Physics Academy of the North East 2026

Testing ΛCDM Mimicking $f(Q)$ Gravity With Inspiral Gravitational Wave Signals

Sikha Gogoi

Department of Physics, Dibrugarh University
Dibrugarh, Assam, 786004

sikhagogoi001@gmail.com

Outline

Introduction

Theoretical Framework: $f(Q)$ Gravity

Modified Propagation of Gravitational Waves

Parameter Estimation: Fisher Analysis Technique

Summary and Conclusion

References

Introduction

- Despite the remarkable success of General Relativity (GR), it has certain unresolved issues, including the cosmic acceleration, cosmological constant problem, Hubble tension, etc.
- This has led to the development of various alternative and modified theories of gravity, which explore different geometrical and dynamical descriptions of gravitational interactions.
- Different theories of gravity can predict modifications in the generation and propagation of GWs, potentially leaving observable signatures in their waveforms.
- Gravitational waves (GWs) provide a new and independent avenue for testing the underlying theory of gravity.

Theoretical Framework: $f(Q)$ Gravity

- $f(Q)$ gravity theory is an alternative theory of gravity (ATGs).
- Gravity is described by non-metricity of flat torsion free spacetime.
- Action is given by,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(Q) + \mathcal{L}_m \right]. \tag{1}$$

- This theory is based on **Symmetric teleparallelism**.
- Q is the non-metricity scalar.

Theoretical Framework: $f(Q)$ Gravity

- We consider the FLRW metric to describe the cosmological background, as given by

$$ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \tag{2}$$

- The non-metricity scalar Q corresponding to the FLRW metric in coincident gauge is releted to Hubble parameter as

$$Q = -6H^2. \tag{3}$$

Theoretical Framework: $f(Q)$ Gravity

- We consider a specific $f(Q)$ gravity model that gives a background evolution similar to that of GR, and is given by,

$$f(Q) = -2\Lambda + Q + \beta\sqrt{-Q}. \tag{4}$$

- β is the free model parameter that characterizes the deviation from GR.
- When $\beta = 0$, the theory reduces to STEGR, based solely on non-metricity scalar Q.
- In this particular gravity model, the Hubble parameter is similar to that of ΛCDM background,

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})}. \tag{5}$$

Modified Propagation of Gravitational Waves

- At the cosmological level, the evolution of tensor perturbation gets modified within the framework of $f(Q)$ gravity theory.
- The modified wave equation on cosmological background is

$$\tilde{h}_\lambda'' + 2\mathcal{H}\left(1 + \frac{d\ln f_Q}{2\mathcal{H}d\tau}\right)\tilde{h}_\lambda' + k^2\tilde{h}_\lambda = 0. \tag{6}$$

- The term $\delta(\tau) = -\frac{1}{2\mathcal{H}}\frac{d\ln f_Q}{d\tau}$ modifies the friction term, because of which the amplitude damping of GWs differs from that of GR.
- In terms of cosmological redshift z, it can be expressed as

$$\delta(z) = \frac{1}{2}(1+z)\frac{d}{dz}\ln\left(1 - \frac{\beta}{2\sqrt{6}H(z)}\right). \tag{7}$$

Modified Propagation of Gravitational Waves

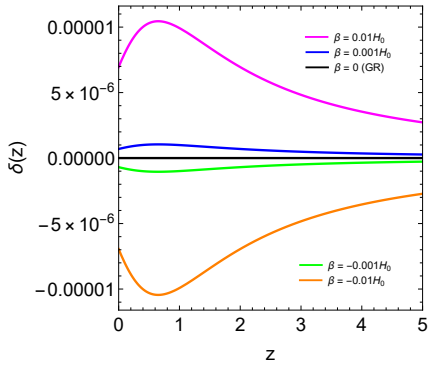

Figure: Variation of modified friction $\delta(z)$ with redshift z.

