



Hexadecapole deformation of ^{176}Yb from quasi-elastic scattering using Bayesian Analysis

Amar Das

1. Reseach Scholar
Department of Physics
Gauahati University
2. Assistant Professor
Department of Physiscs
Suren Das College,
Autonomous
Hajo, Assam

Supervisor

Dr. Kushal Kalita,
Professor, Department Physics
Gauhati University, Guwahati, Assam

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Introduction

Nuclear reaction

a+X= b+Y+Q or X(a,b)Y,

Q= Energy released or absorbed

- **Various nuclear interactions mainly depend on bombarding energy, mass and atomic number.**
- **Shape and size of nuclei which affect the reaction process.**
- **For elastic scattering, Q value is '0' , non-zero for reaction channels**

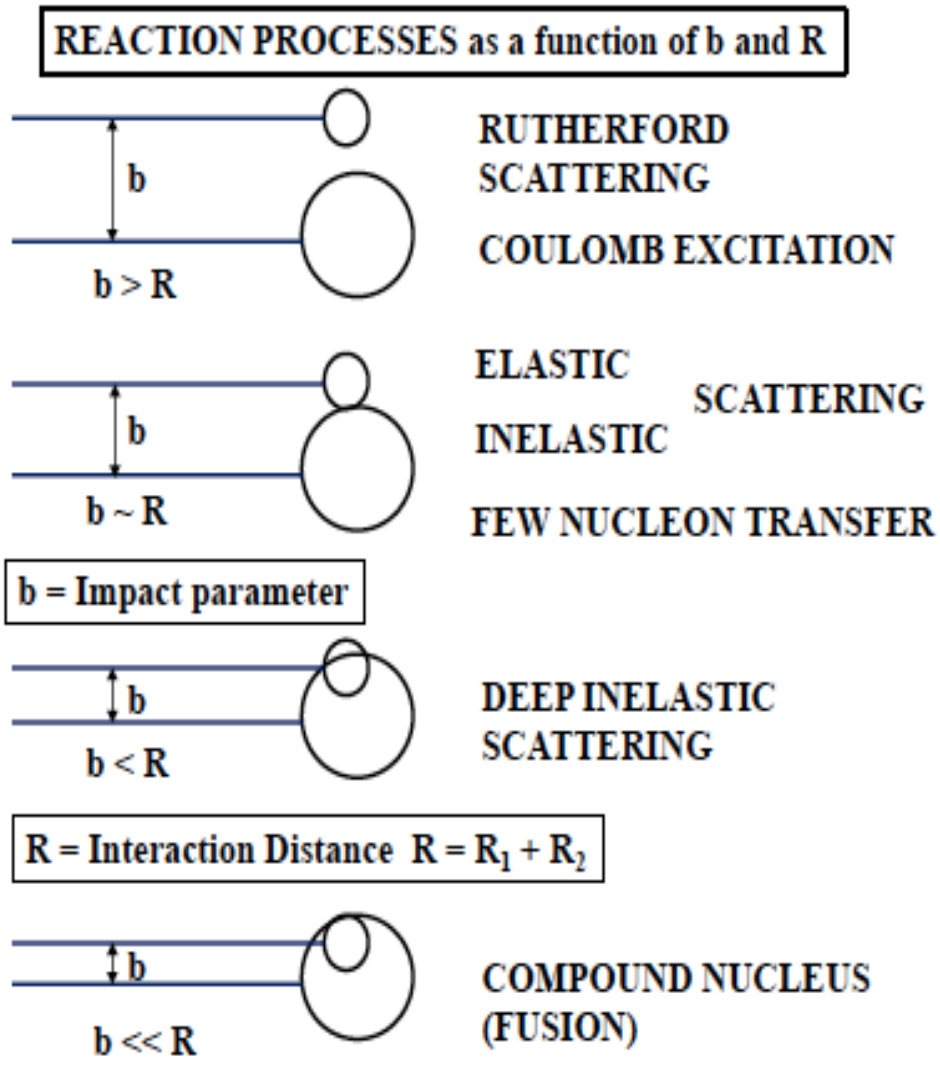


Figure1: Different type of reaction

Introduction

- **The nuclear structure have important role in nuclear fusion reactions near Coulomb barrier which are important topic from the last few decades.**
- **Couple channel frame work.**
- **Barrier distribution(BD).**
- **Fusion BD can be extracted from fusion excitation function.**
- **It requires very precise data as it has double differentiation in its expression.**
- **Another method by which a fusion BD can be extracted is quasi-elastic scattering(QES).**
- **($\sigma_{QES}=\sigma_{Tran}+\sigma_{El}+\sigma_{Inel}$)**

Introduction

- **The advantage of QES method is that one can extract BD with less precise data as it has single differential in its expression.**
- **BD found by QES have lesser error in higher energy region as compared to BD found from fusion method.**
- **Quasielastc scattering BD can be exctracted using**
- **1) Back angle method (beam like particle).**
- **Centrifugal correction in energy or effective energy.**
- **2)Forward angle (the target like particle)**

QES BD also can be extracted by measuring the target like particle in forward angle.

Introduction

- **Couple Channel Effect**

1. Effect of different coupling in quasi-elastic scattering measurement can be seen by using scattered version of couple channel code CCFULL. In this code, Woods-Saxon (WS) potentials were used.

2. In WS formalism, at least six parameters are varied in order to analyze quasi-elastic scattering data and previous experimental results have showed that WS formalism is very good phenomenological model to study nuclear collision.

Introduction

Bay's Theorem : The following formula can be used to determine hexadecapole deformation of a nuclei

$$P(\theta \mid D) = P(D \mid \theta) \cdot P(\theta) / P(D)$$

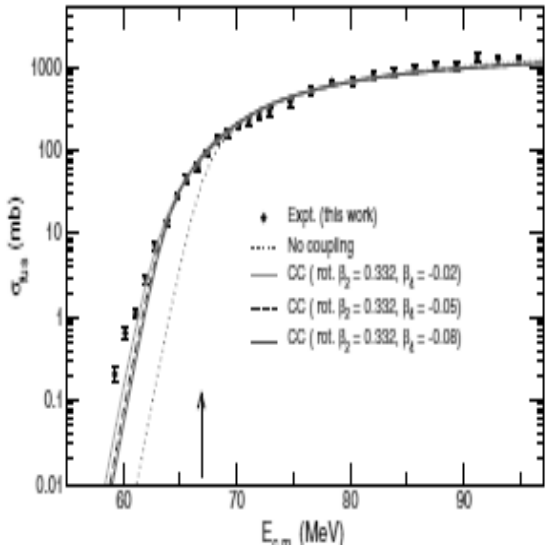
posterior ∝ likelihood × prior

P(θ)	P(D | θ)	P(θ | D)	P(D)
Prior	**Likelihood**	**Posterior**	**Evidence**
What we believe about the parameter before seeing data.	How probable the data are for each value of θ.	Our updated belief after seeing the data.	A normalizing constant that makes the posterior sum to one.

