrac{1}{n}\,\alpha_{-n}\cdot b_{r-\frac{1}{2}}$$
$$Q_{n}{}^{r} := \alpha_{n}\cdot b_{-r+\frac{1}{2}}\,, \quad Q_{nr} := \alpha_{n}\cdot b_{r-\frac{1}{2}}$$

   - super–Virasoro constraints:

$$G_{1/2}F_{\text{NS}} = \sum_{n\geq 1}\Big[n\,Q^{n}{}_{n+1} + Q_{n}{}^{n}\Big]F_{\text{NS}} = 0$$
$$G_{3/2}F_{\text{NS}} = \Big[Q_{11} + \sum_{k\geq 1}\Big(k\,Q^{k}{}_{k+2} + Q_{k+1}{}^{k}\Big)\Big]F_{\text{NS}} = 0$$

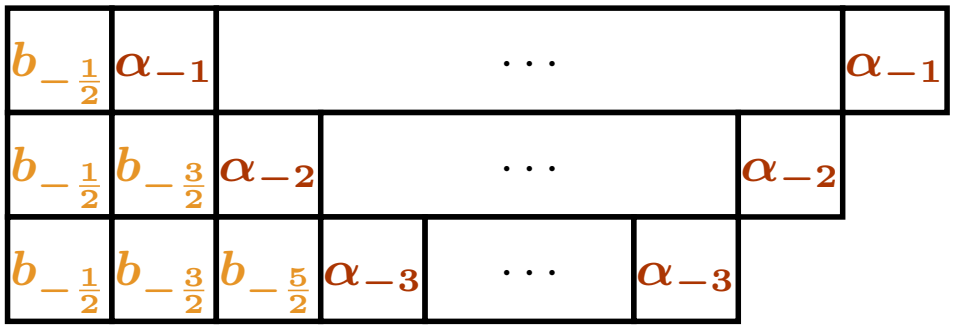
2. Ramond sector: $\mathfrak{osp}(2M+1|2K)$

T. Basile, CM '24

The superstring, super quickly

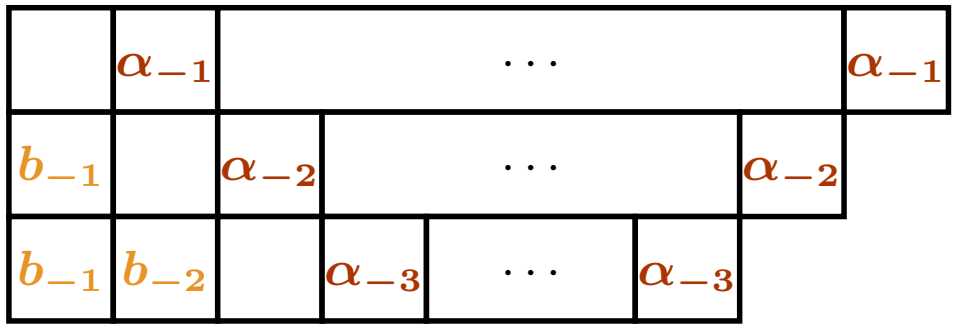
1. Neveu–Schwarz sector:

$w = 0$:



2. Ramond sector: (L: diagonal length)

$w = 0 \;\Rightarrow\; 2^L$ configurations :



ambiguity due to $Q^n{}_n := \frac{1}{n}\,\alpha_{-n} \cdot b_n \Rightarrow$ non–trivial multiplicity!

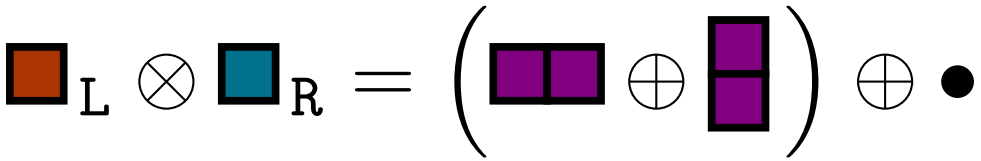
- construct $w > 0$: dress with raising operators of the $\mathfrak{osp}$, apply super–Virasoro constraints

T. Basile, CM ’24

What about closed strings?

Closed string motivation

- graviton: universally a *closed* string state



e.g. bosonic string:

$$|\text{graviton}\rangle = \varepsilon_{\mu\nu}\, \alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1}\, |0;p\rangle \ , \quad p^{\mu}\varepsilon_{\mu\nu} = 0 = \varepsilon^{\mu}{}_{\mu}$$

Scherk, Schwarz 1974, Yoneya 1974, . . .