Key Observations

- The deviation from GR depends on the model parameter β.
- For $\beta = 0$, we recover the STEGR limit.
- At very low ($z \ll 1$) and very high redshifts, the deviation is small, and the theory approaches GR limit.
- The deviation is maximum at intermediate redshifts ($z \sim 0.5$–1).

Modified Propagation of Gravitational Waves

- The luminosity distance for GWs is related to the modified friction term as

$$d_{\rm L}^{\rm GW}(z) = d_{\rm L}^{\rm EM} \exp\Big[- \int_0^z \frac{\delta(z')}{1+z'} dz' \Big], \tag{8}$$

with,

$$d_{\rm L}^{\rm EM}(z) = (1+z) \int_0^z \frac{dz'}{H(z')}. \tag{9}$$

- As a consequence, the luminosity distance for GWs ($d_{\rm L}^{\rm GW}$) is not equal to that for EM radiation ($d_{\rm L}^{\rm EM}$).

Modified Propagation of Gravitational Waves

- The ratio of the luminosity distance for GWs to that for EM radiation for the $f(Q)$ model can be expressed as,

$$\frac{d_{\mathrm{L}}^{\mathrm{GW}}}{d_{\mathrm{L}}^{\mathrm{EM}}} = \sqrt{\frac{f_Q(0)}{f_Q(z)}} = \sqrt{\frac{1-\beta/2\sqrt{6}H_0}{1-\beta/2\sqrt{6}H(z)}}. \tag{10}$$

Modified Propagation of Gravitational Waves

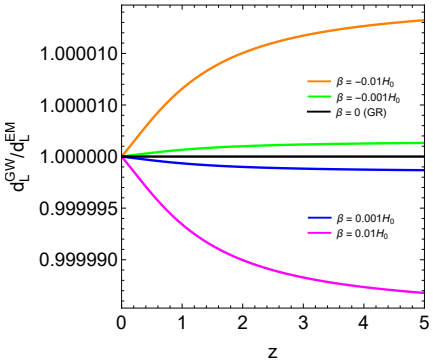

Figure: Luminosity distance ratio d_L^{GW}/d_L^{EM} as a function of redshift z.

Key Observations

- For $\beta > 0$ and hence for $\delta > 0$, $d_L^{GW} < d_L^{EM}$
- For $\beta < 0$ and hence for $\delta < 0$, $d_L^{GW} > d_L^{EM}$
- Since the amplitude of GWs is proportional to $1/d_L^{GW}$, therefore $\beta > 0$ (< 0) induces an increase (decrease) in GW amplitude.
- At $z \ll 1$, $d_L^{GW} \approx d_L^{EM}$

Parameter Estimation: Fisher Analysis Technique

- The bounds on the model parameter β are obtained using GW signals from different binary coalescence sources such as BBH, BNS, BBHNS.
- For this purpose, we employ Fisher analysis technique.
- If $\tilde{h}(\upsilon, \theta_i)$ is the GW, where υ is the frequency and θ_i are the free model parameters, then the root mean squared error on a parameter is,

$$\Delta\theta^i = \sqrt{\Gamma^{-1}_{ii}}, \tag{11}$$

Fisher Analysis

- Γ_{ij} is the Fisher matrix defined as,

$$\Gamma_{ij} = \left(\frac{\partial \tilde{h}}{\partial \theta_i} \middle| \frac{\partial \tilde{h}}{\partial \theta_j} \right). \tag{12}$$

- The notation $(a|b)$ denotes the inner product of two vectors a and b, defined using the detector's power spectral density $S_n(\upsilon)$ as

$$(a|b) := 2 \int_{\upsilon_{\text{low}}}^{\upsilon_{\text{high}}} \frac{a^* b + b^* a}{S_n(\upsilon)} \, d\upsilon, \tag{13}$$