This method have been used to deremine the hexadecapole deformation of any deform nuclei, where we have prior Knowledge of the parameter Ref: C. Kumar, S. Nath, Physics Letters B 862 (2025) 139319

Literature survey :

Quasielastic scattering(QES) measurement for projectile and Hexadecapole deformation measurement has been done by various authors such as



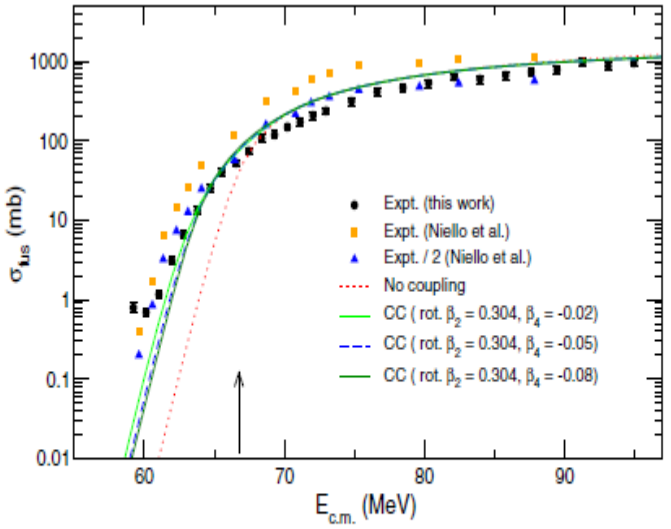


Figure 2: EXF of a)^{16}O+ ^{174}Yb b) ^{16}O+ ^{176}Yb by Tapan Rajbonghs et al., PRC 93, 054622,(2016)

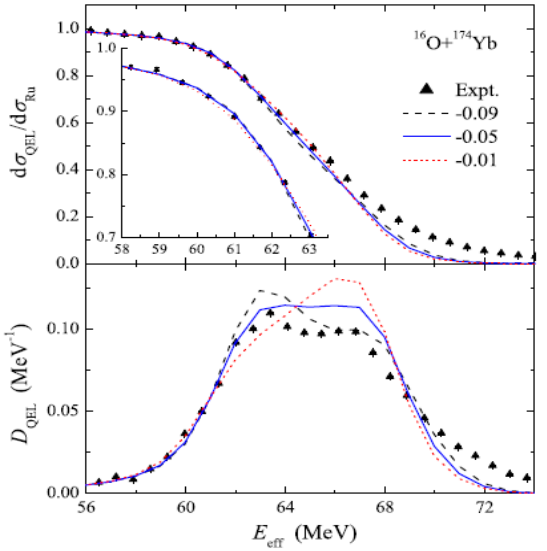


Figure 3: EXF and BD of ^{16}O+^{174}Yb (H.M Jia et al., Phys.Rev. C 90, 031601(R) (2014)

Literature survey :

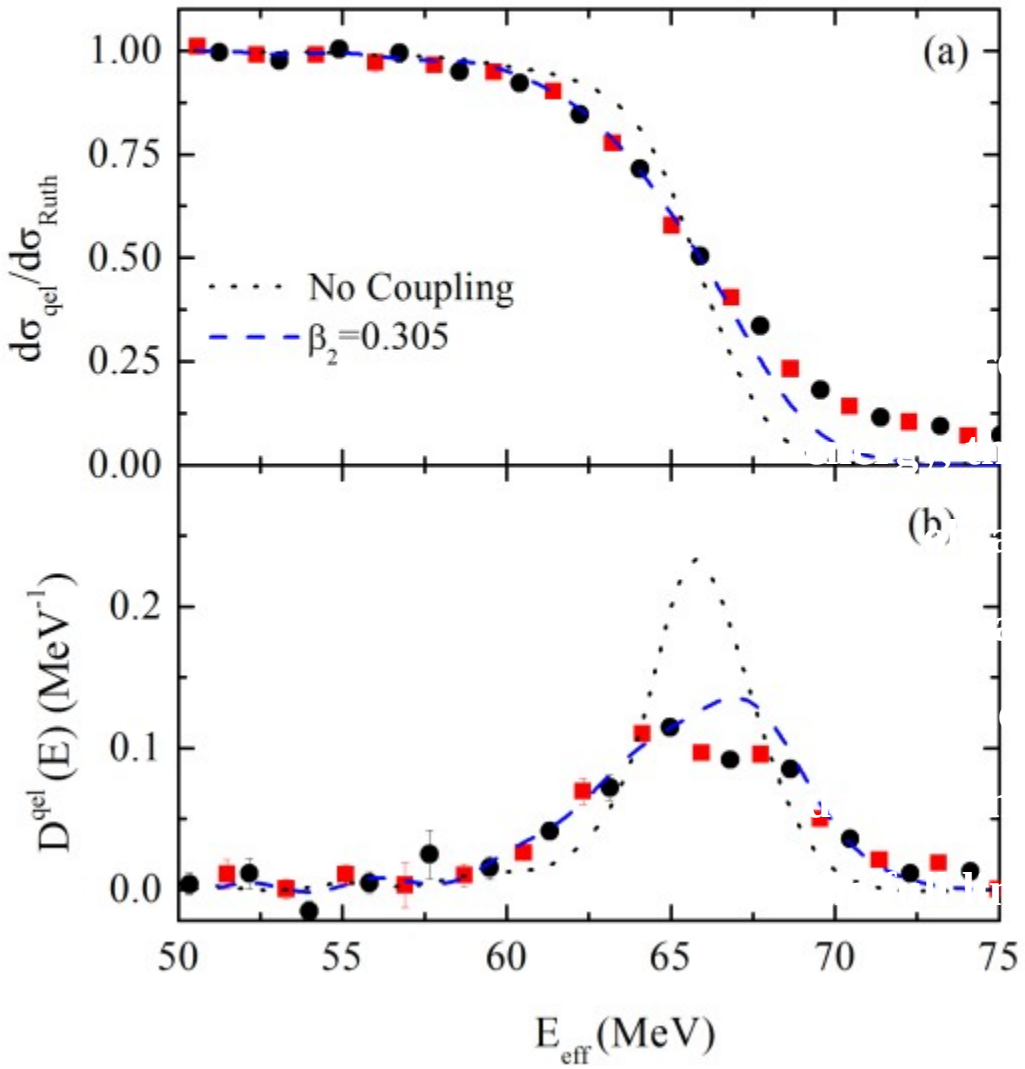


Figure 4: Quasielastic a) EXF and b) BD of $^{16}O+^{176}Yb$ done by G. Mohanto, et al., European Journal of Physics Volume 59, 234, (2023)

