



Paths to Solving the Strong CP Problem and the Footprints Left Behind

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Sydney-CPPC zoom seminar



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θ Term

Lagrangian of **QED** :

$$\mathbf{E}^2 - \mathbf{B}^2 \rightarrow F^{\mu\nu} F_{\mu\nu}$$

Why don't we have the following **θ term** in QED ?

$$\mathbf{E} \cdot \mathbf{B} \rightarrow \theta \frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

Parity or **Time-reversal** : $\mathbf{E} \cdot \mathbf{B} \rightarrow -\mathbf{E} \cdot \mathbf{B}$

This term has no physical consequences
because it's a total derivative :

$$\epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} = \partial_\mu (\epsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta})$$

θ Term

Story is different in Lagrangian of **QCD** !

QCD is a non-Abelian gauge theory.

θ term is still a total derivative, but ...

$$\int d^4x \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G_{\mu\nu} \neq 0$$

There are “instanton” solutions that **do not** drop off fast enough to ignore the surface term.

The QCD theta term has physical consequences !

Strong CP Problem

QCD Lagrangian for strong interactions allows

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

explicitly violating **CP** symmetry.

The physical strong CP phase : $\bar{\theta} \equiv \theta - \arg \det (M_u M_d)$

The current upper bound on the neutron electric dipole moment

$$\Rightarrow |\bar{\theta}| < 10^{-11}$$

Why is $\bar{\theta}$ so small ??

Some shifts of $\bar{\theta}$ would not provide a visible change in our world.

Potential Solutions

- **Massless up quark**

Existing global $U(1)$ axial symmetry can rotate the θ term to zero.

Ruled out by the lattice result.

- **Axion**

Axion dynamically cancels the θ term at the minimum of its potential.

- **Spontaneous CP (or P) violation**

CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum.

CKM phase is generated without reintroducing the θ term.

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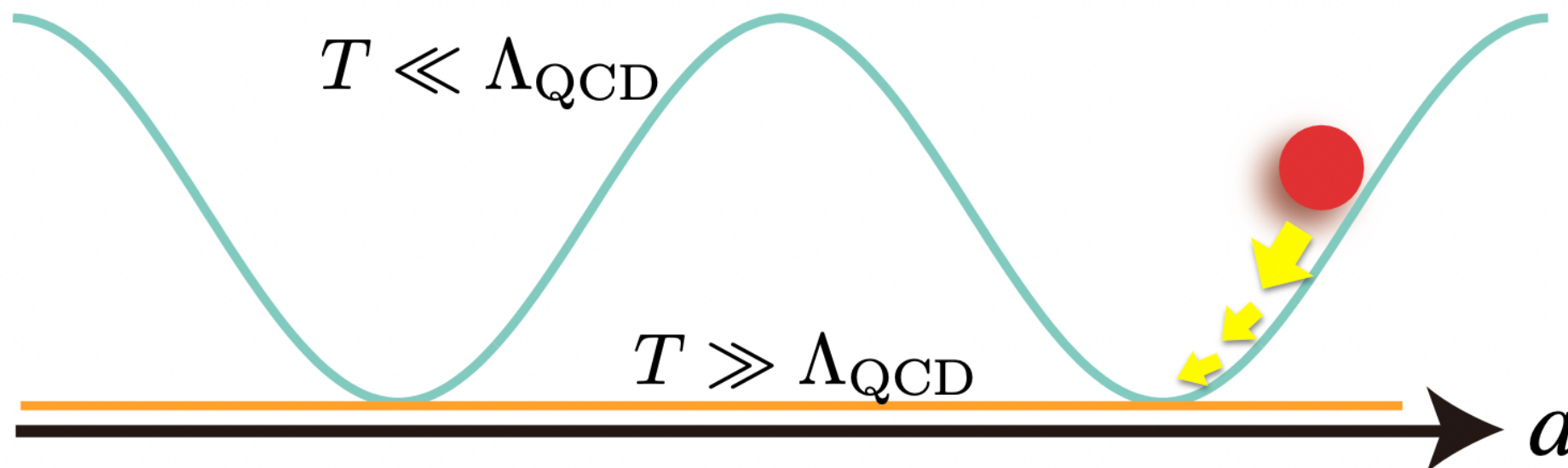
CKM phase is generated without reintroducing the θ term.

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Axion Solution

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



Fuminobu Takahashi slide

The **axion a** dynamically cancels the strong CP phase !

Axion Solution

Axion is a pseudo-Nambu-Goldstone boson associated with spontaneous breaking of a **global U(1)_{PQ} symmetry**.

Non-perturbative QCD effects break the U(1)_{PQ} explicitly and generate the axion potential :

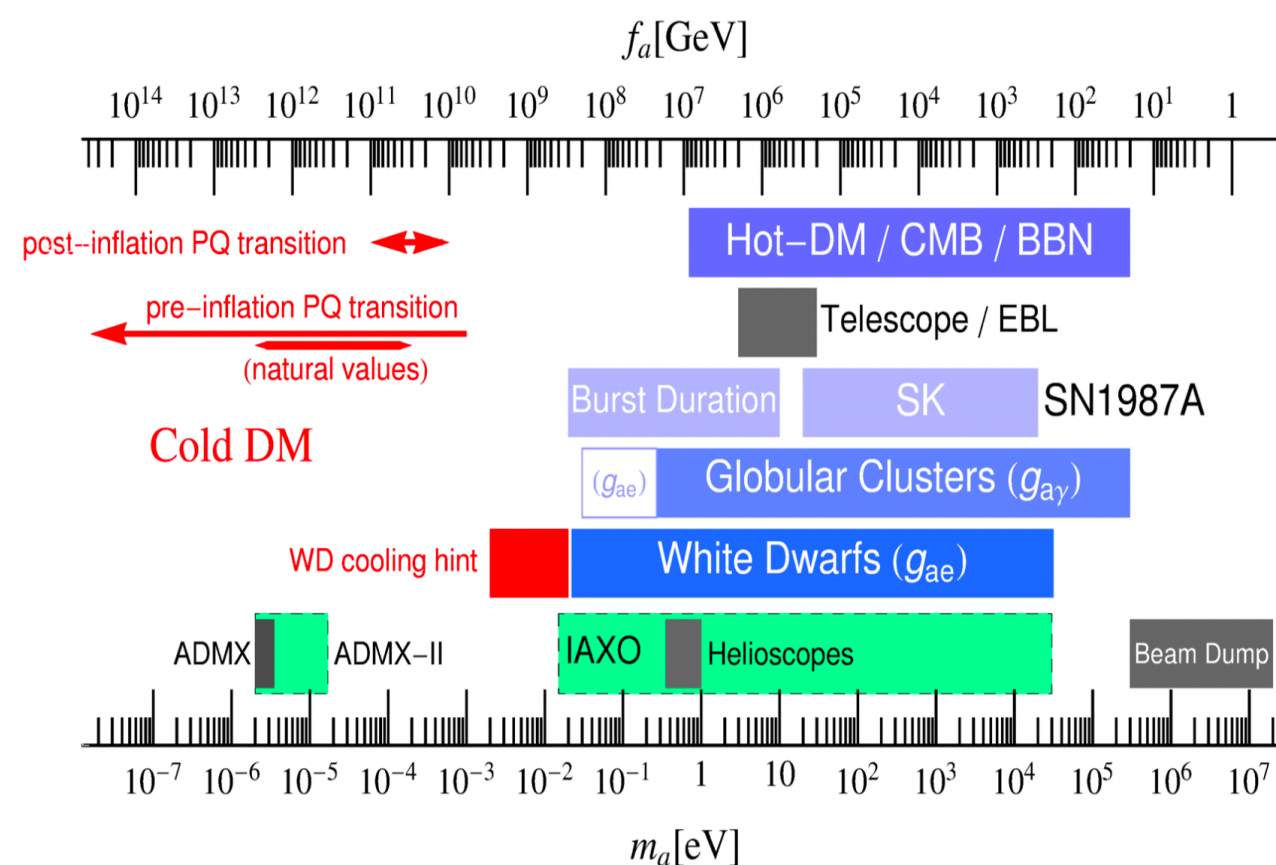
$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

Axion can be a dominant component of **dark matter** :

$$10^8 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$



Naturalness problem of the intermediate scale



PDG

Composite Axion

Axion emerges as **a pion-like bound state** of a new gauge dynamics.

Chiral symmetry breaking :

$$SU(4)_L \times SU(4)_R \rightarrow \underline{SU(4)_V} \\ \supset SU(3)_c$$

One of pNGBs is identified as axion.

Fields	$[SU(3)_c]$	$[SU(N)]$	$U(1)_{PQ}$
Q	3	N	1
η	1	N	-3
$\bar{Q}$	$\bar{3}$	$\bar{N}$	0
$\bar{\eta}$	1	$\bar{N}$	0

Anomaly coefficient :

$$A(U(1)_{PQ} - SU(3)_c^2) = \underline{N} \quad \Rightarrow \quad \text{Axion coupling to QCD}$$


Domain wall number

Dimensional transmutation naturally explains the hierarchically small PQ breaking scale.

Axion Cosmology

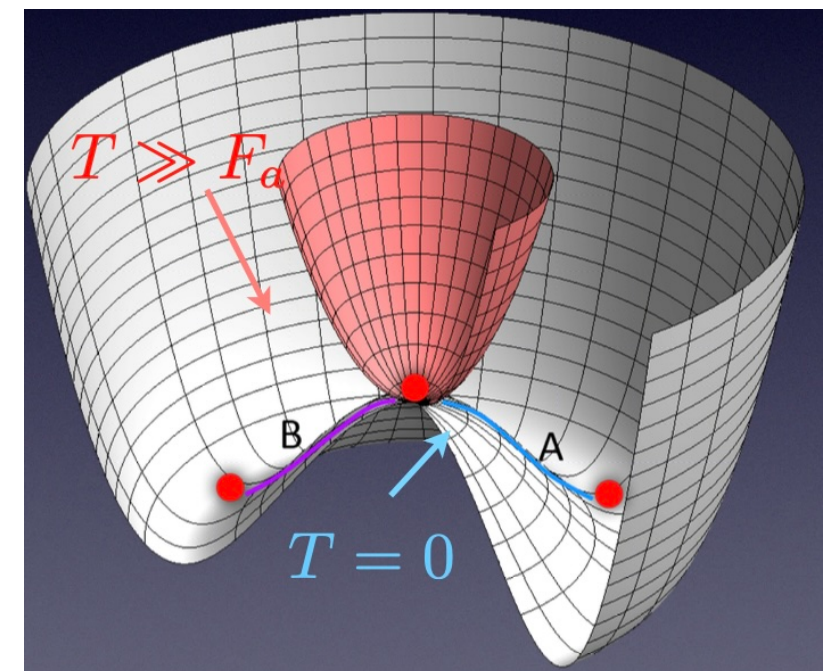
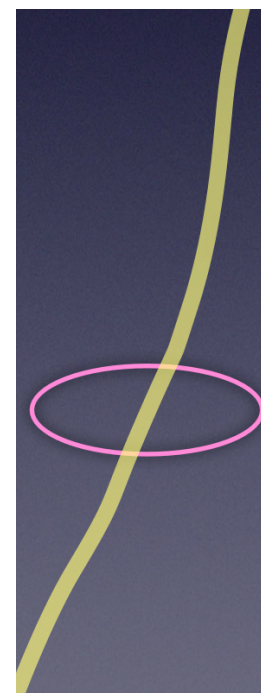
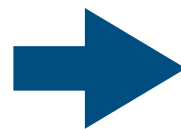
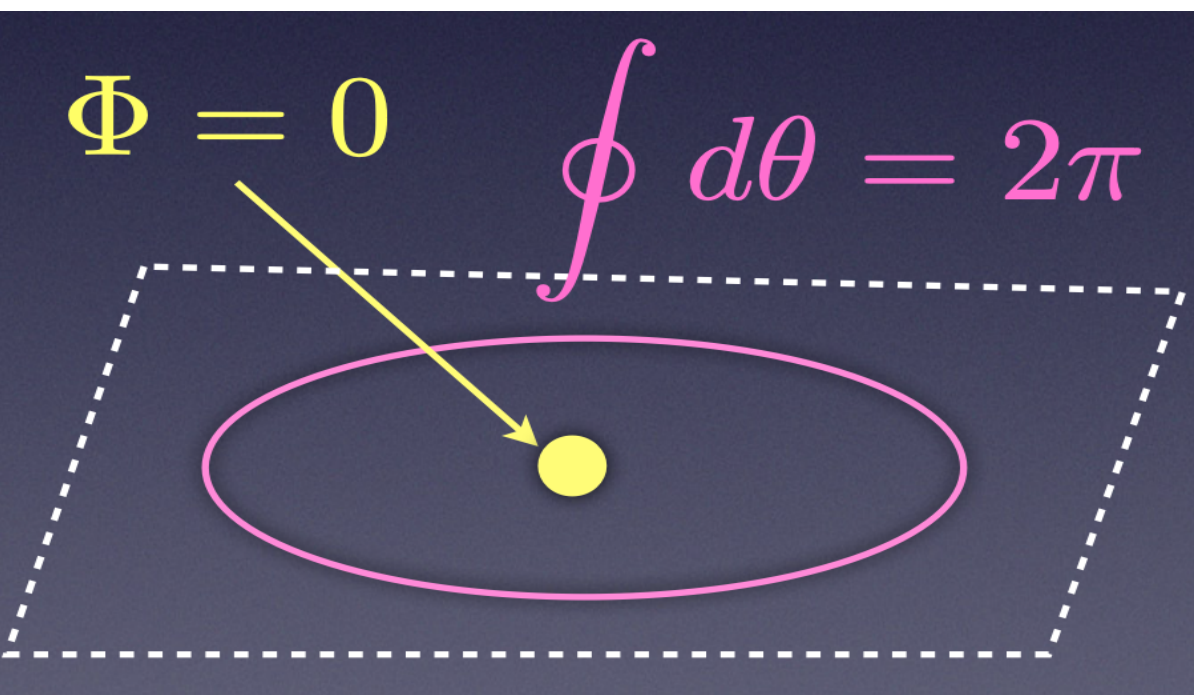
- Cosmological consequences :

