Analytical estimation of thickness of photon rings around effective quantum gravity blackhole

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Overview

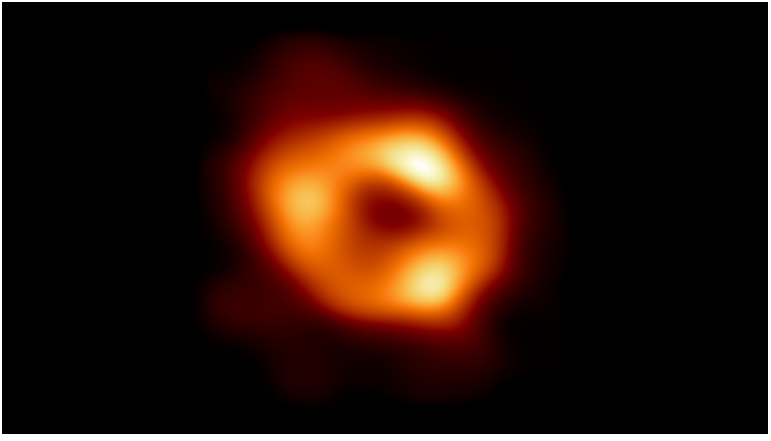


Figure: Image of the blackhole in the milky way galaxy.

Introduction

- Gravitational lensing constitutes one of the most compelling observational manifestations of General Relativity, offering a unique avenue to probe the underlying spacetime geometry in the vicinity of compact objects.
- blackhole emerging from effective quantum gravity frameworks exhibit additional spacetime parameters beyond mass, charge, and angular momentum, potentially imprinting distinctive signatures on gravitational lensing observables.
- In this work, we investigate quantum-gravity-induced modifications to the angular thickness of higher-order images produced by an extended, luminous annular accretion disk surrounding a quantum-corrected blackhole.

Quantum modified Blackhole

- We adopt a quantum-corrected blackhole spacetime that preserves general covariance within the framework of spherically symmetric gravitational systems, described by the general line element:

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + C(r)\,d\Omega^2. \tag{1}$$

$$A(r) = 1 - \frac{2M}{r} + \zeta^2 \frac{M^2}{r^2}\left(1 - \frac{2M}{r}\right), \quad B(r) = A(r)^{-1}, \quad C(r) = r^2. \tag{2}$$

- The parameter ζ characterizes the quantum-gravity corrections to the classical Schwarzschild geometry and is defined as $\zeta_M = \sqrt{4\sqrt{3}\pi\gamma^3 l_p^2/M}$, where, γ denotes the Barbero–Immirzi parameter and l_p represents the Planck length.

- This quantum-corrected metric provides a distinctive geometric framework for exploring departures from classical blackhole predictions.

Experimental setup

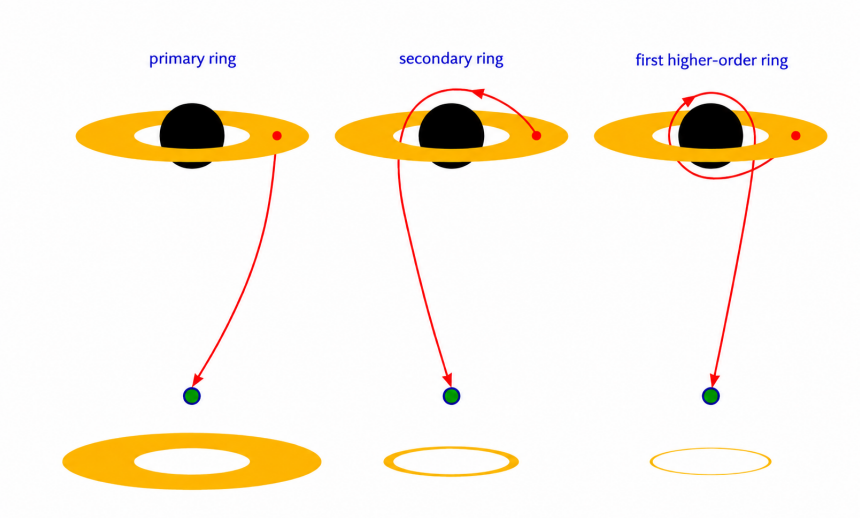


Figure: Representation of the primary, secondary and higher order images of the accretion disk.

