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Same Texture, Different Vacua: Two A_4 Realisations of a Predictive Neutrino Mass Matrix

Sagar Tirtha Goswami, Pralay Chakraborty, Subhankar Roy

Department of Physics, Gauhati University, Guwahati, Assam

Motivation

Textures, symmetries and vacua

- The structure of M_ν is not fixed by experiment: masses, ordering, $\delta_{\rm CP}$ and Majorana phases are still open.
- **Predictive textures**: a few correlations among the entries of $M_\nu \Rightarrow$ testable predictions.
- A texture is usually "explained" by a flavour symmetry (A_4) plus a **vacuum alignment** of flavon fields.
- The usual practice stops when M_ν comes out right. But:

> Is the chosen alignment actually a stable minimum of the scalar potential?

- Here: **one texture, two A_4 realisations, two different vacua**, and the potential decides.

The Texture

Definition

- Majorana neutrinos, with the texture

$$M_\nu = \begin{pmatrix} a & b & b \\ b & 2b & t \\ b & t & d \end{pmatrix}.$$

- Two correlations:

$$(M_\nu)_{12} = (M_\nu)_{13}, \qquad (M_\nu)_{22} = 2\,(M_\nu)_{12}.$$

- This is the **triadic texture** of Ref. [1], written in the phase convention $e \to -e$, i.e. $M_\nu \to P M_\nu P$, $P = \mathrm{diag}(-1, 1, 1)$. All observables are unchanged.

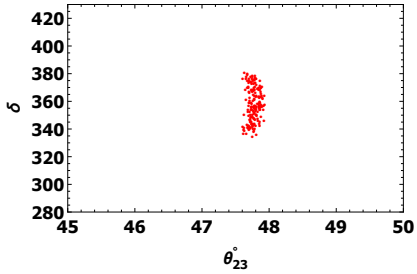
- Diagonalised as $U^T M_\nu U = \mathrm{diag}(m_1, m_2, m_3)$, with $U = V\,\mathrm{diag}(e^{i\alpha}, e^{i\beta}, 1)$.

- The three off-diagonal conditions fix a, t, d (in units of b) from the mixing angles and δ_{CP}; Δm^2_{21} fixes $|b|$. ▸ Analytic a, t, d

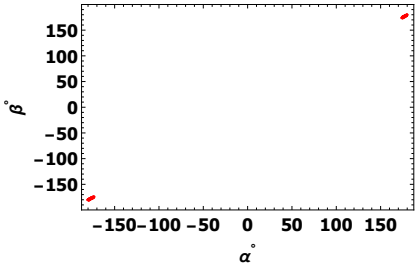
[1] S.T. Goswami, P. Chakraborty, S. Roy, arXiv:2605.18154 [hep-ph] (under review).

Results

Atmospheric angle, Dirac and Majorana phases (NO)



θ_{23} vs $\delta_{\rm CP}$ (degrees)



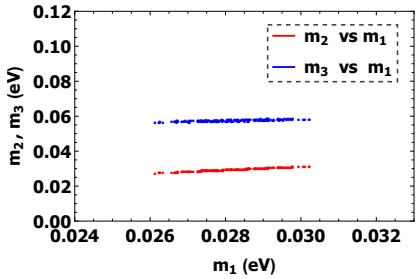
Majorana phases α vs β

- $\theta_{23} \neq 45^\circ$: upper octant, $\theta_{23} \approx 47.6^\circ$–$47.9^\circ$. ▸ Why not 45°?
- $\delta_{\rm CP} \approx 335^\circ$–$380^\circ$: close to CP conservation.
- Majorana phases locked: $\alpha \simeq \beta$ (linear fit: slope 1.00).

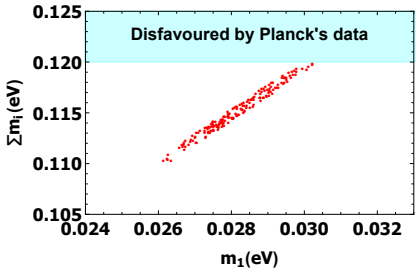
Inputs: NuFIT 6.0 3σ ranges; $\Sigma m_i < 0.12$ eV imposed.

Results

Absolute mass scale (NO)



m_2, m_3 vs m_1



Σm_i vs m_1

- Normal ordering with a non-zero lightest mass: $m_1 \approx 0.026$–0.030 eV.
- $m_3 \approx 0.057$ eV; $\Sigma m_i \approx 0.110$–0.120 eV.
- Sharp target for cosmology (Planck, DESI) and for $0\nu\beta\beta$, since $m_{\beta\beta} = |a|$.

Symmetry Realisation

Field content

	D_L	e_R	μ_R	τ_R	H	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	Φ	ξ	ψ	χ	η	ρ	κ
A_4	3	1	$1''$	$1'$	1	$1''$	1	$1'$	3	3	1	1	1	$1''$	1
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
$U(1)_Y$	-1	-2	-2	-2	$+1$	0	0	0	0	0	0	0	0	0	0
Z_{10}	0	0	0	0	0	0	4	3	0	0	7	6	2	0	7
Z_7	-2	4	4	4	1	6	6	6	0	5	0	0	2	2	2
Z_5	0	2	2	2	0	2	0	0	3	0	2	2	0	1	3
Z_3	ω^2	ω	ω	ω	ω	1	1	1	1	1	1	1	1	1	1

- Flavour group $A_4 \otimes Z_{10} \otimes Z_7 \otimes Z_5 \otimes Z_3$; three RH neutrinos.
- Scalars: Higgs doublet H; A_4-triplet flavons Φ, ξ; singlet flavons $\psi, \chi, \eta, \rho, \kappa$.
- The cyclic groups forbid unwanted lower-dimensional operators.

S.T. Goswami, P. Chakraborty, S. Roy, arXiv:2605.18154 [hep-ph] (under review).

$$
\begin{aligned}
-\mathcal{L}_Y = & \frac{y_{L1}}{\Lambda}(\overline{D_L}\Phi)_1 H e_R + \frac{y_{L2}}{\Lambda}(\overline{D_L}\Phi)_{1'} H \mu_R + \frac{y_{L3}}{\Lambda}(\overline{D_L}\Phi)_{1''} H \tau_R \\
& + \frac{y_1}{\Lambda}(\overline{D_L}\Phi)_{1'} \widetilde{H} \nu_{eR} + \frac{y_2}{\Lambda^2}(\overline{D_L}\Phi)_1 \widetilde{H} \nu_{\mu R} \chi + \frac{y_3}{\Lambda^2}(\overline{D_L}\Phi)_{1''} \widetilde{H} \nu_{\tau R} \psi \\
& + \frac{y_c}{2}(\overline{\nu_{eR}^c}\nu_{eR})_{1'}\rho + \frac{y_b}{2}\left[(\overline{\nu_{eR}^c}\nu_{\tau R})_1 + (\overline{\nu_{\tau R}^c}\nu_{eR})_1\right]\kappa + \frac{y_a}{2}(\overline{\nu_{\mu R}^c}\nu_{\mu R})_1 \eta \\
& + \frac{y_w}{\Lambda^2}\left[(D_L D_L^c)_{3_s}\xi\right]_1 \widetilde{H}\widetilde{H} + \text{h.c.}
\end{aligned}
$$

- Hybrid mechanism: **Type-I seesaw** + **Weinberg-like dimension-six operator** (last line):

$$M_\nu = -M_D M_R^{-1} M_D^T + M_w.$$

- The correlations $(M_\nu)_{12} = (M_\nu)_{13}$, $(M_\nu)_{22} = 2(M_\nu)_{12}$ come from M_w, i.e. from the alignment of ξ.

