Realization of a One-Zero Texture Majorana Neutrino Mass Matrix with Modular A_4 Symmetry in the Minimal Seesaw

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PANE 2026
Bodoland University, Kokrajhar

19th November 2026

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Neutrino Oscillation

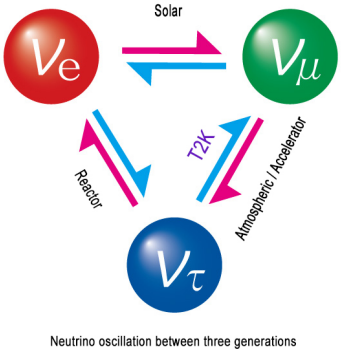


Figure: Neutrinos change their flavor as they travel

- Neutrino oscillations have established non-zero neutrino masses and flavor mixing.
- $$\begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{bmatrix}$$

  $$=> |\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha_i} |\nu_i\rangle$$

- **The PMNS Matrix U** (3×3) contains:
  - 3 mixing angles: θ_{12}, θ_{13}, θ_{23}
  - 1 Dirac CP phase: δ_{CP}
  - 2 Majorana phases: α, β

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Discrete flavor symmetry**:
  - Relates neutrino flavors using finite symmetry groups (e.g.A_4, S_4) in a structured way.
  - Explains neutrino mixing patterns and mass hierarchies through mathematical symmetry.

A_4 Flavor Symmetry

- A_4 is the alternating group of even permutations of 4 objects.
  - Order: 12 elements
  - Generators: $S^2 = (ST)^3 = T^3 = I$
- Irreducible representations:

$$\mathbf{1}, \quad \mathbf{1'}, \quad \mathbf{1''}, \quad \mathbf{3}$$

- A_4 provides a framework for assigning different representations to leptons.
- Early A_4-based models naturally led to the tri-bimaximal (TBM) mixing scheme.

The Modular Symmetry

- Modular symmetry is based on

$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}; ad - bc = 1 \right\}.$$

- It acts on a complex modulus τ, where $\mathrm{Im}(\tau) > 0$

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

- The projective modular group is defined as

$$\overline{\Gamma} = \Gamma / \pm I.$$

- The generators:

$$S : \tau \to -\frac{1}{\tau}, \qquad T : \tau \to \tau + 1.$$

  - Satisfy:

$$S^2 = (ST)^3 = \mathbb{I}.$$

- Principal congruence subgroup :

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}) \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}, \quad N = 1, 2, 3, \ldots$$

- Finite modular groups are obtained by:

$$\Gamma_N = \overline{\Gamma}/\overline{\Gamma}(N).$$

- For different values of N:

$$\Gamma_2 \simeq S_3, \qquad \Gamma_3 \simeq A_4, \qquad \Gamma_4 \simeq S_4.$$

- In this work we consider:

$$\boxed{\Gamma_3 \simeq A_4}.$$

- Modular forms are functions of τ, transform as

$$\boxed{Y(\gamma\tau) = (c\tau + d)^k \rho(\gamma) Y(\tau)}$$

- Here, k is the modular weight and $\rho(\gamma)$ represents the matrix representation of the modular group.
- For $\Gamma_3 \simeq A_4$, the modular forms can be arranged into representations of A_4.
- In flavour models, modular forms play the role of Yukawa couplings, determining the structure of fermion mass matrices.

Global Analysis of Neutrino Oscillation Data

Parameter	Normal Ordering		Inverted Ordering	
	Best Fit	3σ Range	Best Fit	3σ Range
$\sin^2\theta_{12}$	0.307	$0.275 \to 0.345$	0.308	$0.275 \to 0.345$
$\theta_{12}(^\circ)$	33.68	$31.63 \to 35.95$	33.68	$31.63 \to 35.95$
$\sin^2\theta_{23}$	0.561	$0.430 \to 0.596$	0.562	$0.437 \to 0.597$
$\theta_{23}(^\circ)$	48.5	$41.0 \to 50.5$	48.6	$41.4 \to 50.6$
$\sin^2\theta_{13}$	0.02195	$0.02023 \to 0.02376$	0.02224	$0.02053 \to 0.02397$
$\theta_{13}(^\circ)$	8.52	$8.18 \to 8.87$	8.58	$8.24 \to 8.91$
$\delta_{CP}(^\circ)$	177	$96 \to 422$	285	$201 \to 348$
$\Delta m^2_{21}(10^{-5}\ \text{eV}^2)$	7.49	$6.92 \to 8.05$	7.49	$6.92 \to 8.05$
$\Delta m^2_{31}(10^{-3}\ \text{eV}^2)$	$+2.534$	$+2.463 \to +2.606$	-2.510	$-2.584 \to -2.438$

Table: NuFIT 6.0 (2024) Neutrino Oscillation Parameters

Texture Zero

- Texture zeros refer to vanishing entries in the neutrino mass matrix.
- They provide specific constraints on the neutrino mass matrix and its structure.
- Depending on the number of vanishing elements, they can be classified into one-zero , two-zero textures etc.