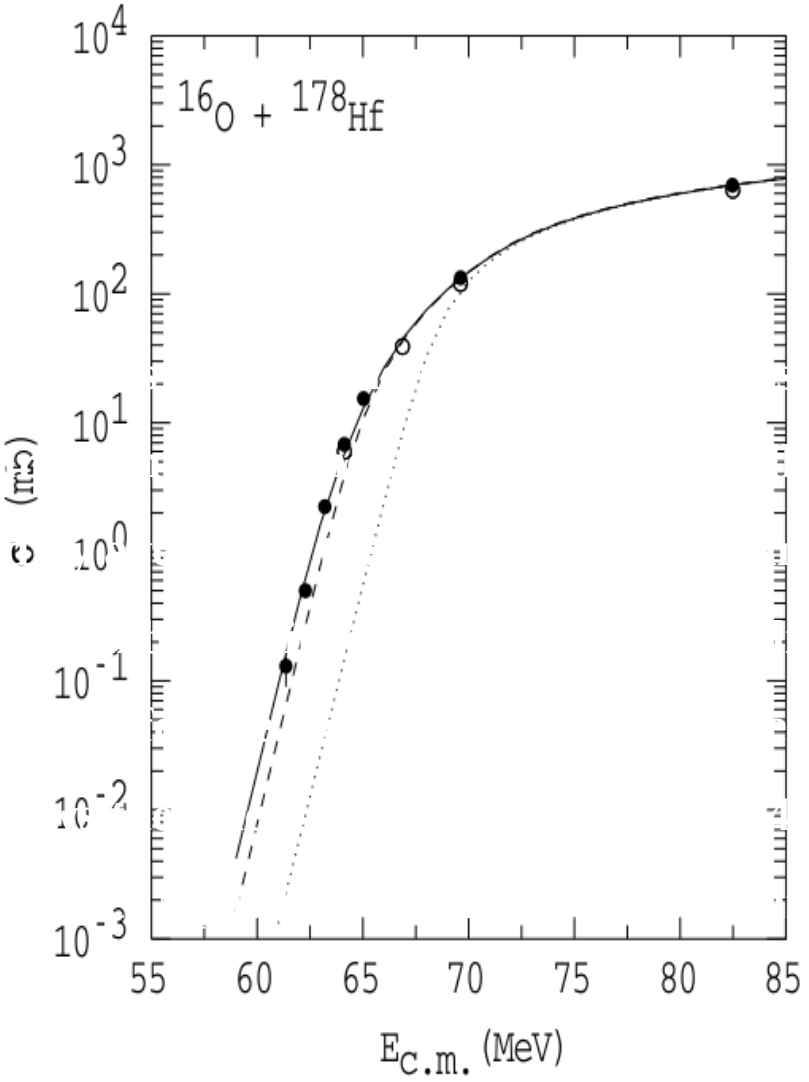


Figure 5: Fusion EXF of $^{16}O+^{178}Hf$ D. J. Hinde, N. Rowley, M. Dasgupta,1 R. D. Butt,C. R. Morton, and A. Mukherjee Phys. Rev. C 68, 044606 (2003)

Motivation

- **Determination of quasi-elastic excitation function and barrier distribution at back angle for $^{16}O+^{176}Yb$ as for this system only fusion measurement have been done.**
- **To see the effect of various transfer channel for the reaction on excitation functions and BDs $^{16}O+^{176}Yb$**
- **Extraction of optical model parameter for $^{16}O+^{176}Yb$ using couple channel code CCFULL (Scattered version)**
- **To find the hexadecapole deformation of ^{176}Yb by fitting the quasi elastic scattering experimental data with theoretical data for the system $^{16}O+^{176}Yb$.**
- **Bayesian Analysis have been perfomed to determine the hexadecapole deformation of ^{176}Yb.**

Experimental Overview

- We have used the HIRA facility for our measurement of quasielastic scattering EXF from below barrier to above barrier for $^{16}O+^{176}Yb$
- ^{16}O (Pulse 2μs) Beam with energy ranges from 57-83 MeV and target ^{176}Yb (160 μg/cm^2) with carbon backing (20 μg/cm^2)
- The energy are taken in step of 1 MeV in above barrier and 2 MeV below barrier for the all systems systems.
- Two Monitoring detector at forward angle and two telescopic detector at backward angles (138^O and 150^O) for QES measurement at back angle.
- We have measured QES cross section for target like particles at 0^o using Method of Rohan Biswas et al., (MWPS: 150 × 50 mm^2 active area (at forward angle))