- closed string spectrum: has a perturbative symmetry
  - generated by linear combinations of

$$\frac{1}{n}\left(A^{i}_{-n}A^{j}_{n} + \bar{A}^{i}_{-n}\bar{A}^{j}_{n}\right)$$

  - contains a Kac–Moody algebra

A, $\bar{A}$: DDF operators, "1–1" with oscillators

Del Giudice, Di Vecchia, Fubini 1972, Brower 1972

Gaberdiel, West 2002, . . .

Level–by–level construction?

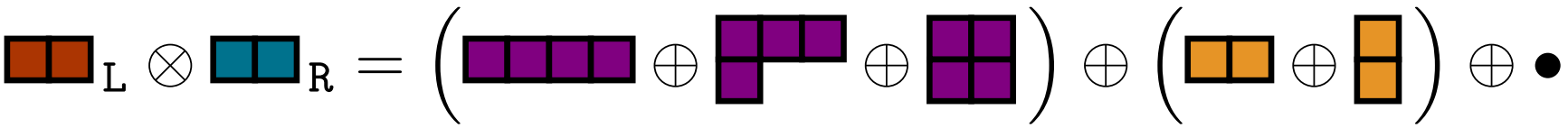
- physicality conditions

  1. Virasoro constraints: $\mathfrak{sp}_{\mathrm{L}}$ $\mathfrak{sp}_{\mathrm{R}}$

$$L_1 \,|\mathrm{phys}\rangle = 0 = \bar{L}_1 \,|\mathrm{phys}\rangle$$
$$L_2 \,|\mathrm{phys}\rangle = 0 = \bar{L}_2 \,|\mathrm{phys}\rangle$$

  2. level–matching: $\displaystyle\sum_{k\geq 1} k\, T^k{}_k = \sum_{k\geq 1} k\, \bar{T}^k{}_k$

- e.g. $N = 2 = \bar{N}$ (use Littlewood–Richardson rule):



$$|\text{spin–2}\rangle = \varepsilon_{\mu\nu}\,\Big(T^{11}\bar{\alpha}^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} + \bar{T}^{11}\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} - 2(D-1)\,\tau^{11}\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1}\Big)\,|0\rangle$$

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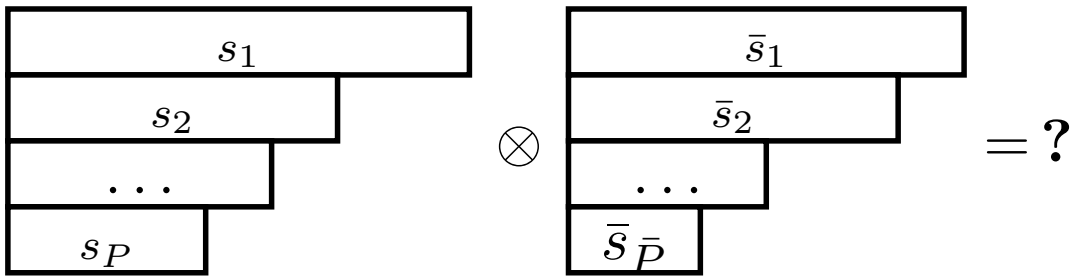
A bigger symmetry

- *trajectory* tensor product?

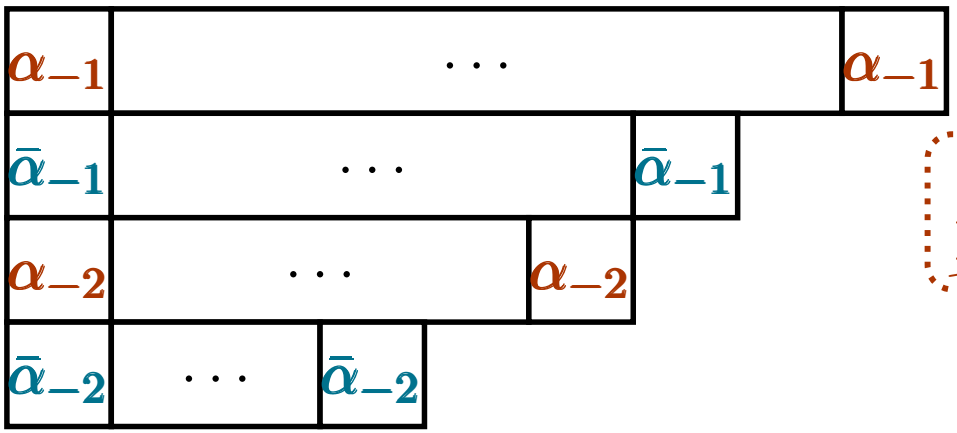


∄ general formula due to *complexity*

- now $\mathfrak{sp}(2\boldsymbol{K} + 2\bar{\boldsymbol{K}})$ generated by $\{T, \bar{T}, \tau\}$!