Realisation I

Altarelli–Feruglio (A–F) basis

- VEVs: $\langle \Phi \rangle = v_\Phi (1,0,0)^T$, $\langle \xi \rangle = v_\xi (0,-1,-1)^T$; singlets $v_\psi, v_\chi, v_\eta, v_\rho, v_\kappa$.
- $M_L = \frac{v_H v_\Phi}{\Lambda} \mathrm{diag}(y_{L1}, y_{L2}, y_{L3})$ is diagonal.

$$M_D = \begin{pmatrix} 0 & \frac{y_2 v_H v_\Phi v_\chi}{2\sqrt{2}\Lambda^2} & 0 \\ \frac{y_1 v_H v_\Phi}{2\Lambda} & 0 & 0 \\ 0 & 0 & \frac{y_3 v_H v_\Phi v_\psi}{2\sqrt{2}\Lambda^2} \end{pmatrix}, \qquad M_R = \frac{1}{2\sqrt{2}} \begin{pmatrix} y_c v_\rho & 0 & y_b v_\kappa \\ 0 & y_a v_\eta & 0 \\ y_b v_\kappa & 0 & 0 \end{pmatrix},$$

$$M_w = \frac{y_w v_H^2 v_\xi}{6\sqrt{2}\Lambda^2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{pmatrix}.$$

- In the convention $e \to -e$ $\big(U_L = U_R = \mathrm{diag}(-1,1,1)\big)$:

$$M_\nu = U_L^T \big(- M_D M_R^{-1} M_D^T + M_w \big) U_L,$$

the texture, with $b = -\frac{y_w v_H^2 v_\xi}{6\sqrt{2}\Lambda^2}$. ▸ A_4 rules

Realisation II

Ma–Rajasekaran (M–R) basis

- VEVs: $\langle\Phi\rangle = v_\Phi(1,1,1)^T$, $\langle\xi\rangle = v_\xi(1,-1,1)^T$; same singlets and M_R.
- M_L is no longer diagonal: $U_L^\dagger M_L U_R$ diagonal with

$$U_L = \tfrac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},\ U_R = \mathbf{1}, \quad M_w = \frac{y_w v_\xi v_H^2}{2\sqrt{2}\Lambda^2}\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

- **Permuted charged-lepton correction (PCLC)** plus the $e \to -e$ phase:

$$U_L'' = U_L(R_1 \leftrightarrow R_2)\,\mathrm{diag}(-1,1,1),$$

$$M_\nu = U_L''^{\,T}\big(-M_D M_R^{-1} M_D^T + M_w\big)U_L''.$$

- $\Rightarrow$ the same texture, with $b = -\dfrac{y_w v_H^2 v_\xi}{3\sqrt{2}\Lambda^2}$. ▸ a, t, d ▸ A_4 rules

PCLC: S.T. Goswami, S. Roy, Nucl. Phys. B 1022 (2026) 117275.

The Vacuum Decides

The scalar potential

- Doublet H; A_4 triplet flavons Φ, ξ; singlet flavons $\psi, \chi, \eta, \rho, \kappa$.
- Complete renormalisable potential, invariant under the gauge and flavour symmetries; no cubic term is allowed:

$$V = V_2 + V_4, \qquad V_4 = V_H^{(4)} + V_\Phi^{(4)} + V_\xi^{(4)} + V_S^{(4)} + V_{\text{mix}}^{(4)} + V_{\text{add}}^{(4)}, \quad \text{e.g.}$$

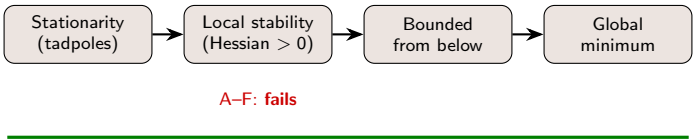
$$V_\Phi^{(4)} = \lambda_{\Phi 1}\left[(\Phi^\dagger\Phi)_1\right]^2 + \lambda_{\Phi 2}(\Phi^\dagger\Phi)_{1'}(\Phi^\dagger\Phi)_{1''} + \lambda_{\Phi 3}\left[(\Phi^\dagger\Phi)_{3_s}(\Phi^\dagger\Phi)_{3_s}\right]_1 + \lambda_{\Phi 4}\left[(\Phi^\dagger\Phi)_{3_a}(\Phi^\dagger\Phi)_{3_a}\right]_1.$$

Full potential: ▸ backup

The Vacuum Decides

Reproducing the texture is not enough

- With this one potential, the chosen VEVs must pass four separate tests:



- **A–F:** tadpoles can be solved, but the 12×12 CP-even Hessian has rank 11:

$$S^{(0)}_{\mathrm{AF}} = \tfrac{1}{\sqrt{2}}\,(\xi_{R3} - \xi_{R2}), \qquad m^2_{S^{(0)}_{\mathrm{AF}}} = 0.$$

A physical **CP-even zero mode** (not a Goldstone) $\Rightarrow$ A–F alignment ✗. ▸ tadpoles

- The $e \to -e$ convention does not touch the scalar sector: the verdict is convention independent.

The Vacuum Decides

M–R: local stability, CP-even sector

- **Stationarity:** 8 tadpoles fix the 8 μ^2; the rest only need $\mathcal{C}_{\rm MR} = 0$ and $\lambda_{\Phi\Phi\xi\rho} = 0$. ▸ μ^2 ▸ rest
- CP-even 12×12 Hessian $\to$ 8 radial $\oplus$ 4 transverse; transverse masses analytic:

$$m^2_{T\pm} = \tfrac{1}{2}\Big[a + b \pm \sqrt{(a-b)^2 + 4(c^2+d^2)}\Big] \quad (\times 2).$$

- Radial block positive via Schur complement. ▸ radial ▸ transverse
- The Higgs boson is identified with the radial eigenstate that is dominantly the doublet:

$$H_{Rh} = \sum_{a=1}^{8} (O_R)_{ah}\, r_a, \qquad m_{H_{Rh}} \simeq 125\ \mathrm{GeV}, \qquad |(O_R)_{1h}|^2 \approx 1,$$

with $r = (h, \Phi_R, \xi_R, \psi_R, \chi_R, \eta_R, \rho_R, \kappa_R)$.