★ $U(1)_{PQ}$ breaking **before** inflation ➡ **Isocurvature perturbations**
Constraint on the inflation scale

- ★ $U(1)_{PQ}$ breaking **after** inflation

Axion field acquires **spatial variations** across the Universe.

$$\Phi \sim v_{PQ} e^{i\theta_a}$$



Masahiro Kawasaki slide

Formation of
cosmic strings

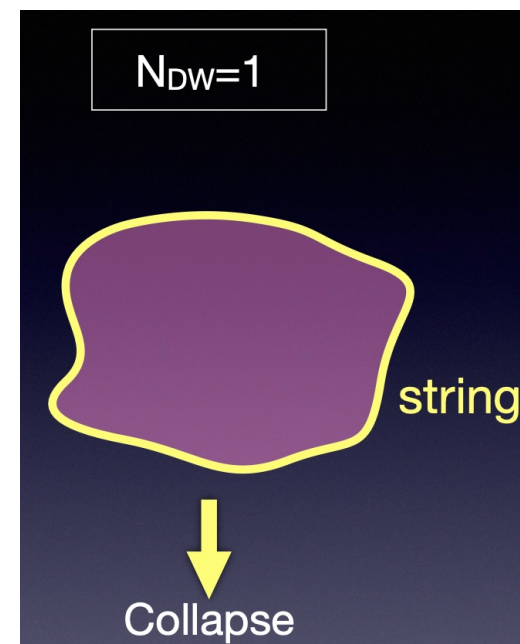
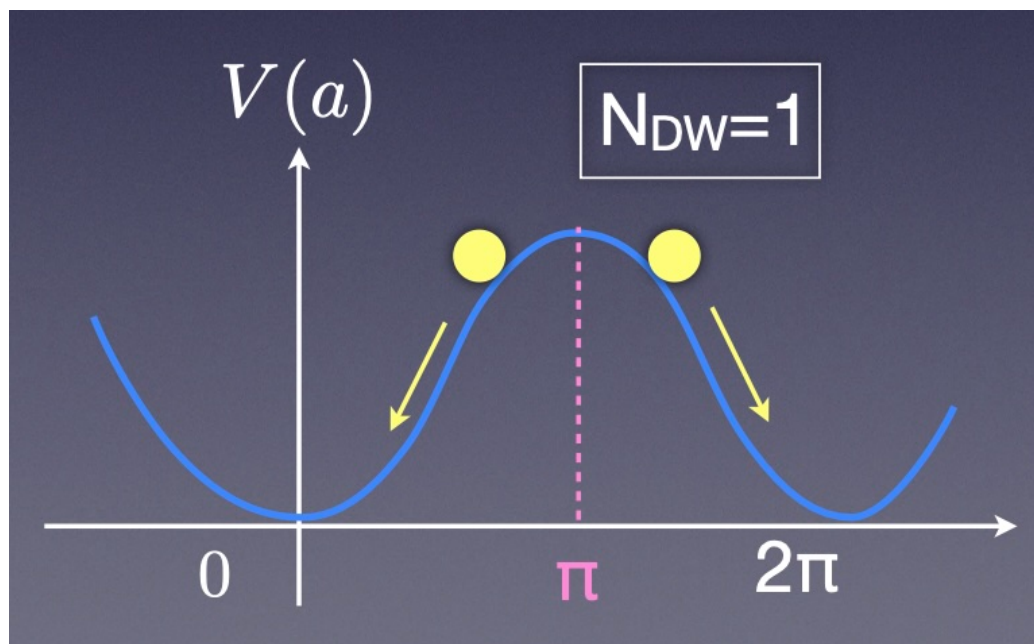
Domain Walls

As the Universe expands and cools,
the axion gets a potential with N_{DW} minima.

Domain wall number (associated with color anomaly)

$$N_{\text{DW}} = 1$$

Domain walls form as disk-like structures attached to strings and eventually collapse due to their tension.



Masahiro Kawasaki slide

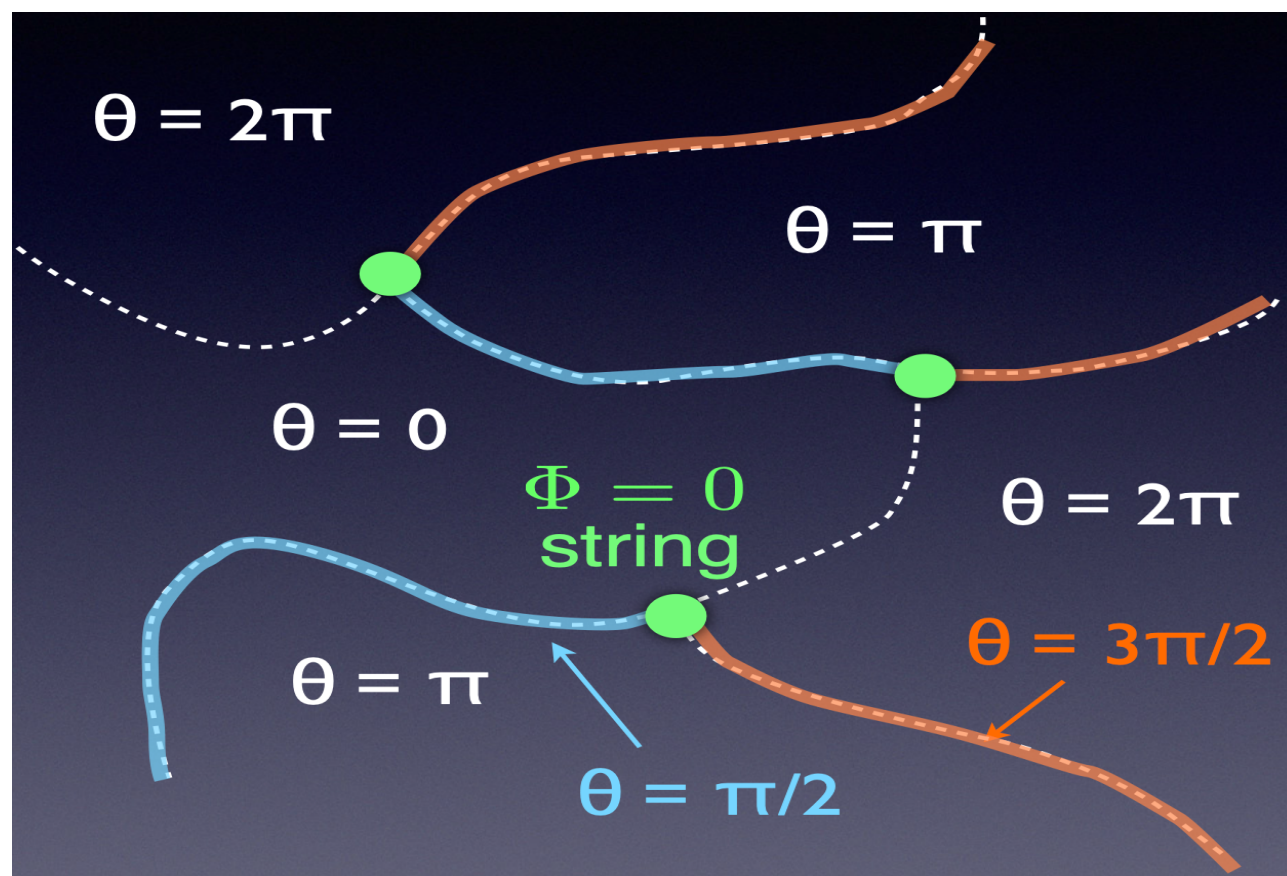
Decay of the string-domain wall network produces a lot of axions
which dominate the axion DM abundance.

Domain Walls

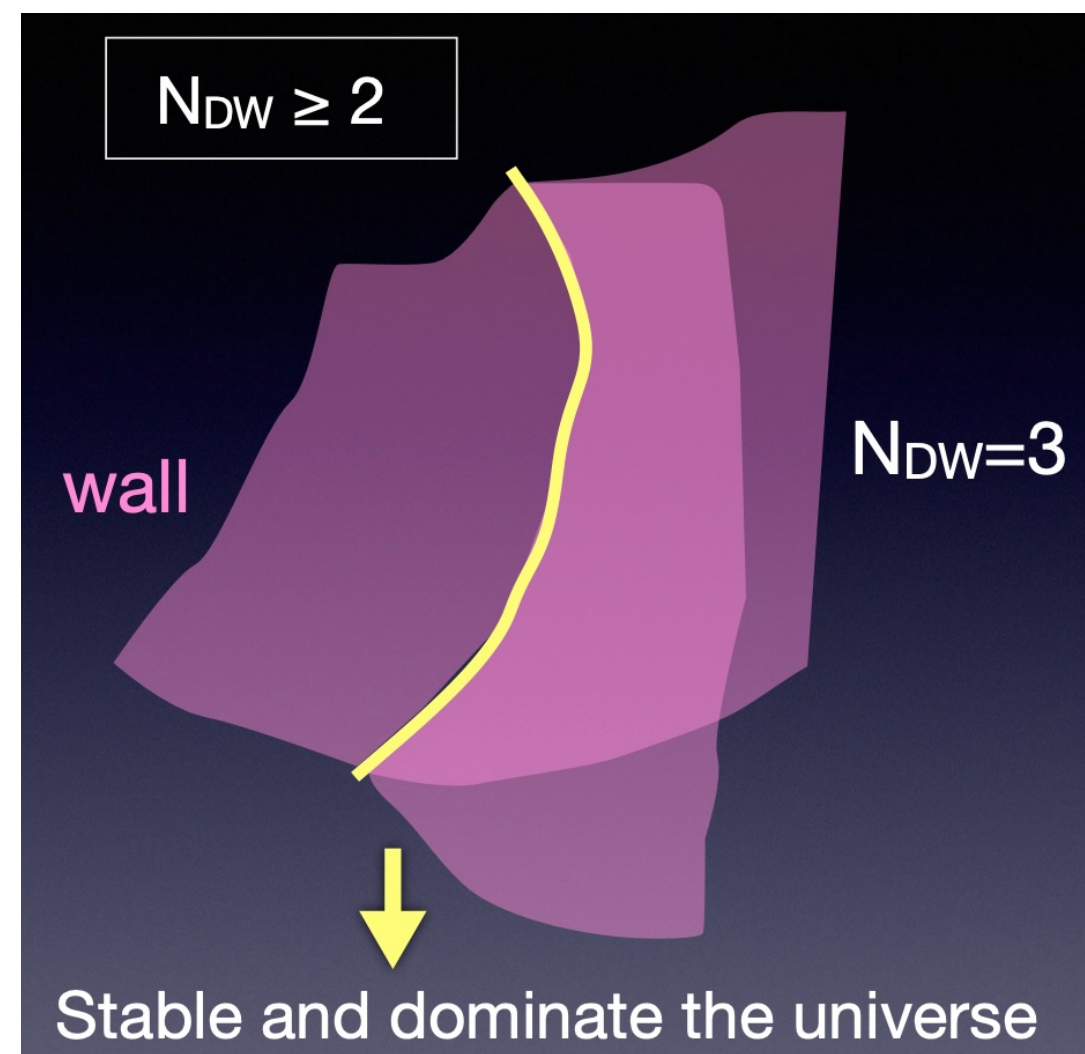
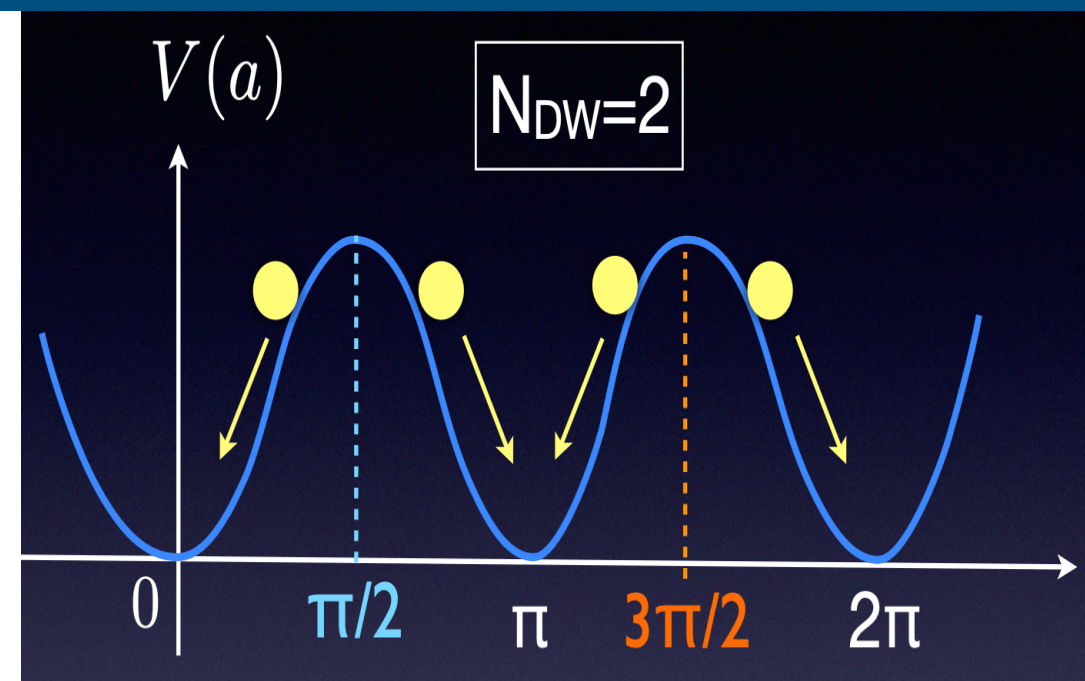
$$N_{\text{DW}} > 1$$

Domain walls form a **stable** network that eventually dominates the Universe.