Gravitational Waveform

- The inspiral phase of compact binary coalescences can be described within the framework of post-Newtonian (PN) formalism.
- We consider the TaylorF2 PN waveform with correction upto 3.5PN order, to describe the inspiral phase of compact binary coalescences with aligned spins as

$$\tilde{h}(v) = A v^{-7/6} \exp\left[i\left(2\pi v t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta v^5}\left(1 + \sum_{i=2}^{7} \alpha_i v^i\right)\right)\right], \tag{14}$$

where the amplitude A is given by

$$A \propto \mathcal{M}_c^{5/6} / d_L^{\mathrm{GW}}, \tag{15}$$

Gravitational Waveform

Definitions

- α's are functions of spins χ_1 and χ_2 of the members of the binaries
- m_1, m_2: component masses of the binary
- $M = m_1 + m_2$: **total mass** of the system
- $\eta = \dfrac{m_1 m_2}{M^2}$: **symmetric mass ratio**
- $\mathcal{M}_c = (1+z)\, M\, \eta^{3/5}$: **redshifted chirp mass**
- $v = (\pi M \upsilon)^{1/3}$: **inspiral reduced frequency**

Free parameters:

$$\theta_i = (M,\, \eta,\, \chi_1,\, \chi_2,\, t_c,\, \phi_c,\, \beta/H_0)$$

Sensitivity of Einstein Telescope

- Einstein Telescope (ET) is a third-generation ground-based GW detector.
- ET is expected to cover the frequency band $1–10^4$ Hz.
- ET has improved amplitude sensitivity about an order of magnitude compared to current detectors.

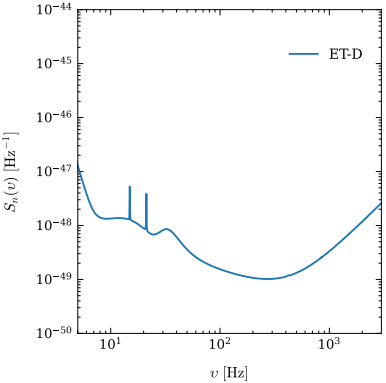

Figure: Power spectral density (PSD) of Einstein Telescope as a function of frequency.

Simulation of GW Events

- In our simulation, we consider three types of compact binary systems:

  1) **Binary black holes** (BBH) with uniform mass distribution in the range $[10-30]M_{\odot}$, with $m_1 \gtrsim m_2$ and $\eta < 0.25$ and uniform distribution of spin magnitudes in the range $[-1, 1]$.

  2) **Binary neutron stars** (BNS) with uniform mass distribution in the range $[1, 2]M_{\odot}$, with $m_1 \gtrsim m_2$ and $\eta < 0.25$. In our simulation, we consider $\chi_1 = \chi_2 = 0$.

  3) **Black hole neutron star binaries** (BBHNS), where the BH masses are chosen to be uniform in the range $[10-30]M_{\odot}$ with χ_{BH} in the range $[-1, 1]$ and the neutron star mass range is chosen to be $[1, 2]M_{\odot}$ with $\chi_{\text{NS}} = 0$.

Simulation of GW Events

- The redshift distribution function is considered to be of the form,

$$P(z) \propto \frac{4\pi d_c^2(z)\, r(z)}{H(z)(1+z)}, \tag{16}$$

where $d_c(z) \equiv \int_0^z \frac{dz'}{H(z')}$ is the comoving distance between the source and detector, and $r(z)$ describes the time evolution of the burst rate, defined as

$$r(z) = \begin{cases} 1+2z, & z \leq 1, \\ \frac{3}{4}(5-z), & 1 < z \leq 5, \\ 0, & z > 5. \end{cases} \tag{17}$$

Signal to Noise ratio

- For the simulated events we calculate the signal-to-noise ratio (SNR) using the formula,

$$\rho^2 := (\tilde{h}|\tilde{h}) = 4\,\mathrm{Re}\int_{\upsilon_{\mathrm{low}}}^{\upsilon_{\mathrm{high}}} \frac{|\tilde{h}(\upsilon)|^2}{S_n(\upsilon)}\,d\upsilon. \tag{18}$$