The Vacuum Decides

M–R: local stability, CP-odd sector

- CP-odd 12×12 Hessian: 8 zero eigenvalues $= 1\ G^0\ (\to Z) + 7$ physical.

$$7 = \underbrace{4}_{\text{Goldstones of accidental } U(1)\text{s}} + \underbrace{3}_{\text{phase-flat directions}}$$

- The four accidental global $U(1)$ symmetries survive the Yukawa restrictions.
- Only phases are flat; magnitudes and alignment are fixed. No negative eigenvalue: the M–R vacuum is a local minimum; the seven massless states are lifted by soft terms that leave the vacuum and the texture unchanged. ▸ spectrum ▸ $U(1)$s ▸ soft terms

The Vacuum Decides

M–R: bounded from below and global minimum

- Split the fields into magnitudes $\mathbf{z}$ and triplet directions: $V = V_{\mathrm{MR}}(\mathbf{z}) + \Delta V_{\mathrm{ang}}$.
- Directions: $\Delta V_{\mathrm{ang}} \geq 0$; magnitudes: $Q_{\mathrm{MR}} > 0$. ▸ angular ▸ radial

$$B_{\Phi,\xi} > 0, \quad P_{\Phi,\xi} > 0, \quad C_T^2 + 12\lambda_{\Phi\xi 14}^2 < B_\Phi B_\xi, \quad \Delta^2 < 4 P_\Phi P_\xi.$$

  The **same inequalities** as local stability $\Rightarrow$ bounded from below.

- **Global minimum**, with no extra condition ▸ backup:

$$V - V_{\mathrm{vac}} = (\mathbf{z} - \mathbf{z}_0)^T Q_{\mathrm{MR}} (\mathbf{z} - \mathbf{z}_0) + \Delta V_{\mathrm{ang}} \geq 0.$$

- Stationarity, stability, BFB and global minimality: **one consistent set of inequalities**.

Phenomenology of the Viable Model

Brief overview

- **CLFV:** neutral flavons transverse to $\langle\Phi\rangle$ carry off-diagonal charged-lepton couplings; heavy RH neutrinos contribute via $W^{\pm}$ loops. Present bounds constrain scalar propagators and Dirac entries.
- **Non-unitarity:** the same active–heavy mixing Θ gives $\tilde{\eta} = \frac{1}{2}\Theta\Theta^{\dagger}$, tested against global fits.
- **Leptogenesis:** the conventional Type-I CP asymmetries vanish at leading order: a definite feature of the Yukawa and M_R structure.
- **Domain walls:** the residual discrete symmetry leaves a 64-fold degenerate vacuum orbit; a small quadratic bias lifts it without moving the vacuum or the texture.

S.T. Goswami, P. Chakraborty, S. Roy, arXiv:2605.18154 [hep-ph] (under review).

Summary

- A predictive Majorana texture with $(M_\nu)_{12} = (M_\nu)_{13}$ and $(M_\nu)_{22} = 2(M_\nu)_{12}$: the triadic texture in the $e \to -e$ convention.
- Predictions (NO): $\theta_{23} \approx 47.8^\circ$, $\delta_{\rm CP}$ near 360°, $\alpha \simeq \beta$, $\Sigma m_i \approx 0.11$–0.12 eV.
- Realised with $A_4 \otimes Z_{10} \otimes Z_7 \otimes Z_5 \otimes Z_3$ via Type-I seesaw + Weinberg-like operator, in two bases.
- The scalar potential separates them: A–F has a CP-even zero mode, M–R is a viable, bounded-below minimum.

Reproducing a texture is not sufficient: the vacuum must also be tested.

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Thank You

Backup

Backup

- Analytic solution: a, t, d; masses; why $\theta_{23} \neq 45^\circ$
- A_4 product rules: A–F and M–R bases
- M–R texture parameters
- Complete scalar potential
- A–F vacuum: tadpoles, Hessian
- M–R vacuum: tadpoles
- M–R CP-even sector: radial and transverse blocks
- M–R CP-odd sector, accidental $U(1)$s, soft terms
- Boundedness from below and global minimum

Backup: Analytic Solution of the Texture

a, t, d from the mixing parameters

- $M_\nu = b \left(\begin{smallmatrix} A & 1 & 1 \\ 1 & 2 & T \\ 1 & T & D \end{smallmatrix}\right)$, $(A, T, D) = (a, t, d)/b$; $V =$ standard PDG matrix $(s_{ij}, c_{ij},\ \delta \equiv \delta_{\rm CP})$.
- Off-diagonal entries of $V^T M_\nu V$ vanish $\Rightarrow$ a *linear* system, unique for $\mathcal{D}_{23} \equiv c_{23}^2 + e^{-2i\delta} s_{23}^2 \neq 0$:

$$A = -\frac{\sin 2\theta_{23}\, q_1 + s_{23}^2\, q_2 + c_{23}^2\, q_3}{\mathcal{D}_{23}}, \qquad T = \frac{(c_{23}^2 - e^{-2i\delta} s_{23}^2) q_1 + \frac{1}{2}\sin 2\theta_{23}(q_2 - e^{-2i\delta} q_3)}{\mathcal{D}_{23}},$$

$$D = \frac{-\sin 2\theta_{23}(1 + e^{-2i\delta}) q_1 + \cos 2\theta_{23}(q_2 - e^{-2i\delta} q_3)}{\mathcal{D}_{23}},$$

$$q_1 = -\sin 2\theta_{23} - \tan\theta_{13}\,(c_{23} - s_{23})\, e^{-i\delta}, \qquad q_2 = -2s_{23}^2 + 2\cot 2\theta_{13}\,(c_{23} + s_{23})\, e^{-i\delta},$$

$$q_3 = -2c_{23}^2 - \tan\theta_{13}\,(c_{23} + s_{23})\, e^{i\delta} + \tfrac{2(c_{23} - s_{23})}{c_{13}} \cot 2\theta_{12}.$$

- A, T, D are **not free**: fixed by $(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$. Also $m_{\beta\beta} = |a|$.

