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Holographic study of one-point correlations in the expanding universe

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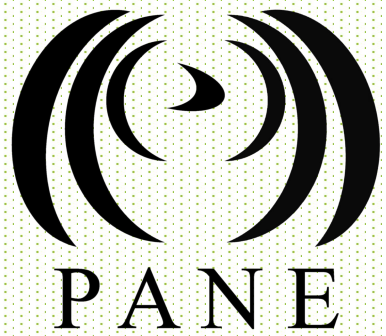

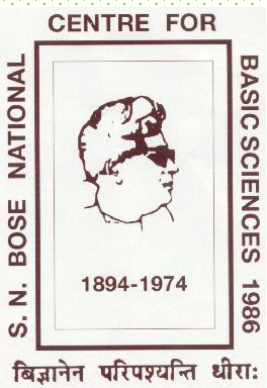



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Outline

- Previous studies
- Braneworld model and Isreal junction condition
- p-brane gas geometry
- Holographic calculation of one-point function
- Holographic one-point function in a single component universe
- Holographic one-point function in a multi-component universe
- Summary
- Referances

Previous studies

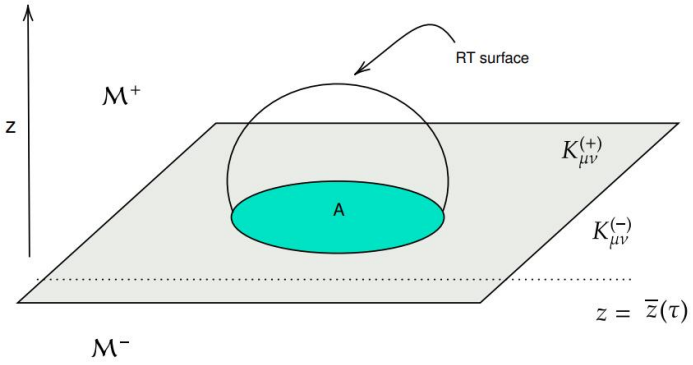
- In **Phys. Rev. D 101 (2020) 126006** , braneworld model has been used to calculate the entanglement entropy (EE) of the universe using the holographic principle.

- In a series of works (**JHEP 08 (2025) 164, PTEP 2026 (2026) 7, 073B11**), we have extended this idea to compute the EE and complexity of various single and multi-component universe.

- Recently in **Mod.Phys.Lett.A 41 (2026) 04n05, 2550233** the authors have studied the time evolution of two-point correlation functions of local operators in an expanding universe.

Braneworld model of cosmology

- According to the Randall-Sundrum braneworld model our universe is situated on a brane which is embedded in a higher dimensional spacetime.



- The Bulk action is given by

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}(\mathcal{R} - 2\Lambda^{(\pm)}) + S_M^{(\pm)} - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}K^{(\pm)}$$

- The variation of the boundary terms of the above action gives

$$\delta S = -\frac{1}{16\pi G}\int_{\partial M} d^4x\sqrt{-\gamma}(K^{(\pm)}_{\mu\nu} - \gamma_{\mu\nu}K^{(\pm)})\delta\gamma^{\mu\nu}$$

Phys. Rev. Lett. 83 (1999) 4690

Braneworld model of cosmology

- Hence the stress tensor of the boundary theory is given by

$$t_{\mu\nu}^{(\pm)} = -\frac{1}{16\pi G}(K_{\mu\nu}^{(\pm)} - \gamma_{\mu\nu}K^{(\pm)})$$

Because of the $\mathcal{Z}_2$ symmetry, the extrinsic curvature tensor of the two bulk spacetimes satisfy $K_{\mu\nu}^{(+)} = -K_{\mu\nu}^{(-)}$. Therefore, if we denote $K_{\mu\nu} = K_{\mu\nu}^{(-)}$, the energy momentum tensor can be written as

$$t_{\mu\nu}^{(\pm)} = \pm\frac{1}{16\pi G}(K_{\mu\nu} - \gamma_{\mu\nu}K)$$

- Now the two boundary stress tensors derived in $\mathcal{M}^{\pm}$ are not the same. This is because of the brane's own nonvanishing stress tensor. Consequently, the brane's stress tensor must cancel the discrepancy of the two boundary stress tensors.
- If we denote $T_{\mu\nu}^{b}$ to be the brane's energy momentum tensor then the Isreal junction condition tells us

$$t_{\mu\nu}^{(+)} - t_{\mu\nu}^{(-)} = T_{\mu\nu}^{b}$$

Braneworld model of cosmology

- In this analysis, the brane is in its ground state with constant energy density and pressure. In this case, the brane action is given by

$$S^b = -\frac{\sigma}{4\pi G}\int_{\partial M} d^4x\sqrt{-\gamma} \longrightarrow T^b_{\mu\nu} = \frac{1}{\sqrt{-\gamma}}\frac{\delta S^b}{\delta \gamma_{\mu\nu}} \longrightarrow T^b_{\mu\nu} = \frac{\sigma}{8\pi G}\gamma_{\mu\nu}$$

