

XVI NExT Ph.D. Workshop – Cosener's House 24-26 August 2026

3 lectures on:

Neutrino Physics and Particle Cosmology

Pasquale Di Bari

(<http://www.hep.phys.soton.ac.uk/~pdb1d08/teaching.html>)

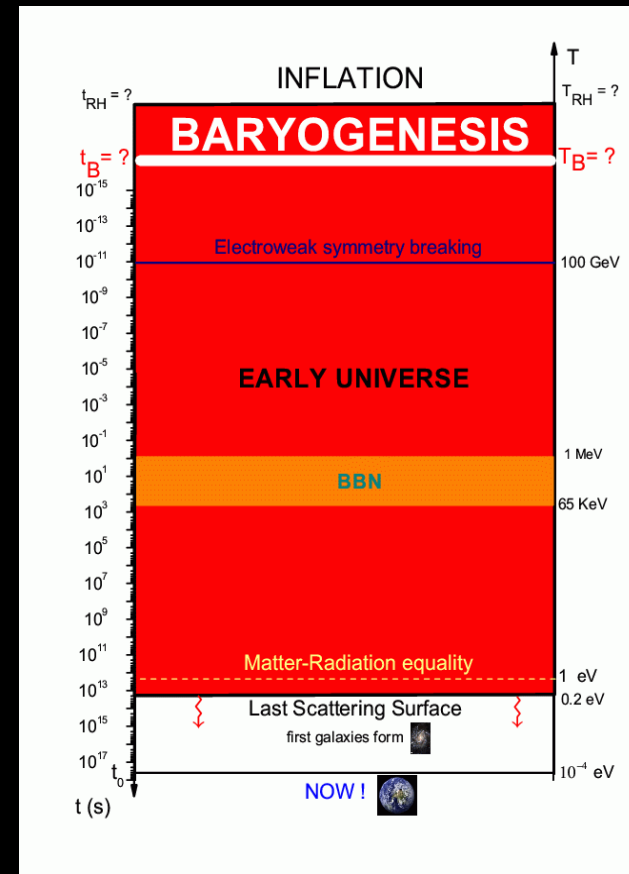


Plan

- 1) Neutrinos
- 2) Cosmology
- 3) Neutrino cosmology

References

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- D.E.Morrissey and M.J. Ramsey-Musolf, *Electroweak Baryogenesis*, arXiv:1206.2942
- R. Allahverdi and A.Mazumdar, *Affleck-Dine Baryogenesis*, NJP 14, 125013
- H.Davoudiasl and R.Mohapatra, *On Relating the Genesis of Cosmic Baryons and Dark Matter*, arXiv:1203.1247
- Steve Blanchet and Pasquale Di Bari, *The minimal scenario of leptogenesis*, arXiv:1211.0512.
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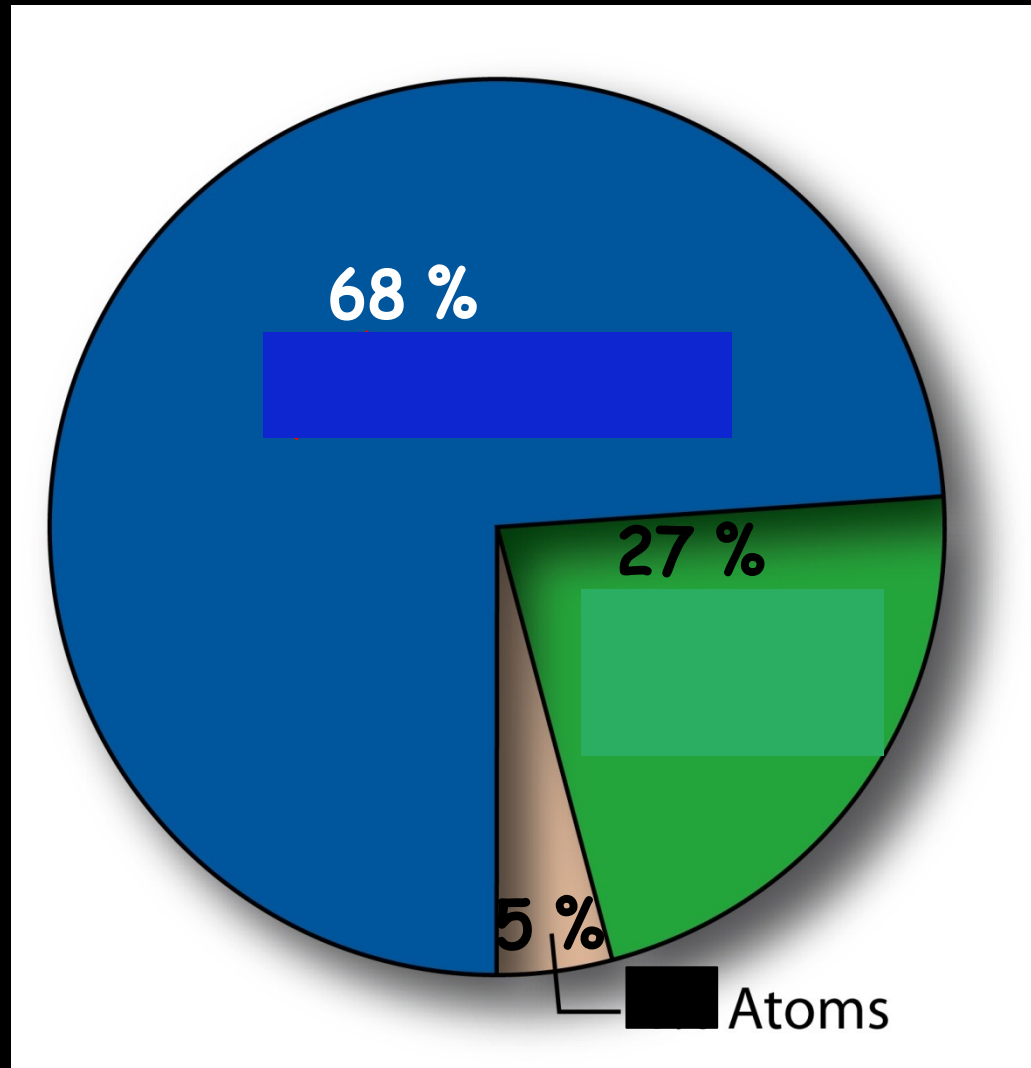