- Since ET is composed of three independent interferometers, the combined SNR for the network of these independent interferometers is given by

$$\rho = \sqrt{\sum_{j=1}^{3} \left[\rho^{(j)}\right]^2}. \tag{19}$$

Signal to Noise Ratio

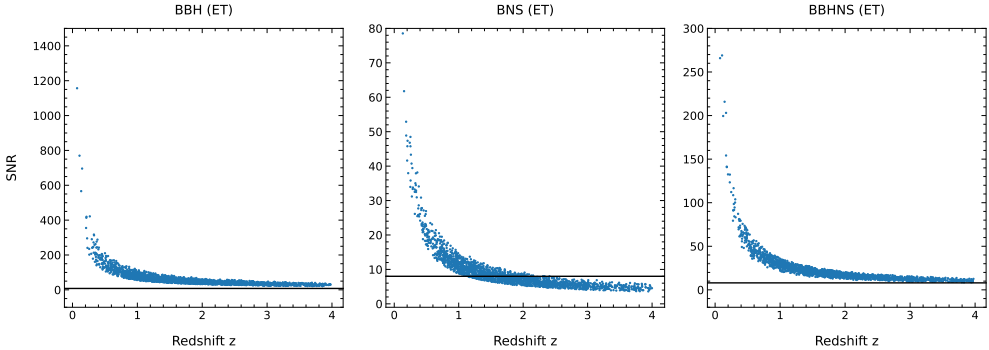

Figure: Variation of signal-to-noise ratio with redshift z for several simulated BBH, BNS and BBHNS events as observed by Einstein Telescope. The horizontal black line represents the detection threshold SNR = 8.

Signal to Noise Ratio

Key Observations

- In this work, we restricted our analysis upto $z = 4$.
- BBH and BBHNS events can be detected upto highest redshift value considered here with appreciable SNR.
- In case of BNS events, ET is sensitive upto $z \sim 1.5$, beyond which the SNR drops below the typical detection threshold ($\rho = 8$).

Parameter Estimation for Einstein Telescope

- Following the calculation of SNR, we investigate the parameter estimation capability of ET.
- We simulated about $O(10^4)$ compact binary events for each class of source.
- Using Fisher analysis we obtain the 1σ uncertainties in the model parameter β/H_0 as follows:
  - For BBH, ET can constrain β/H_0 upto $O(10^{-4})$.
  - For BNS, ET can constrain β/H_0 upto $O(10^{-3})$.
  - For BBHNS, ET can constrain β/H_0 upto $O(10^{-3})$.

Parameter Estimation for Einstein Telescope

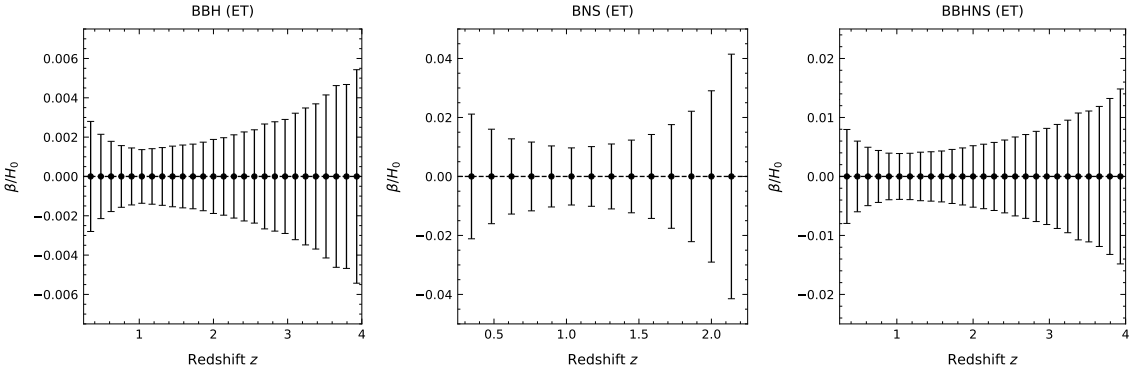

Figure: Constraints on β/H_0 obtained by simulating several BBH (left), BNS (middle), and BBHNS (right) events in each redshift bin, from the perspective of ET power spectral density sensibility.