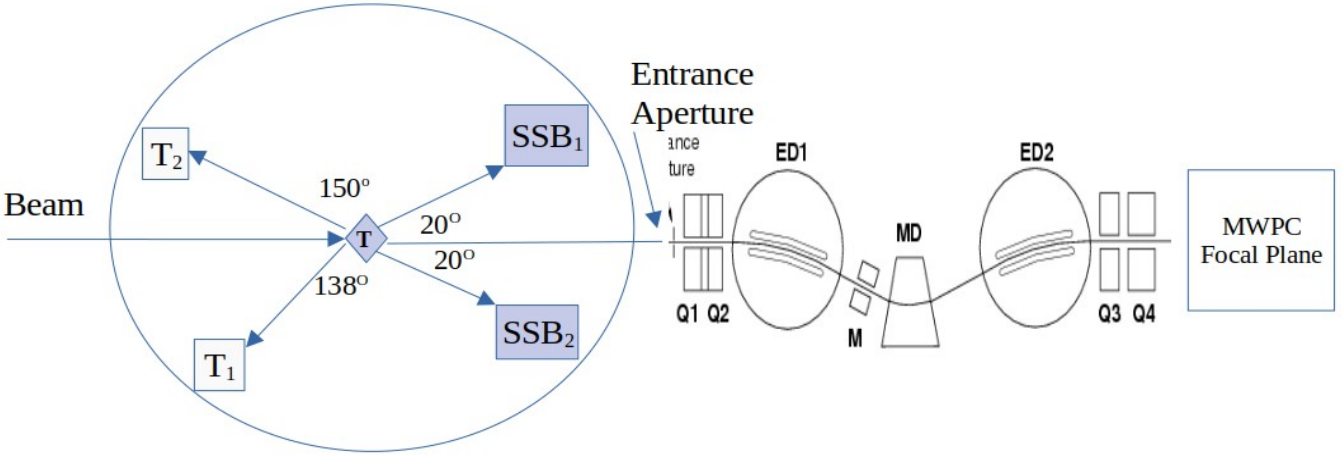


Figure 6: Experimental setup

Experimental Overview

^{116}Sn

^{178}Hf

^{176}Yb

Quarz



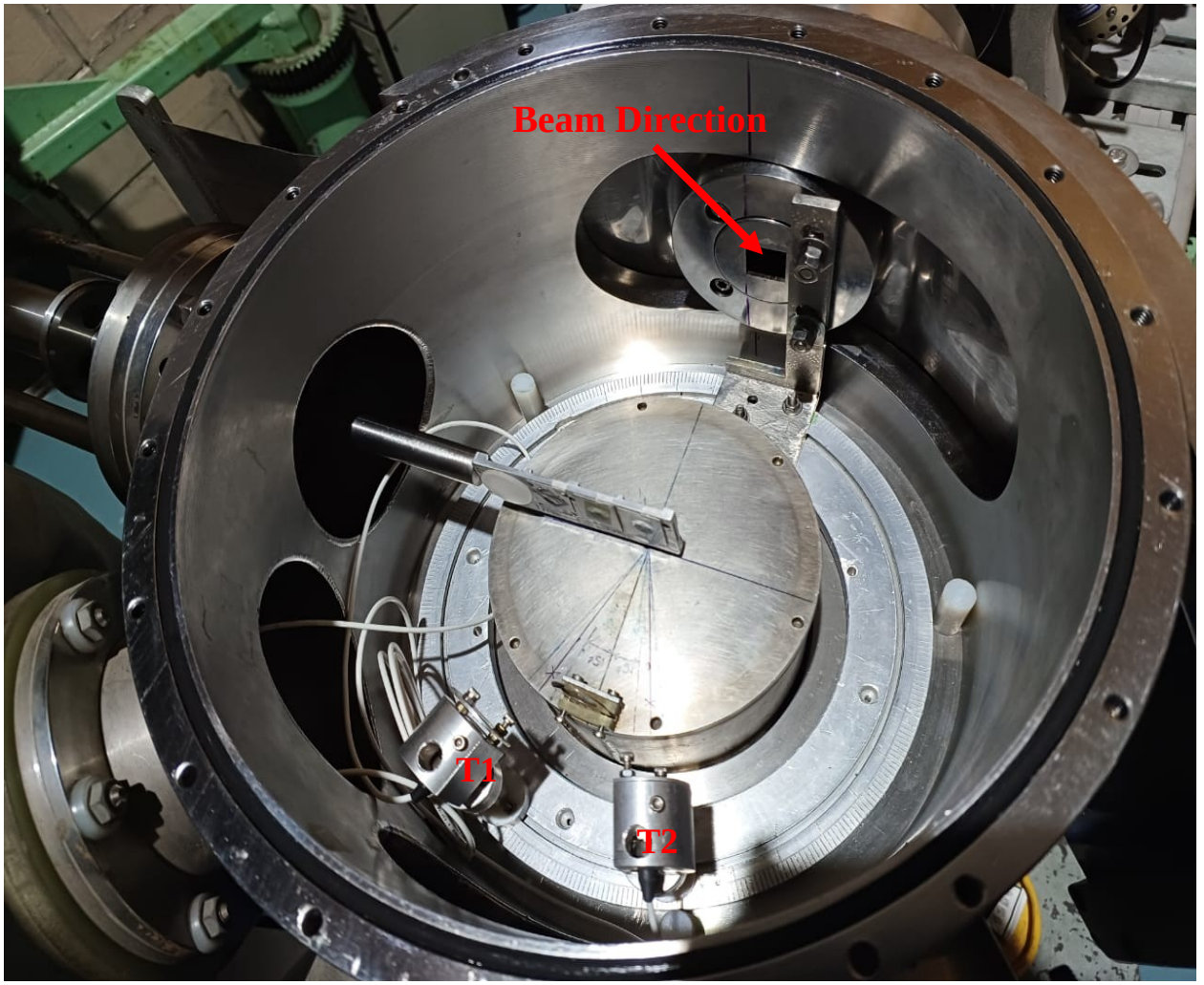


Figure 7: a) Target leather **b)Target Chamber of our Experimental Setup**

Experimental Details

- **Detector specifications are as follows:**

  T1: 40 µm-300 µm (IUAC)

  T2: 15 µm-300 µm (Gauhati University)

  MWPS: 150 × 50 mm^2 active area (at forward angle)

- **Two monitors were placed at $+20^0$ and -20^0 on the both sides of beam direction.**
- **Monitor collimator size= 1mm**
- **Telescope detector's collimator size= 5mm.**
- **Solid angle = 3.67 msr**
- **Back angle: 138 ^{0}**

MWPS detector at 0 ^{0}

- **The products of reactions were collected in the Control Room of IUAC using NIAS-MARS (Multi-parameter Acquisition with Root base Storage) software.**