◂ Backup index ▸ Masses and θ_{23} ◂ Back to talk

Backup: Masses and $\theta_{23} \neq 45^\circ$

Analytic consequences of the texture

- With A, T, D inserted, $V^T M_\nu V = b\,\mathrm{diag}(F_1, F_2, F_3)$:

$$F_1 = A - \tan\theta_{13}(c_{23}+s_{23})e^{i\delta} - \tfrac{c_{23}-s_{23}}{c_{13}}\tan\theta_{12}, \qquad F_3 = A\,e^{-2i\delta} + \cot\theta_{13}(c_{23}+s_{23})e^{-i\delta},$$
$$F_2 = A - \tan\theta_{13}(c_{23}+s_{23})e^{i\delta} + \tfrac{c_{23}-s_{23}}{c_{13}}\cot\theta_{12}.$$

- Masses: $m_i = |b|\,\rho_i$, $\rho_i \equiv |F_i|$; the solar splitting fixes the scale, $|b|^2 = \dfrac{\Delta m^2_{21}}{\rho_2^2 - \rho_1^2}$.

- The key difference: $\quad F_2 - F_1 = -\dfrac{2\sqrt{2}\,\sin\!\left(\theta_{23} - \frac{\pi}{4}\right)}{c_{13}\,\sin 2\theta_{12}}$.

$\theta_{23} = 45^\circ \;\Rightarrow\; F_1 = F_2 \;\Rightarrow\; m_1 = m_2 \;\Rightarrow\; \Delta m^2_{21} = 0$ ✗ Hence $\theta_{23} \neq 45^\circ$ (both orderings).

- Near 45°: $\rho_2^2 - \rho_1^2 \to 0 \Rightarrow |b|$ and Σm_i grow. $\Sigma m_i < 0.12$ eV pushes θ_{23} away from maximal.

◂ a, t, d ◂ Backup index ◂ Back to results

Backup: The A_4 Group

Altarelli–Feruglio basis (T diagonal)

- Irreps $\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}$; generators $S^2 = T^3 = (ST)^3 = e$; singlets: $1 \otimes 1 = 1$, $1' \otimes 1'' = 1$, $1'' \otimes 1' = 1$.

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad \omega = e^{2\pi i/3}.$$

$$\begin{aligned}
(ab)_1 &= a_1 b_1 + a_2 b_3 + a_3 b_2, \\
(ab)_{1'} &= a_3 b_3 + a_1 b_2 + a_2 b_1, \\
(ab)_{1''} &= a_2 b_2 + a_1 b_3 + a_3 b_1,
\end{aligned}$$

$$(ab)_{3_s} = \frac{1}{3}\begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}, \qquad (ab)_{3_a} = \frac{1}{2}\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_1 b_3 - a_3 b_1 \end{pmatrix}.$$

Back to talk · M–R basis · Backup index

Backup: The A_4 Group

Ma–Rajasekaran basis (S diagonal)

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}.$$

$$\begin{aligned} (ab)_1 &= a_1 b_1 + a_2 b_2 + a_3 b_3, \\ (ab)_{1'} &= a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\ (ab)_{1''} &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\ (ab)_{3_s} &= \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}, \qquad (ab)_{3_a} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}. \end{aligned}$$

- U_ω relates the two triplet bases: same group, different coordinates.

◂ Back to talk ▸ A–F basis ◂ Backup index

Backup

M–R basis: texture parameters ($e \to -e$ convention)

$$a = \frac{v_H^2\big(-9v_\Phi^2 v_\chi^2 y_2^2 + 2v_\eta v_\xi y_a y_w \Lambda^2\big)}{6\sqrt{2}\, v_\eta y_a \Lambda^4}, \qquad b = -\frac{v_H^2 v_\xi y_w}{3\sqrt{2}\Lambda^2},$$

$$t = -\frac{v_H^2\big(18v_\Phi^2 v_\psi y_1 y_3 + \sqrt{2}\, v_\kappa v_\xi y_b y_w \Lambda\big)}{12 v_\kappa y_b \Lambda^3},$$

$$d = \frac{v_H^2\big(18\omega v_\rho v_\Phi^2 v_\psi^2 y_3^2 y_c - 8v_\kappa^2 v_\xi y_b^2 y_w \Lambda^2\big)}{12\sqrt{2}\, v_\kappa^2 y_b^2 \Lambda^4}.$$

- a, t, d are the same as in Ref. [1]; only b changes sign.

◂ Backup index

Backup: Triadic Scalar Sector

Complete scalar potential (I)

$V = V_2 + V_4$, with H, the A_4 triplets Φ, ξ and the singlets $s = \psi, \chi, \eta, \rho, \kappa$; no renormalisable cubic term is allowed.

$$V_2 = -\mu_H^2 (H^\dagger H) - \mu_\Phi^2 (\Phi^\dagger \Phi)_1 - \mu_\xi^2 (\xi^\dagger \xi)_1 - \sum_s \mu_s^2 |s|^2, \qquad V_H^{(4)} = \lambda_H (H^\dagger H)^2,$$

$$V_\Phi^{(4)} = \lambda_{\Phi 1} \left[(\Phi^\dagger \Phi)_1\right]^2 + \lambda_{\Phi 2} (\Phi^\dagger \Phi)_{1'} (\Phi^\dagger \Phi)_{1''} + \lambda_{\Phi 3} \left[(\Phi^\dagger \Phi)_{3_s} (\Phi^\dagger \Phi)_{3_s}\right]_1 + \lambda_{\Phi 4} \left[(\Phi^\dagger \Phi)_{3_a} (\Phi^\dagger \Phi)_{3_a}\right]_1,$$

$$V_\xi^{(4)} = \lambda_{\xi 1} \left[(\xi^\dagger \xi)_1\right]^2 + \lambda_{\xi 2} (\xi^\dagger \xi)_{1'} (\xi^\dagger \xi)_{1''} + \lambda_{\xi 3} \left[(\xi^\dagger \xi)_{3_s} (\xi^\dagger \xi)_{3_s}\right]_1 + \lambda_{\xi 4} \left[(\xi^\dagger \xi)_{3_a} (\xi^\dagger \xi)_{3_a}\right]_1,$$

$$V_S^{(4)} = \sum_s \lambda_s |s|^4 + \sum_{s<t} \lambda_{st} |s|^2 |t|^2 \quad \text{(all ten singlet pairs)},$$

$$V_{\text{mix}}^{(4)} = \lambda_{H\Phi} (H^\dagger H)(\Phi^\dagger \Phi)_1 + \lambda_{H\xi} (H^\dagger H)(\xi^\dagger \xi)_1 + \sum_s \left[\lambda_{Hs} (H^\dagger H) + \lambda_{\Phi s} (\Phi^\dagger \Phi)_1 + \lambda_{\xi s} (\xi^\dagger \xi)_1\right] |s|^2 + V_{\Phi\xi}^{(4)}.$$

▸ $V_{\Phi\xi}^{(4)}$ and the extra term

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Backup: Triadic Scalar Sector