- Therefore the Isreal junction condition becomes

$$K_{\mu\nu} - \gamma_{\mu\nu}K = \sigma\gamma_{\mu\nu} \xrightarrow[\text{compute K and put it back}]{\text{Take trace with boundary metric,}} K_{\mu\nu} = -\frac{\sigma}{3}\gamma_{\mu\nu}$$

- Let us consider an arbitrary spacetime metric in (4+1)-dimensions

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)\delta_{ij}dx^i dx^j$$

- Using the above spacetime metric and the junction condition, one can calculate the special component of the extrinsic curvature, which reads

$$K_{ij} = -\frac{\sqrt{AB}}{B}\frac{C'}{C}\frac{1}{\sqrt{A - B\dot{r}^2}}\gamma_{ij}$$

$$\frac{C'}{C} = \frac{\sigma}{3}\sqrt{\frac{B}{A}}\sqrt{A - B\dot{r}^2}$$

Braneworld model of cosmology

- In order to study the time dynamics of the universe we must introduce the cosmological time (τ). To do so we will do the following reparametrization

$$-d\tau^2 = -A(r)dt^2 + B(r)dr^2$$

- Identifying C(r) as the scale factor of the universe (a(τ))

$$ds_b^2 = -d\tau^2 + a(\tau)^2\delta_{ij}dx^i dx^j$$

- Finally the rate of chanage of brane's radial position with cosmic time becomes

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{\sigma}{3}\right)^2\left(\frac{C}{C'}\right)^2 - \frac{1}{B}$$

- The above equation will be useful to determine the time-dependent radial position of the brane for different matter-dominated universes.

p-brane gas geometry

- The action governing the p-branes distributed in the (d + 1)-dimensional AdS space is given by

$$S = \frac{1}{16\pi G}\int d^{d+1}x\sqrt{-g}(\mathcal{R}-2\Lambda) + T_p N_p \int d^{p+1}\xi\sqrt{-h}\partial^{\alpha}x_{\mu}h_{\alpha\beta}\partial^{\beta}x_{\nu}g^{\mu\nu}$$

- One can derive Einstein's equations as follows

$$R_{MN} - \frac{1}{2}g_{MN}R + \Lambda g_{MN} = 16\pi G T_{MN}$$

$$T_{MM} = -\frac{T_p n_p R^{d-p}}{r^{d-p}}\left\{g_{tt}, \frac{p-1}{d-1}g_{11}, \frac{p-1}{d-1}g_{22}, \ldots, \frac{p-1}{d-1}g_{(d-1)(d-1)}, g_{rr}\right\}$$

- Now to solve Einstein's equation let us take the ansatz for the spacetime metric to be AdS black brane

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{R^2}{r^2 f(r)}$$ where, i, j = 2, 3 . . . , d.

- This ansatz along with Einstein's equation leads to the following expression of the brane's lapse function

$$f(r) = 1 - \frac{\rho_p}{r^{4-p}} - \frac{C}{r^4}$$

where ρ_p is the energy density of the p-brane gas

$$\rho_p = 8\pi G c_p n_p T_p R^{d-p+2}$$

Junction conditions

Junction condition

$$\frac{1}{r^2}\left(\frac{dr}{d\tau}\right)^2 = \frac{\rho_p}{r^{4-p}}$$

Friedmann Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^{3(1+\omega)}}$$

$$p = 4 - 3(1+\omega)$$

Type of universe	Time-dependent brane position
Pure AdS	$\bar{r}(\tau) = r_0 e^{H\tau}$
Radiation dominated universe	$\bar{r}(\tau) = \frac{\sqrt{2}m^{1/4}}{\sqrt{R}}\tau^{1/2} + r_i$
Matter dominated universe	$\bar{r}(\tau) = \left(\frac{3}{2}\right)^{2/3}\frac{\mathscr{v}^{1/3}}{R^{2/3}}\tau^{2/3} + r_i$
Exotic matter dominated universe	$\bar{r}(\tau) = \frac{\sqrt{\delta}}{R}\tau + r_i$

