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Effects of Dark Matter Distribution on the Dynamical Motion and Evolution Around the Black Hole

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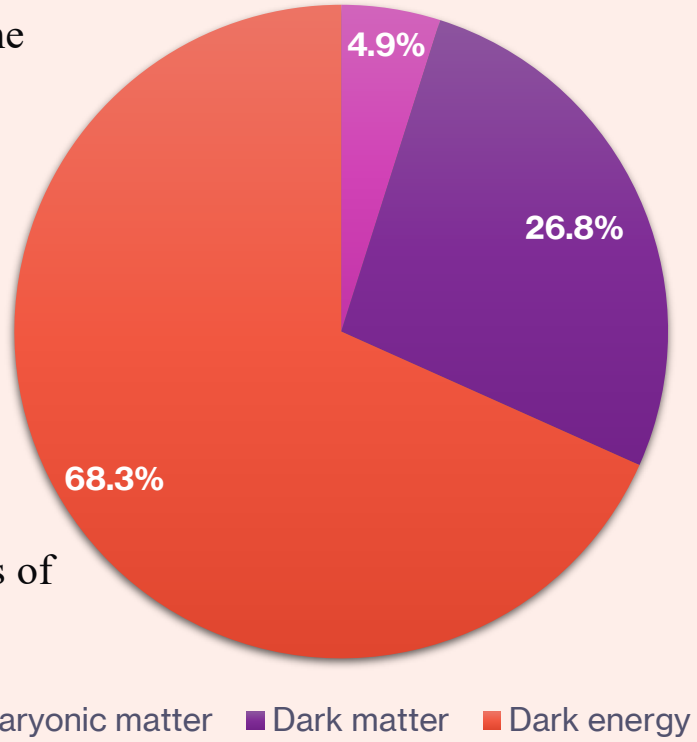
Introduction

- The universe is largely invisible to us; ordinary atoms make up only about 4.9% of everything that exists. The remaining 95% consists of two mysterious components: **Dark Matter and Dark Energy**.
- These mysterious elements of our universe significantly influence the structure of galaxies and the orbits of particles and stars in galaxies. **It acts as a gravitational framework.**
- Dark matter surrounds the supermassive black hole in many galaxies in the form of a halo structure.
- Here, in the study I will discuss the *Effects of dark matter distribution on the orbital dynamics of test particles around a* ***rotating, charged black hole*** *also called a Kerr-Newman black hole.*
- **Since Bound precession orbits and circular orbits** play a foundational role in the classical tests of gravitational theories.



e.g. **Extreme Mass Ratio Inspirals (EMRI)**

Rotating black hole spacetime in dark matter halo

- Consider spherically symmetric spacetime metric (black hole surrounded by dark matter halo)

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{G(r)} + d\Omega^2,$$

where $d\Omega^2 = h^2(d\theta^2 + \sin^2\theta\, d\phi^2)$ *with* $h^2 = r^2, F(r) = A(r) + f(r)$ *and* $G(r) = B(r) + g(r)$ *in which* $A(r)$ *and* $B(r)$*represent the metric coefficients of the pure dark matter halos and the unknown functions* $f(r)$ *and* $g(r)$ *are the blackhole and dark matter parameters to be determined.*

Using Newman-Janis Method, spherically symmetric black hole solution to convert into the rotating black hole surrounded by dark matter halo.

- *The effective space metric for a rotating black hole with dark matter halo profile yields*:

$$ds^2 = -\left(1 - \frac{r^2 - G(r)r^2)}{\rho^2}\right)dt^2 + \frac{2a\sin^2\theta\left(r^2 - G(r)r^2\right)}{\rho^2}dtd\phi + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)^2 - \Delta a^2\sin^2\theta]d\phi^2, \longrightarrow (1)$$

where $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = a^2 + G(r)r^2$ *with* $\boldsymbol{G(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$ *for the rotating charged blackhole (Kerr-Newman)*

Here, Q represents the charge of the black hole and a = L/M represents the angular momentum per unit mass of the black hole.