➔ **Domain wall problem**



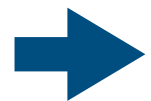
Masahiro Kawasaki slide



Bias Term

A potential solution to the domain wall problem is to introduce a small explicit breaking of the $U(1)_{PQ}$ symmetry (**Bias term**).

Bias term lifts the vacuum degeneracy !

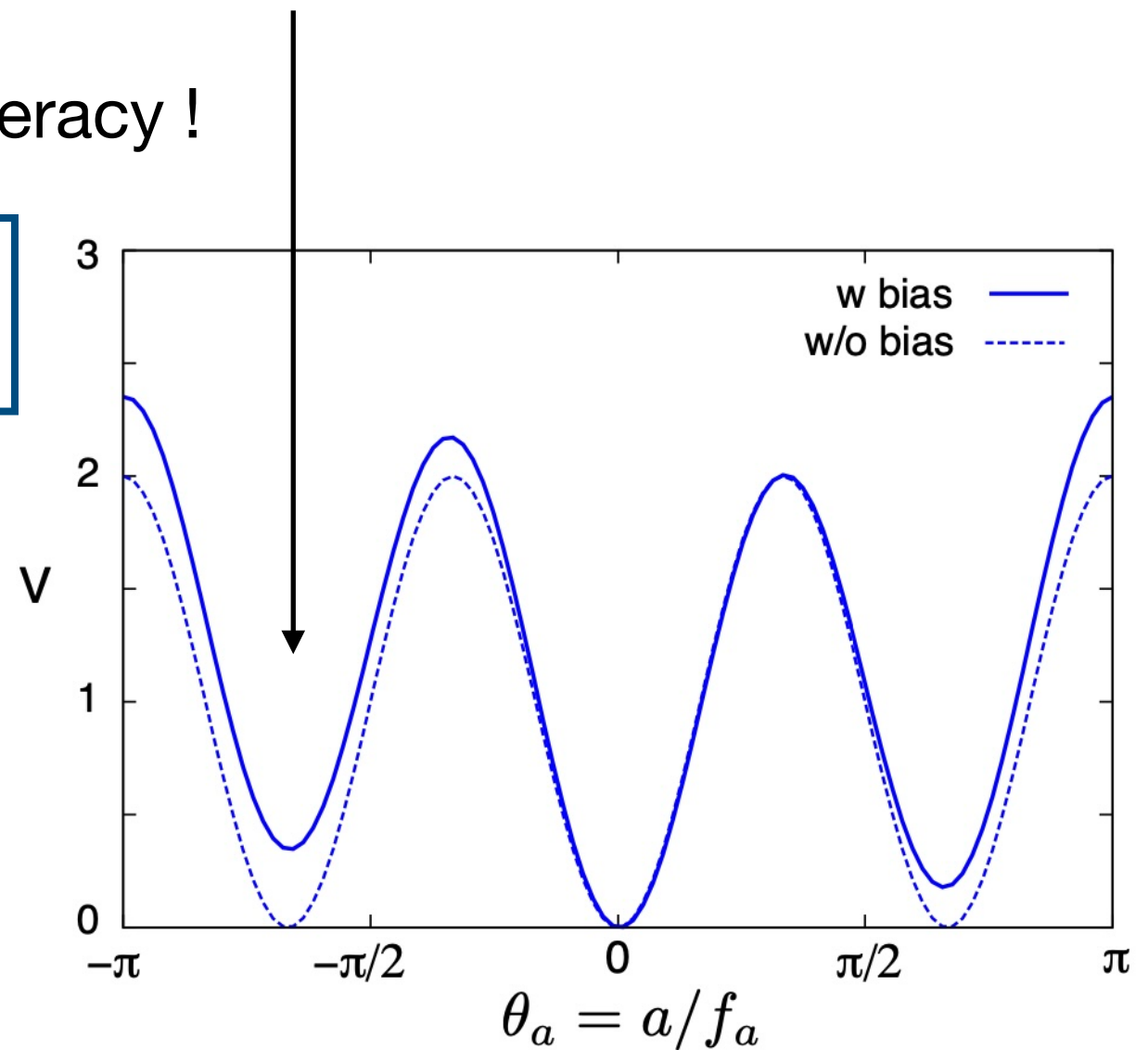


Domain walls collapse before they dominate the Universe.

★ Axion DM abundance is changed.

However ...

★ **Ad hoc introduction** of the bias term is unsatisfactory.



Potential Solutions

- **Special embedding**

Hor, YN, Suzuki, Xu (2025)

The SM gauge group is identified as a **special subgroup** of a UV group to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

- **Bias with a timer**

Hao, Nakagawa, YN, Suzuki (2025)

New scalar field interaction with the axion provides the origin of a (time-dependent) bias term.

- **PQ symmetry non-restoration**

Nakagawa, YN, Qiu, Wang, Wang (2025)

PQ symmetry remains broken in the entire history of the Universe, avoiding the formation of axion strings and domain walls.

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Potential Solutions

- **Special embedding**

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The SM gauge group is identified as a **special subgroup** of a UV group to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

— **Our focus** —

- **Bias with a timer**

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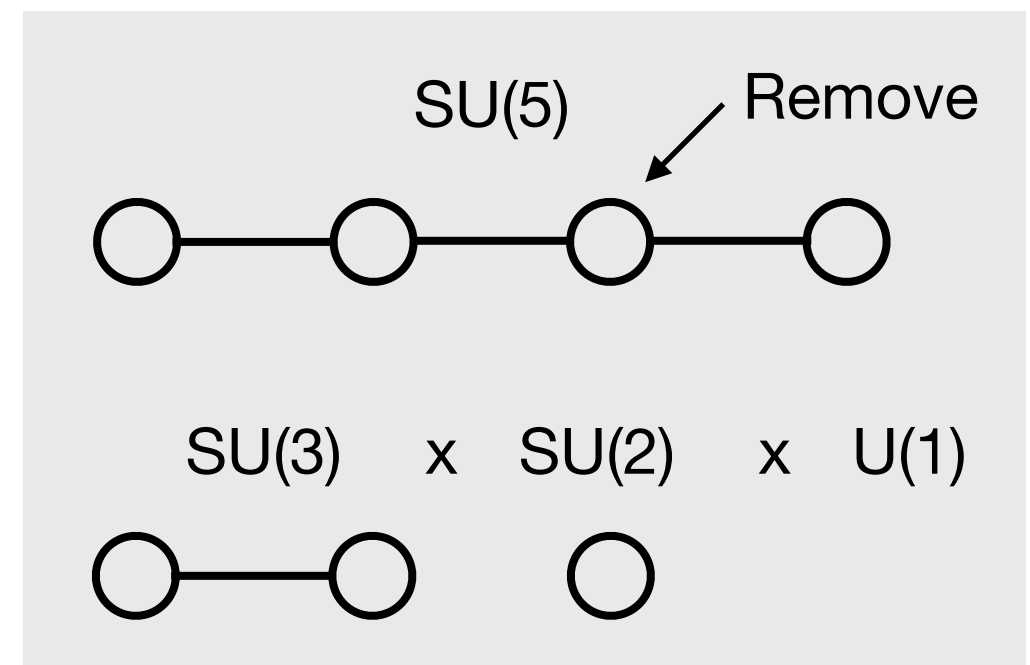
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Special Subalgebra

Simple Lie algebras possess not only **regular** subalgebras but also **special** subalgebras.

Regular subalgebras : systematically obtained by removing nodes from Dynkin diagrams.

Special subalgebras :
do not follow this scheme !



➡ Representation contents and group-theoretic indices
are **reorganized** in a non-trivial manner.

Special Embedding

Consider a gauge sym. breaking : $SU(MN) \rightarrow SU(M) \times SU(N)$

Embed **bi-fundamental** representation of $SU(M) \times SU(N)$ into **fundamental** representation of $SU(MN)$.

$$\mathbf{v} \rightarrow e^{i\beta_M^a \hat{T}_M^a} e^{i\beta_N^\alpha \hat{\tau}_N^\alpha} \mathbf{v}$$

$$\hat{T}_M^a \equiv T_M^a \otimes 1_{N \times N} = \begin{pmatrix} T_M^a & \mathbf{0} & \cdots \\ \mathbf{0} & T_M^a & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad \text{Generators of } SU(M) \text{ in fundamental rep.}$$

$$\text{tr}(T_M^a T_M^b) = \frac{1}{2} \delta^{ab}$$

$$\hat{\tau}_N^\alpha \equiv 1_{M \times M} \otimes \tau_N^\alpha = \begin{pmatrix} 1_{M \times M}(\tau_N^\alpha)_{11} & 1_{M \times M}(\tau_N^\alpha)_{12} & \cdots \\ 1_{M \times M}(\tau_N^\alpha)_{21} & 1_{M \times M}(\tau_N^\alpha)_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Generators of $SU(N)$ in fundamental rep.

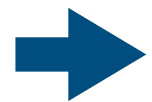
$$\text{tr}(\hat{T}_M^a \hat{\tau}_N^\alpha) = 0$$

Special Embedding

Embed $SU(M) \times SU(N)$ generators into those of $SU(MN)$:

$$\hat{T}_M^a = \underbrace{\mathcal{O}_M^{am}}_{\text{Coefficients}} T_{UV}^m, \quad \hat{\tau}_N^\alpha = \underbrace{\mathcal{O}_N^{\alpha m}}_{\text{Coefficients}} T_{UV}^m$$

$$\begin{aligned} \text{tr}(\hat{T}_M^a \hat{T}_M^b) &= N \cdot \frac{1}{2} \delta^{ab} & \text{tr}(\hat{\tau}_N^\alpha \hat{\tau}_N^\beta) &= M \cdot \frac{1}{2} \delta^{\alpha\beta} \\ \text{tr}(T_{UV}^m T_{UV}^n) &= \frac{1}{2} \delta^{mn} \end{aligned}$$



$$\begin{aligned} \mathcal{O}_M^{am} \mathcal{O}_M^{bn} \delta_{mn} &= c_M \delta^{ab}, & c_M &= N \\ \mathcal{O}_N^{\alpha m} \mathcal{O}_N^{\beta n} \delta_{mn} &= c_N \delta^{\alpha\beta}, & c_N &= M \end{aligned}$$

Special Embedding

Consider **a fermion** ψ which transforms as fundamental rep. of $SU(MN)$ and bi-fundamental rep. of $SU(M) \times SU(N)$:

$$\begin{aligned}\mathcal{D}_\mu \psi &= \partial_\mu \psi - ig_{UV} A_{UV,\mu}^m T_{UV}^m \psi \\ &\supset \partial_\mu \psi - ig_{IR,M} A_{IR,M,\mu}^a \hat{T}_M^a \psi - ig_{IR,N} A_{IR,N,\mu}^\alpha \hat{\tau}_N^\alpha \psi\end{aligned}$$

Matching the gauge fields :

$$A_{UV,\mu}^m = \frac{g_{IR,M}}{g_{UV}} \underline{A_{IR,M,\mu}^a} \mathcal{O}_M^{am}, \quad A_{UV,\mu}^m = \frac{g_{IR,N}}{g_{UV}} \underline{A_{IR,N,\mu}^\alpha} \mathcal{O}_N^{\alpha m}$$

Canonically normalized kinetic term



$$g_{IR,M} = \frac{g_{UV}}{\sqrt{c_M}}, \quad g_{IR,N} = \frac{g_{UV}}{\sqrt{c_N}}$$

Special Embedding

Consider a **fermion** ψ which transforms as fundamental rep. of $SU(MN)$ and bi-fundamental rep. of $SU(M) \times SU(N)$:

$$\mathcal{D}_\mu \psi = \partial_\mu \psi - ig_{UV} A_{UV,\mu}^m T_{UV}^m \psi$$

$$\supset \partial_\mu \psi - ig_{IR,M} A_{IR,M,\mu}^a \hat{T}_M^a \psi - ig_{IR,N} A_{IR,N,\mu}^\alpha \hat{\tau}_N^\alpha \psi$$

Theta term

$$\int \frac{g_{UV}^2}{8\pi^2} \text{tr}(F_{UV} \wedge F_{UV}) \supset \int c_N \frac{g_{IR,N}^2}{8\pi^2} \text{tr}(F_{IR,N} \wedge F_{IR,N}) + \int c_M \frac{g_{IR,M}^2}{8\pi^2} \text{tr}(F_{IR,M} \wedge F_{IR,M})$$

Domain wall # = 1 in UV gauge theory

➡ Nontrivial domain wall # in IR gauge theory !