Lecture III

Neutrino Cosmology

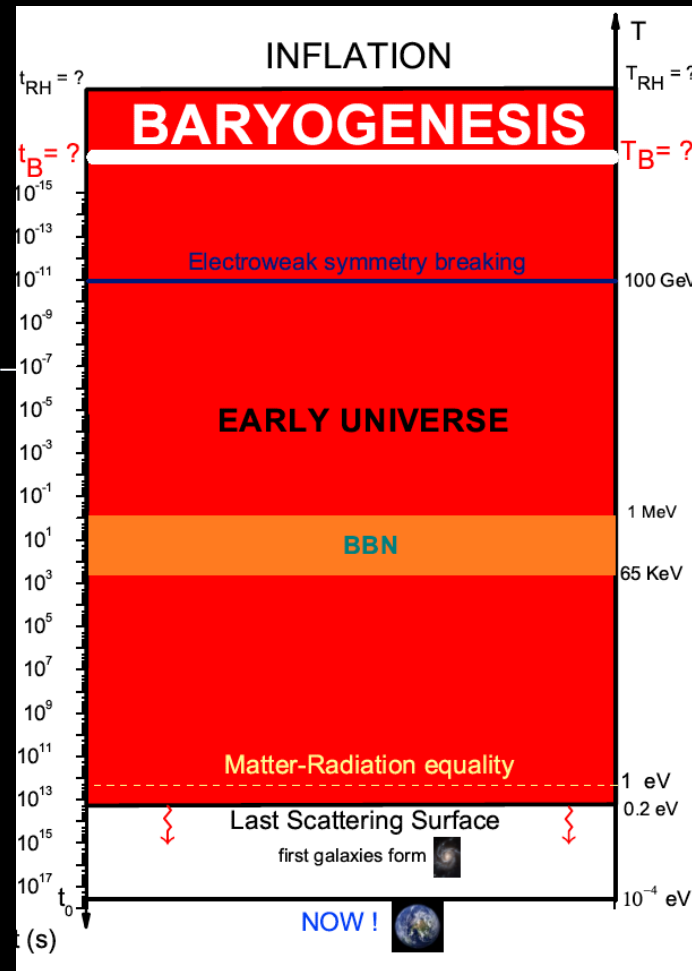
- Upper bound on neutrino masses
- Seesaw mechanism
- Leptogenesis
- Dark matter
- How neutrinos can help solving cosmological anomalies and tensions

The matter-energy budget in the Λ CDM model at t_0



Cosmological puzzles

dark
matter
production



It is reasonable to think that the same extension of the SM necessary to explain neutrino masses and mixing might also address the cosmological puzzles:

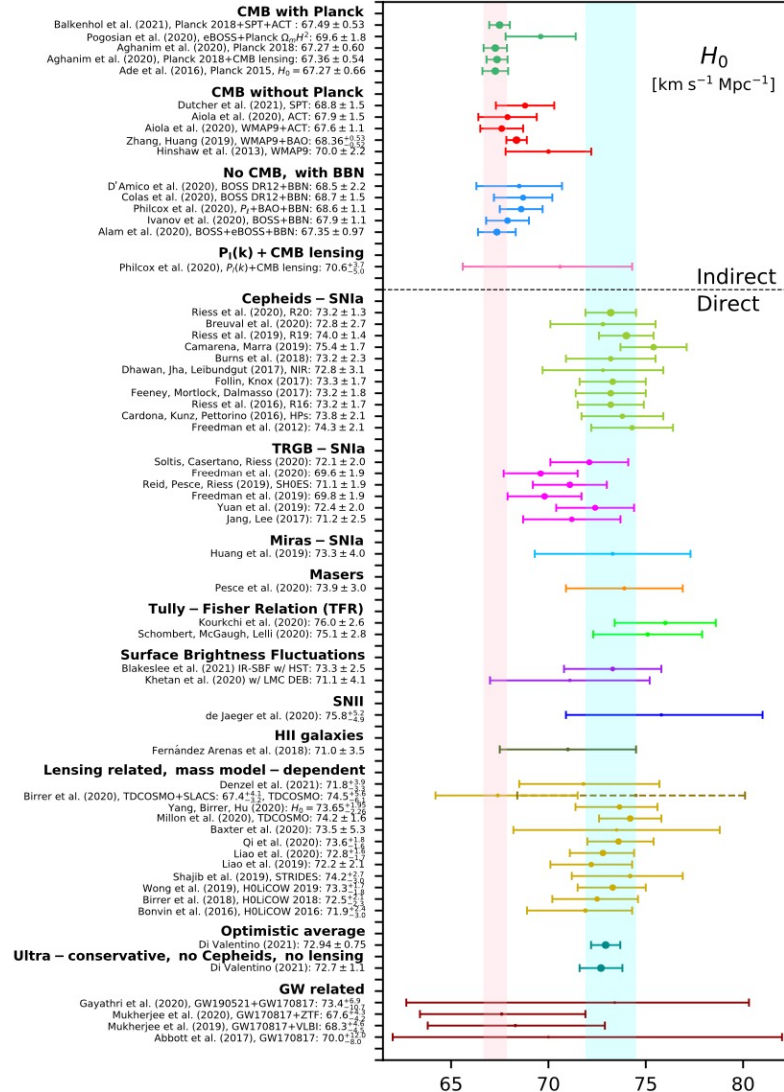
- **Leptogenesis**,
- **RH neutrino as dark matter**

Hubble tension

(E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D.F. Mota, A.G. Riess, J. Silk 2103.01183)

In the Realm of the Hubble tension – a Review of Solutions

10

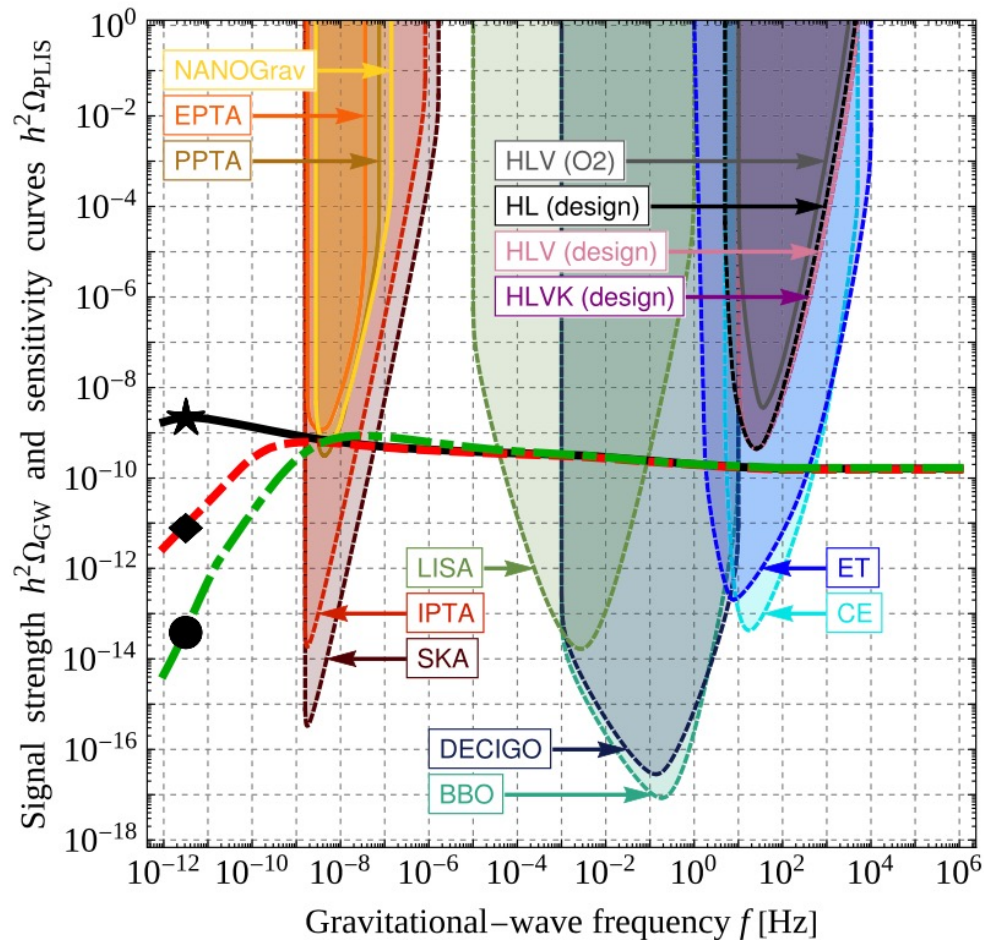


Has NANOGrav Found First Evidence for Cosmic Strings?