The Schwarchild black hole metric is restored when $\boldsymbol{a = 0, Q = 0}$ ***and*** $\boldsymbol{k = 0}$.

- *The covariant components of the metric are identified as*

$$g_{tt} = \frac{r^2 - \mathrm{G(r)r^2}}{\rho^2} - 1, \qquad g_{t\phi} = g_{\phi t} = \frac{a(r^2 - \mathrm{G(r)r^2})\sin^2\theta}{\rho^2}, \qquad g_{rr} = \frac{\rho^2}{\Delta}, g_{\theta\theta} = \rho^2,$$
$$g_{\phi\phi} = \frac{\sin^2\theta\left((a^2 + r^2)^2 - a^2\Delta\sin^2\theta\right)}{\rho^2}$$

For rotational Black Hole confined in confined to a plane, $\theta = \pi/2, \dot{\theta} = 0$

- *The radial equation of motion*

$$\dot{r}^2 + V_{\mathrm{ef}} = 0,$$

where V_{ef} *represents the effective potential of the system* which is given by

$$V_{\mathrm{ef}} = \frac{g_{t\phi}^2 - 2EL_\phi g_{t\phi} - g_{\mathrm{tt}}L_\phi^2 - g_{\phi\phi}(E^2 + g_{\mathrm{tt}})}{g_{\mathrm{rr}}(g_{t\phi}^2 - g_{\mathrm{tt}}g_{\phi\phi})}$$

Substituting covariant components of the metric into the effective potential gives:

$$V_{\mathrm{ef}} = \frac{a^2(GE^2 - 2E^2 + 1) + 2aE(G-1)L_\phi + G\left(L_\phi^2 + r^2\right) - E^2r^2}{r^2}$$

- For the Navarro–Frenk–White (NFW) Halo profile, we have

$$\boldsymbol{G_{\mathrm{NFW}}(r) = \left(1 + \frac{r}{h}\right)^{\frac{-8\pi\kappa}{r}} - \frac{2M}{r}}$$

Here, ***h*** *represents the* ***characteristic radius*** *of the dark matter halo and* ***κ*** *estimates the* ***mass of the dark matter***.

- ***Interestingly, in the limit*** $\kappa = 0$ ***and*** $h \to \infty, a \to 0$ ***and*** $Q \to 0$ ***in the above profile, the metric*** (1) ***reduces to Schwarzschild metric.***

Also

$$\lim_{r\to\infty} V_{ef} = 1 - E^2$$

Geodesic Equations for the dark matter halo profile

- The geodesic equations of motion for the test particle can be studied from the radial and azimuthal equations of motion given by

$$\dot{r}^2 = -V_{\text{ef}},$$
$$\dot{\phi} = -\frac{E g_{t\phi} + g_{tt} L_\phi}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}$$

Combining the above equations and differentiating both sides with respect to ϕ, we obtain a second-order linear differential equation

$$\frac{d^2 r(\phi)}{d\phi^2} = \frac{a^2 + r^2 G(r)}{2r^3\big(aE - G(r)(aE + L)\big)^3}(A + B + C + D),$$

where

$$A = 2\big(a^2 + r^2 G(r)\big)\big(aE - G(r)(aE + L)\big)\big(-E^2(2a^2 + r^2) + a^2 + G(r)\big((aE + L)^2 + r^2\big) - 2aEL\big),$$
$$B = -2r\big(a^2 + r^2 G(r)\big)(aE + L)G'(r)\big(-E^2(2a^2 + r^2) + a^2 + G(r)\big((aE + L)^2 + r^2\big) - 2aEL\big),$$
$$C = -2r^2\big(rG'(r) + 2G(r)\big)\big(aE - G(r)(aE + L)\big)\big(-E^2(2a^2 + r^2) + a^2 + G(r)\big((aE + L)^2 + r^2\big) - 2aEL\big),$$
$$D = -r\big(a^2 + r^2 G(r)\big)\big(aE - G(r)(aE + L)\big)\big(G'(r)\big((aE + L)^2 + r^2\big) + 2rG(r) - 2E^2 r\big)$$

Here, the prime denotes the derivative with respect to r.