Complete scalar potential (II): the Φ–ξ part

$$V_{\Phi\xi}^{(4)} = \lambda_{\Phi\xi 1} J_1 + \lambda_{\Phi\xi 2} J_2 + \lambda_{\Phi\xi 4} J_4 + \lambda_{\Phi\xi 5} J_5 + \lambda_{\Phi\xi 8} J_8 + \lambda_{\Phi\xi 9} J_9 + \lambda_{\Phi\xi 11} J_{11} + \lambda_{\Phi\xi 12} J_{12} + \lambda_{\Phi\xi 14} J_{14},$$

$$\begin{aligned}
J_1 &= (\Phi^\dagger\Phi)_1(\xi^\dagger\xi)_1, & J_8 &= (\Phi^\dagger\xi)_1(\xi^\dagger\Phi)_1,\\
J_4 &= \left[(\Phi^\dagger\Phi)_{3_s}(\xi^\dagger\xi)_{3_s}\right]_1, & J_5 &= \left[(\Phi^\dagger\Phi)_{3_a}(\xi^\dagger\xi)_{3_a}\right]_1,\\
J_{11} &= \left[(\Phi^\dagger\xi)_{3_s}(\xi^\dagger\Phi)_{3_s}\right]_1, & J_{12} &= \left[(\Phi^\dagger\xi)_{3_a}(\xi^\dagger\Phi)_{3_a}\right]_1,\\
J_2 &= (\Phi^\dagger\Phi)_{1'}(\xi^\dagger\xi)_{1''} + (\Phi^\dagger\Phi)_{1''}(\xi^\dagger\xi)_{1'}, & J_9 &= (\Phi^\dagger\xi)_{1'}(\xi^\dagger\Phi)_{1''} + (\Phi^\dagger\xi)_{1''}(\xi^\dagger\Phi)_{1'},\\
J_{14} &= \left[(\Phi^\dagger\xi)_{3_s}(\xi^\dagger\Phi)_{3_a}\right]_1 - \left[(\Phi^\dagger\xi)_{3_a}(\xi^\dagger\Phi)_{3_s}\right]_1.
\end{aligned}$$

- One further CP-even quartic is allowed by the flavour symmetry ($\lambda_{\Phi\Phi\xi\rho}$ real):

$$V_{\text{add}}^{(4)} = \lambda_{\Phi\Phi\xi\rho}(\mathcal{O}_\rho + \mathcal{O}_\rho^\dagger), \qquad \mathcal{O}_\rho = \left[(\Phi\Phi)_{3_s}\xi^\dagger\right]_{1''}\rho^\dagger;$$

the stationarity conditions then force $\lambda_{\Phi\Phi\xi\rho} = 0$.

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Backup: A–F Vacuum

Tadpoles and μ^2 solutions

- Residual tadpoles (nonzero VEVs) require

$$\lambda_{\xi 2} = \tfrac{4}{9}\lambda_{\xi 3}, \qquad \lambda_{\Phi\xi 11} = -\tfrac{3}{2}\lambda_{\Phi\xi 14} + \tfrac{9}{2}\lambda_{\Phi\xi 2} - \lambda_{\Phi\xi 4} + \tfrac{9}{2}\lambda_{\Phi\xi 9}, \qquad \lambda_{\Phi\Phi\xi_\rho} = 0.$$

- Quadratic parameters ($\mathcal{S} = \{\psi, \chi, \eta, \rho, \kappa\}$):

$$\begin{aligned}
\mu_H^2 &= \tfrac{1}{2}\Big[2\lambda_H v_H^2 + \lambda_{H\Phi} v_\Phi^2 + 2\lambda_{H\xi} v_\xi^2 + \textstyle\sum_s \lambda_{Hs} v_s^2\Big], \\
\mu_\Phi^2 &= \tfrac{1}{36}\Big[18\lambda_{H\Phi} v_H^2 + (36\lambda_{\Phi 1} + 16\lambda_{\Phi 3}) v_\Phi^2 + 18\textstyle\sum_s \lambda_{\Phi s} v_s^2 \\
&\quad + v_\xi^2(36\lambda_{\Phi\xi 1} + 4\lambda_{\Phi\xi 11} - 9\lambda_{\Phi\xi 12} + 12\lambda_{\Phi\xi 14} - 8\lambda_{\Phi\xi 4} + 36\lambda_{\Phi\xi 9})\Big], \\
\mu_\xi^2 &= \tfrac{1}{72}\Big[36\lambda_{H\xi} v_H^2 + (144\lambda_{\xi 1} + 36\lambda_{\xi 2} + 48\lambda_{\xi 3}) v_\xi^2 + 36\textstyle\sum_s \lambda_{\xi s} v_s^2 \\
&\quad + v_\Phi^2(36\lambda_{\Phi\xi 1} + 4\lambda_{\Phi\xi 11} - 9\lambda_{\Phi\xi 12} + 12\lambda_{\Phi\xi 14} - 8\lambda_{\Phi\xi 4} + 36\lambda_{\Phi\xi 9})\Big], \\
\mu_s^2 &= \tfrac{1}{2}\Big[\lambda_{Hs} v_H^2 + \lambda_{\Phi s} v_\Phi^2 + 2\lambda_{\xi s} v_\xi^2 + 2\lambda_s v_s^2 + \textstyle\sum_{t\neq s} \lambda_{st} v_t^2\Big].
\end{aligned}$$

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Backup: A–F Vacuum

The CP-even Hessian

Basis $(h, \phi_{R1}, \phi_{R2}, \phi_{R3}, \xi_{R1}, \xi_{R2}, \xi_{R3} \,|\, \psi_R, \chi_R, \eta_R, \rho_R, \kappa_R)$, after imposing the A–F tadpoles:

$$(M^2_{\text{even}})_{\text{AF}} = \begin{pmatrix} A_{\text{AF}} & Y_{\text{AF}} \\ Y^T_{\text{AF}} & S_{\text{AF}} \end{pmatrix}, \quad A_{\text{AF}} = \begin{pmatrix} \alpha_H & p_{H\Phi} & 0 & 0 & 0 & -p_{H\xi} & -p_{H\xi} \\ p_{H\Phi} & \alpha_\Phi & 0 & 0 & 0 & u & u \\ 0 & 0 & K_{\Phi 2} & C & D_1 & 0 & 0 \\ 0 & 0 & C & B & D_2 & 0 & 0 \\ 0 & 0 & D_1 & D_2 & E & 0 & 0 \\ -p_{H\xi} & u & 0 & 0 & 0 & F & F \\ -p_{H\xi} & u & 0 & 0 & 0 & F & F \end{pmatrix}, \quad Y_{\text{AF}} = \begin{pmatrix} \mathbf{p}_H^T \\ \mathbf{p}_\Phi^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ -\mathbf{p}_\xi^T \\ -\mathbf{p}_\xi^T \end{pmatrix}$$

$$p_{H\xi} = \lambda_{H\xi} v_H v_\xi, \quad F = \tfrac{2}{9}(9\lambda_{\xi 1} + 4\lambda_{\xi 3}) v_\xi^2, \quad (\mathbf{p}_\xi)_s = \lambda_{\xi s} v_\xi v_s,$$

$$u = -\tfrac{v_\Phi v_\xi}{12}\left[12\lambda_{\Phi\xi 1} - 3\lambda_{\Phi\xi 12} + 2\lambda_{\Phi\xi 14} + 6\lambda_{\Phi\xi 2} - 4\lambda_{\Phi\xi 4} + 18\lambda_{\Phi\xi 9}\right].$$