Simone Blasi,^{1,*} Vedran Brdar,^{1,†} and Kai Schmitz^{2,‡}

¹Max-Planck-Institut für Kernphysik, 69117 Heidelberg, Germany

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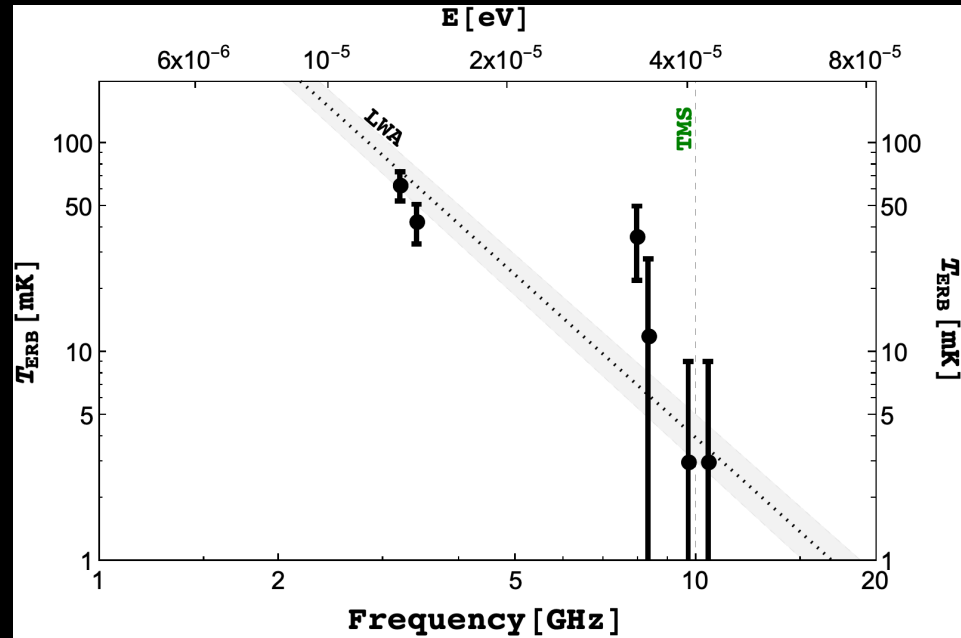


ARCADE 2: Results

(Fixsen, Kogut, Levin, Limon, Lubin, Mirel, Seiffert, Singal, Wollack, Villela, Wuensche 0901.0555)

$$T_{\text{ERB}} \equiv T_{\gamma 0}(\nu) - T_{\text{CMB},0}$$

↓
effective temperature



(Figure courtesy of Rishav Roshan)

- The ARCADE 2 measurement of the CMB temperature is in excellent agreement with the FIRAS instrument above 10 GHz
- Below 10 GHz, the detected radio background is brighter than expected ($\sim 5\sigma$ deviation)
- The Long Wavelength Array (LWA) measured the diffuse radio background in the 40-80 MHz, also finding an excess. In combination with ARCADE 2 data, they find a good fit in terms of a power-law:

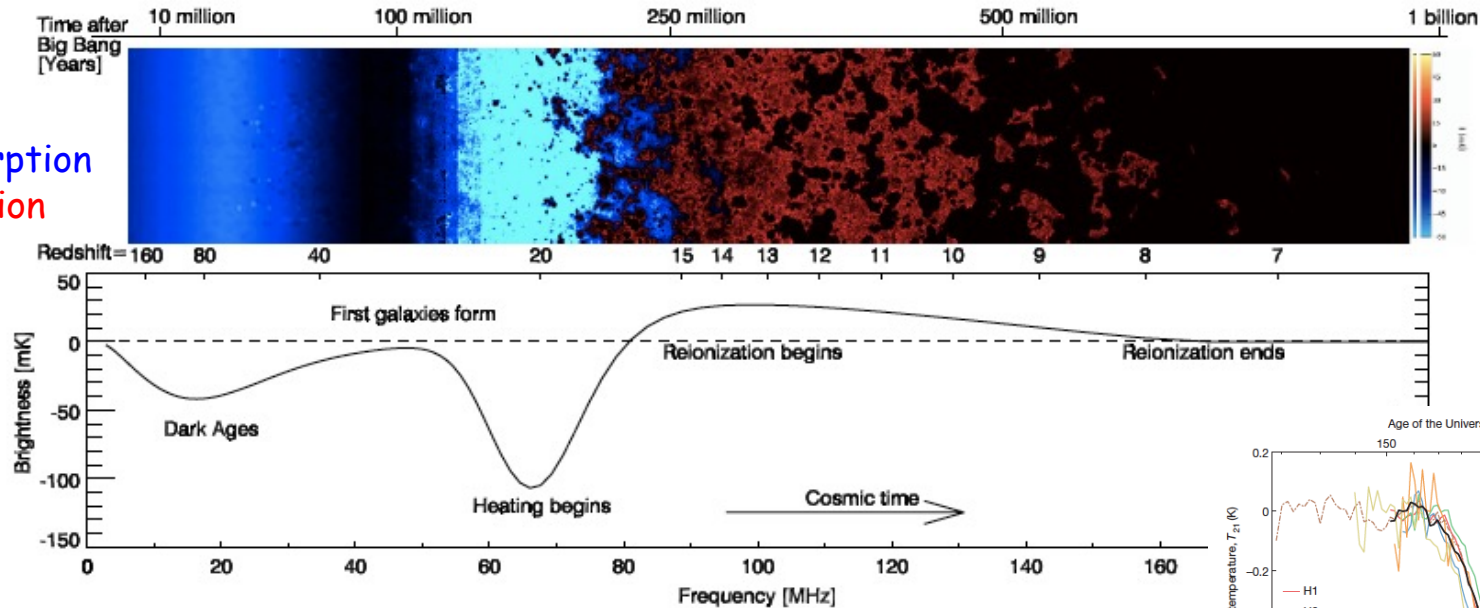
$$T_{\text{ERB}} = (30.4 \pm 2.6) \left(\frac{\nu}{310 \text{ MHz}} \right)^{-2.58 \pm 0.05} \text{ K.}$$

21 cm cosmology (global signal)