UV Completion

Hor, YN, Suzuki, Xu (2026)

Gauge symmetry breaking :

$$\underline{SU(4N)} \times SU(3)_2 \times SU(N)_2 \times U(1)_X \rightarrow \underline{SU(N) \times SU(3)_c \times U(1)_Y}$$

$\supset SU(4)_1 \times SU(N)_1$: **Special embedding**

Diagonal subgroup

- All SM matter fields are charged under $SU(3)_2 \times U(1)_X$
- **A vector-like pair of PQ-charged fermions** transform as (anti-)fundamental reps. under $SU(4N)$, so that **$N_{DW} = 1$** .

After gauge symmetry breaking ...

$$\Psi : (\mathbf{N}, \mathbf{3}) + (\mathbf{N}, \mathbf{1}) , \quad \bar{\Psi} : (\bar{\mathbf{N}}, \bar{\mathbf{3}}) + (\bar{\mathbf{N}}, \mathbf{1})$$

- The apparent vacuum degeneracy is lifted by **small instanton effects** on the axion potential that operates as a PQ-violating bias term.

Small Instanton

QCD θ -vacuum : superposition of n -vacua (energy degenerate but topologically distinct)

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle = \cdots + |0\rangle + e^{-i\theta} |1\rangle + \cdots$$


Instanton describes the tunneling effect between degenerate n -vacua

Instanton : localized object in Euclidean spacetime, satisfying Euclidean EOM with non-trivial topology and minimize Euclidean action

SU(2) BPST instanton solution with $Q = 1$:

$$\frac{g^2}{32\pi^2} \int d^4x F^{a\mu\nu} \tilde{F}_{\mu\nu}^a \Big|_{\text{inst.}} = Q \quad (Q \in \mathbb{Z})$$

$$A_{\mu}^a(x) \Big|_{1-\text{inst.}} = 2\eta_{a\mu\nu} \frac{(x - x_0)_{\nu}}{(\underbrace{x - x_0}_{\text{Position}})^2 + \underbrace{\rho^2}_{\text{Instanton size}}}$$

Position

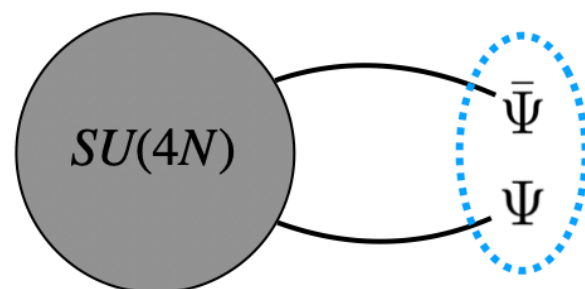
Instanton size

Small Instanton

Instantons contribute to the axion potential $\propto \exp\left(-\frac{8\pi^2}{g^2(1/\rho)}\right)$

In QCD, large-size instantons dominate the axion potential due to asymptotic freedom.

UV gauge interaction beyond QCD $\rightarrow$ **Small instanton effects**



$$\sim \exp \begin{pmatrix} i \frac{a(x)}{f_{PQ}} \mathbf{1}_{3 \times 3} & 0 \\ 0 & -i \frac{3a(x)}{f_{PQ}} \end{pmatrix}$$

't Hooft vertex

$\rightarrow V_{4N} \sim \Lambda_{\text{cut}} \Psi \bar{\Psi} e^{-8\pi^2 / (g_{SU(4N)} (\Lambda_{\text{cut}}))^2}$

Λ_{cut} : cut-off scale

Post-Inflation Axion

PQ symmetry is spontaneously broken after reheating.

We focus on the scenario where the axion field starts to oscillate due to the axion mass originated from **the bias term** :

$$m_{\text{bias}}^2 = \frac{1}{f_{\text{PQ}}^2} \frac{\partial^2 V_{\text{bias}}}{\partial \theta_a^2}$$

Temperature when the oscillation starts :

$$m_a(T_1) \approx m_{\text{bias}} = 3H(T_1) \quad H(T_1)^2 \approx \frac{\pi^2}{90M_{\text{Pl}}^2} g_* T_1^4$$

$$T_1 > \underline{T_{1,\text{QCD}}} \equiv 0.98 \text{ GeV} \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{-0.19}$$

↑
Temperature when the axion would start to oscillate with non-perturbative QCD effects if there was no bias term

Domain walls decay in a similar way as the standard $N_{\text{DW}} = 1$ case.

Post-Inflation Axion

Axion abundance :

$$\Omega_a h^2 \approx (2_{-1}^{+2}) \times 10^{-12} \frac{f_{\text{PQ}}}{T_1}$$

A large bias term shifts the axion potential minimum :

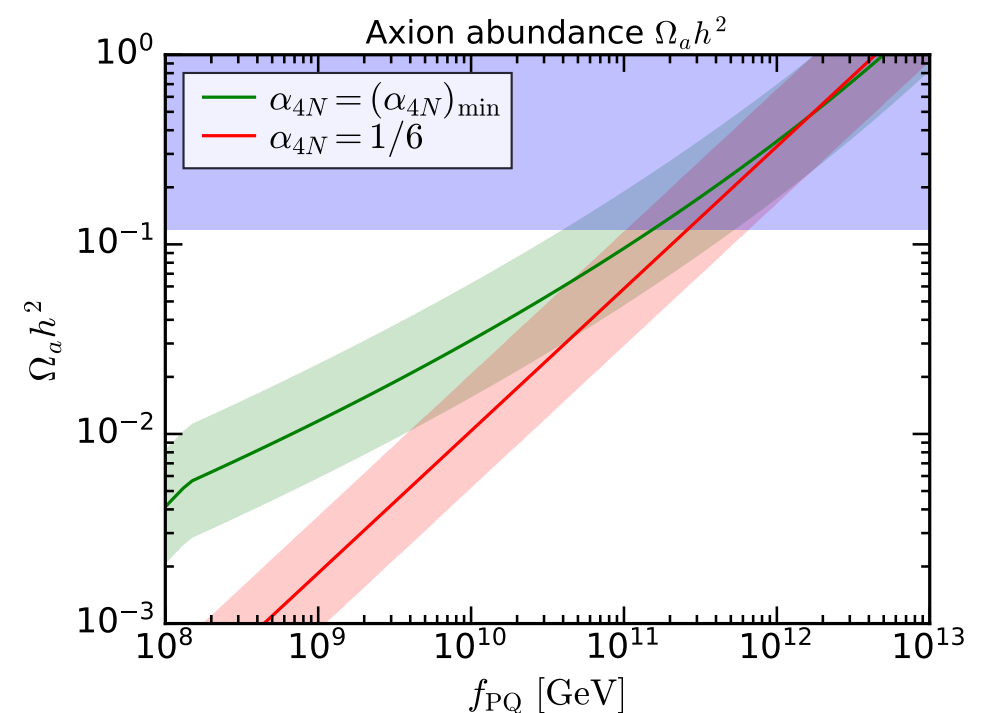
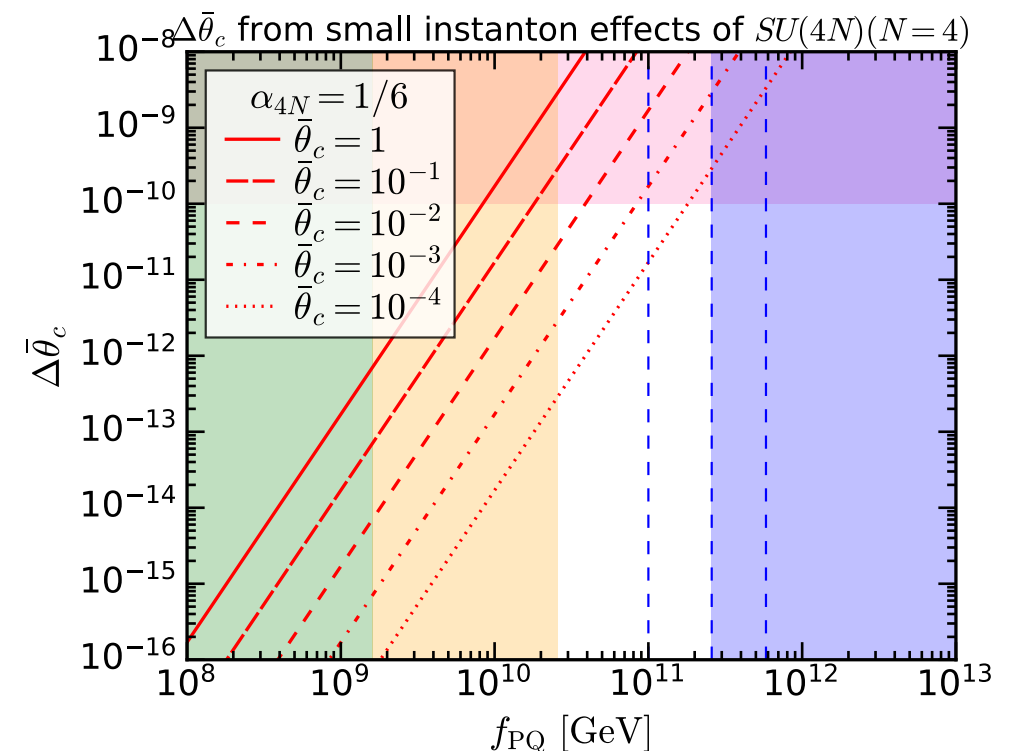
$$\Delta \bar{\theta}_c \approx \frac{V_{\text{bias}}}{V_{\text{QCD}}} \bar{\theta}_c \quad \text{Relative phase}$$

Neutron EDM

$$\Rightarrow \Delta \bar{\theta}_c \lesssim 10^{-10}$$

Axion does not appear to explain the correct DM abundance...

$$\text{Pink: } \Delta \bar{\theta}_c > 10^{-10} \quad \text{Green: } f_a < 4 \times 10^8 \text{ GeV} \quad \text{Orange: } T_1 < T_{1, \text{QCD}} \quad \text{Purple: } \Omega_a h^2 > 0.12$$



Potential Solutions

- **Special embedding**

Hor, YN, Suzuki, Xu (2025)

The SM gauge group is identified as a **special subgroup** of a UV group to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

- **Bias with a timer**

Hao, Nakagawa, YN, Suzuki (2025)

New scalar field interaction with the axion provides the origin of a (time-dependent) bias term.

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PQ symmetry remains broken in the entire history of the Universe, avoiding the formation of axion strings and domain walls.

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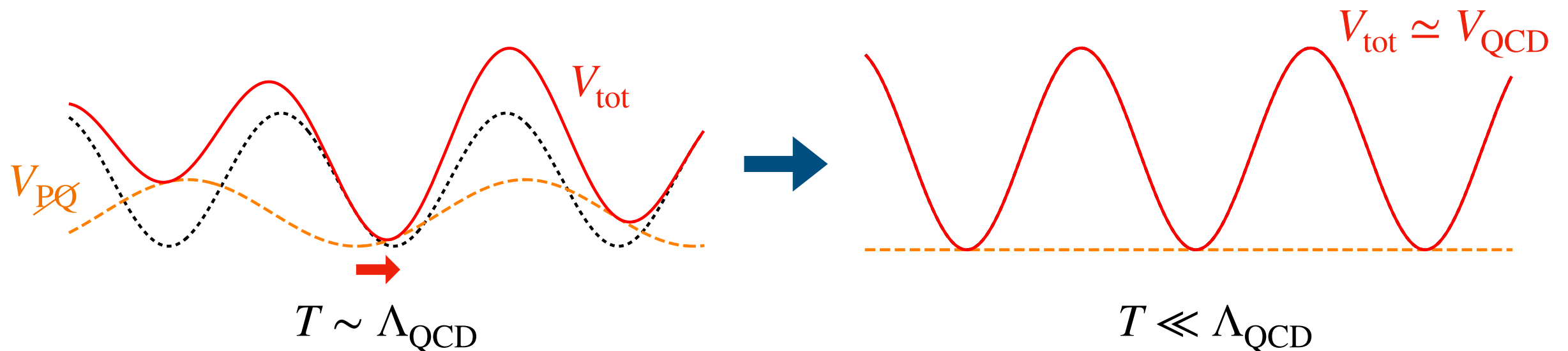
Bias with a Timer

Hao, Nakagawa, YN, Suzuki (2025)

The **new scalar field** acquires **a nonzero VEV during inflation**, inducing an effective PQ-violating interaction.

$$V_{\text{PQ}}(P, S) \supset \lambda_P \left(|P|^2 - \frac{v_{\text{PQ}}^2}{2} \right)^2 + \frac{1}{(n!)^2} \frac{\lambda_S^2}{M_{\text{Pl}}^{2n-4}} |S|^{2n} \\ + m_S^2 |S|^2 + \left(\frac{\lambda}{m! \ell! M_{\text{Pl}}^{m+\ell-4}} S^m P^\ell + \text{h.c.} \right) \quad l, m, n: \text{integers}$$

As the scalar field relaxes to the origin, the PQ violation is turned off.