Trajectory equation determines the geometry of geodesics in a plane

MBO AND ISCO

- **Marginally Bound Orbit (MBO)** ⟹ Critical threshold separating bound and unbound particle trajectories.
- At this orbit, a particle possesses exactly the energy required to escape to infinity with zero kinetic energy.
- This orbit is not stable, and any slight additional perturbation will cause the particle to either escape to infinity from the black hole's gravitational field or spiral into the black hole.
- The corresponding conditions of the **MBO** are given by

$$V_{ef} = 0, \qquad \frac{dV_{ef}}{dr} = 0,$$

- **Innermost Stable Circular Orbit (ISCO)** ⟹ Determines the inner boundary of stable circular motion.
- The condition for the **inner most stable circular orbit** is given by

$$V_{ef} = 0, \quad \frac{dV_{ef}}{dr} = 0, \quad \frac{d^2V_{ef}}{dr^2} = 0$$

- We emphasis the properties of periodic orbits embedded in different dark matter halo which satisfy the relation, $L_{ISCO} \leq L$ and $E_{ISCO} \leq E \leq E_{MBO}$

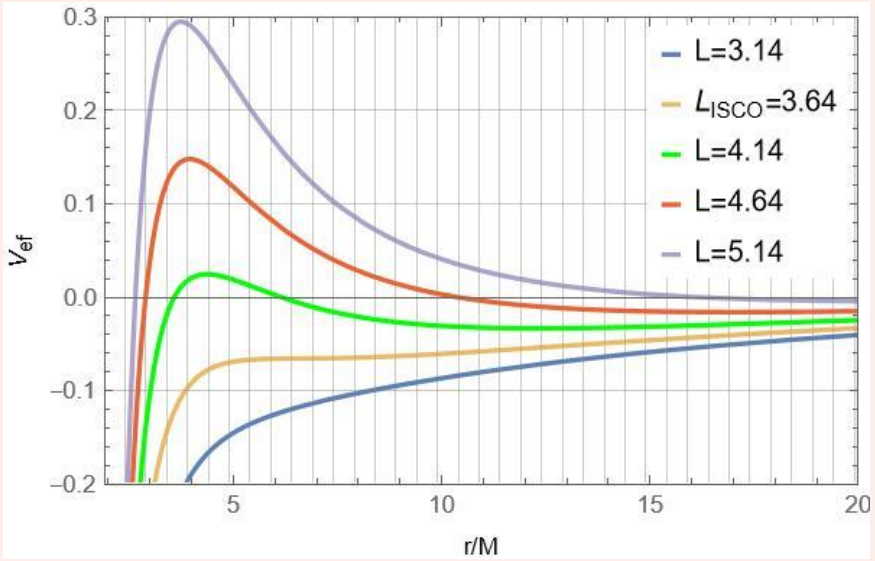


Figure 1. Plot showing relationship diagram between the effective potential V_{ef} and the radial radius r at a=0.1, Q=0.49, E=0.968, k=10^4, h=10^7

- As the orbital angular momentum decreases, the two extreme values of the effective potential gradually approach each other and finally converge at one point. This line corresponds to the innermost stable circular orbit (ISCO) of the particle's motion **(Last stable circular orbit)** as shown in Figure 1 when L_{ISCO}=3.64.
- When $E \geq 1$, there is no bound orbit and the particle can move to infinity; when $E < 1$, there is a bound orbit for the motion of the test particle.

Violations of this inequality result in either the capture of particles into black hole or the escape of particles into infinity.