lowest weight states:

$$\tau^{m\,n} := \frac{1}{m\,n}\,\alpha_{-m}\cdot\bar{\alpha}_{-n}\,, \quad \tau_{m\,n} := \alpha_m\cdot\bar{\alpha}_n\,,$$
$$\tau^{m}{}_{n} := \frac{1}{m}\,\alpha_{-m}\cdot\bar{\alpha}_n\,, \quad \tau_{m}{}^{n} := \frac{1}{n}\,\alpha_m\cdot\bar{\alpha}_{-n}$$



generically ~~level-matching~~!

T. Basile, CM '26

Idea: a new "tiling"

- juxtapose to superstring technology of T. Basile, CM '24 :
  - similarities:
    1. two types of oscillators, $(\alpha, \bar{\alpha})$ or (α, b)
    2. $\mathfrak{sp}(2K + 2\bar{K})$ or Ramond $\mathfrak{osp}$: lowest weight states not sufficient for $w = 0$ trajectories
  - disparity: $\nexists$ mixed generators τ in physicality constraints

$$\Downarrow$$

need new, alternative approach . . .

- realisation:

all $\mathfrak{sp}_{\mathrm{L}}$ (or $\mathfrak{sp}_{\mathrm{R}}$) lowest weight states

pass half the closed string Virasoro

$$F_{\mathrm{L}}^{w=0} = \varepsilon_{\mu(s_1),\nu(s_2),\ldots,\lambda(s_P)}\, \alpha_{-1}^{\mu(s_1)} \alpha_{-2}^{\nu(s_2)} \ldots \alpha_{-P}^{\lambda(s_P)}$$

T. Basile, CM '26

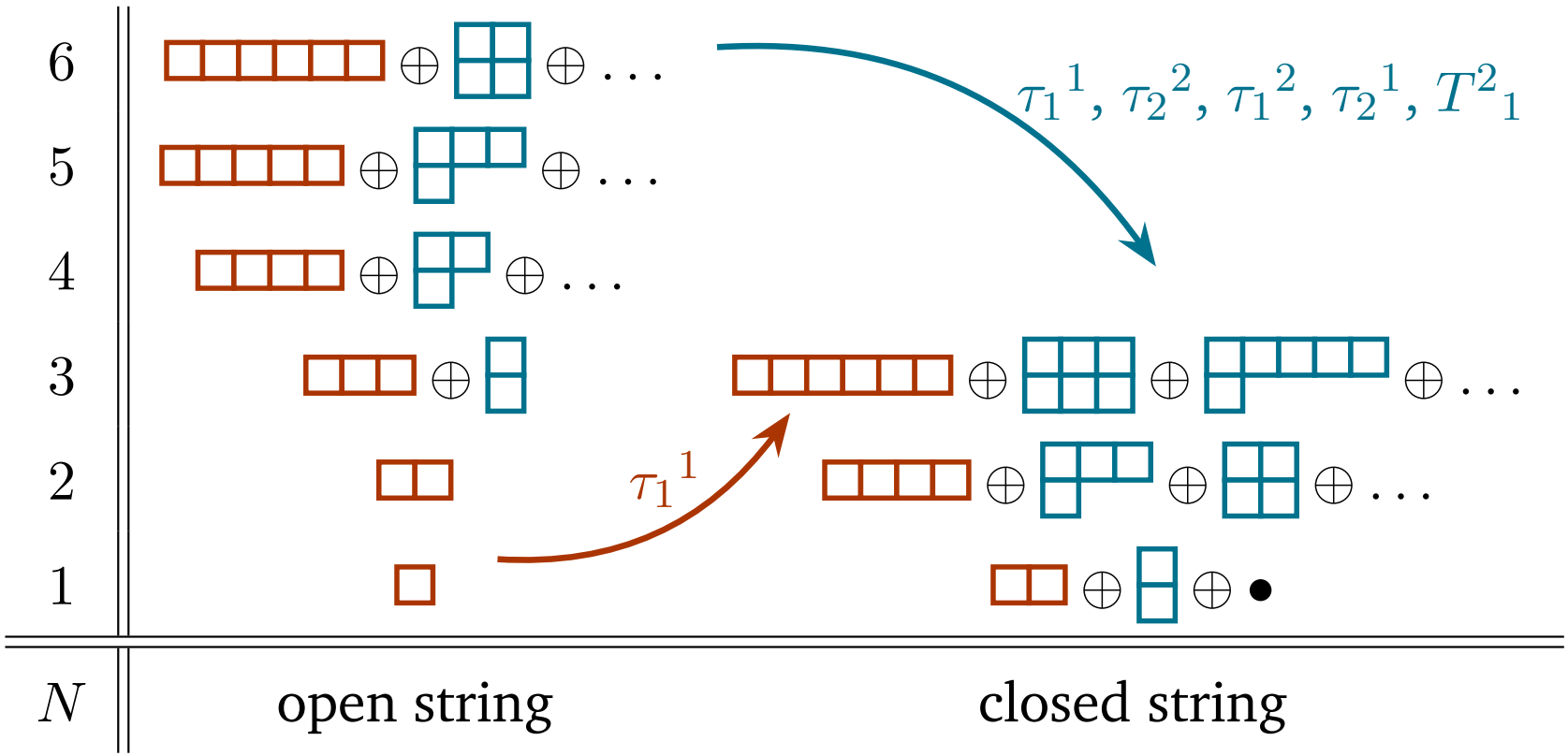
Idea: a new "tiling"