Bias with a Timer

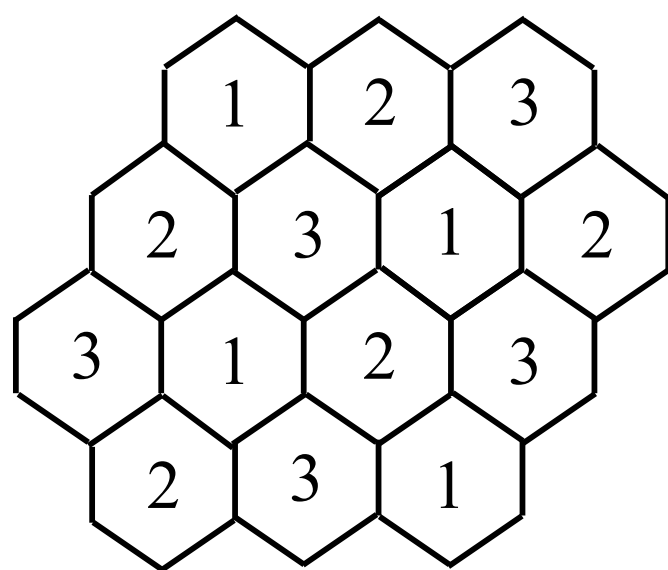
$(N_{\text{DW}}, l) = (2, 3) \rightarrow$ New way of domain wall decays !

Structural instability

$$V_{\cancel{PQ}} = -\frac{1}{\ell^2} m_{\cancel{PQ}}^2 v_{\cancel{PQ}}^2 \cos \left(\boxed{\ell} \frac{a}{v_{\cancel{PQ}}} + \delta' \right)$$

DW for $V_{\cancel{PQ}}$

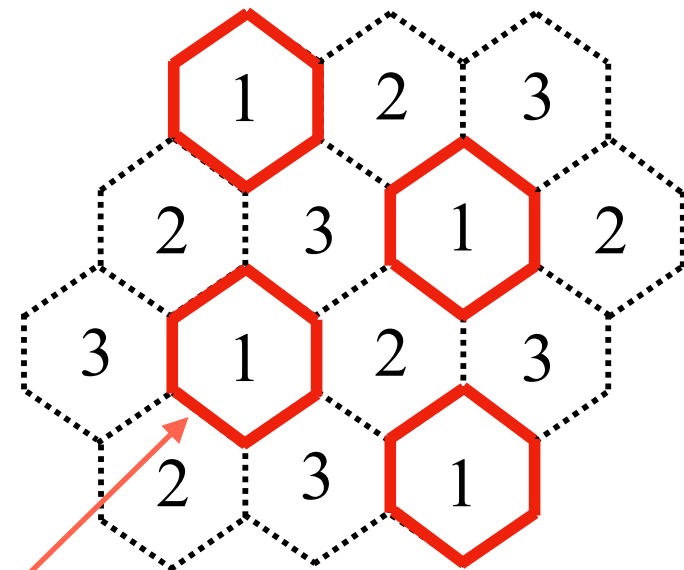
$l (= 3)$ walls are attached with the axion string.



V_{QCD} becomes dominant.



Vacua 2 and 3 are integrated.



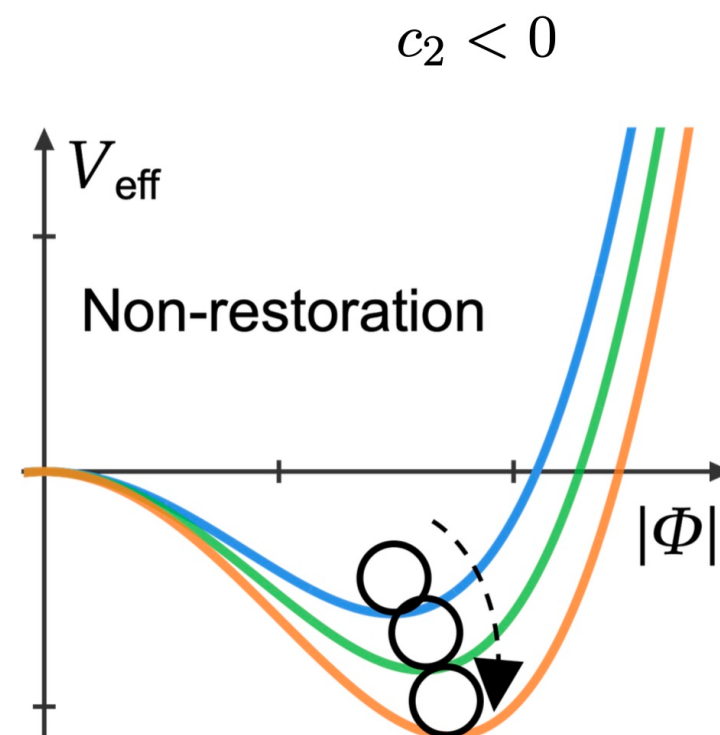
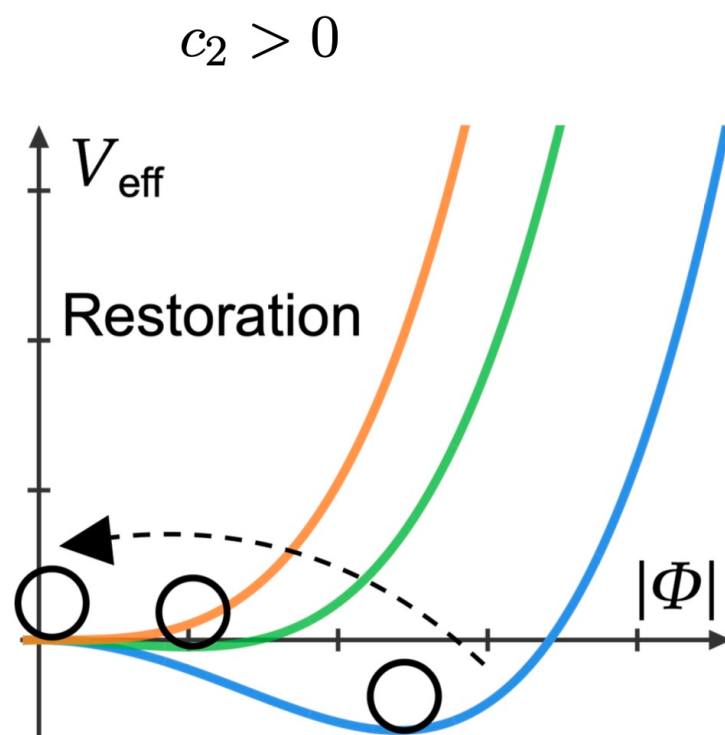
The domain wall is isolated and collapses by the tension !

PQ Non-restoration

Nakagawa, YN, Qiu, Wang, Wang (2025)

PQ symmetry remains broken in the entire history of the Universe, avoiding the formation of axion strings and domain walls.

$$V^\beta(|\Phi|) = c_2|\Phi|^2T^2 + \mathcal{O}(T^3)$$



Non-restoration can be realized by **additional scalar(s)** coupled to PQ breaking field(s).

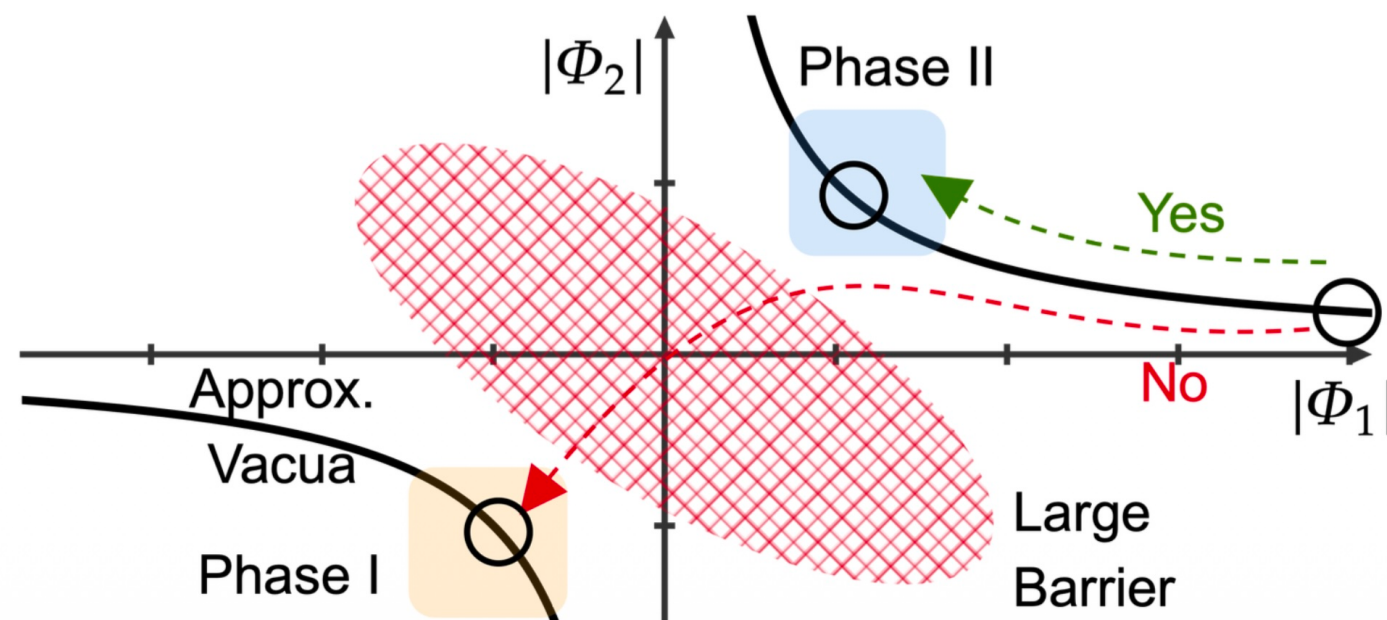
PQ Non-restoration

★ $U(1)_{PQ}$ breaking **during** inflation ➡ **Isocurvature problem**

Linde's solution (1991)

During inflation, PQ fields are stabilized at **large field values**, $f_{inf} \gg f_a$.

After inflation, PQ fields roll along a flat direction, which is stabilized around f_a .

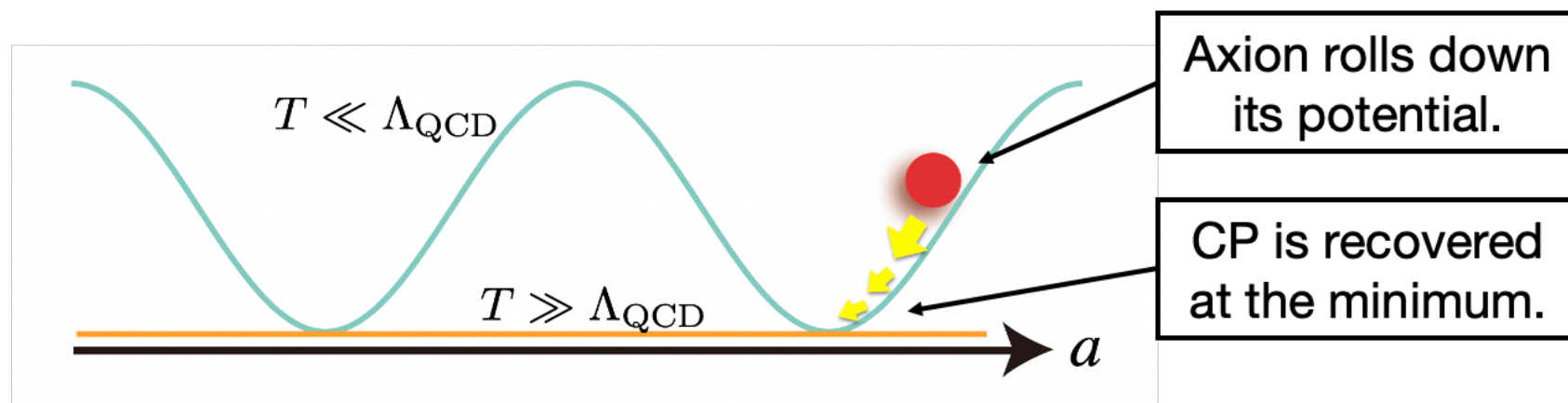


When reheating completed, thermal correction **never** restores PQ symmetry thanks to **the additional scalars**.

Axion Quality Problem

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



The **axion a** dynamically cancels the strong CP phase.

Axion arises from spontaneous breaking of a global symmetry which must be preserved to an extraordinary high degree.



Incompatible with quantum gravity effects (**axion quality problem**)

Potential Solutions

- **Massless up quark**

Existing global $U(1)$ axial symmetry can rotate the θ term to zero.

Ruled out by the lattice result.

- **Axion**

Axion dynamically cancels the θ term at the minimum of its potential.

- **Spontaneous CP (or P) violation**

CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum.

CKM phase is generated without reintroducing the θ term.

...