- Rows 6 and 7 (ξ_{R2}, ξ_{R3}) are **identical** $\Rightarrow \operatorname{rank} = 11$ for generic couplings.
- Null vector $u^{(0)}_{\text{AF}} = (0,0,0,0,0,-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0,0,0,0,0)^T$. Remaining entries ($\alpha_{H,\Phi}$, $K_{\Phi 2}$, B, C, $D_{1,2}$, E, S_{AF}): thesis appendix.

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Backup: M–R Vacuum

Tadpoles and μ^2 solutions

- Independent real tadpoles: $T_h = T_{\phi_{R1}} = T_{\xi_{R2}} = T_{\psi_R} = T_{\chi_R} = T_{\eta_R} = T_{\rho_R} = T_{\kappa_R} = 0$ fix the 8 μ^2:

$$
\begin{aligned}
\mu_H^2 &= \tfrac{1}{2}\Big[2\lambda_H v_H^2 + 3\lambda_{H\Phi} v_\Phi^2 + 3\lambda_{H\xi} v_\xi^2 + \textstyle\sum_s \lambda_{Hs} v_s^2\Big], \\
\mu_\Phi^2 &= \tfrac{1}{2}\Big[\lambda_{H\Phi} v_H^2 + 2(3\lambda_{\Phi 1} + 4\lambda_{\Phi 3}) v_\Phi^2 \\
&\qquad + (3\lambda_{\Phi\xi 1} + 4\lambda_{\Phi\xi 11} + 4\lambda_{\Phi\xi 4} + 3\lambda_{\Phi\xi 8}) v_\xi^2 + \textstyle\sum_s \lambda_{\Phi s} v_s^2\Big], \\
\mu_\xi^2 &= \tfrac{1}{2}\Big[\lambda_{H\xi} v_H^2 + 2(3\lambda_{\xi 1} + 4\lambda_{\xi 3}) v_\xi^2 \\
&\qquad + (3\lambda_{\Phi\xi 1} + 4\lambda_{\Phi\xi 11} + 4\lambda_{\Phi\xi 4} + 3\lambda_{\Phi\xi 8}) v_\Phi^2 + \textstyle\sum_s \lambda_{\xi s} v_s^2\Big], \\
\mu_s^2 &= \tfrac{1}{2}\Big[\lambda_{Hs} v_H^2 + 3\lambda_{\Phi s} v_\Phi^2 + 3\lambda_{\xi s} v_\xi^2 + 2\lambda_s v_s^2 + \textstyle\sum_{t\neq s} \lambda_{st} v_t^2\Big], \quad s \in \{\psi, \chi, \eta, \rho, \kappa\}.
\end{aligned}
$$

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Backup: M–R Vacuum

Remaining tadpoles: $\mathcal{C}_{\rm MR} = 0$, $\lambda_{\Phi\Phi\xi\rho} = 0$

- Remaining real tadpoles share one combination:

$$\mathcal{C}_{\rm MR} = \lambda_{\Phi\xi 11} + \lambda_{\Phi\xi 12} + 2\lambda_{\Phi\xi 4} + \lambda_{\Phi\xi 8} - \lambda_{\Phi\xi 9},$$

$$T_{\phi_{R2}} \propto -v_\xi^2 v_\Phi \, \mathcal{C}_{\rm MR}, \quad T_{\xi_{R1}}, T_{\xi_{R3}} \propto v_\xi v_\Phi^2 \, \mathcal{C}_{\rm MR} \;\Rightarrow\; \mathcal{C}_{\rm MR} = 0.$$

- Imaginary-direction tadpoles from $\mathcal{O}_\rho = [(\Phi\Phi)_{3_s}\xi^\dagger]_{1''}\rho^\dagger$:

$$\partial_{\phi_{I1}} V^{(4)}_{\rm add}\big|_0 \propto -\sqrt{3}\,\lambda_{\Phi\Phi\xi\rho} v_\Phi v_\xi v_\rho,$$

$$\partial_{\rho_I} V^{(4)}_{\rm add}\big|_0 \propto +\sqrt{3}\,\lambda_{\Phi\Phi\xi\rho} v_\Phi^2 v_\xi \;\Rightarrow\; \lambda_{\Phi\Phi\xi\rho} = 0.$$

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Backup: M–R Vacuum

M–R: CP-even radial block (I)

- Vacuum-adapted rotation: $P_{8+4}^T U_{\rm trip}^T M^2_{\rm even} U_{\rm trip} P_{8+4} = M_R^2 \oplus M_T^2$, with

$$U_\Phi = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad U_\xi = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

  (third columns = VEV directions).

- Radial basis $\mathbf{r} = (h, \Phi_R, \xi_R, \psi_R, \chi_R, \eta_R, \rho_R, \kappa_R)$, $M_R^2 = \begin{pmatrix} M_{3R}^2 & X_R \\ X_R^T & M_S^2 \end{pmatrix}$:

$$M_{3R}^2 = \begin{pmatrix} 2\lambda_H v_H^2 & \sqrt{3}\lambda_{H\Phi} v_H v_\Phi & \sqrt{3}\lambda_{H\xi} v_H v_\xi \\ \sqrt{3}\lambda_{H\Phi} v_H v_\Phi & 2A_\Phi v_\Phi^2 & C_R v_\Phi v_\xi \\ \sqrt{3}\lambda_{H\xi} v_H v_\xi & C_R v_\Phi v_\xi & 2A_\xi v_\xi^2 \end{pmatrix},$$

$$(X_R)_{1s} = \lambda_{Hs} v_H v_s, \;\; (X_R)_{2s} = \sqrt{3}\lambda_{\Phi s} v_\Phi v_s, \;\; (X_R)_{3s} = \sqrt{3}\lambda_{\xi s} v_\xi v_s, \;\; (M_S^2)_{ss} = 2\lambda_s v_s^2, \;\; (M_S^2)_{st} = \lambda_{st} v_s v_t.$$

$$A_\Phi = 3\lambda_{\Phi 1} + 4\lambda_{\Phi 3}, \quad A_\xi = 3\lambda_{\xi 1} + 4\lambda_{\xi 3}, \quad C_R = 3\lambda_{\Phi\xi 1} + 4\lambda_{\Phi\xi 11} + 4\lambda_{\Phi\xi 4} + 3\lambda_{\Phi\xi 8}.$$