Strong deflection limit

- In the strong-field regime, photons executing successive revolutions around a blackhole give rise to a sequence of higher-order relativistic images, encoding the intricate structure of the surrounding spacetime.
- We employ the Bozza–Scarpetta formalism to analytically determine the deflection angles of photons originating from arbitrary locations.
- This formalism yields an analytical expression for the deflection angle, exhibiting a logarithmic divergence near the photon sphere given as

$$\Delta\phi = -\bar{a}\log[\frac{\epsilon}{\eta_o\eta_s}] + \bar{b} + \pi \tag{3}$$

where,

$$\epsilon = \frac{u}{u_m} - 1 \tag{4}$$

$$\bar{a} = r_m \sqrt{\frac{B_m}{A_m \beta_m}}, \qquad \bar{b} = \bar{a} \log\left(\frac{2\beta_m}{b_m^2}\right) + b_O + b_S - \pi,$$

$$\beta_m = \frac{1}{2} r_m^2 \frac{C_m'' A_m - A_m'' C_m}{A_m^2},$$

$$\eta_O = 1 - \frac{r_m}{r_O}, \qquad \eta_S = 1 - \frac{r_m}{r_S}, \qquad \eta = 1 - \frac{r_m}{r},$$

$$b_i = \int_0^{\eta_i} g_1(\eta)\, d\eta = \int_0^{\eta_i} \mathrm{Sign}(\eta)\, g_1(\eta)\, d\eta,$$

$$g_1(\eta) = u_m \sqrt{\frac{B(\eta)}{C(\eta)}}\, [R(\eta, u_m)]^{-1/2} \frac{r_m}{(1-\eta)^2} - \frac{u_m}{\sqrt{\beta_m}} \sqrt{\frac{B_m}{C_m}} \frac{r_m}{|\eta|},$$

$$R(r, u) = \frac{C(r)}{A(r)} - u^2.$$

Strong Gravitational Lensing Results

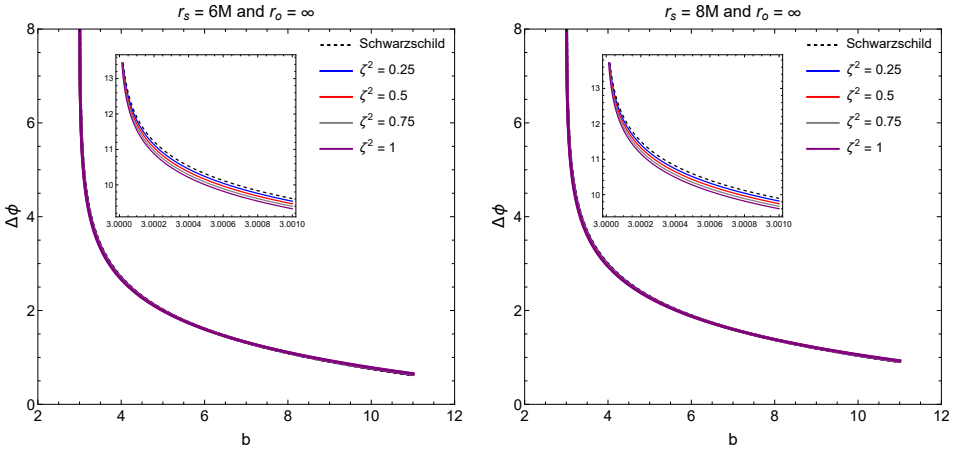


Figure: Strong deflection angle of light with source distance $r_s = 6M$ (left) and $r_s = 8M$ (right).

Photon Ring Thickness

- The angular thickness of a photon ring is determined by the separation between the image positions of photons originating from the inner and outer boundaries of the accretion disk.

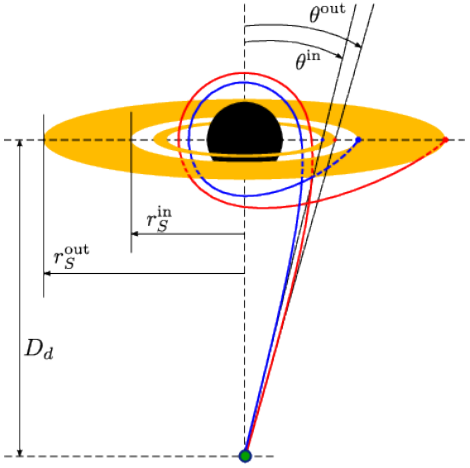


Figure: Graphical representation of the $n = 2$ photon ring of an annular accretion disk around a BH.