Spontaneous CPV

- Spontaneous CP violation (SCPV) provides **an axionless solution** to the strong CP problem.
- CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum

➡ Generation of the CKM phase without reintroducing a nonzero strong CP phase

Nelson-Barr mechanism

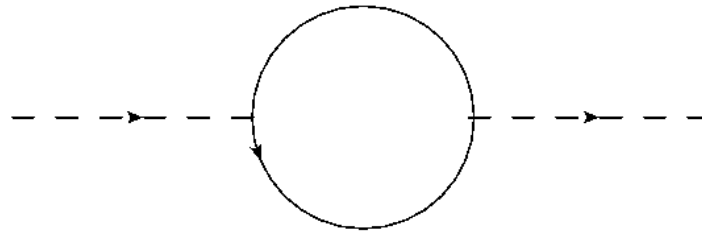
A vector-like pair of heavy quarks is introduced, so that the extended quark mass matrix transmits SCPV into the CKM matrix.

$$\mathcal{L} = \mu \bar{q}q + a_{a\bar{f}} \bar{\eta}_a \bar{d}_{\bar{f}} q + y_{f\bar{f}} H Q_f \bar{d}_{\bar{f}} \quad \Rightarrow \quad \mathcal{M} = \begin{pmatrix} \mu & a_{a\bar{f}} \bar{\eta}_a \\ 0 & m_d \end{pmatrix}$$

Heavy quarks CP breaking field

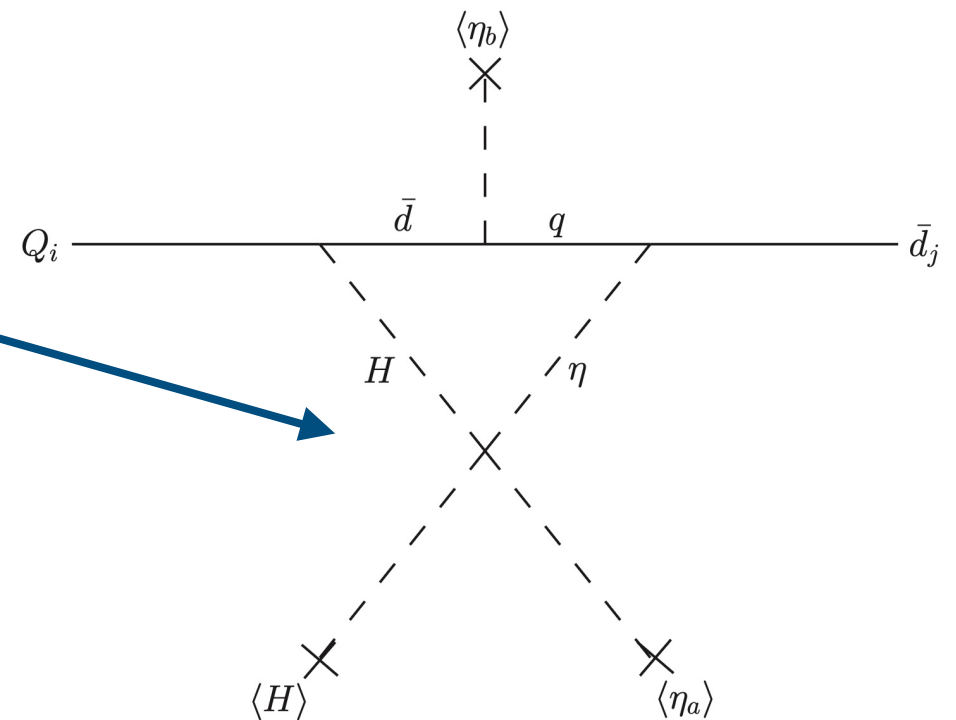
Challenges for SCPV

- **Naturalness problem**: the mechanism requires new scalar fields whose VEVs break CP much below the Planck scale.



- Sensitivity to **higher-dim. operators** and **radiative corrections**, regenerating a strong CP phase.

$$\mathcal{L} \supset \frac{\eta_b^*}{\Lambda} \eta_a \bar{q} q + \frac{\eta_a^*}{\Lambda} H Q \bar{q} + \underline{\gamma_{ab} \eta_a^* \eta_b H^\dagger H}$$



Dine, Draper (2015)

Challenges for SCPV

- **Naturalness problem:** the mechanism requires new scales whose VEVs break CP much below the Planck scale.

Supersymmetry (SUSY) offers a natural framework to address these difficulties !

- Sensitivity to **higher-dim. operators** and **radiative corrections**, regenerating a strong CP phase.

- ✓ SUSY stabilizes the SCPV scale in much the same way it stabilizes the EW scale.
- ✓ SUSY can forbid or strongly suppress dangerous higher-dim. operators.

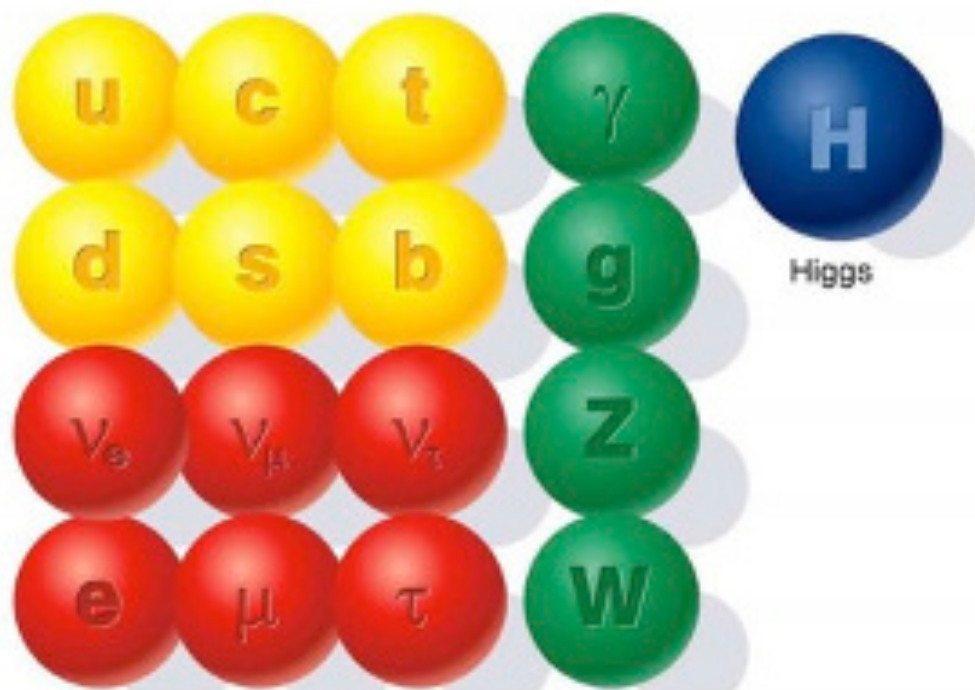
It's reasonable to consider SCPV in SUSY.

For non-SUSY approach, see *e.g.* Girmohanta, Lee, YN, Suzuki (2022).

Supersymmetry

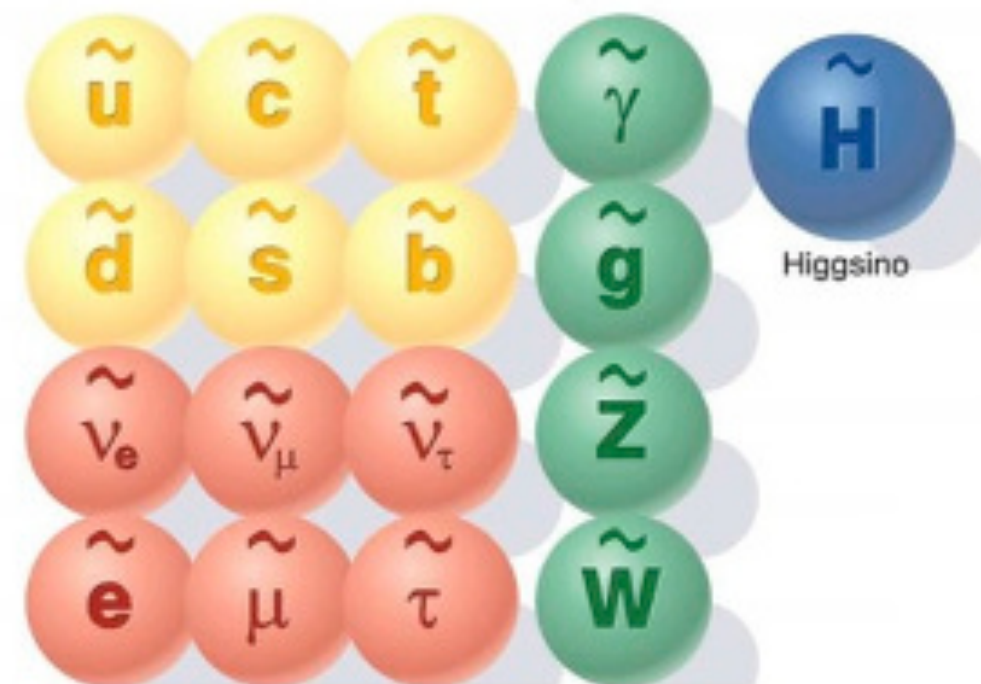
- The symmetry to exchange a boson with a fermion.
- Every particle has a partner with the same quantum numbers but a different spin.

Standard Model particles



- quarks
- leptons
- force carriers

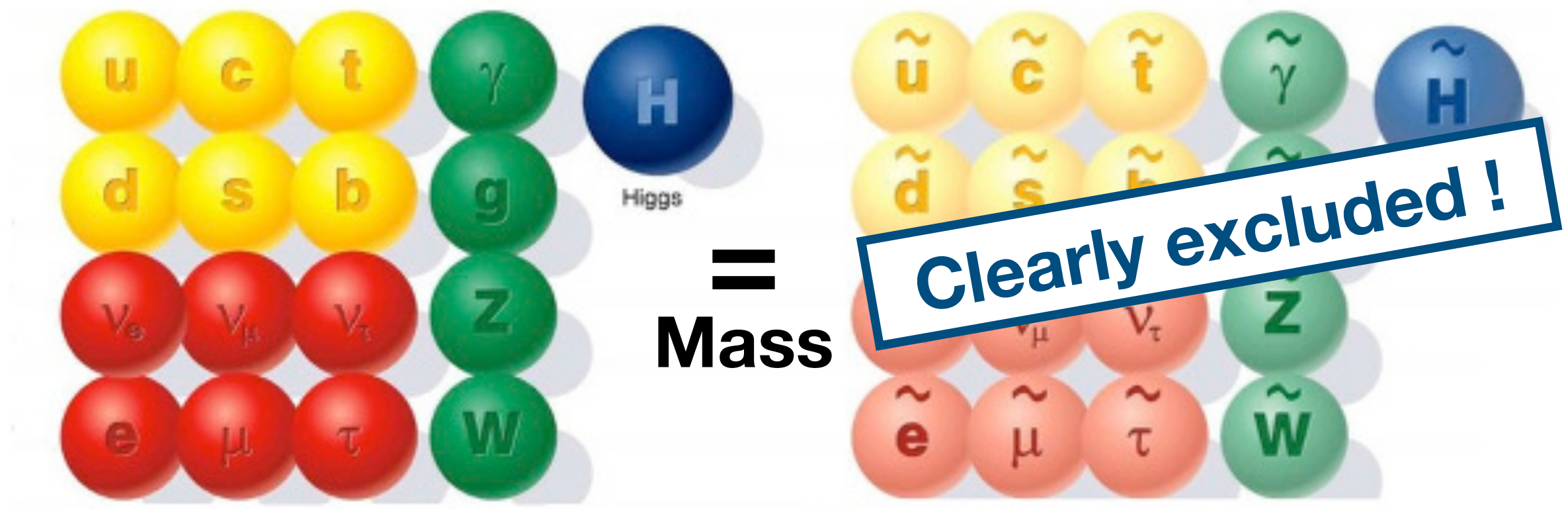
SUSY particles



- squarks
- sleptons
- SUSY force carriers

Supersymmetry

Unbroken SUSY requires superpartners with the same mass as the SM particles.

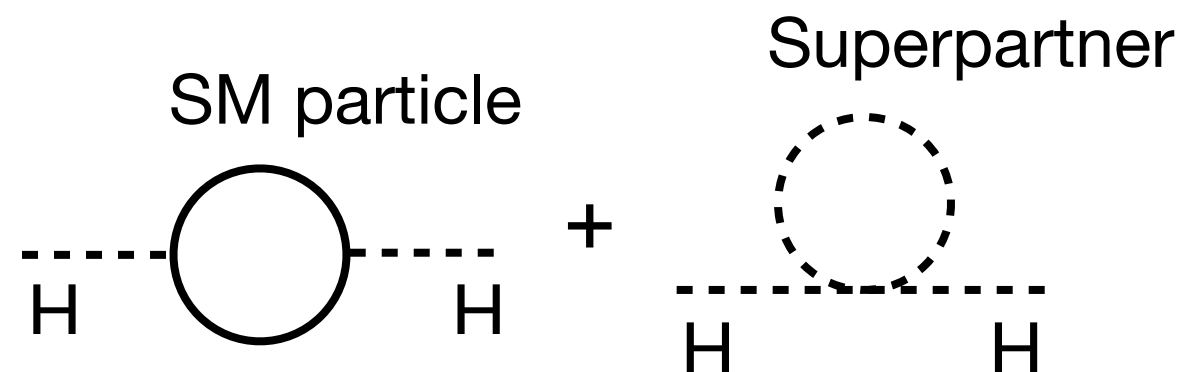


If SUSY is realized in nature, it must be broken.

➡ Rich varieties of superpartner mass spectrum.

Supersymmetry

Quantum effects of the SM particles are cancelled by the opposite-sign effects of superpartners.



The weak scale is stable

Natural

Cancellation works well

➔ Superpartner mass must be near the weak scale.

The LHC can find SUSY ?



Supersymmetry

To ensure proton stability, a discrete symmetry called “R-parity” is imposed.

SM particles : even, Supersymmetric particles : odd

The lightest supersymmetric particle (**LSP**) becomes stable.