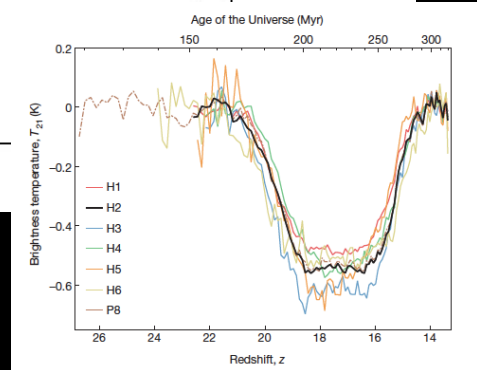
- 21 cm line (emission or absorption) is produced by hyperfine transitions between the two energy levels of 1s ground state of Hydrogen atoms. The energy splitting between the two level is $E_{21}=5.87\mu\text{eV}$
- The **21cm brightness temperature** parametrises the brightness contrast :

$$T_{21}(z) \simeq 23 \text{ mK} (1 + \delta_B) x_{H_I}(z) \left(\frac{\Omega_B h^2}{0.02} \right) \left[\left(\frac{0.15}{\Omega_m h^2} \right) \left(\frac{1+z}{10} \right) \right]^{1/2} \left[1 - \frac{T_\gamma(z)}{T_S(z)} \right]$$

spin
temperature



EDGES (anomalous) signal →



Ordinary neutrinos as hot dark matter

From neutrino oscillations

$$m_\nu \equiv \sum_i m_i \geq m_{\text{atm}} + m_{\text{sol}} \simeq 0.06 \text{ eV}$$

At present time

$$\Omega_{\nu 0} = \frac{\varepsilon_{\nu 0}}{\varepsilon_{c0}} \simeq \frac{m_\nu n_{\nu 0}}{\varepsilon_{c0}} \simeq \frac{m_\nu}{93.14 h^2 \text{ eV}}$$

$$\Omega_{\nu 0} h^2 \leq \Omega_{\text{DM}} h^2 \simeq 0.12$$

$\Rightarrow$

$$\sum_i m_i \leq 11 \text{ eV}$$

However a much more restrictive upper bound can be derived considering that neutrinos behave like HOT Dark Matter while we want it to be COLD:

$$\Omega_{\nu 0} h^2 \leq f_\nu \Omega_{\text{DM}} h^2 \simeq 0.12 f_\nu$$

$\Rightarrow$

$$\sum_i m_i \leq 11 \text{ eV } f_\nu$$

The fraction of hot dark matter is severely constrained from CMB+BAO data. From the final *Planck* data release (combined with BAO data):

$$f_\nu < 0.01 \Rightarrow \sum_i m_i < 0.11 \text{ eV (95\% C.L.)}$$

Massive neutrinos and cosmological perturbations

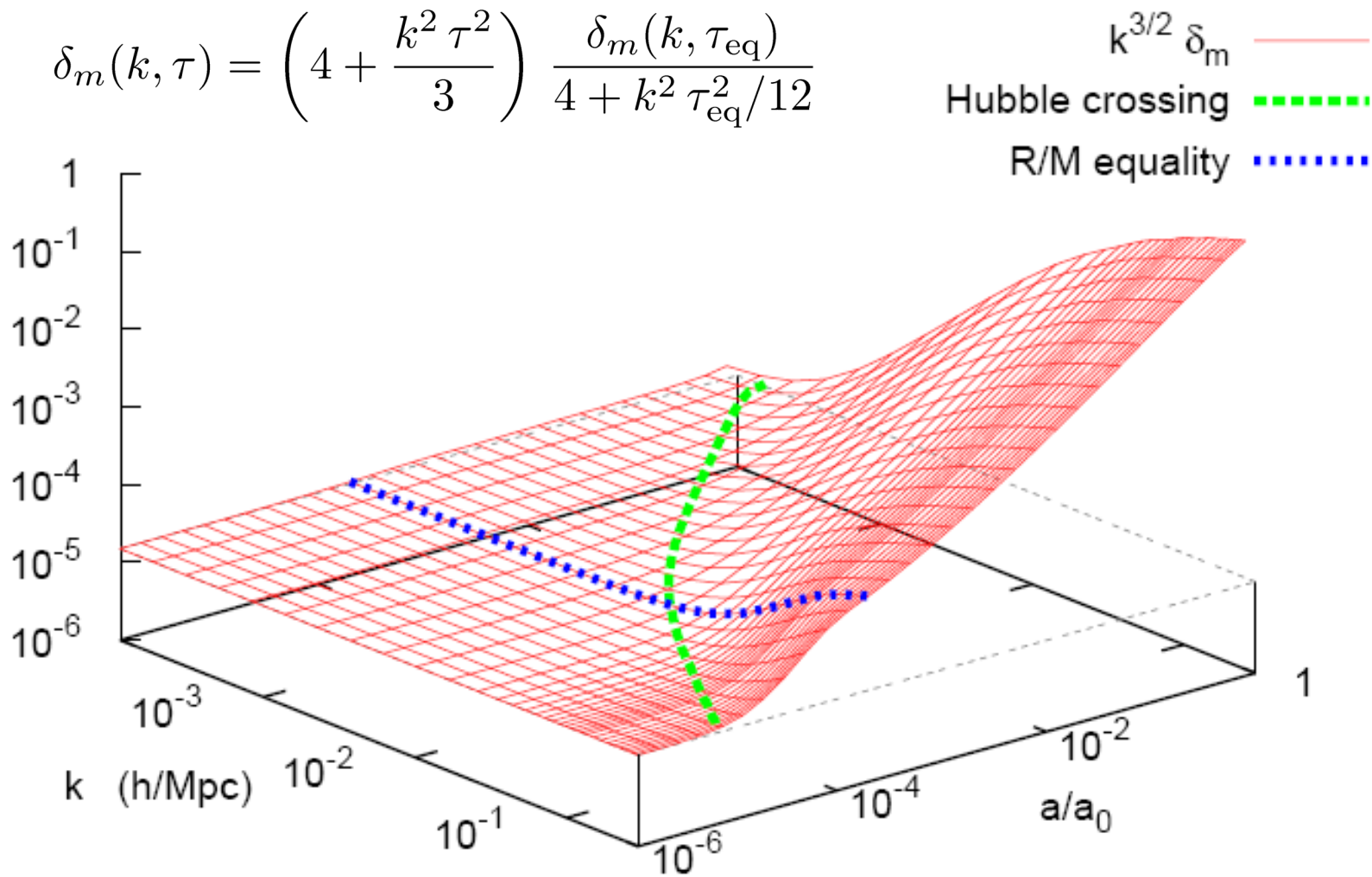
$$\begin{aligned}\delta g_{\mu\nu}(\mathbf{x}, t) &= g_{\mu\nu}(\mathbf{x}, t) - \bar{g}_{\mu\nu}(t) \ , \\ \delta T_{\mu\nu}(\mathbf{x}, t) &= T_{\mu\nu}(\mathbf{x}, t) - \bar{T}_{\mu\nu}(t) \ ,\end{aligned}$$

great advantage of linear theory is, as usual, to obtain independent equations of evolution for each Fourier mode. Note that the Fourier decomposition must be performed with respect to the comoving coordinate system: so, the quantity $(2\pi/k)$ is the *comoving wavelength* of a perturbation of wavevector $\mathbf{k}$, while the *physical wavelength* is given by