Parameter Estimation for Einstein Telescope

Key Observations

- For BBH and BBHNS, the parameter is well constrained at the intermediate redshift ($z \sim 0.5 - 2$).
- The upgradation in uncertainties at low redshifts ($z << 1$) is due to the small number of detected events in that region.
- At the high redshift regions, the constraints worsen primarily due to reduction in SNR values.
- Overall, the constraints for BBHNS are weaker in comparison to BBH systems.
- BNS systems have the weakest constraint as compared to BBH and BBHNS , which is due to the low SNR value.

Summary and Conclusion

- In this work we forecast constraints on the model parameter β/H_0 of a $f(Q)$ model that mimics ΛCDM background using Fisher analysis.
- We consider ET, a future third generation ground based detector, whose sensitivity is expected to be significantly better than current detectors.
- We found that, ET can constrain the model parameter β/H_0 upto order $\mathcal{O}(10^{-4})$.
- Overall our results show the potential of future GW detector to test ATGs with high precision.

References

- S. Gogoi, U. D. Goswami, *Probing ΛCDM-mimicking $f(Q)$ gravity model using gravitational waves from compact binary coalescences*, Eur. Phys. J. C **86**, 1291 (2026).
- L. Heisenberg, *Review on $f(Q)$ Gravity*, Phys. Rept. **1066**, 1 (2024).
- S. Chakraborty *et al.*, *Reproducing ΛCDM-like solutions in f(Q) gravity: a comprehensive study across all connection branches*, JCAP **05**, 098 (2025).
- J. B. Jiménez *et al.*, *Cosmology in f(Q) geometry*, Phys. Rev. D **101**, 103507 (2020).
- E. Belgacem, Y. Dirian, S. Foffa, and M. Maggiore, *Modified gravitational-wave propagation and standard sirens*, Phys. Rev. D **98**, 023510 (2018).

References

- C. Cutler and E. Flanagan, *Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral waveform?*, Phys. Rev. D **49**, 2658 (1994).
- A. H. Nitz *et al.*, *Accuracy of gravitational waveform models for observing neutron-star–black-hole binaries in Advanced LIGO*, Phys. Rev. D **88**, 124039 (2013).
- S. Hild *et al.*, *Sensitivity studies for third-generation gravitational wave observatories*, Class. Quant. Grav. **28**, 094013 (2011).
- R. C. Nunes, M. E. S. Alves and J. C.N. de Araujo, *Forecast constraints on f(T) gravity with gravitational waves from compact binary coalescences*, Phys. Rev. D **100**, 064012 (2019).

Eur. Phys. J. C (2026) 86:1291
https://doi.org/10.1140/epjc/s10052-026-16564-5

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Probing ΛCDM-mimicking $f(Q)$ gravity model using gravitational waves from compact binary coalescences

Sikha Gogoi[a], **Umananda Dev Goswami**[b]

Department of Physics, Dibrugarh University, Dibrugarh, Assam 786004, India

Received: 18 June 2026 / Accepted: 18 October 2026
© The Author(s) 2026

Abstract The direct detection of gravitational waves (GWs) is a very significant achievement in the history of physics and has opened a new window to probe the possible deviations of physics from that of general relativity (GR). In this work, we forecast constraints on the free parameter of an $f(Q)$ gravity model that mimics a ΛCDM background at the level of cosmic expansion. We consider modified gravitational wave signals from inspiraling of compact binary sys-

of prediction, on September 14, 2015, GWs were finally detected by the Laser Interferometer Gravitational Observatory (LIGO) system, which for the first time detected a GW signal, named as GW150914, from the merger of two black holes [2]. This event is marked as a historical achievement in science in general and physics in particular. Another remarkable achievement in support of GR is the first and

Physics Academy of the North East 2026

Thank You!

Sikha Gogoi
sikhagogoi001@gmail.com