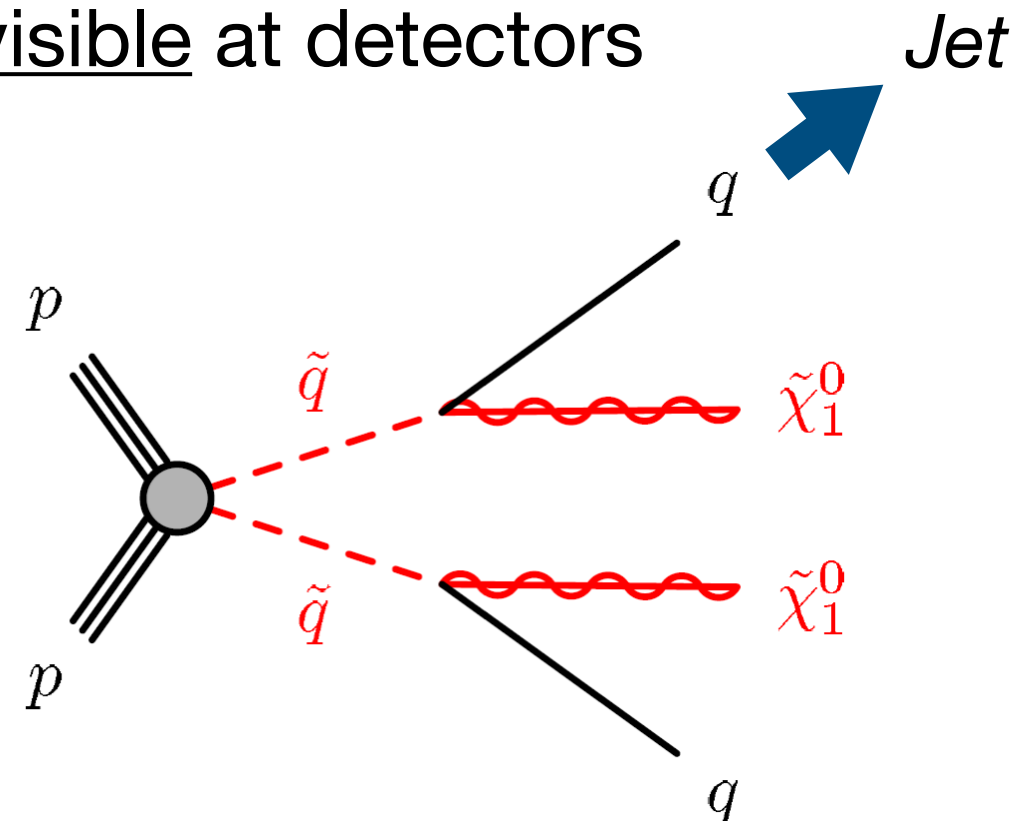
➔ **A candidate of dark matter**

Electrically neutral and invisible at detectors

SUSY searches at the LHC

Find “**Jet + missing energy**” signals !

$\tilde{q}$: Squark
 $\tilde{\chi}_1^0$: LSP



Supersymmetry

Strength of gauge boson couplings changes with energy.

Virtual superpartners change their behaviors.

S. Dimopoulos, S. Raby, F. Wilczek (1981).

PHYSICAL REVIEW D VOLUME 24, NUMBER 6 15 SEPTEMBER 1981

Supersymmetry and the scale of unification

S. Dimopoulos

*Institute for Theoretical Physics, Stanford University, Stanford, California 94305,
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106,
and University of Michigan, Ann Arbor, Michigan 48109*

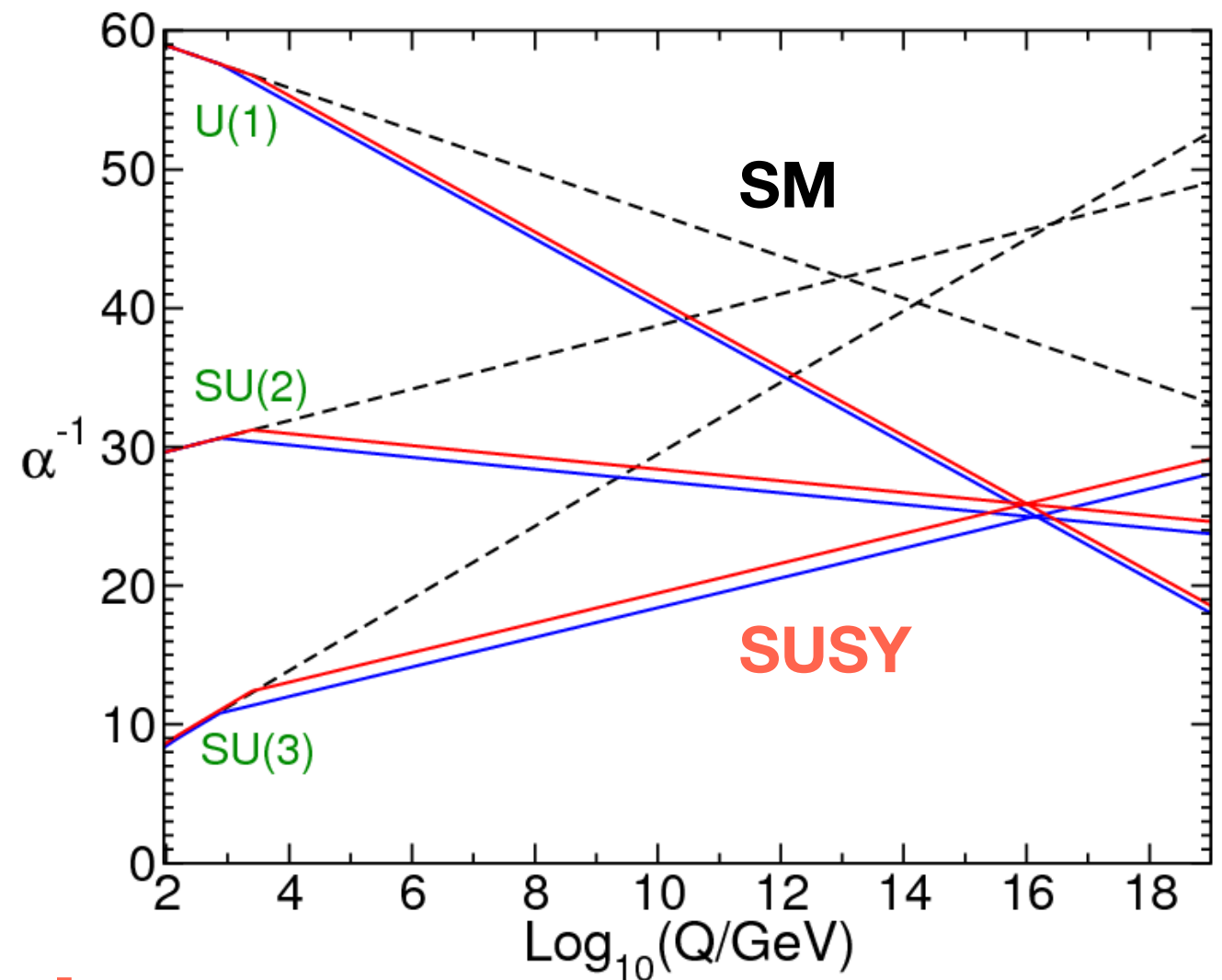
S. Raby

*Institute for Theoretical Physics, Stanford University, Stanford, California 94305
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

Frank Wilczek

*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 20 April 1981)*

Unified theories which are supersymmetric down to energies $\sim 10^3$ – 10^5 GeV have been proposed as possible solutions to the gauge-hierarchy problem. The additional particles then required have significant effects on renormalization of coupling constants. The previous successful calculation of the weak mixing angle is only slightly changed, but the scale of unification is moved significantly higher, into the range of the Planck mass. This may be suggestive of an eventual unification including gravity, and markedly reduces the predicted rate of nucleon decay.



**Three couplings unify
precisely at a high scale !**

SCPV in SUSY

The physical strong CP phase :

$$\bar{\theta} \equiv \theta - \arg \det (M_u M_d) - 3 \arg (\underline{m_{\tilde{g}}})$$

Gluino mass

To set the cosmological constant to zero

$$\Rightarrow \underline{\langle W \rangle} \sim \underline{m_{3/2}} M_{\text{Pl}}^2$$



Gravitino mass

Complex phase generates a gluino mass phase via 1-loop anomaly med.

Constraint on $\bar{\theta} \Rightarrow \frac{\alpha_s}{4\pi} \frac{m_{3/2}}{m_{\tilde{g}}} < 10^{-10}$

Gravitino mass must be sufficiently small.

SCPV in SUSY

- Flat directions are ubiquitous in SUSY.

Valleys in field space along which the potential is exactly flat.

- Flat directions naturally contain a point to spontaneously break CP.

How to stabilize such a point ?

Giving positive masses for all scalar fields around the minimum.

- Two qualitatively different ways for the stabilization:
 - ✓ Purely in a **supersymmetric** way ← No low-energy mode ...
 - ✓ Through **SUSY breaking effects**

SCPV in SUSY

- **Flat directions** are ubiquitous in SUSY.

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- Two qualitatively different ways for the stabilization:
 - ✓ Purely in a **supersymmetric** way
 - ✓ Through **SUSY breaking effects**  **Let's focus !**

SCPV via ~~SUSY~~

Liu, Nakagawa, YN, Wang (2025)

- **Supersymmetric potential**

$$V_F = \lambda^2 |\phi_1 \phi_2 - v^2|^2 + \lambda^2 |X|^2 (|\phi_1|^2 + |\phi_2|^2)$$

Real parameter

➡ $\langle X \rangle = 0, \quad \langle \phi_1 \rangle = v_1 e^{i\theta}, \quad \langle \phi_2 \rangle = v_2 e^{-i\theta} \quad v_1 v_2 \equiv v^2$

Not uniquely determined **SCPV scale**

$$\phi_1(x) = \left(v_1 + \frac{\sigma_1(x)}{\sqrt{2}} \right) \exp \left[i \left(\theta + \frac{\pi_1(x)}{\sqrt{2}v_1} \right) \right], \quad \phi_2(x) = \left(v_2 + \frac{\sigma_2(x)}{\sqrt{2}} \right) \exp \left[i \left(-\theta + \frac{\pi_2(x)}{\sqrt{2}v_2} \right) \right]$$

Massless modes :

$$s(x) = \frac{1}{f_a} (v_1 \sigma_1(x) - v_2 \sigma_2(x)), \quad a(x) = \frac{1}{f_a} (v_1 \pi_1(x) - v_2 \pi_2(x))$$

$f_a \equiv \sqrt{v_1^2 + v_2^2}$

SCPV via ~~SUSY~~

Two terms with different periodicity are needed, *e.g.* $V = \mathcal{C} \cos \theta + \mathcal{D} \cos(2\theta)$

- **SUSY breaking**

$$V_{\text{soft}} = \left(\frac{1}{2} b_1 \phi_1^2 + \frac{1}{2} b_2 \phi_2^2 + \text{h.c.} \right) + m_1^2 \phi_1^* \phi_1 + m_2^2 \phi_2^* \phi_2$$

The potential is bounded from below. $\Rightarrow |b_1| \leq m_1^2, \quad |b_2| \leq m_2^2$

- **Non-perturbative effect**

SU(N) dark QCD with quarks coupled to SCPV fields.

$$\Rightarrow V_{\text{dyn}} = \frac{(\kappa_1^2 + \kappa_2^2) \Lambda^{6 - \frac{2}{N}}}{|\kappa_1 \phi_1 + \kappa_2 \phi_2|^{2 - \frac{2}{N}}}$$

Real coupling constants

Λ : dark QCD dynamical scale

SCPV via ~~SUSY~~

$$V_{\text{tot}} = V_F + V_{\text{soft}} + V_{\text{dyn}}$$

$$= b_1 \left(v_1^2 + \frac{b_2}{b_1} \frac{v^4}{v_1^2} \right) \frac{\cos(2\theta)}{\cos(2\theta)} + m_1^2 \left(v_1^2 + \frac{m_2^2}{m_1^2} \frac{v^4}{v_1^2} \right) + \frac{(\kappa_1^2 + \kappa_2^2) \Lambda^{6-\frac{2}{N}}}{[\kappa_1^2 v_1^2 + \kappa_2^2 v^4 / v_1^2 + 2\kappa_1 \kappa_2 v^2 \cos(2\theta)]^{1-\frac{1}{N}}}$$

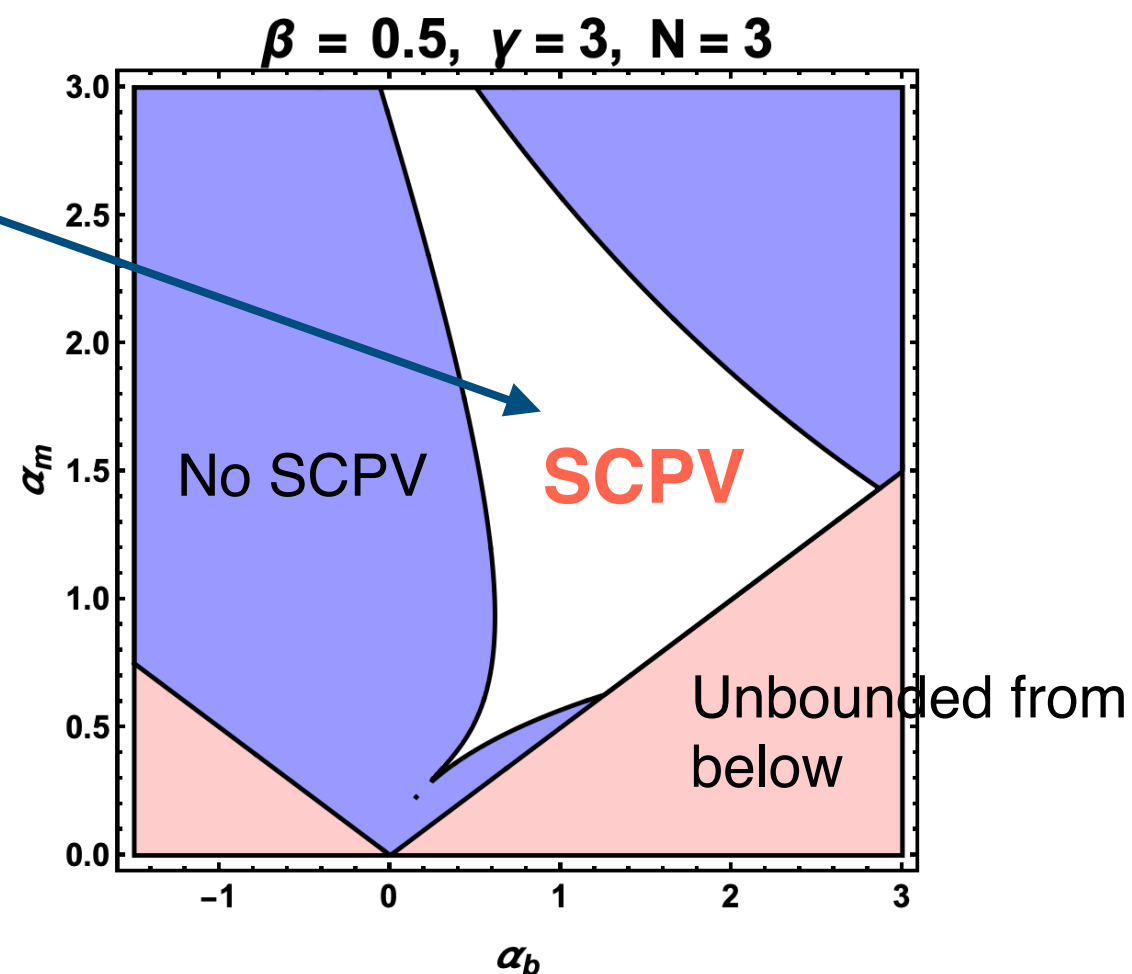
Stabilized at a non-trivial value ($\neq 0, \pi$)

All directions properly stabilized.