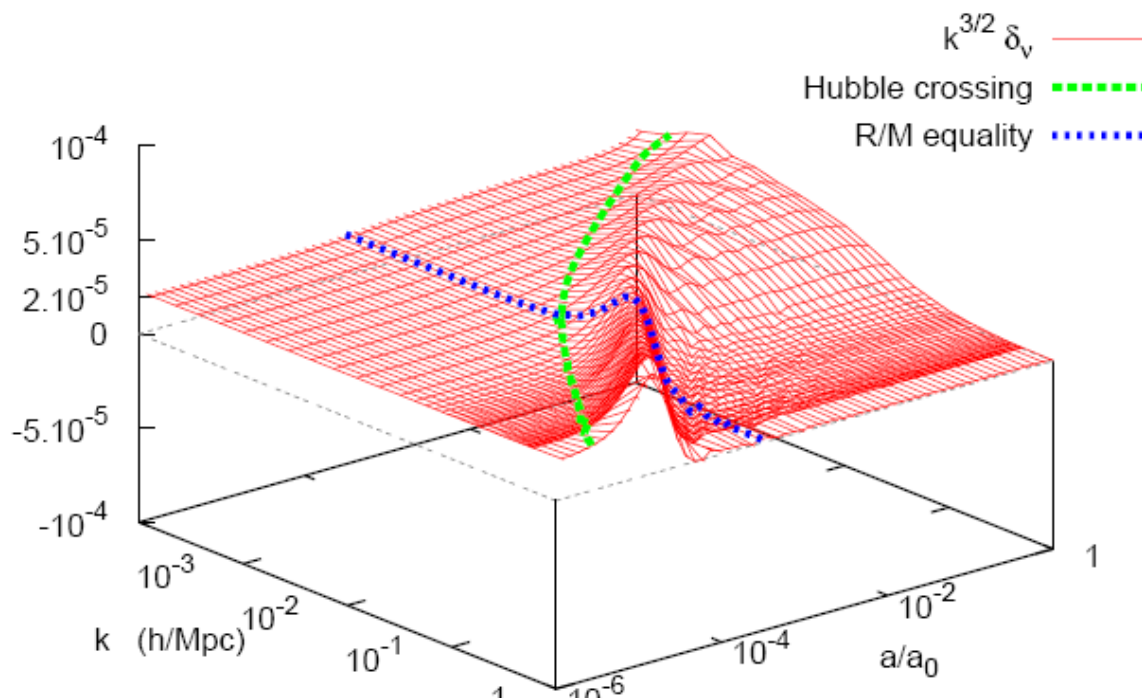
$$\lambda = a(t) \frac{2\pi}{k} \ , \tag{15}$$

where $a(t)$ is the scale factor of the Universe. For each mode $\mathbf{k}$, the perturbation amplitudes evolve under some equations of motion (which depend only on the modulus k , since the background is isotropic), and on top of this evolution, the physical wavelength is stretched according to the Universe expansion.

$$\delta_m(k, \tau) = \left(4 + \frac{k^2 \tau^2}{3}\right) \frac{\delta'_m(k, \tau_{\text{eq}})}{4 + k^2 \tau_{\text{eq}}^2 / 12}$$



The physical effect of free-streaming is to damp small-scale neutrino density fluctuations: neutrinos cannot be confined into (or kept outside of) regions smaller than the free-streaming length, for obvious kinematic reasons. We will see that the metric perturbations are also damped on those scales by gravitational back-reaction. On scales much larger than the free-streaming scale, the neutrino velocity can be effectively considered as vanishing, and after the non-relativistic transition the neutrino perturbations behave like cold dark matter perturbations. In particular, modes with $k < k_{\text{nr}}$ are never affected by free-streaming and evolve like in a pure Λ CDM model.



$$k_{nr} = 0.018 \Omega_m^{1/2} \left(\frac{m}{1 \text{ eV}} \right)^{1/2} h \text{ Mpc}$$

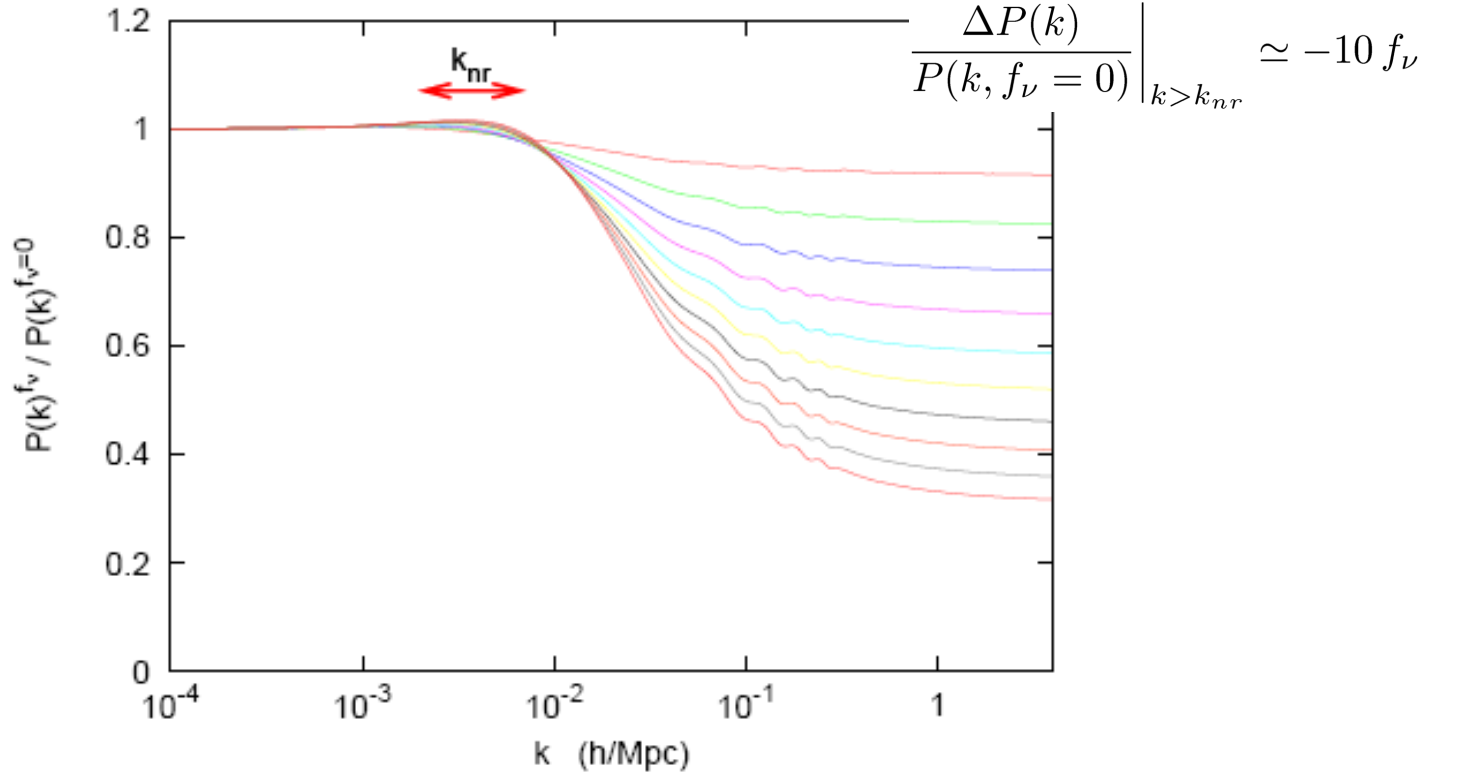
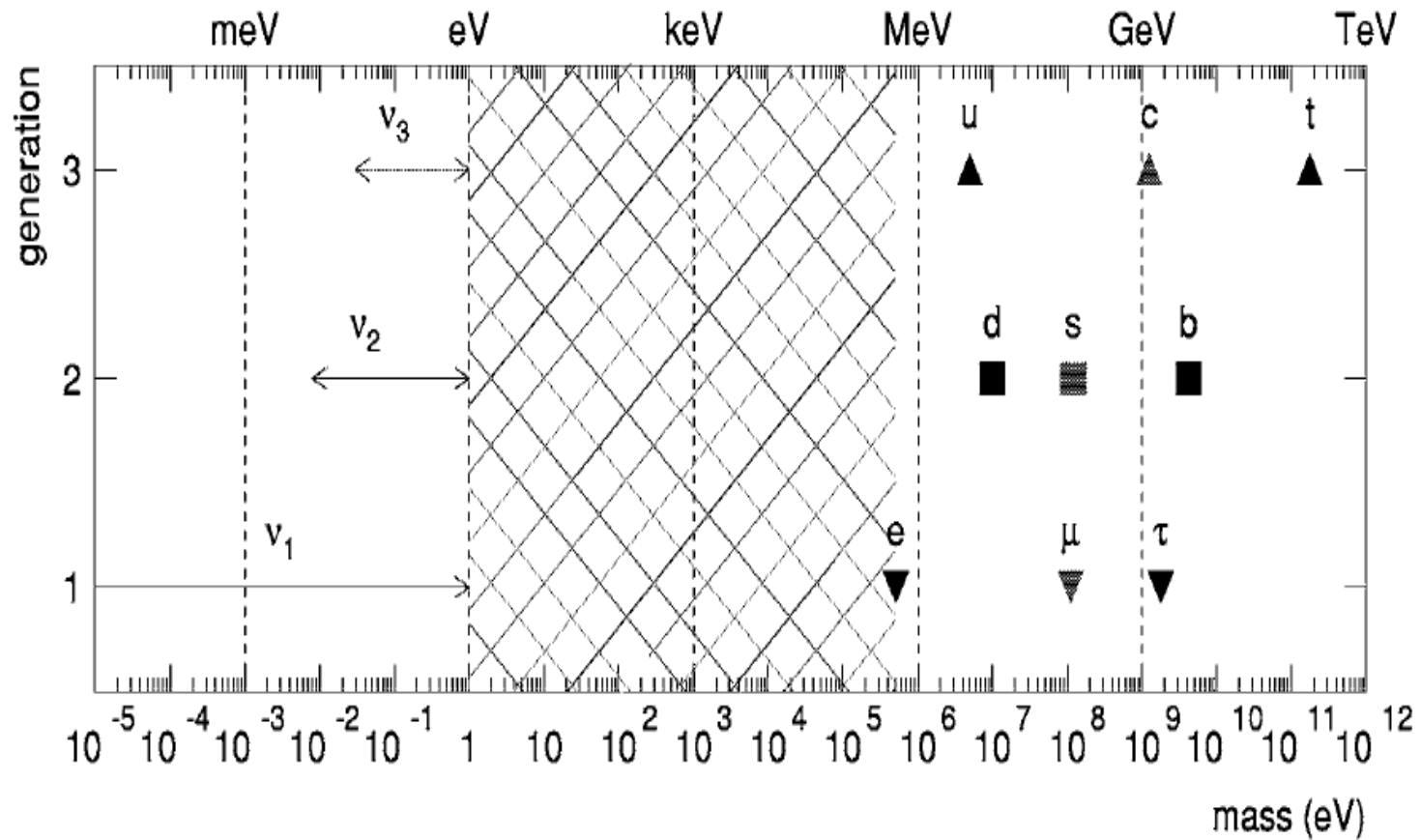