Orbital periodicity and stability

- Precession and periodic orbits are important tools for studying the dynamical properties of test particles around black holes.
- Precession parameter q :

$$q = \frac{2}{\pi}\int_{r_1}^{r_2} \frac{\dot{\phi}}{\dot{r}} dr - 1,$$

where

$$\dot{\phi} = -\frac{E g_{t\phi} + g_{tt} L_\phi}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}, \quad \dot{r} = \sqrt{-\frac{g_{t\phi}^2 - 2E L_\phi g_{t\phi} - g_{\mathrm{tt}} L_\phi^2 - g_{\phi\phi}(E^2 + g_{\mathrm{tt}})}{g_{\mathrm{rr}}\left(g_{t\phi}^2 - g_{\mathrm{tt}} g_{\phi\phi}\right)}}$$

Rewrite as follows:

$$\boldsymbol{q = \frac{-2}{\pi}\int_{r_1}^{r_2} \frac{\left(E g_{t\phi} + g_{tt} L_\phi\right)\sqrt{g_{rr}}}{\sqrt{\left(-g_{t\phi}^2 + g_{tt} g_{\phi\phi}\right)\left(g_{t\phi}^2 - g_{\mathrm{tt}} g_{\phi\phi} + 2E L_\phi g_{t\phi} + g_{\mathrm{tt}} L_\phi^2 + g_{\phi\phi} E^2\right)}} dr - 1 = w + \frac{v}{z}.}$$

- The characteristics of periodic orbits around BHs done using categorization technique put forward by Janna Levin et al. (2008).
- Three numbers (z, w, v) are used in this classification approach to construct a rational parameter q.
- The numbers z, w, and v represent the **zoom number, whirl number, and vertex number** of the orbit respectively.
- When **q is an irrational number,** the test **particle does not return to its original position** after completing one orbit around the black hole.
- When ***q* is a rational number, the particle's trajectory exhibits periodicity**.

Typical natures of the effective potential

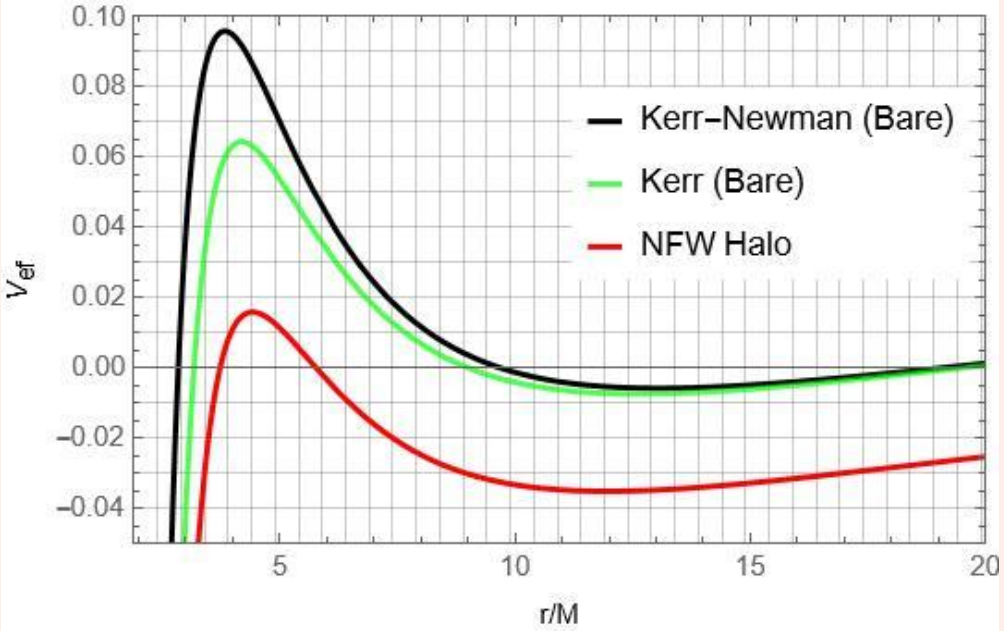


Figure 2. Plot showing the influence of dark matter on the effective potential of particles at a=0.365, L=4.1, Q=0.49, E=0.963, k=10^4, h=10^7