Junction conditions (multi-component universe)

- For a multi-component universe with coexsiting radiation and matter the junction condition becomes

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{m}{r^2 R^2} + \frac{\mathscr{e}}{R^2 r} \qquad r_t = m/\mathscr{e},$$

- In the early time limit ($\tau \to 0, r \ll r_t$), the above equation, after integration gives

$$\left(\frac{r^2}{2} - \frac{r_i^2}{2}\right) - \left(\frac{r^3}{6r_t} - \frac{r_i^3}{6r_t}\right) = \tau\sqrt{m}$$

- Upon inverting this perturbatively, we get

$$r \simeq r_i + \frac{\sqrt{\tilde{m}}\,\tau}{r_i} + \frac{\sqrt{\tilde{m}}\,\tau}{2r_t} \quad \Longrightarrow \quad \bar{z} \simeq z_i\left(1 - \sqrt{\tilde{m}}\,\tau\left(z_i^2 + \frac{z_i z_t}{2}\right)\right)$$

- In the early time limit ($\tau \to \infty, r \gg r_t$), the junction integral equation, can be written as

$$\tau = \int_{r_i}^{r_t} dr \frac{r}{\sqrt{\tilde{m} + \tilde{\mathscr{e}}\, r}} + \int_{r_t}^{r} d\tilde{r} \frac{\sqrt{\tilde{r}}}{\sqrt{\tilde{\mathscr{e}}}}\left(1 - \frac{r_t}{2\tilde{r}}\right)$$

Junction conditions (multi-component universe)

- After integrating perturbatively and performing a series inversion, we get

$$r \simeq \left(\frac{3}{2}\right)^{2/3} \tilde{z}^{1/3}\tau^{2/3}\left(1-\left(\frac{2}{3}\right)^{2/3}\frac{r_t}{\tilde{z}^{1/3}}\tau^{-2/3}+\frac{2}{3\sqrt{\tilde{m}}}\frac{1}{\tau}\left(\frac{2r_t^2}{3}-\frac{r_i^2}{2}+\frac{r_i^3}{6r_t}\right)\right)$$

$$\bar{z} \simeq \left(\frac{2}{3}\right)^{2/3} \tilde{z}^{-1/3}\tau^{-2/3}\left(1+\left(\frac{2}{3}\right)^{2/3}\frac{r_t}{\tilde{z}^{1/3}}\tau^{-2/3}-\frac{2\sqrt{z_t}}{3\sqrt{\tilde{z}}}\frac{1}{\tau}\left(\frac{2}{3z_t^2}-\frac{1}{2z_i^2}+\frac{z_t}{6z_i^3}\right)\right)$$

Holographic calculation of one-point function

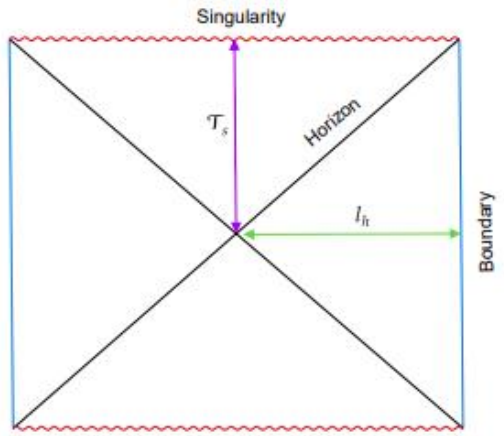


Figure 1: Penrose diagram of an eternal AdS-Schwarzschild black hole. Here, the blue lines indicate the AdS boundary where the conformal field theory lives. The wiggly red lines denote the black hole singularity. Here l_h is the regularised geodesic length from the boundary to the horizon and $\mathcal{T}_s$ denotes the time-like distance between the black hole horiozn to singularity.