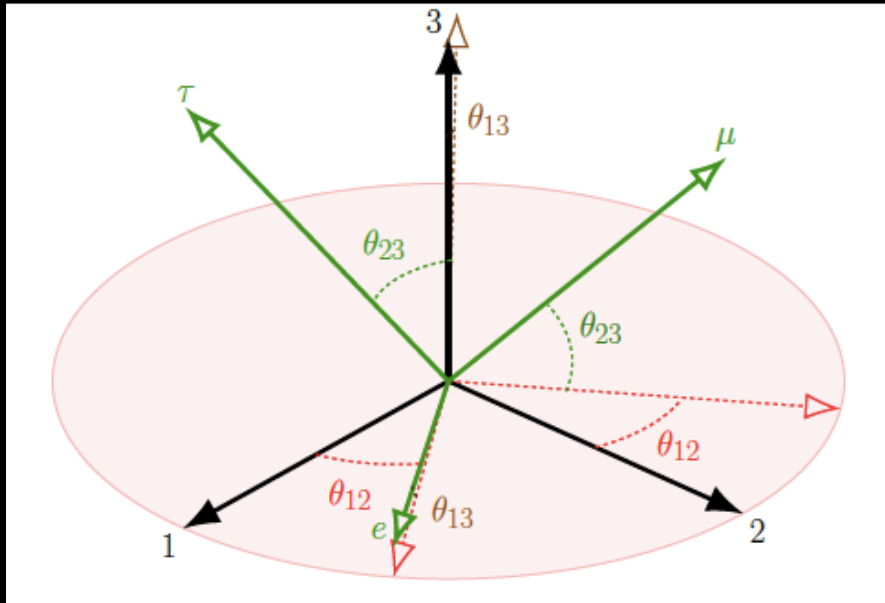
Fig. 13. Ratio of the matter power spectrum including three degenerate massive neutrinos with density fraction f_ν to that with three massless neutrinos. The parameters $(\omega_m, \Omega_\Lambda) = (0.147, 0.70)$ are kept fixed, and from top to bottom the curves correspond to $f_\nu = 0.01, 0.02, 0.03, \dots, 0.10$. The individual masses m_ν range from 0.046 eV to 0.46 eV, and the scale k_{nr} from $2.1 \times 10^{-3} h \text{ Mpc}^{-1}$ to $6.7 \times 10^{-3} h \text{ Mpc}^{-1}$ as shown on the top of the figure. k_{eq} is approximately equal to $1.5 \times 10^{-2} h \text{ Mpc}^{-1}$.

Neutrinos are much lighter than all other fermions:

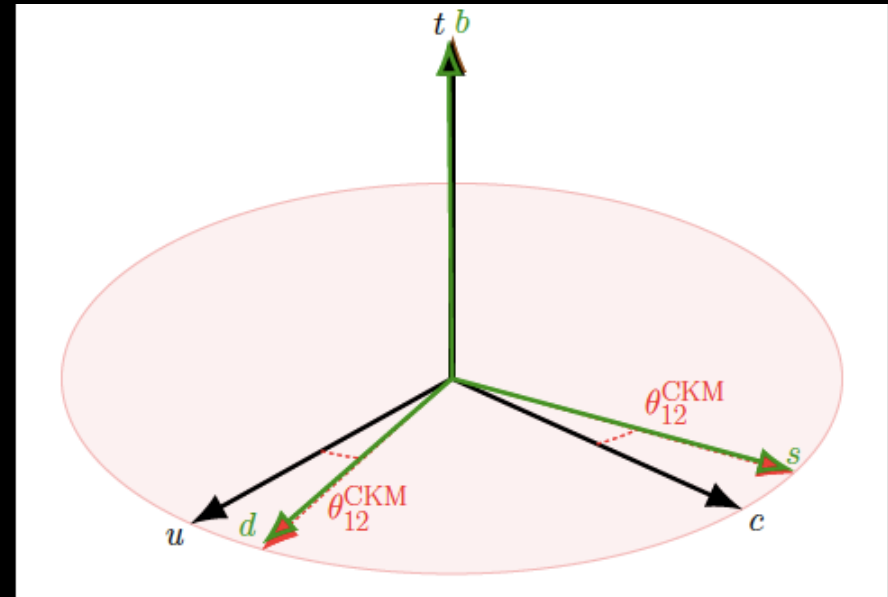


Leptonic mixing angles are much larger than quark mixing angles

lepton mixing angles



quark mixing angles



Minimally extended SM

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_Y^\nu$$

$$-\mathcal{L}_Y^\nu = \overline{\nu}_L h^\nu \nu_R \phi \Rightarrow -\mathcal{L}_{\text{mass}}^\nu = \overline{\nu}_L m_D \nu_R$$