- **idea: dress open string trajectories**

  with $T^{k>\ell}{}_{\ell}\,, T^{k\ell}\,, \bar{T}^{k>\ell}{}_{\ell}\,, \bar{T}^{k\ell}\,, \tau^{k\ell}\,, \tau_{\ell}{}^{k}$

- example: *first* diagram apparitions

and impose Virasoro & level–matching



T. Basile, CM '26

What about interactions?

Reminder: basics of string scattering

- general string scattering amplitude:

$$\mathcal{A}_N = \text{worldsheet integral of } \langle : V_1 : \ \ldots \ : V_N : \rangle$$

- universal Koba–Nielsen factor: e.g. open string

$$\mathcal{E}_N := \left\langle \prod_{i=1}^{N} : e^{ip_i \cdot X_i} : \right\rangle_{\mathbb{D}_2} = \prod_{i<j}^{N} |z_{ij}|^{2\alpha' \, p_i \cdot p_j}$$

  example: Veneziano = worldsheet integral of $\mathcal{E}_4$

- (open string) massless vector $\Rightarrow$ Yang–Mills

Neveu, Scherk 1971, ...

- (closed string) massless spin–2 $\Rightarrow$ Einstein–Hilbert

Scherk, Schwarz 1974, Yoneya 1974, ...

Massive string scattering

- not a lot was/is known
- example: $\exists$ lightest massive spin–2 ($\in$ subleading Regge)

$$V^{(-1,-1)}(z,\bar{z},\alpha,k) = e^{-\phi(z)-\phi(\bar{z})}\,\alpha_{\mu\nu}\,\psi^{\mu}(z)\mathcal{J}(z)\,\widetilde{\psi}^{\nu}(\bar{z})\widetilde{\mathcal{J}}(\bar{z})\,e^{ik\cdot X(z,\bar{z})}$$

that reproduces *bimetric vertices* (3–points, non–abelian $\mathcal{J}$)

Lüst, CM, Mazloumi, Stieberger '21, '23

[bimetric theory $\sim$ interacting graviton and massive spin–2 in Minkowski]

de Rham, Gabadadze, Tolley '10, '11, Hassan, Rosen '11

- entire *trajectories*? leading Regge $\Rightarrow$ $\mathcal{A}_3$ & $\mathcal{A}_4$ known

Sagnotti, Taronna '10, Schlotterer '10

what about deeper trajectories?

Scattering trajectories

- example: leading Regge

$$\varepsilon_{\mu(s)}\partial X^{\mu_1}\dots\partial X^{\mu_s} \quad \Rightarrow \quad (\varepsilon\cdot\partial X)^s \quad \Rightarrow \quad \left(\varepsilon\cdot\frac{\partial}{\partial\xi}\right)^s e^{\xi\cdot\partial X}\Big|_{\xi=0}$$

e.g. Kawai, Lewellen and Tye '85

- for subleading trajectories: need ε_I and $\xi^{(n)}$

$$V = \text{differential operator acting on } e^{i\,p\cdot X+\xi^{(1)}\cdot\partial X+\xi^{(2)}\cdot\partial^2 X+\dots}$$