- In an asymptotically AdS_{d+1} backgroumd minimally coupled scalar field φ of mass m is dual to a boundary operator $\mathcal{O}$ in one lower dimension. The boundary operator has a conformal dimension

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 \tilde{R}^2}$$

- In JHEP 03 (2021) 131 , Grinberg and Maldacena had derived a geodesic formula for the holographic one point function for a boundary operator, which is given by

$$\langle \mathcal{O} \rangle \sim e^{-\Delta(l_{hor} + i\mathcal{T}_s)} .$$

l_h : Regularised spacelike geodesic length from AdS boundary to the horizon

$\mathcal{T}_s$: The proper time the particle takes to reach the singularity from the horizon

One-point function in Braneworld model

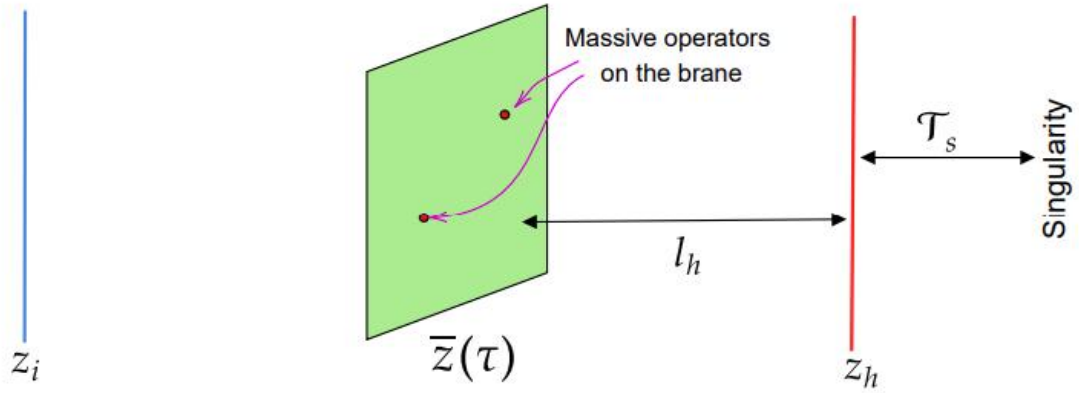


Figure 2: A schematic representation of the location of massive operators and the spacelike geodesic length from the brane to the horizon in the braneworld model and the timelike geodesic length from the horizon to the singularity. Here, z_i, z_h, and $\bar{z}(\tau)$ are the initial brane position, horizon location, and the current brane position at a fixed cosmological time τ, respectively. The massive operators with conformal dimension Δ are denoted by red dots on the brane.

One-point function in Braneworld model

- We will start by calculating the regularised spacelike length from infinity to the horizon, which is given by

$$l_h = \int_{\bar{z}}^{z_h} \frac{dz}{z\sqrt{f(z)}}$$

- Now, in order to evaluate the above integral, we will use the following binomial expansion

$$l_h = \int_{\bar{z}}^{z_h} \frac{dz}{z} \sum_{n=0}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\Gamma(n+1)} \left(\frac{z}{z_h}\right)^{4n-np} .$$

- Now, separating out the leading order term corresponding to n = 0 from all of the other terms, we get

$$l_h = \int_{\bar{z}}^{z_h} dz \left[\frac{1}{z} + \frac{1}{z} \sum_{n=1}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\Gamma(n+1)} \left(\frac{z}{z_h}\right)^{4n-np}\right]$$

$$= \log\left(\frac{z_h}{\bar{z}}\right) + \sum_{n=1}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\Gamma(n+1)} \frac{1}{(4n-np)} + \dots .$$

- After evaluating the sum in the second term, we get

$$l_h = \log\left(\frac{z_h}{\bar{z}}\right) + \left(\frac{\log 4}{4-p}\right) .$$

One-point function in Braneworld model

- The proper time the particle takes to reach the singularity from the horizon is given by the following integral

$$\mathcal{T}_s = \int_{z_h}^{\infty} \frac{dz}{z\sqrt{\left(\frac{z}{z_h}\right)^{4-p} - 1}} \, .$$

- After doing binomial expansion, we get

$$\mathcal{T}_s = \int_{z_h}^{\infty} \frac{dz}{z_h} \sum_{n=0}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\Gamma(n+1)} \left(\frac{z_h}{z}\right)^{4n-np+3-\frac{p}{2}} = \sum_{n=0}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\Gamma(n+1)} \frac{1}{(4n-np+2-\frac{p}{2})} \, .$$

- Evaluating the above summation, we get the final expression for the proper time required to reach the singularity from the horizon

$$\mathcal{T}_s = \frac{\pi}{(4-p)} \, .$$

- Using the geodesic formlua for the one-point function

$$\langle \mathcal{O} \rangle_p \approx \left(\frac{\bar{z}}{z_h}\right)^{\Delta} 2^{\frac{-2\Delta}{4-p}} \times e^{\frac{-i\Delta\pi}{4-p}} \, .$$