Experimental Overview

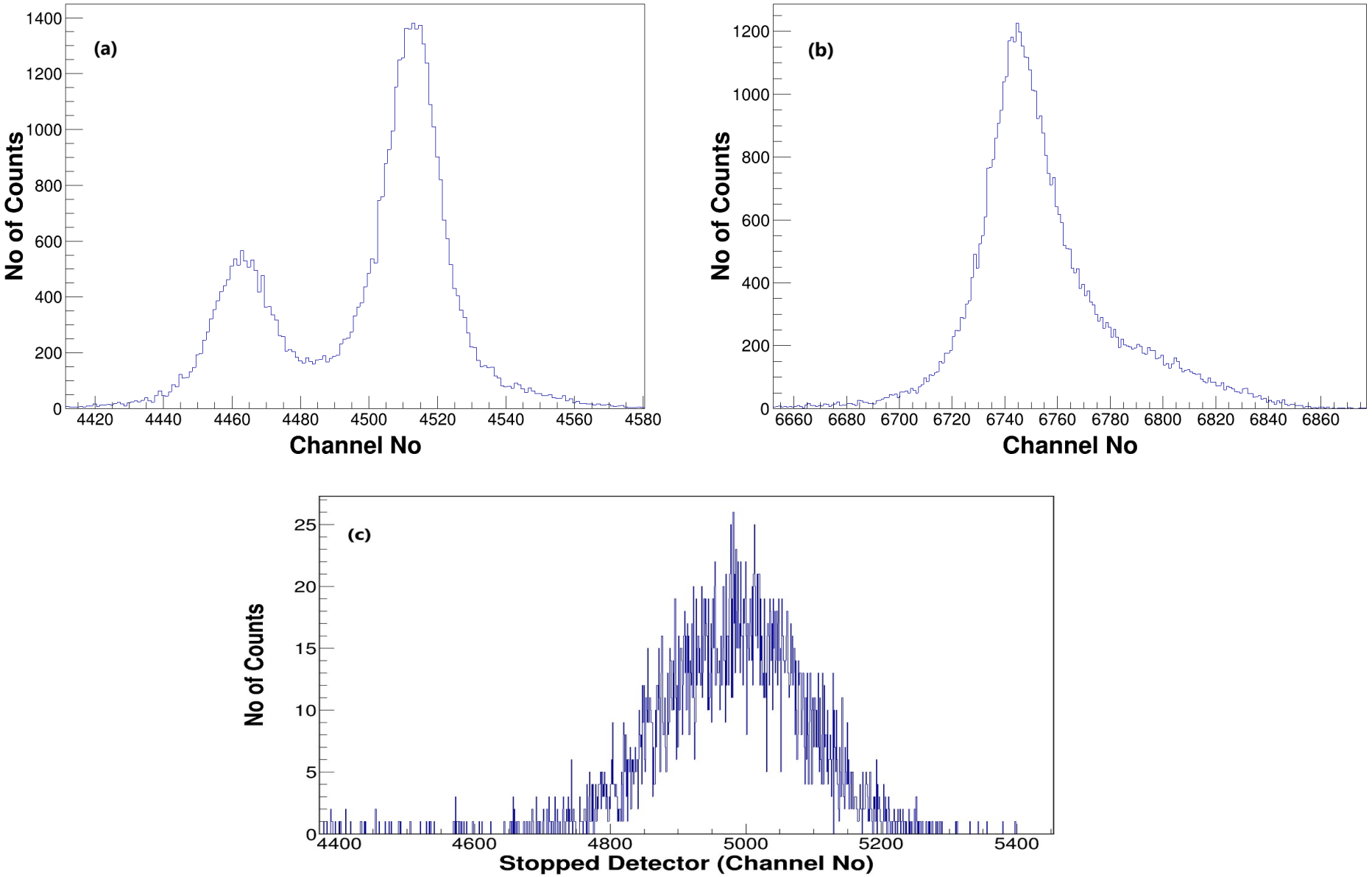


Figure 8: Spectra of a) Monitor detector at 20 w.r.t beam b) Monitor detector at -20 w.r.t beam c) Back angle $^{16}O+^{176}Yb$ at 74MeV

Data analysis (Back angle)

The experimental quasielastc scattering cross section for back angle can be found by using the formula given below [Saumyajit Biswas et al.,]

$$\frac{d\sigma_{\mathrm{qel}}}{d\sigma_{\mathrm{R}}}(\theta_{\mathrm{tel}}) = \left[\frac{N_{\mathrm{tel}}(\theta_{\mathrm{tel}})}{N_{\mathrm{mon}}(\theta_{\mathrm{mon}})}\right]\left[\frac{\frac{d\sigma_{\mathrm{R}}}{d\Omega}(\theta_{\mathrm{mon}})}{\frac{d\sigma_{\mathrm{R}}}{d\Omega}(\theta_{\mathrm{tel}})}\right]\left[\frac{\Delta\Omega_{\mathrm{mon}}}{\Delta\Omega_{\mathrm{tel}}}\right].$$

Where

$$\left[\frac{\Delta\Omega_{\mathrm{mon}}}{\Delta\Omega_{\mathrm{tel}}}\right] = \left[\frac{N^{\mathrm{Nd}}_{\mathrm{mon}}(\theta_{\mathrm{mon}})}{N^{\mathrm{Nd}}_{\mathrm{tel}}(\theta_{\mathrm{tel}})}\right]\left[\frac{\frac{d\sigma^{\mathrm{Nd}}_{\mathrm{R}}}{d\Omega}(\theta_{\mathrm{tel}})}{\frac{d\sigma^{\mathrm{Nd}}_{\mathrm{R}}}{d\Omega}(\theta_{\mathrm{mon}})}\right]$$

$$D_{\mathrm{QEL}}(E) = -\frac{d}{dE}\left[\frac{d\sigma_{\mathrm{QEL}}/d\Omega}{d\sigma_{\mathrm{Ruth}}/d\Omega}\right]$$

- **Wood Saxon parameters are extracted by matching the experimental cross-section with theoretical data. Theoretical analysis is carried out using the Scattered version of couple channel code CCFUL. The interacting nuclei have potential which is consists of nuclear and Coulomb potentials as.**

$$V_{Coul}(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Z_P Z_T e^2}{r}, r \geq R_C \\ \frac{1}{4\pi\varepsilon_0}\frac{Z_P Z_T e^2}{2R_C}(3 - \frac{r^2}{R_C^2}), r \leq R_C \end{cases}$$

Coulombic Potential

$$V_N(r) = \frac{-V_0}{1+\exp(\frac{r - R_{0R}}{a_R})} + i\frac{-W_0}{1+\exp(\frac{r - R_{0I}}{a_I})}$$

Nuclear Potential

Data analysis

During the fitting of theoretical EXF and BD with experimental EXF and BD the projectile and targets are considered to be inert and best fitted Wood Saxon parameters for present systems are given below