- to calculate any $\mathcal{A}_N^{tree}$: act on the "*generalized*" Koba–Nielsen

$$\mathcal{Z}_N := \Big\langle \prod_i^N : \exp\left(\sum_{n_i=0}^{\infty}\partial^{n_i}X_i\cdot\xi_i^{(n_i)}\right) : \Big\rangle$$

with the external legs' differential operators

CM, Skvortsov '23

Scattering trajectories

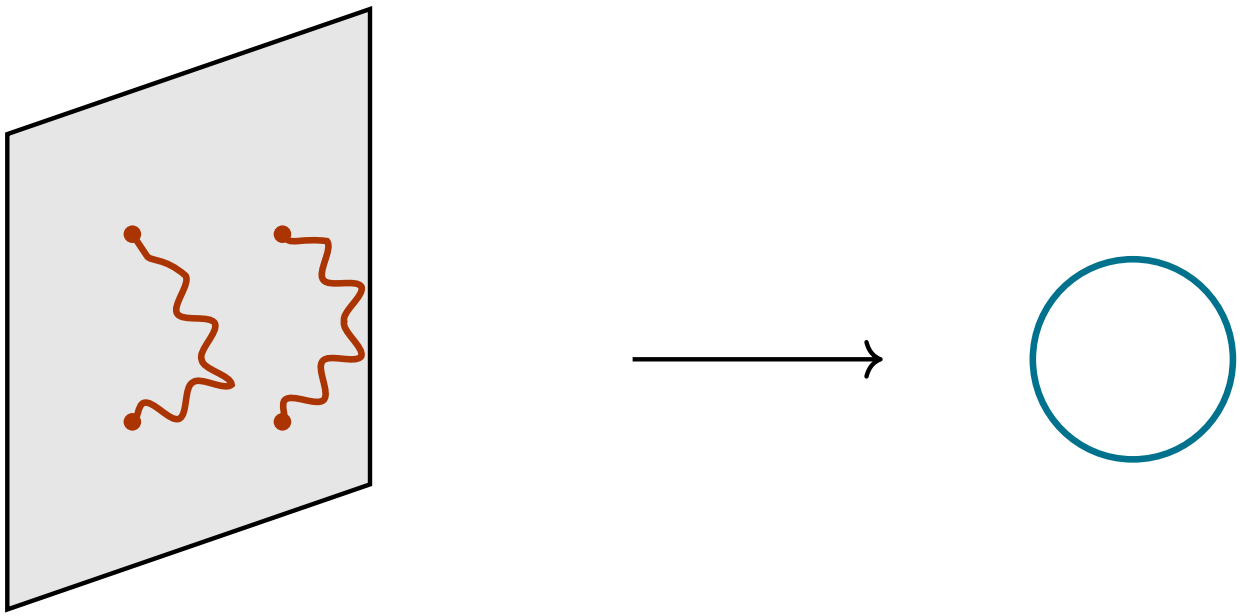
- example: spin s and two tachyons (open string)

$$\overline{\mathcal{A}}_3^{w=2} = (-D + 4s + 31)\overline{\mathcal{A}}_3^{w=0} \quad \text{where} \quad \overline{\mathcal{A}}_3^{w=0} = (\epsilon \cdot p_2)^s$$

CM, Skvortsov '23

$\Rightarrow$ can we predict the 3–point couplings *without* acting on $\mathcal{Z}_N$?

- not all 3–point amplitudes are "trivial":



Mixed amplitudes and black holes

1. spacetime–filling brane: genuine 3–point amplitude
   - graviton and light massive spin-2

     Lüst, CM, Mazloumi, Stieberger '21
   - graviton and *leading* Regge

     Bossard, Casagrande, CM *to appear*

2. non spacetime–filling brane $\Rightarrow$ amplitude looks like $\mathcal{A}_4$!

   $\Rightarrow$ toy model for Hawking radiation emission
   - graviton and gluons

$$\mathcal{A}^{000} = \frac{\Gamma(-2t)}{\Gamma(1-t)^2} \times \text{kinematic factor}$$

     Hashimoto, Klebanov 1996
   - graviton and *(sub)leading* Regge

     Juliano, Leone, CM *to appear*

Conclusions

- ongoing research programme:
  - $\mathfrak{sp}(2K + 2\bar{K})$ related to spectrum algebra of Gaberdiel, West 2002 ?
  - BCJ double copy and KLT from open string "tiling"?
  - black hole modelling from deep string scattering
  - technology around other vacua
  - spectrum & amplitudes of tensionless strings? see also Delfante, CM '26

How well do we understand the underlying principles of string theory?

The key to some of its deep mysteries may lie in its **spectrum**