Time dependence through brane position

$$\langle \mathcal{O} \rangle_p(\tau) \approx \left(\frac{\bar{z}(\tau)}{z_h}\right)^{\Delta} 2^{\frac{-2\Delta}{4-p}} \times e^{\frac{-i\Delta\pi}{4-p}} \, .$$

Radiation dominated universe

- The radiation dominated universe is obtained through the backreaction of bulk 0-brane gas in the braneworld model. For p=0, the one-point function becomes

$$\langle \mathcal{O} \rangle_0 = \left(\frac{\bar{z}}{z_h}\right)^{\Delta} e^{-\Delta/2}$$

- In the early time ($\tau \to 0$), the one-point function of the radiation dominated universe becomes

$$\langle \mathcal{O} \rangle_0^{(early)} = \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \frac{\sqrt{2} z_i}{z_h} \sqrt{\tau}\right)^{\Delta} e^{-\frac{\Delta}{2}} \approx \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \frac{\Delta \sqrt{2} z_i}{z_h} \sqrt{\tau}\right) e^{-\frac{\Delta}{2}}$$

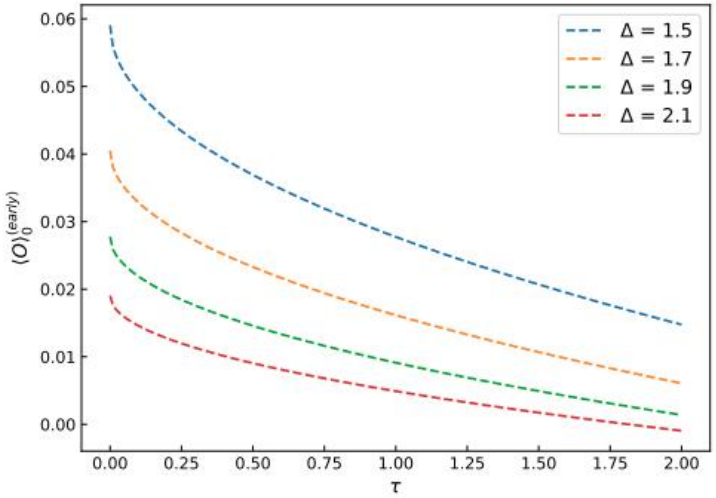
- Similarly, using the late-time expression for the brane position for a radiation-dominated universe, one gets the following

$$\langle \mathcal{O} \rangle_0^{(late)} = \frac{\tau^{-\frac{\Delta}{2}}}{2^{\frac{\Delta}{2}}} e^{-\frac{\Delta}{2}}$$

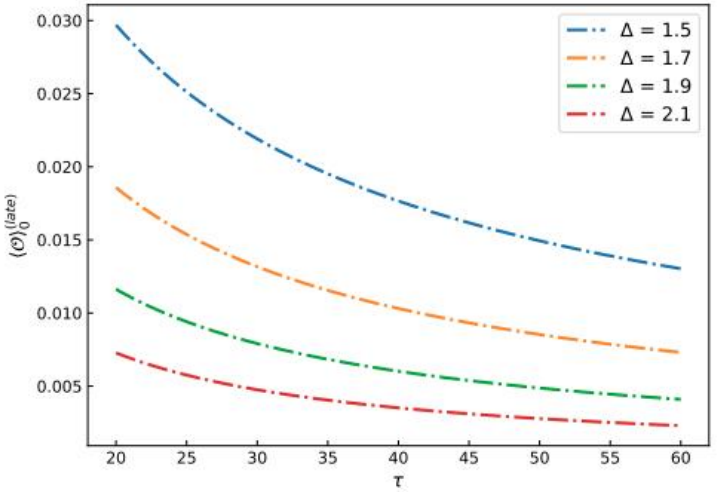
Radiation dominated universe

Early time leading behavior: $\sqrt{\tau}$

Late time leading behavior: $\tau^{-\frac{\Delta}{2}}$



(a) $\langle O \rangle_0^{(early)}$ vs τ.



(b) $\langle O \rangle_0^{(late)}$ vs τ.