Table1: Wood Saxon parameter for $^{16}O+ ^{176}Yb$

Systems	V_0 (MeV)	r_r (fm)	a_r (fm)	W_0 (MeV)	r_i (fm)	a_i (fm)
$^{16}O+^{176}Yb$	63.53	1.17	0.75	30	1	0.50

Table2: Wood Hexadecapole deformationand excited energy of nuclei for $^{16}O+ ^{176}Yb$

• Nuclei	• β_2	• E_{2+} (MeV)	• E_{3-} (MeV)	• E_{4+} (MeV) • -
• ^{16}O	• 0.362	• 6.917	• 6.129	
• ^{176}Yb	• 0.267	• 0.093	• 0.270	• 0.306

Data analysis

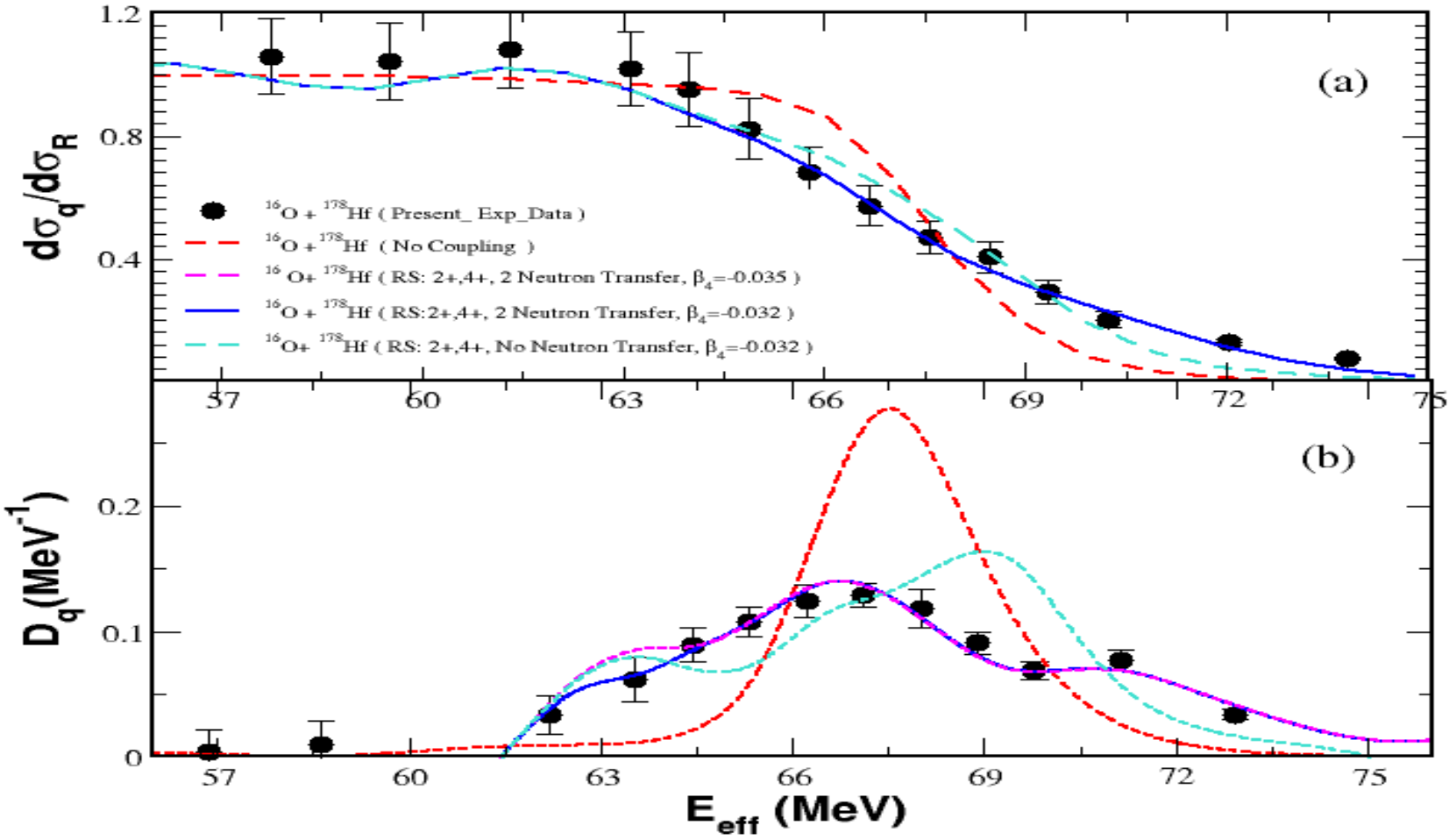


Figure 9: The experimentally measured excitation function of the $^{16}O + ^{176}Yb$ system which is compared with the different modes of coupling between interacting partners using CCFULL. QE cross sections with the inelastic couplings between projectile and target, one pair of neutron transfer and deformation are shown by curves of different colours. (b) The corresponding extracted barrier distribution curves with the best channels.

Data analysis

^{176}Yb targets is deformed so, we have extracted the hexadecapole deformation of ^{176}Yb by fitting the experimental and theoretical EXF and BDs.

For best fitting chi square χ^2 minimization have been done for the BDs for respective systems. χ^2 value can be found by

$$\chi^2_\nu = \frac{1}{\nu}\sum_{i=1}^{N}\frac{(O_i - E_i)^2}{\sigma_i^2}$$

- **Where v is the no of energy points or degree of freedom, O_i, E_i and σ_i are the experimental value of BD, theoretical value of BD experimental uncertainity associated with the experimental BD.**

Data analysis

The figure plot of chi square minimization is shown below, during χ^2 test the $ß_4$ (Hexadecapole) were varied from -0.10 to 0.10 for the system as many authors found it different values. The EXF and BDs with different values of hexadecapole deformations are shown in figure 10. The minimum value of χ^2 were obtain at $ß_4$=-0.019±0.006 for ^{176}Yb and form factor =0.3MeV.