One Zero Texture:

- One of the six independent elements of the Majorana neutrino mass matrix is zero.
- Less restrictive compared with other texture zero structures.

$$G_1 : \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad G_2 : \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad G_3 : \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix},$$

$$G_4 : \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad G_5 : \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad G_6 : \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix}$$

Table: The six independent one-zero texture patterns of the effective Majorana neutrino mass matrix

Minimal Seesaw Framework

- The minimal seesaw mechanism introduces N_1^c and N_2^c.
- The Dirac mass matrix and right-handed Majorana mass matrix are given by

$$M_D : 3 \times 2, \qquad M_R : 2 \times 2.$$

- The effective light neutrino mass matrix is obtained through the seesaw relation:

$$\boxed{M_\nu = -M_D M_R^{-1} M_D^T}$$

- One light neutrino remains massless:

$$\begin{cases} m_1 = 0 & \text{Normal Ordering,} \\ m_3 = 0 & \text{Inverted Ordering.} \end{cases}$$

The Model

- Under $SU(2)_L \times U(1)_Y$, A_4 and the modular weights the field contents of the model transform as

Symmetry	L_e	L_μ	L_τ	e_R^c	μ_R^c	τ_R^c	N_1^c	N_2^c	H_d	H_u	ϕ
$SU(2)_L$	2	2	2	1	1	1	1	1	2	2	1
$U(1)_Y$	-1/2	-1/2	-1/2	1	1	1	0	0	-1/2	1/2	0
A_4	1	$1'$	$1''$	1	$1''$	$1'$	$1'$	$1''$	1	1	1
$-k_I$	-2	-2	-2	-4	-4	-4	-2	-2	0	0	-4

- The transformation properties of the higher-weight modular forms is

Y^n_m	Y^4_1	$Y^4_{1'}$	Y^6_1	Y^8_1	$Y^8_{1'}$	$Y^8_{1''}$
A_4	1	$1'$	1	1	$1'$	$1''$
$-k_I$	4	4	6	8	8	8

- The superpotential is given by

$$
\begin{aligned}
W = {} & \alpha_1(L_e H_d e^c_R Y^6_1) + \alpha_2(L_\mu H_d \mu^c_R Y^6_1) + \alpha_3(L_\tau H_d \tau^c_R Y^6_1) \\
& + g_1(L_e H_u N^c_2 Y^4_{1'}) + g_2(L_\mu H_u N^c_1 Y^4_{1'}) + g_3(L_\tau H_u N^c_2 Y^4_1) + g_4(L_\tau H_u N^c_1 Y^4_1) \\
& + y_{\phi_1}\phi N^c_1 N^c_1 Y^8_{1'} + y_{\phi_2}\phi N^c_2 N^c_2 Y^8_{1''}.
\end{aligned}
$$

- **Charged lepton sector:**
  - $W_\ell = \alpha_1(L_e H_d e_R^c Y_1^6) + \alpha_2(L_\mu H_d \mu_R^c Y_1^6) + \alpha_3(L_\tau H_d \tau_R^c Y_1^6)$.
  - Charged lepton mass matrix

$$M_\ell = \frac{v_d}{\sqrt{2}} Y_1^6 \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}.$$

- **Neutrino sector:**

$$W_\nu = g_1 \left(L_e H_u N_2^c Y_{1'}^4\right) + g_2 \left(L_\mu H_u N_1^c Y_{1'}^4\right) + g_3 (L_\mu H_u N_2^c Y_1^4) + g_4 \left(L_\tau H_u N_1^c Y_1^4\right) + \left(y_{\phi_1} \phi N_1^c N_1^c Y_{1'}^8 + y_{\phi_2} \phi N_2^c N_2^c Y_{1''}^8\right).$$

- The Dirac and Majorana mass matrices

$$M_D = \frac{v_u}{\sqrt{2}} \begin{bmatrix} 0 & g_1 Y_{1'}^4 \\ g_2 Y_{1'}^4 & g_3 Y_1^4 \\ g_4 Y_1^4 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & g_1' \\ g_2' & g_3' \\ g_4' & 0 \end{bmatrix}, \quad M_R = \begin{bmatrix} y_{\phi_1} v_\phi Y_{1'}^8 & 0 \\ 0 & y_{\phi_2} v_\phi Y_{1''}^8 \end{bmatrix} \equiv \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}.$$