Dirac
Mass

(in a basis where charged lepton mass matrix is diagonal)

diagonalising m_D :

$$m_D = V_L^\dagger D_{m_D} U_R$$

$$D_{m_D} \equiv \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix}$$

$\Rightarrow$ neutrino masses: $m_i = m_{Di}$

leptonic mixing matrix: $U = V_L^\dagger$

But many unanswered questions:

- Why neutrinos are much lighter than all other fermions?
- Why large mixing angles (differently from CKM angles)?
- Cosmological puzzles?
- Why not a Majorana mass term as well?

Majorana mass from new physics: effective theory approach

(Weinberg 1979)

Allowing for lepton number violation:

$$\mathcal{L}_{\text{Weinberg}} = (\overline{L}_\alpha^c \tilde{\phi}^*) \frac{g_{\alpha\beta}}{\Lambda} (\tilde{\phi}^\dagger L_\beta) + h.c.$$

After EWSSB this generates a left-left neutrino Majorana mass term:

$$\Rightarrow -\mathcal{L}_{\text{Majorana}}^\nu = \overline{\nu_{L\alpha}^c} m_{\nu\alpha\beta} \nu_{L\beta} \quad \text{with} \quad m_{\nu\alpha\beta} = \frac{g_{\alpha\beta} v^2}{\Lambda} \propto \frac{m_D^2}{\Lambda}$$

(seesaw formula)

Minimal seesaw mechanism (type I)

• Dirac + (right-right) Majorana mass terms

(Minkowski '77; Gell-mann, Ramond, Slansky; Yanagida; Mohapatra, Senjanovic '79)

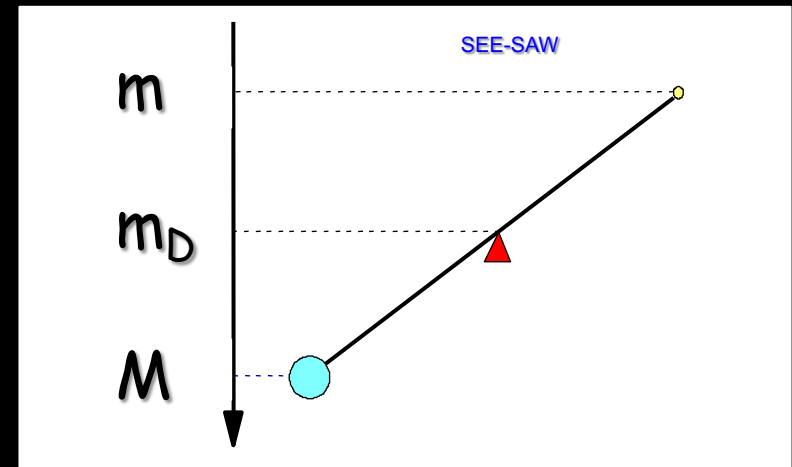
$$-\mathcal{L}_{mass}^{\nu} = \bar{\nu}_L m_D \nu_R + \frac{1}{2} \bar{\nu}_R^c M \nu_R + h.c. = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

In the *see-saw limit* ($M \gg m_D$) the mass spectrum splits into 2 sets:

- 3 light **Majorana neutrinos** with masses (seesaw formula): $m_\nu = -m_D M^{-1} m_D^T \Rightarrow \text{diag}(m_1, m_2, m_3) = -U^\dagger m_\nu U^*$
- 3 (?) very heavy Majorana neutrinos N_1, N_2, N_3 with $M_3 > M_2 > M_1 \gg m_D$

1 generation toy model :

$$\begin{aligned} m_D &\sim m_{\text{top}}, \\ m &\sim m_{\text{atm}} \sim 50 \text{ meV} \\ \Rightarrow M &\sim 10^{15} \text{ GeV} \end{aligned}$$



Seesaw models with three generations: two limits

In the flavour basis (both charged lepton mass and Majorana mass matrices are diagonal):

$$-\mathcal{L}_{\text{mass}}^{\nu+\ell} = \overline{\alpha_L} m_\alpha \alpha_R + \overline{\nu_{L\alpha}} m_{D\alpha I} \nu_{RI} + \frac{1}{2} \overline{\nu_{RI}^c} M_I \nu_{RI} + \text{h.c.}$$

$$\alpha = e, \mu, \tau$$

$$I = 1, 2, 3$$

bi-unitary parameterisation: $m_D = V_L^\dagger D_{m_D} U_R$ $D_{m_D} \equiv \text{diag}(m_{D1}, m_{D2}, m_{D3})$

FIRST (EASY) LIMIT: ALL MIXING FROM THE LEFT-HANDED SECTOR

• $U_R = I \Rightarrow$ again $U = V_L^\dagger$ and neutrino masses: $m_i = \frac{m_{Di}^2}{M_I}$

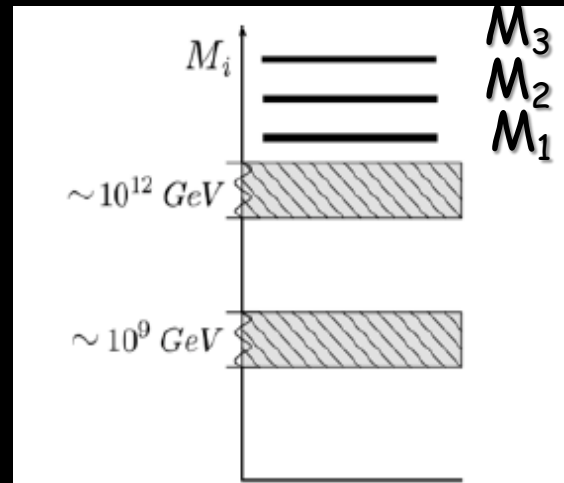
If also $m_{D1} = m_{D2} = m_{D3} = \lambda$ then simply: $M_I = \frac{\lambda^2}{m_i}$

Exercise: $\lambda \sim 100 \text{ GeV}$

$$m_1 \sim 10^{-4} \text{ eV} \Rightarrow M_3 \sim 10^{17} \text{ GeV}$$

$$m_2 = m_{\text{sol}} \sim 10 \text{ meV} \Rightarrow M_2 \sim 10^{15} \text{ GeV}$$

$$m_3 = m_{\text{atm}} \sim 50 \text{ meV} \Rightarrow M_1 \sim 10^{14} \text{ GeV}$$