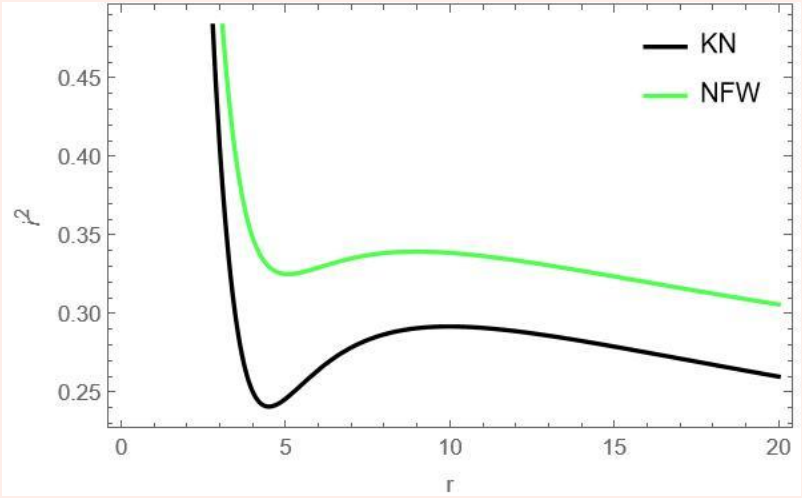


Figure 3.Relationship diagram of $\dot{r}^2$ - r at a=0.1, L=3.8, Q=0.29, E=0.963, k=10^4, h=10^7

- The Peak represents an unstable circular orbit (MBO)
- Particles located exactly at this turning point will either plunge into the black hole or escape outward if slightly perturbed.
- The Valley represents a stable circular orbit (ISCO)
- Particles remain in stable, repeating orbits around the black hole.
- Rotation alone provides a smaller potential barrier than the combined effects of rotation and charge.
- Introducing an electric charge to a rotating black hole creates an additional repulsive component in the effective potential.
- Makes the potential barrier higher and steeper, meaning incoming particles require higher energies to penetrate the barrier and plunge into the black hole.
- Dark Matter inclusion adds a massive, continuous gravitational pull.
- This broadens the region where stable circular orbits can exist, meaning the **ISCO is shifted outward.**

- The NFW curve remains above the KN curve. Thus, at a given radius, the allowed radial speed is greater for NFW profile than for Kerr-Newman.

$$(\dot{r}^2 = -V_{ef})$$

- Here, the larger radial velocity in the NFW model indicates a comparatively lower effective radial potential in the considered radial range.
- At small radial distances, $\dot{r}^2$is relatively large and decreases rapidly to a minimum around r~4-5 indicating a region of strongest effective gravitational influence whereas at larger radii, it decreases slowly.
- This non-monotonic behaviour indicates that the radial dynamics are strongly dependent on the gravitational field.
- Overall, the differences highlight how central compact objects produce sharper potential wells ,while extended halo profile smooth out the velocity variation.