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Backup: M–R Vacuum

M–R: CP-even radial block (II): positivity

- $M_R^2 = D_v K_R D_v$; exact positivity: $K_3 > 0$ and $K_S - P^T K_3^{-1} P > 0$ (Schur), where $K_3 > 0$ means

$$\lambda_H > 0, \quad 4\lambda_H A_\Phi - 3\lambda_{H\Phi}^2 > 0,$$

$$4\lambda_H A_\Phi A_\xi - 3A_\Phi \lambda_{H\xi}^2 - 3A_\xi \lambda_{H\Phi}^2 - \lambda_H C_R^2 + 3C_R \lambda_{H\Phi} \lambda_{H\xi} > 0.$$

- Higgs = eigenstate of M_R^2 with $m \simeq 125$ GeV and $|(O_R)_{1h}|^2 \simeq 1$.

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Backup: M–R Vacuum

M–R: CP-even transverse block (I)

- Basis $\mathbf{t} = (\Phi_{T1}, \Phi_{T2}, \xi_{T1}, \xi_{T2})$, with

$$B_\Phi = 3\lambda_{\Phi 2} - 4\lambda_{\Phi 3}, \quad B_\xi = 3\lambda_{\xi 2} - 4\lambda_{\xi 3}, \quad C_T = -2\lambda_{\Phi\xi 11} + 3\lambda_{\Phi\xi 2} - 2\lambda_{\Phi\xi 4} + 3\lambda_{\Phi\xi 9},$$

$$a = B_\Phi v_\Phi^2, \quad b = B_\xi v_\xi^2, \quad c = C_T v_\Phi v_\xi, \quad d = 2\sqrt{3}\lambda_{\Phi\xi 14} v_\Phi v_\xi,$$

$$M_T^2 = \begin{pmatrix} a & 0 & c & -d \\ 0 & a & -d & -c \\ c & -d & b & 0 \\ -d & -c & 0 & b \end{pmatrix}.$$

- Eigenvalues (each doubly degenerate):

$$m_{T\pm}^2 = \tfrac{1}{2}\Big[a + b \pm \sqrt{(a-b)^2 + 4(c^2 + d^2)}\Big].$$

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Backup: M–R Vacuum

M–R: CP-even transverse block (II)

- Rotation: $s_T = \sqrt{c^2 + d^2}$, $\cos 2\gamma = c/s_T$, $\sin 2\gamma = -d/s_T$; $\cos 2\beta = (a-b)/D_T$, $\sin 2\beta = 2s_T/D_T$, $D_T = \sqrt{(a-b)^2 + 4s_T^2}$; $O_T^T M_T^2 O_T = \mathrm{diag}(m_{T+}^2, m_{T+}^2, m_{T-}^2, m_{T-}^2)$.
- Full CP-even: $U_{\mathrm{even}} = U_{\mathrm{trip}} P_{8+4}\,\mathrm{diag}(O_R, O_T)$; positivity of the transverse masses:

$$B_\Phi > 0, \qquad B_\xi > 0, \qquad C_T^2 + 12\lambda_{\Phi\xi 14}^2 < B_\Phi B_\xi .$$

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Backup: M–R Vacuum

M–R: CP-odd sector (I): basis and couplings

- Adapted phases (same U_Φ, U_ξ): $\Phi_{IT1} = \frac{-\phi_{I1}+\phi_{I3}}{\sqrt{2}}$, $\Phi_{IT2} = \frac{-\phi_{I1}+2\phi_{I2}-\phi_{I3}}{\sqrt{6}}$, $\Phi_{I0} = \frac{\phi_{I1}+\phi_{I2}+\phi_{I3}}{\sqrt{3}}$; similarly for ξ with $\xi_{I0} = \frac{\xi_{I1}-\xi_{I2}+\xi_{I3}}{\sqrt{3}}$.
- Couplings: $P_\Phi = -(\lambda_{\Phi 3} + \lambda_{\Phi 4})$, $P_\xi = -(\lambda_{\xi 3} + \lambda_{\xi 4})$, $\Delta = \lambda_{\Phi\xi 4} - \lambda_{\Phi\xi 5} + \lambda_{\Phi\xi 8} - \lambda_{\Phi\xi 9}$; $A = 6P_\Phi v_\Phi^2$, $B = 6P_\xi v_\xi^2$, $D = \Delta v_\Phi v_\xi$.

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Backup: M–R Vacuum

M–R: CP-odd sector (II): spectrum

- Two 2×2 blocks:

$$M_1^2 = \begin{pmatrix} A & D \\ D & B \end{pmatrix}, \qquad M_2^2 = \begin{pmatrix} A & 3D \\ 3D & B \end{pmatrix},$$

$$m_{1\pm}^2 = \tfrac{1}{2}\Big[A + B \pm \sqrt{(A-B)^2 + 4D^2}\Big], \quad m_{2\pm}^2 = \tfrac{1}{2}\Big[A + B \pm \sqrt{(A-B)^2 + 36D^2}\Big],$$

$$\tan 2\theta_1 = \frac{2D}{A-B}, \quad \tan 2\theta_2 = \frac{6D}{A-B},$$

$$A_{i+} = c_i \Phi_{ITi} + s_i \xi_{ITi}, \quad A_{i-} = -s_i \Phi_{ITi} + c_i \xi_{ITi}.$$

- Spectrum: $\{m_{1\pm}^2, m_{2\pm}^2\}$ + **8 zero modes** $= G^0 + (\Phi_{I0}, \xi_{I0}, \psi_I, \chi_I, \eta_I, \rho_I, \kappa_I)$.
- Positivity: $P_\Phi > 0, \qquad P_\xi > 0, \qquad \Delta^2 < 4 P_\Phi P_\xi$.