Matter dominated universe

- The radiation dominated universe is obtained through the backreaction of bulk 0-brane gas in the braneworld model. For p=1, the one-point function becomes

$$\langle \mathcal{O} \rangle_1 = \left(\frac{\bar{z}}{z_h}\right)^{\Delta} e^{-2\Delta/3}$$

- In the early time ($\tau \to 0$), the one-point function of the matter dominated universe becomes

$$\langle \mathcal{O} \rangle_1^{(early)} = \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \left(\frac{3}{2}\right)^{\frac{2}{3}} \tilde{\dot{\mathcal{V}}}^{\frac{1}{3}} \tau^{\frac{2}{3}} z_i\right)^{\Delta} e^{-2\Delta/3} \approx \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \Delta \left(\frac{3}{2}\right)^{\frac{2}{3}} \tilde{\dot{\mathcal{V}}}^{\frac{1}{3}} \tau^{\frac{2}{3}} z_i\right) e^{-2\Delta/3}$$

- Using the late-time expression for the brane position for a matter-dominated universe, one gets the following

$$\langle \mathcal{O} \rangle_1^{(late)} = \left(\frac{\left(\frac{2}{3}\right)^{\frac{2}{3}}}{\tilde{\dot{\mathcal{V}}}^{\frac{1}{3}} z_h \tau^{\frac{2}{3}}}\right)^{\Delta} e^{-2\Delta/3}$$

Exotic matter dominated universe

- The radiation dominated universe is obtained through the backreaction of bulk 0-brane gas in the braneworld model. For p=2, the one-point function becomes

$$\langle \mathcal{O} \rangle_2 = \left(\frac{\bar{z}}{z_h}\right)^{\Delta} e^{-\Delta}$$

- In the early time ($\tau \to 0$), the one-point function of the exotic matter dominated universe becomes

$$\langle \mathcal{O} \rangle_2^{(early)} = \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \sqrt{\tilde{\delta}} z_i \tau\right)^{\Delta} 2^{-\Delta} \approx \left(\frac{z_i}{z_h}\right)^{\Delta} \left(1 - \Delta \sqrt{\tilde{\delta}} z_i \tau\right) 2^{-\Delta}$$

- Using the late-time expression for the brane position for a exotic matter-dominated universe, one gets the following

$$\langle \mathcal{O} \rangle_2^{(late)} = \left(\frac{1}{\sqrt{\tilde{\delta}} \tau z_h}\right)^{\Delta} 2^{-\Delta}$$

Multi-component universe

- For a universe with coexisting radiation and matter, the black brane lapse function in the braneworld model is given by

$$f(z) = 1 - \tilde{\boldsymbol{\imath}} z^3 - \tilde{m} z^4$$

- Using this lapse function, we can compute the spacelike geodesic length from the brane up to the horizon, which reads

$$l_h = \int_{\bar{z}}^{z_h} \frac{dz}{z\sqrt{1 - \tilde{\boldsymbol{\imath}} z^3 - \tilde{m} z^4}}$$

- We can solve the above integral perturbatively.

$$l_h = \int_{\bar{z}}^{z_h} \frac{dz}{z}\left[1 + \frac{\tilde{\boldsymbol{\imath}}}{2} z^3 + \frac{\tilde{\tilde{m}}}{2} z^4\right] + \mathcal{O}(\tilde{\tilde{m}}^2) + \mathcal{O}(\tilde{\boldsymbol{\imath}}^2) + \dots$$

$$\approx \log\left(\frac{z_h}{\bar{z}}\right) + \frac{\tilde{\boldsymbol{\imath}}}{6}(z_h^3 - \bar{z}^3) + \frac{\tilde{m}}{8}(z_h^4 - \bar{z}^4) \ .$$

- Similarly the time-like geodesic extending from the horizon to the singularity can be computed through the following integral

$$\mathcal{T}_s = \int_{z_h}^{\infty} \frac{dz}{z\sqrt{\tilde{\boldsymbol{\imath}} z^3 + \tilde{m} z^4 - 1}}$$

Multi-component universe

- After evaluating the integral perturbatively

$$\mathcal{T}_s = \frac{\pi}{4} + \frac{\tilde{\mathscr{r}} z_h^3}{8}\left[B\left(\frac{1}{2},-\frac{1}{2}\right) - B\left(\frac{3}{4},-\frac{1}{2}\right)\right]$$