Influences of the halo parameters on the periodic orbits

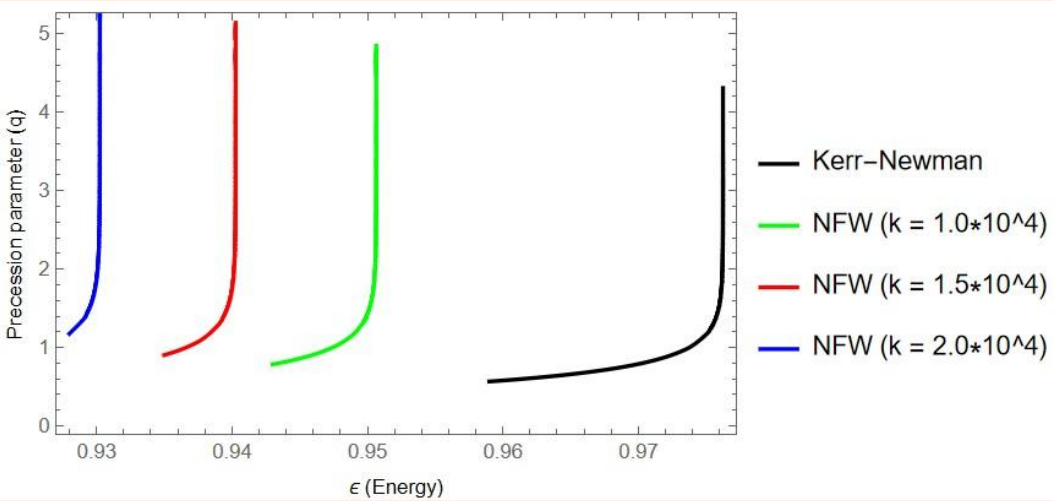


Figure 4. Precession parameter q changes with energy E, at a=0.1, Q=0.29, L=3.85, h= 10^7 at various dark matter masses (k)

- Divergence of the precession parameter
- Kerr–Newman curve diverges at a substantially higher energy than the NFW curves.
- Strength of the dark matter halo significantly influence the orbital stability and relativistic precession characteristics.
- Increasing k shifts the divergence toward lower orbital energy.
- Shift in the critical energy suggests a modification of the ISCO location.
- Decreasing k, the curve approaches the Kerr–Newman.

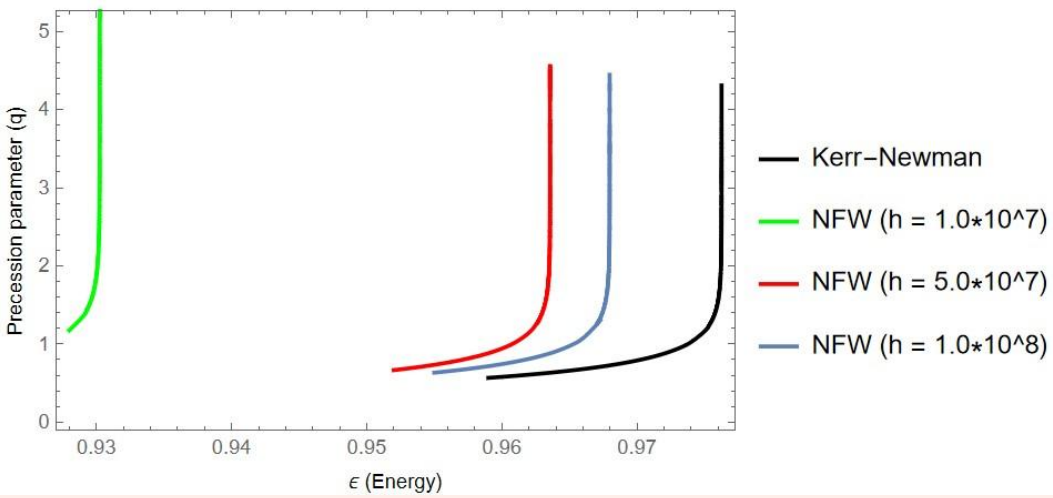
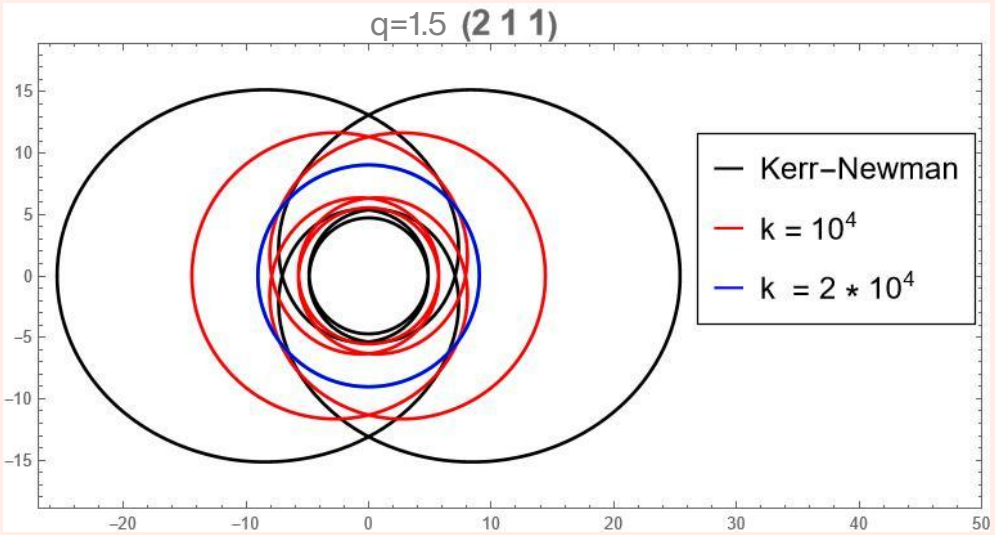