Formulation of ring thickness

- The corresponding image positions are obtained by inverting the deflection angle relation to determine the impact parameter of the respective photon trajectories.
- Finally, for a distant observer located at a distance D_d, the angular positions of the corresponding images are determined through the relation

$$\theta = u/D_d \tag{5}$$

- For the n^{th}-order relativistic image, the deflection angle satisfies the condition

$$\Delta\phi = \pi\left(n + \frac{1}{2}\right) \tag{6}$$

- Inverting the deflection angle formula:

$$\Delta\phi = -\bar{a}\log\left[\frac{u}{u_m} - 1\right] + \log\left[f(r_s)\right] \tag{7}$$

$$u = u_m\left[1 + f(r_s)e^{-\Delta\phi/\bar{a}}\right]. \tag{8}$$

- The function $f(r_s)$ is given by

$$f(r_s) = \frac{6(27 + 2\zeta^2)(r_s - 3)}{r_s(27 + \zeta^2)} e^{-2\bar{a}} e^{b_o + b_s}. \tag{9}$$

- The correction to regular path contributions are given as

$$b_o + b_s = 0.316 + 2\log\left[\frac{2\sqrt{\frac{3r_s}{6+r_s}}}{1+\sqrt{\frac{3r_s}{6+r_s}}}\right] + \frac{\zeta^2}{1620}\left[54 + 120\log\left(\frac{1+\sqrt{\frac{3r_s}{6+r_s}}}{2\sqrt{\frac{3r_s}{6+r_s}}}\right) + \frac{\sqrt{3}\left(3-\frac{15r_s}{6+r_s}\right)}{\left(\frac{r_s}{6+r_s}\right)^{5/2}}\right]. \tag{10}$$

- Inserting the condition on deflection angle for higher order images as $\Delta\phi = \pi(n+1/2)$ gives:

$$u = u_m\left[1 + f(r_s)e^{-\bar{a}}e^{-\pi(n+1/2)}\right] \tag{11}$$

- At last, the definition of the image angle gives:

$$\theta = \frac{u}{D_d} = \frac{u_m}{D_d}\left[1 + f(r_s)e^{-\bar{a}}e^{-\pi(n+1/2)}\right]. \tag{12}$$

- By considering the image angles of photons originating from the inner and outer edges of the accretion disk, denoted by $\theta_{\rm in}$ and $\theta_{\rm out}$, respectively, we can define the angular thickness of the $n^{\rm th}$-order ring as:

$$\begin{aligned}\Delta\theta_n &= \theta_{\rm out} - \theta_{\rm in} \\ &= \frac{u_m}{D_d}\left[f(r_s^{\rm out}) - f(r_s^{\rm in})\right]e^{-\bar{a}}e^{-\pi(n+1/2)}.\end{aligned} \tag{13}$$

Results

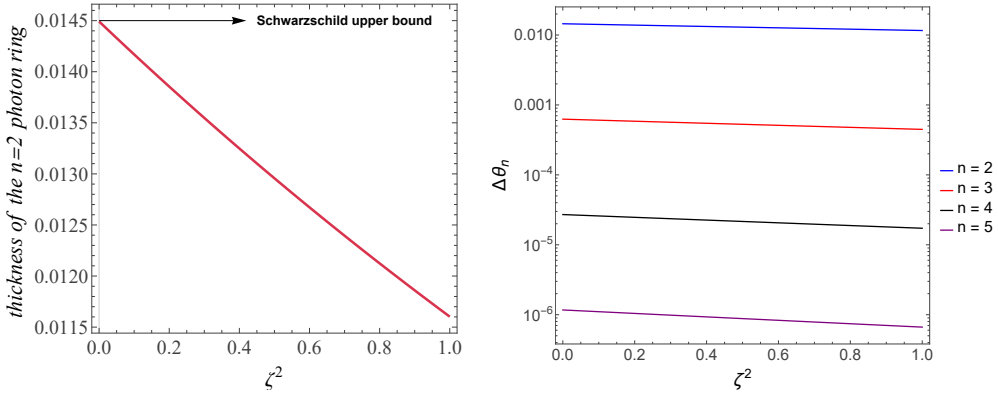


Figure: Quantum correction to $n = 2$ and other higher order photon rings of accretion disk around Schwarzschild BH. We have taken $r_s^{in} = 6M$ and $r_s^{out} = 30M$ and plotted the ring thickness against he quantum parameter ζ^2.

conclusion

- We have analytically derived the angular thickness of photon rings in a quantum-corrected spacetime and investigated its dependence on the quantum parameter.
- We initiated our analysis using the Strong Deflection Limit (SDL) for sources at different locations and observe only marginal quantum-induced corrections to the deflection angle.
- Subsequently, we mapped the angular positions of the inner and outer edges of the accretion disk by determining the corresponding impact parameters of the emitted light rays.
- Despite negligible corrections to the deflection angle, we identify appreciable quantum-induced modifications in the angular thickness of photon rings.
- Future high-resolution observations of blackhole photon rings may reveal subtle corrections to their thickness, providing a promising avenue for constraining quantum parameters and testing the viability of quantum-corrected blackhole theories.

References

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Thank You!