- The neutrino mass matrix is $M_\nu = -M_D M_R^{-1} M_D^T = \begin{bmatrix} \frac{g_{1'}^2}{M_2} & \frac{g_3' g_1'}{M_2} & 0 \\ \frac{g_3' g_1'}{M_2} & \frac{(g_2')^2}{M_1} + \frac{(g_3')^2}{M_2} & \frac{g_4' g_2'}{M_1} \\ 0 & \frac{g_4' g_2'}{M_1} & \frac{(g_4')^2}{M_1} \end{bmatrix}.$

- We construct a Hermitian matrix:

$$H_\nu = M_\nu^\dagger M_\nu$$

- It is diagonalized using a unitary transformation:

$$\boxed{U_\nu^\dagger H_\nu U_\nu = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}}$$

- Since the charged lepton mass matrix is diagonal:

$$U_{\mathrm{PMNS}} = U_\nu$$

Numerical Analysis

- The complex modulus τ is varied within the fundamental domain:

$$\mathrm{Im}(\tau) > 0, \quad |\,\mathrm{Re}(\tau)| \leq \frac{1}{2}, \quad |\tau| \geq 1.$$

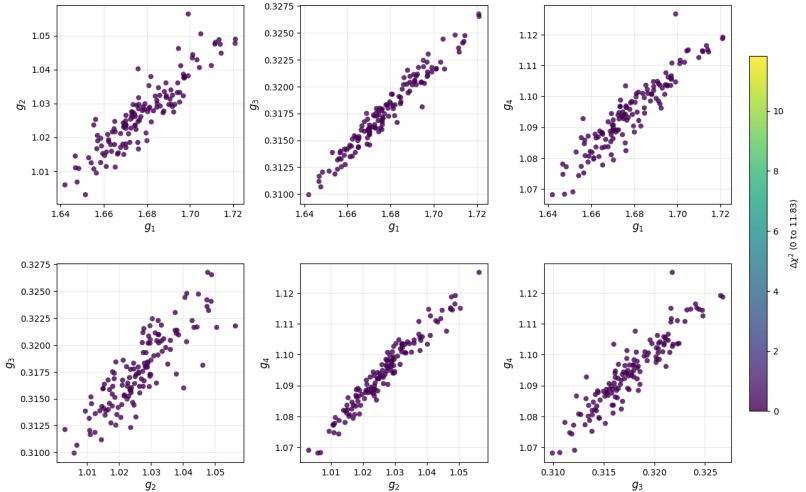
- The model parameters and coupling constants are varied to reproduce the experimental observables.
- A χ^2 minimization is performed using the Differential Evolution algorithm.
- Both normal ordering (NO) and inverted ordering (IO) are investigated.

Numerical Results

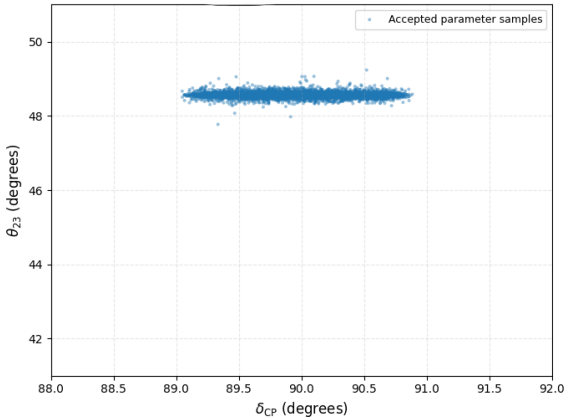
Table: Best-fit results for the model in NO and IO.

Parameter	NO	IO
$\mathrm{Re}(\tau)$	0.49	0.04
$\mathrm{Im}(\tau)$	1.37	1.08
θ_{12}	36.35°	33.64°
θ_{13}	7.85°	8.57°
θ_{23}	55.67°	48.56°
$\Delta m^2_{21}\,(10^{-5}\,\mathrm{eV}^2)$	8.33	7.49
$\Delta m^2_{3\ell}\,(10^{-3}\,\mathrm{eV}^2)$	2.49	-2.50
$m_1\,(\mathrm{eV})$	0.00	0.05
$m_2\,(\mathrm{eV})$	0.009	0.05
$m_3\,(\mathrm{eV})$	0.05	0.00
$\sum m_i\,(\mathrm{eV})$	0.06	0.10

Pairwise coupling correlations for IO



Correlation between the δ phase and θ_{23} for IO



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Thank You