- The thermal one-point function in a universe with a coexisting radiation and matter is given by

$$\langle \mathcal{O} \rangle_{rad+mat} = \left(\frac{\bar{z}}{z_h}\right)^{\Delta} e^{-\Delta\left[\frac{\tilde{\mathscr{r}}}{6}(z_h^3-\bar{z}^3)+\frac{\tilde{m}}{8}(z_h^4-\bar{z}^4)\right]}$$

Early time:

$$\langle \mathcal{O} \rangle^{(early)}_{rad+mat} \approx \left(\frac{z_i}{z_h}\right)^{\Delta} e^{-\Delta\left[\frac{\tilde{\mathscr{r}}}{6}(z_i^3-z_h^3)+\frac{\tilde{m}}{8}(z_i^4-z_h^4)\right]} \times \left[1 - \Delta\sqrt{\tilde{m}}\tau\left(z_i^2 + \frac{z_i z_t}{2}\right)\right]$$

Late time:

$$\langle \mathcal{O} \rangle^{(late)}_{rad+mat} = \left(\frac{\left(\frac{2}{3}\right)^{\frac{2}{3}}\tilde{\mathscr{r}}^{-\frac{1}{3}}}{z_h}\right)^{\Delta} \tau^{-\frac{2\Delta}{3}}\left[1 + \frac{\Delta\left(\frac{2}{3}\right)^{\frac{2}{3}}\tilde{\mathscr{r}}^{-\frac{1}{3}}\tau^{-\frac{2}{3}}}{z_t}\right] e^{-\Delta\left[\frac{\tilde{\mathscr{r}}}{6}z_h^3+\frac{\tilde{m}}{8}z_h^4\right]} \times e^{\left[\frac{\Delta\tilde{\mathscr{r}}\tau^{-2}\left(\frac{2}{3}\right)^2\tilde{\mathscr{r}}^{-1}\tau^{-2}}{6} + \frac{\Delta\tilde{m}\left(\frac{2}{3}\right)^{\frac{8}{3}}\tilde{\mathscr{r}}^{-\frac{4}{3}}\tau^{-\frac{8}{3}}}{8}\right]}.$$

Summary

- A short review on cosmological braneworld model and Isreal junction condition is discussed.
- We have discussed avout the geodesic formula to evaluate the thermal one point function of a boundary operator .
- Using the RS-II braneworld model, we have studied the time dynamics of heavy one-point operators in single and multi-component expanding universe.
- Time dynamics of the one point function for multi-component scenario is consistent with the thermal history of the universe.

Future directions

Higher curvature corrections or anisotropy can be included. Also OPFs in the intermediate regiem can be studied.

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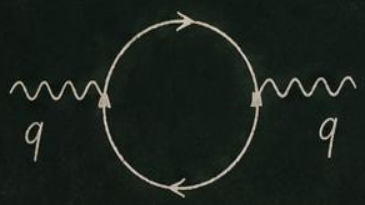
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 \, d\Phi_n$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0$$



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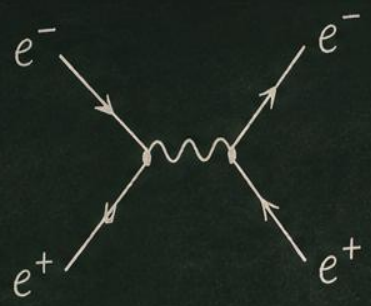
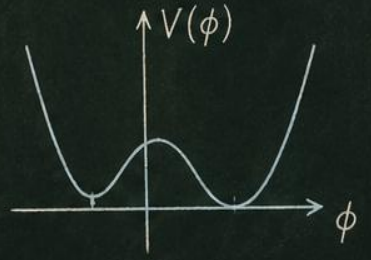
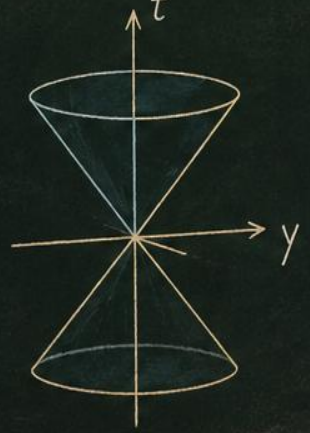


$$D_\mu F^{\mu\nu} = J^\nu$$

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i\not{D} - m)q$$



$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\epsilon}$$

$$Z = \int D\phi \, e^{iS[\phi]}$$

$$[\hat{x}^\mu, \hat{p}_\nu] = i\hbar \, \delta^\mu_\nu$$