Existence of light modes :

$$m_a^2 \sim m_s^2 \lesssim m_{\text{soft}}^2$$

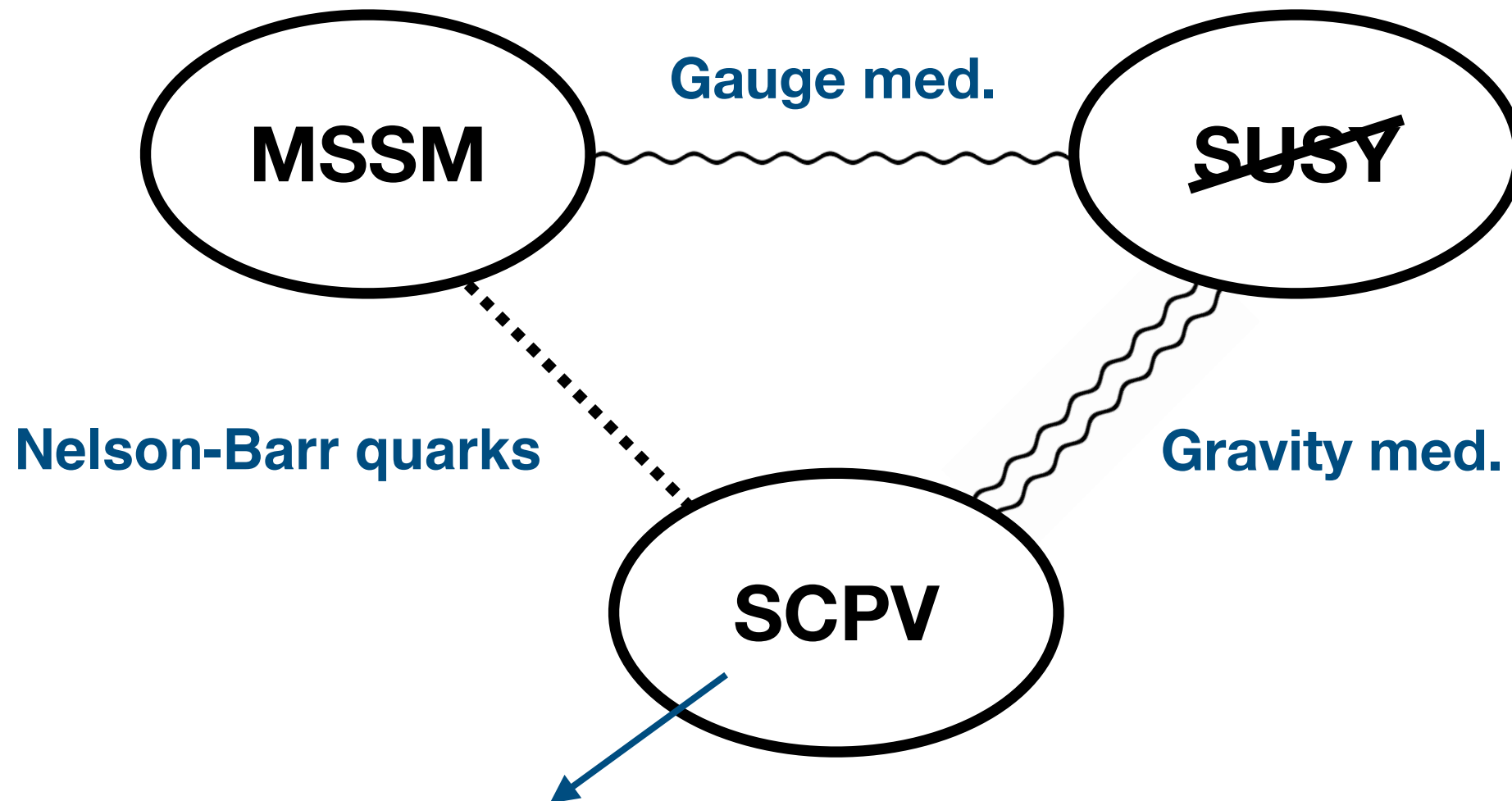
Much smaller than SCPV scale !



$$\alpha_b \equiv b_2/b_1, \alpha_m \equiv m_2^2/m_1^2, \beta \equiv b_1/m_1^2, \gamma \equiv \Lambda^{6-\frac{2}{N}}/(m_1^2 v^{4-\frac{2}{N}})$$

SCPV via ~~SUSY~~

Solving the strong CP problem via SCPV



$$m_a^2 \sim m_s^2 \sim m_{3/2}^2 < (10 - 100 \text{ keV})^2$$

Light particles feebly interacting with SM are predicted !

Detailed phenomenology & cosmology will be explored in a future study.

Baryogenesis

Considering SCPV ...

Fujikura, YN, Sato, Yamada (2022)

- What is **the source of CPV** for baryogenesis?
- High reheating temperature for thermal leptogenesis causes **the overproduction of gravitinos**.

Affleck-Dine (AD) mechanism

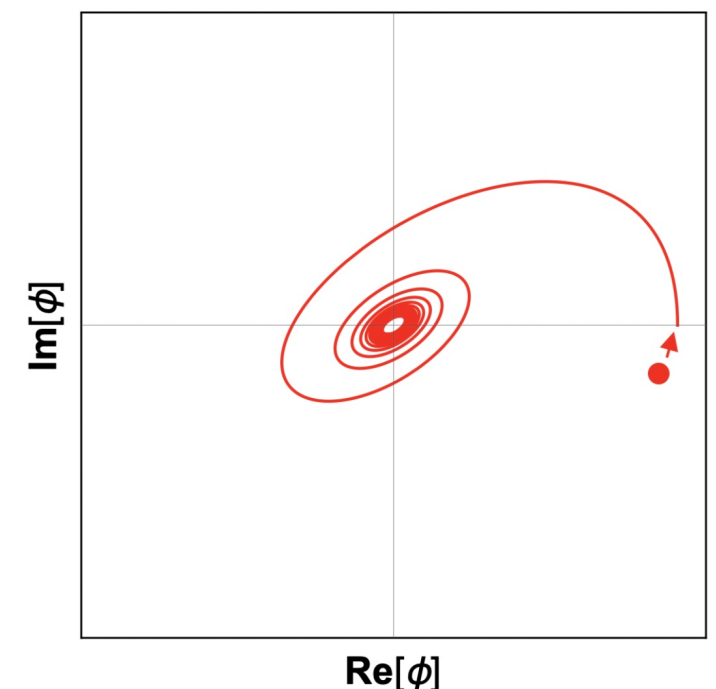
realized in SUSY and compatible with a low reheating temperature.

- ✓ **AD field** (ϕ) coherent rotational motion leads to baryon asymmetry.

$$\phi^3 \approx Q\bar{q}L$$

$$\phi = \frac{\varphi}{\sqrt{2}} e^{i\theta}, \quad n_{B-L} = q_{B-L} \varphi^2 \dot{\theta}$$

- ✓ Explicit CPV is not needed.



Baryogenesis

Fujikura, YN, Sato, Yamada (2022)

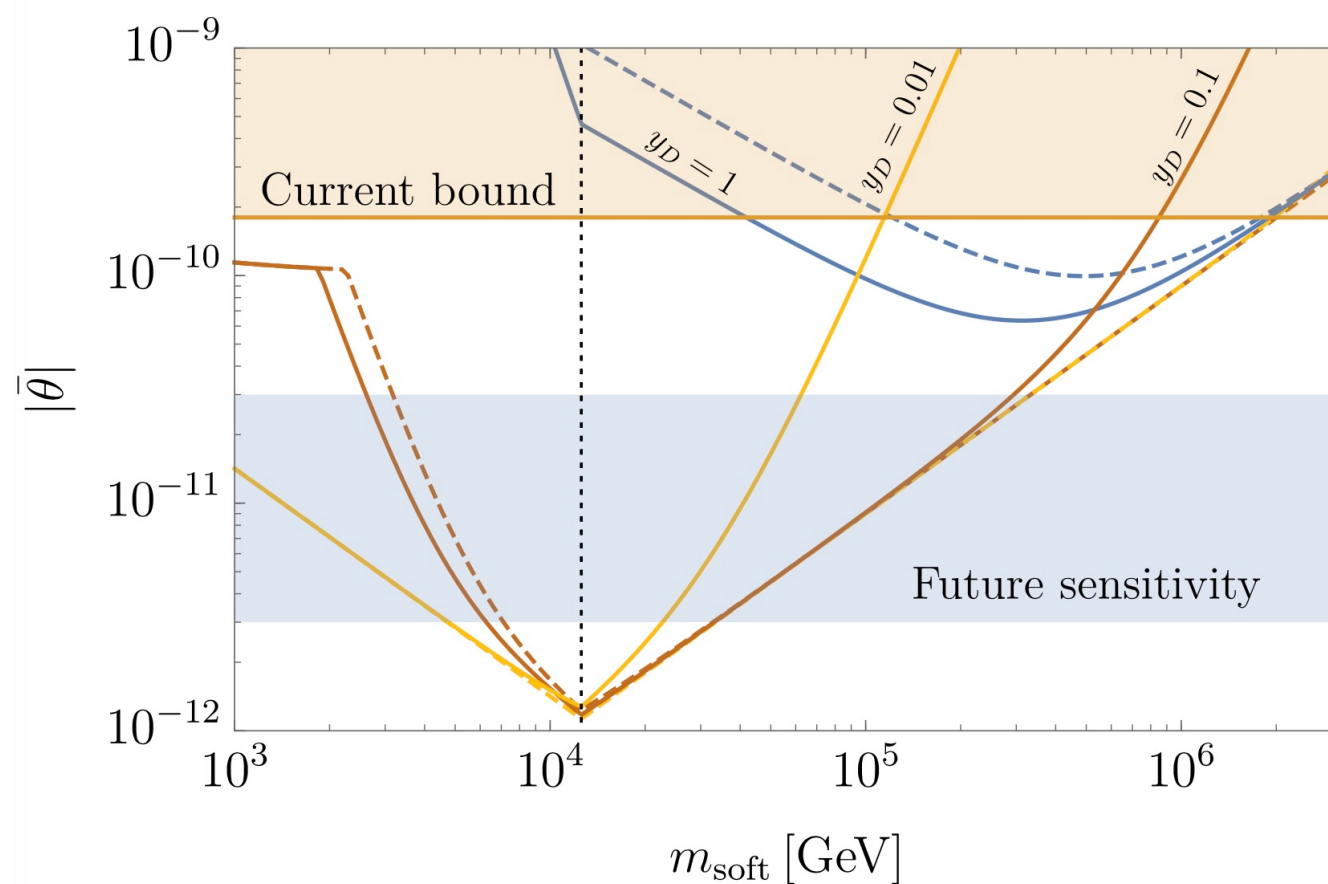
If **gravitino** gives DM ...

Lyman- α constraint $\rightarrow m_{3/2} \gtrsim 5.3 \text{ keV}$

Smallest value of $\bar{\theta}$

SCPV scale and reheating temperature are chosen to obtain the observed asymmetry and DM.

Consistency with SCPV via ~~SUSY~~ will be explored in a future study.



Neutron EDM is within the reach of near-future experiments !

Summary

Composite axion

- UV completion via **special embedding** leads to domain wall # = 1.
- **Small instanton effects** provide a bias term for the domain wall decay.

Spontaneous CP violation (SCPV) in SUSY

- SCPV vacuum stabilization via **SUSY breaking** predicts **light particles feebly interacting with SM**.
- **Neutron EDM** is within the reach of near-future experiments.

Thank you.