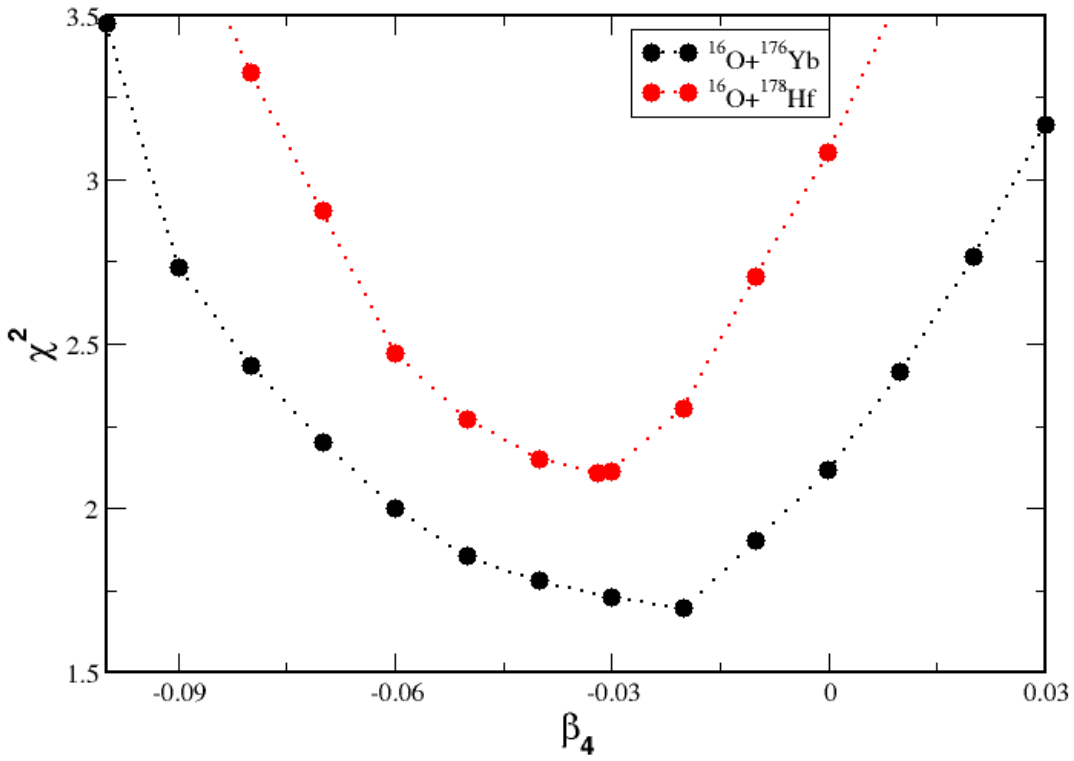


Figure 10: The Chi Square minimization of ^{16}O+^{176}Yb at form factor 0.3MeV

Data Analysis

Estimating P_{tr} and β_4 with Bayesian MCMC

POSTERIOR

$$P(P_{tr}, \beta_4 \mid \text{data}) \propto \exp[-\tfrac{1}{2}\,\chi^2(P_{tr}, \beta_4)] \times P(P_{tr}, \beta_4)$$

Likelihood Gaussian, built from the χ^2 statistic

Prior uniform within set bounds

01 · PRIOR

Parameter ranges:

Form factor (P_{tr}): 0 to 0.45 MeV

β_4 are varied from -0.10 to 0.10

Uniform prior inside these bounds

02 · SAMPLER

emcee sampler

1,000 walkers

10,000 steps each

500 burn-in steps

Affine-invariant ensemble

03 · CONVERGENCE

Autocorrelation time

35 steps for P_{tr}

39 steps for β_4

Longest values, oblate shape, quasielastic reactions

Bayesian Analysis to determine the ß$_4$:

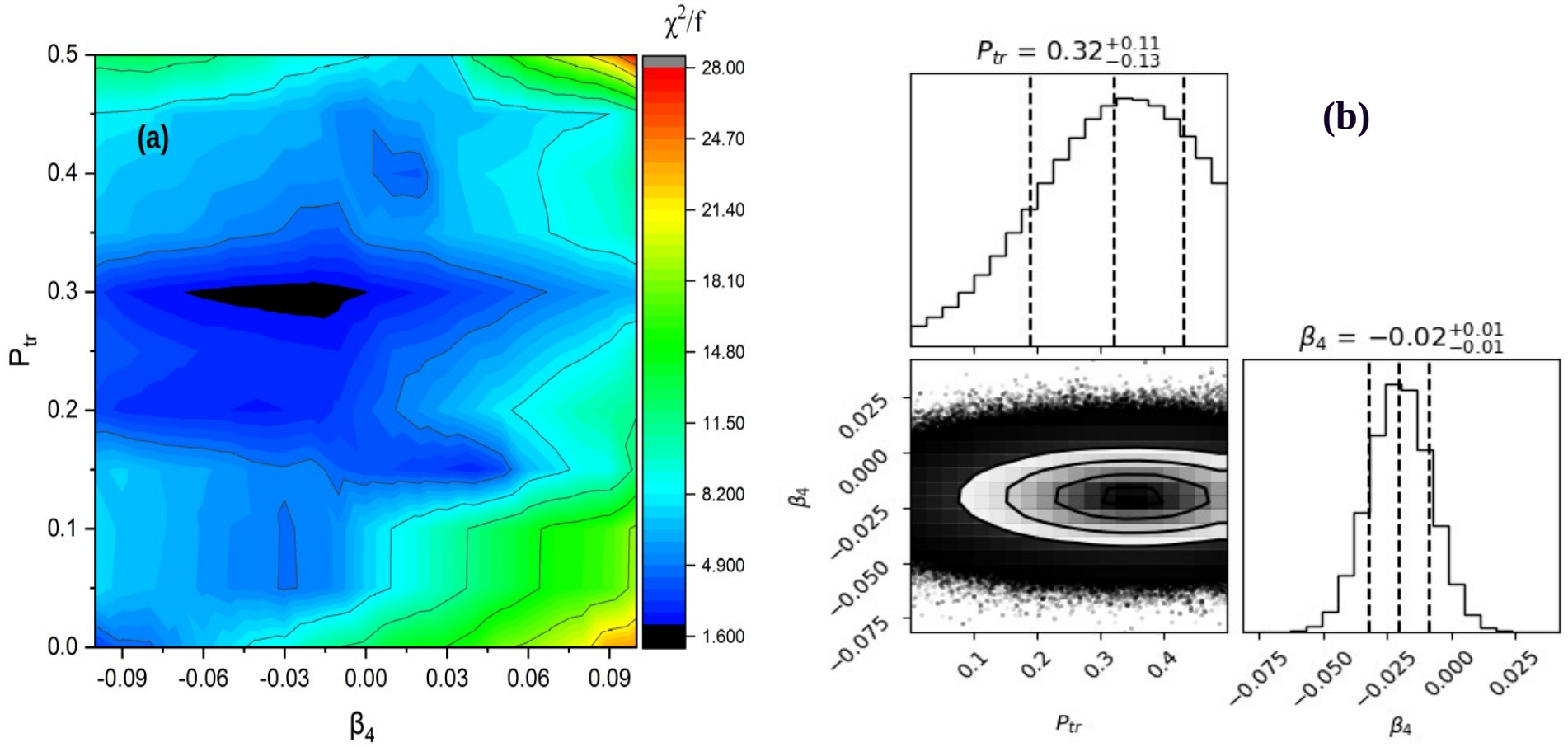


Figure:11 Chi Square and Bayesian analysis where ß$_4$ is veried from -0.10 to 0.10 and form factor is varied frm 0 to 0.45 MeV

Data analysis (Back angle)

Comparison of present data with existing data with similar systems:

1) $^{16}O+^{176}Yb$

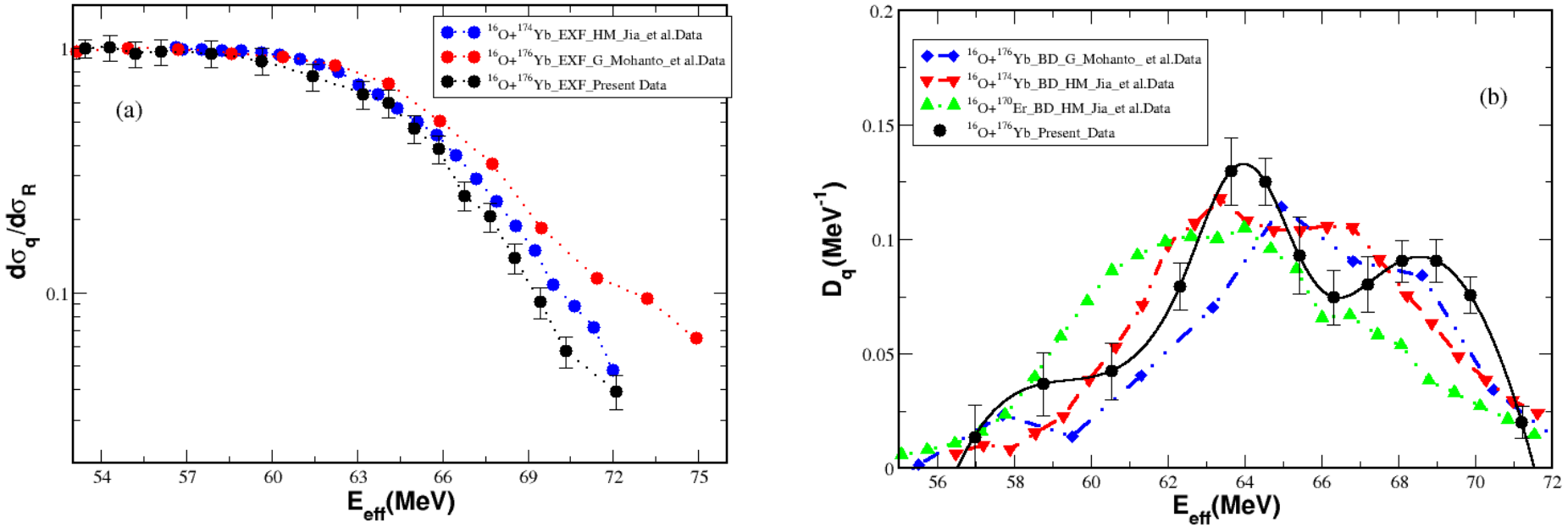


Figure:12 Comparison of the QE functions of $^{16}O+^{174}Yb$, ^{176}Yb (a) EXF b) BDs

Data analysis

Observation from the analysis:

- **We have measured quasielastic EXF and BDs for ^{16}O+ ^{176}Yb**
- **The best fitting were obtained when we consider neutron transfer with form factor 0.3MeV for the deformed target.**
- **Bayesian Analysis have been done which also confirms the hexadecapole deformation value as mention above.**
- **Effect of neutron transfer have been seen for ^{16}O+ ^{176}Yb. The $ß_4$ value for present work is shown below:**

Parameters	Phys. Rev. C 113, 014601	Present Work
P_{tr}	0.30	0.32+0.11-0.13
$ß_4$	0.019±0.006	0.020±0.01

Table 3: Comparison of present hexadecapole value with my Previous work.

Publication:

1. Extracting the hexadecapole deformation of ^{176}Yb and ^{178}Hf from quasielastic scattering, **Phys. Rev. C 113, 014601 – Published 5 January, 2026**,

2. Quasi-elastic scattering and barrier distribution of ^{16}O+^{116}Sn using a recoil mass spectrometer **under communication** (Nuclear Physics, Section A)

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Our Collaborators

[1,2]Amar Das, [1]K. Kalita*, [1,3]Harun Al Rashid, [1]Lakhyajit Sarma, [1,4]Nabendu K. Deb, [1]Taniya M. Sonowal, [1]A. Tamang, [5]S. Nath, [5]J. Gehlot, [5]N Madhavan, [5]Gonika, [5]Chandra Kumar, [6]R. K. Sahoo, [7]Bhargav Baruah, [7]Rasna Baruah, [8]A. Parihari, [9]M. Swain, [10]M. Bhuyan

[1]Department of Physics, Gauhati University, Guwahati-781014, Assam, India

[2]Department of Physics, Suren Das College (Autonomous), Hajo, Assam, India

[3]Physics Department,Baosi Banikanta Kakati College, Nagaon, Barpeta, Assam, India

[4]Physics Department,Kamrup College, Chamata, Nalbari, Assam, India

[5]Inter University Accelerator Centre, New Delhi, India-110067

[6]Physics Department,Central University of Jharkhand, India

[7]Physics Department,Cotton University, Guwahati, Assam, India

[8]Physics Department,Rajdhani College, New Delhi, India

[9]Physics Department,Cenurion University of Technology and Management, Odisha

[10] Physics Department, Rangia College, Kamrup, Assam, India

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Thank You