Figure 5. Precession parameter q changes with energy E, at a=0.1, Q=0.29, L=3.85, k= 2.0*10^4 at various dark matter halo scale (h)

- Increasing halo scale parameter (h), shifts the divergence of the precession parameter q toward higher orbital energy.
- Since increasing h, produce more diffuse dark matter distribution.
- Here, critical orbital energy is sensitive to the NFW halo parameter.
- Increasing h, the curve approaches the Kerr–Newman.

Influences of the halo parameters on the periodic orbits

- When q is a rational number, the particle's trajectory exhibits periodicity. Such periodic orbits mean that the particle can return to its original position within a finite amount of time, forming a completely closed trajectory and thus presenting a periodic structure.



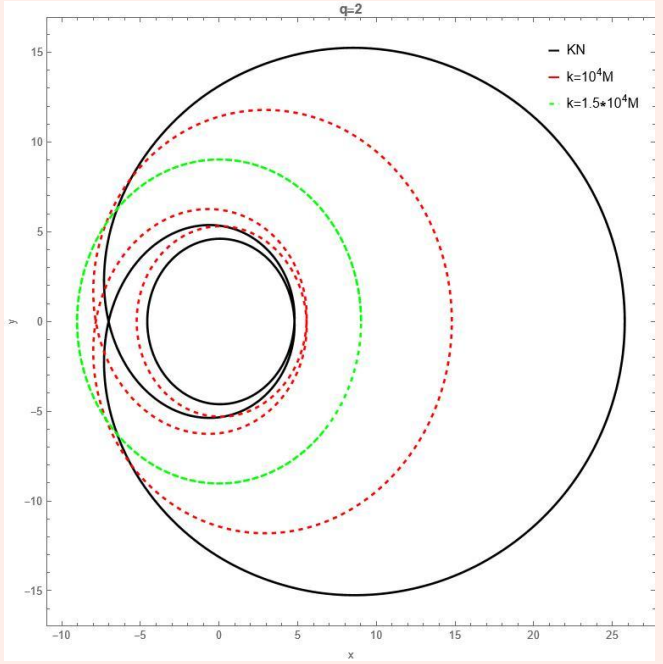


Figure 6. Periodic Orbit around black hole embedded in NFW dark matter distribution for various dark matter masses ($k = 10^4$, $k = 2 * 10^4$) in various (z w v) orientations. Kerr-Newman curve is a baseline reference independent of dark matter distribution.

- Increasing the dark matter mass, the outer leaf shrinks inwards due to the increasing gravitational pull.
- The added dark matter mass suppresses the centrifugal potential barrier.
- Decreasing the dark matter mass, alters the spacetime geometry which pushes the centrifugal barrier outwards to a higher radius, in turn making the particle whirl at a larger radius to maintain a periodic orbit.