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Backup: M–R Vacuum

M–R: Accidental $U(1)$ symmetries

- Rephasing charges q_F leaving the retained Yukawa/Majorana operators invariant:

$$\begin{aligned} -q_D + q_\Phi + q_H + q_{e,\mu,\tau} &= 0, & -q_D + q_\Phi - q_H + q_{\nu_e} &= 0, \\ -q_D + q_\Phi - q_H + q_{\nu_\mu} + q_\chi &= 0, & -q_D + q_\Phi - q_H + q_{\nu_\tau} + q_\psi &= 0, \\ 2q_{\nu_e} + q_\rho &= 0, & q_{\nu_e} + q_{\nu_\tau} + q_\kappa &= 0, \\ 2q_{\nu_\mu} + q_\eta &= 0, & -2q_D - 2q_H + q_\xi &= 0. \end{aligned}$$

- 15 charges, 10 independent equations $\Rightarrow$ 5-dimensional solution space; minus hypercharge $\Rightarrow$ **4 accidental global** $U(1)$**s**.
- Broken by the vacuum $\Rightarrow$ 4 Goldstones; the other 3 zero modes are tree-level phase-flat directions (no protecting symmetry):

$$8 = 1\ G^0 + \underbrace{4\ \text{Goldstones} + 3\ \text{flat}}_{7\ \text{physical}}.$$

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Backup: Soft Breaking

Soft breaking: lifting the zero modes

- Why: exactly massless CP-odd states can be restrictive (stellar, astrophysical, early-Universe bounds).
- The 7 physical CP-odd zero modes are lifted by 7 dimension-two soft terms backup:

$$\Delta V_{\text{soft}} = s_\Phi \sum_i \phi_{Ii}^2 + s_\xi \sum_i \xi_{Ii}^2 + \sum_{X=\psi,\chi,\eta,\rho,\kappa} s_X X_I^2 .$$

- They vanish (with first derivatives) on the M–R vacuum:
  - ▷ vacuum, BFB and global minimum unchanged;
  - ▷ $\Delta M_\ell = \Delta M_D = \Delta M_R = \Delta M_w = 0 \Rightarrow$ texture unchanged.

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Backup: Soft Breaking

Symmetry content of the soft terms

Term	A_4	Z_{10}	Z_7	Z_5	Z_3	$U(1)$ broken
s_Φ	✓	✓	✓	✗	✓	A, C
s_ξ	✓	✓	✗	✓	✓	A
s_ψ	✓	✗	✓	✗	✓	C, D
s_χ	✓	✗	✓	✗	✓	B, C
s_η	✓	✗	✗	✓	✓	B
s_ρ	✗	✓	✗	✗	✓	C
s_κ	✓	✗	✗	✗	✓	D

- All seven preserve the electroweak group, Z_3 and CP (real s_X); dimension two only $\Rightarrow$ soft.

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Backup: Boundedness from Below

Angular part (I)

- Radial variables $\mathbf{z} = (z_H, z_\Phi, z_\xi, z_\psi, z_\chi, z_\eta, z_\rho, z_\kappa)$, $z_H = H^\dagger H$, $z_\Phi = (\Phi^\dagger\Phi)_1$, $z_s = |s|^2$; $\Phi = \sqrt{z_\Phi}\, n_\Phi$, $\xi = \sqrt{z_\xi}\, n_\xi$.
- Triplet self-terms, $U_X = \frac{(X^\dagger X)_{1'}(X^\dagger X)_{1''}}{z_X^2} \geq 0$, $W_X = -\frac{[(X^\dagger X)_{3_a}(X^\dagger X)_{3_a}]_1}{z_X^2} \geq 0$:

$$V_X^{(4)} = z_X^2\Big(\frac{A_X}{3} + \frac{B_X}{3}U_X + P_X W_X\Big),$$

  with $U_X = W_X = 0$ on the M–R directions.
- Mixed quartic (recoupling + stationarity):

$$V_{\Phi\xi}^{(4)} = \frac{C_R}{3}J_1 + \frac{C_T}{3}J_2 - \Delta J_5 + \lambda_{\Phi\xi 14}J_{14},$$

$$j_2^2 + \tfrac{3}{4}j_{14}^2 = 4U_\Phi U_\xi, \quad |j_5| \leq \sqrt{W_\Phi W_\xi}.$$

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Backup: Boundedness from Below

Angular part (II)

- Cauchy–Schwarz:

$$\left|\tfrac{C_T}{3} j_2 + \lambda_{\Phi\xi 14} j_{14}\right| \le \tfrac{2}{3}\sqrt{C_T^2 + 12\lambda_{\Phi\xi 14}^2}\sqrt{U_\Phi U_\xi}, \qquad |\Delta j_5| \le |\Delta|\sqrt{W_\Phi W_\xi}.$$

- Hence the **same** inequalities as the local spectra, $C_T^2 + 12\lambda_{\Phi\xi 14}^2 < B_\Phi B_\xi$, $\Delta^2 < 4 P_\Phi P_\xi$ (with $B_X, P_X > 0$), give $\Delta V_{\rm ang} \ge 0$ for *all* triplet directions.

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Backup: Boundedness from Below

Radial part

- Along M–R directions: $V_{4,\mathrm{rad}} = \mathbf{z}^T Q_{\mathrm{MR}} \mathbf{z}$, $Q_{\mathrm{MR}} = \begin{pmatrix} Q_3 & R \\ R^T & Q_S \end{pmatrix}$:

$$Q_3 = \begin{pmatrix} \lambda_H & \lambda_{H\Phi}/2 & \lambda_{H\xi}/2 \\ \lambda_{H\Phi}/2 & A_\Phi/3 & C_R/6 \\ \lambda_{H\xi}/2 & C_R/6 & A_\xi/3 \end{pmatrix},$$

$$R_{1s} = \tfrac{\lambda_{Hs}}{2}, \quad R_{2s} = \tfrac{\lambda_{\Phi s}}{2}, \quad R_{3s} = \tfrac{\lambda_{\xi s}}{2}, \quad (Q_S)_{ss} = \lambda_s, \quad (Q_S)_{st} = \tfrac{\lambda_{st}}{2}.$$

- Exact criterion: copositivity ($z_i \geq 0$). Sufficient: $Q_{\mathrm{MR}} > 0 \iff Q_3 > 0,\ Q_S - R^T Q_3^{-1} R > 0$.
- Q_{MR} and the radial CP-even Hessian are related by a non-singular VEV rescaling.

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Backup: Boundedness from Below

Global minimum

- **Global minimum** (tadpoles used):

$$V(\mathbf{z}, n_\Phi, n_\xi) - V(\mathbf{z}_0, n_\Phi^{\mathrm{MR}}, n_\xi^{\mathrm{MR}}) = (\mathbf{z} - \mathbf{z}_0)^T Q_{\mathrm{MR}} (\mathbf{z} - \mathbf{z}_0) + \Delta V_{\mathrm{ang}} \geq 0.$$

- Sufficient (not necessary) region:

$$Q_{\mathrm{MR}} > 0, \quad B_\Phi, B_\xi, P_\Phi, P_\xi > 0, \quad C_T^2 + 12\lambda_{\Phi\xi 14}^2 < B_\Phi B_\xi, \quad \Delta^2 < 4 P_\Phi P_\xi.$$

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