Influences of the halo parameters on the periodic orbits

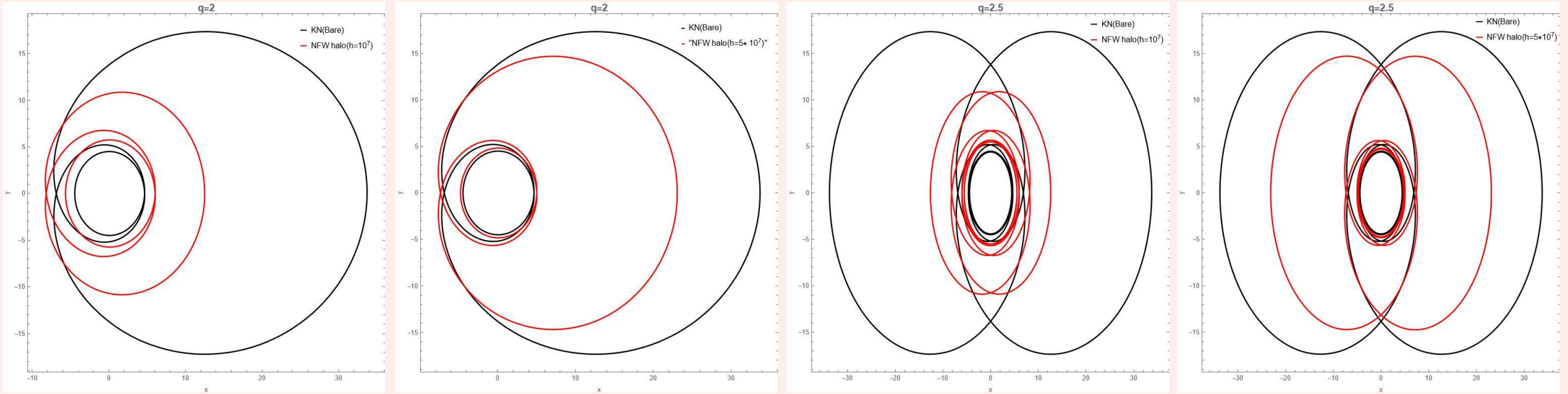


Figure 7. Periodic Orbit around black hole embedded in NFW dark matter distribution for various dark matter halo scales ($h = 10^7$, $h = 5 * 10^7$) in various (z w v) orientations. Kerr-Newman curve is a baseline reference independent of dark matter distribution.

- As the dark matter halo scale increases, the dark matter density decreases thus lowering the gravitational pull, pushing the centrifugal potential barrier outwards.
- Making the outer leaf expand in radius while the inner whirl can now shrink to maintain a periodic orbit.
- Similarly, decreasing the halo scale, the spacetime geometry the outer leaf shrinks inwards due to the increasing gravitational pull, thus suppresses the centrifugal potential barrier.

Conclusion

- The addition of dark matter envelope around the supermassive black hole, leads to an increase in total mass, thus deepening the gravitational well while simultaneously lowering the centrifugal barrier. This shows that the **orbital dynamics is highly sensitive to dark matter.**
- When the black hole is wrapped in dark matter, the **potential barrier is lowered, thus causing the particle to enter the "Zoom-Whirl" phase at much lower energies.**
- As we increase the dark matter mass, the Zoom-Whirl phase is achieved at correspondingly lower energies as observed in Figure 4.
- At lower h values, the particle achieves "Zoom-Whirl" at lower energies, while for higher h values, particle achieves "Zoom-Whirl" at higher energies as observed in Figure 5.
- Increasing the dark matter mass, the outer leaf shrinks inwards due to the increasing gravitational pull. Since, the added dark matter mass suppresses the centrifugal potential barrier as observed in Figure 6.
- As the dark matter halo scale increases, the dark matter density decreases thus lowering the gravitational pull, pushing the centrifugal potential barrier outwards, making the outer leaf expand in radius while the inner whirl can now shrink to maintain a periodic orbit as observed in Figure 7.

THANK YOU