Typically RH neutrino mass spectrum emerging in simple discrete flavour symmetry models

A SECOND LIMIT: ALL MIXING FROM THE RH SECTOR

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03; PDB, Riotto '08; PDB, Re Fiorentin '12)

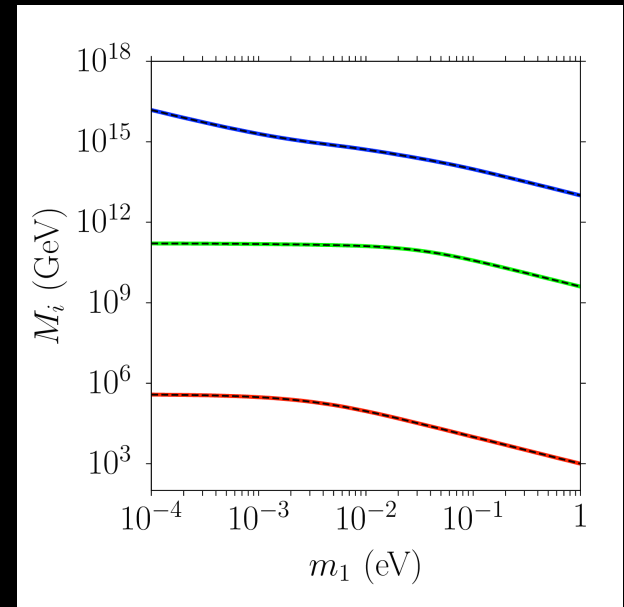
$$\bullet \quad V_L = I \Rightarrow M_1 = \frac{m_{D1}^2}{m_{\beta\beta}}; \quad M_2 = \frac{m_{D2}^2}{m_1 m_2 m_3} \frac{m_{\beta\beta}}{|(m_v^{-1})_{\tau\tau}|}; \quad M_3 = m_{D3}^2 |(m_v^{-1})_{\tau\tau}|$$

If one also imposes (SO(10)-inspired models)

$$m_{D1} = \alpha_1 m_{up}; \quad m_{D2} = \alpha_2 m_{charm}; \quad m_{D3} = \alpha_3 m_{top}; \quad \alpha_i = O(1)$$

one obtains a very hierarchical
RH neutrino mass spectrum
(barring very fine-tuned solutions)

Combining discrete flavour + grand
unified symmetries one can obtain
mass spectra between
these two limits



WHAT CAN HELP TESTING THE EXISTENCE OF HEAVY RH
NEUTRINOS AND THEIR MASS SPECTRUM?

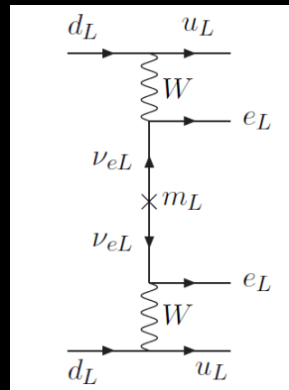
Neutrinos are predicted to be Majorana particles

Both light and heavy neutrinos are Majorana neutrinos:

$$\nu_i \equiv \nu_{iL} + \nu_{iL}^c$$

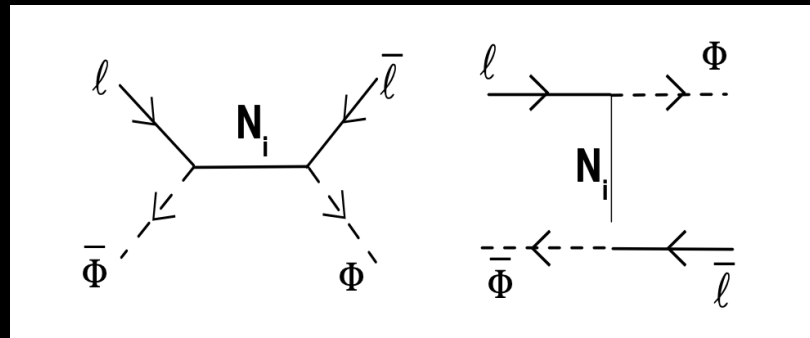
$$N_I \equiv N_{IR} + N_{IR}^c$$

- For light neutrinos:



$0\nu\beta\beta$
decay

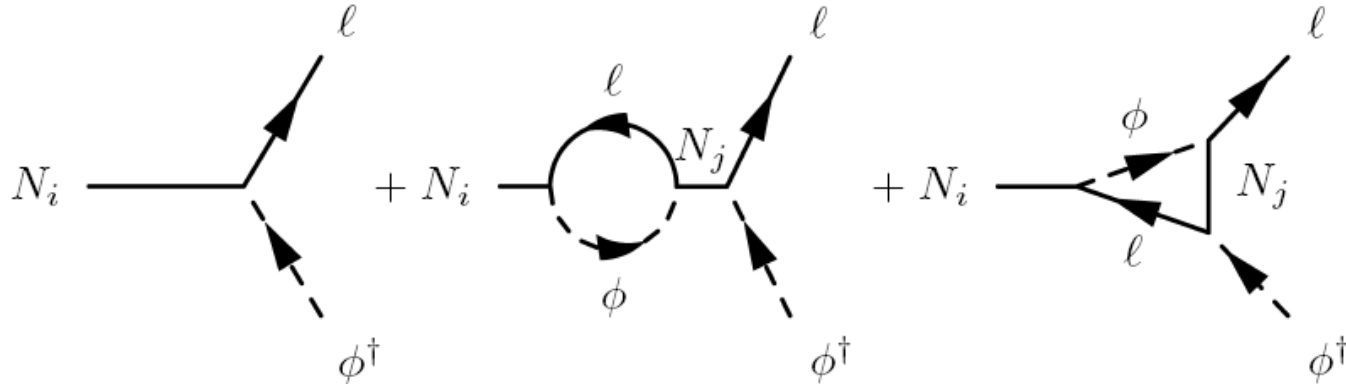
- For heavy neutrinos:



They are both $\Delta L = 2$ processes

Total CP asymmetries

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

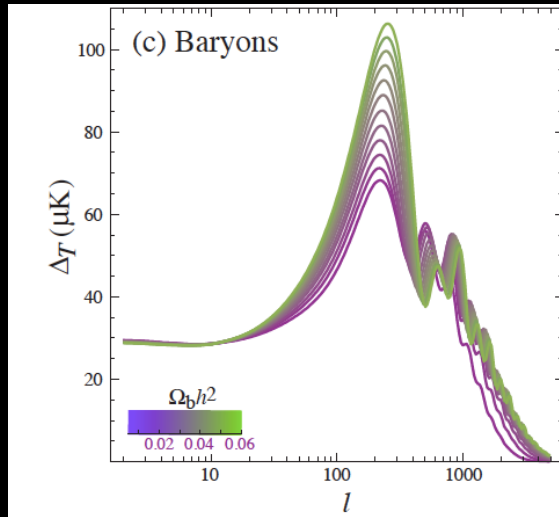


$$\varepsilon_i \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \text{Im} \left[(m_D^\dagger m_D)_{ij}^2 \right] \times \left[f_V \left(\frac{M_j^2}{M_i^2} \right) + f_S \left(\frac{M_j^2}{M_i^2} \right) \right]$$

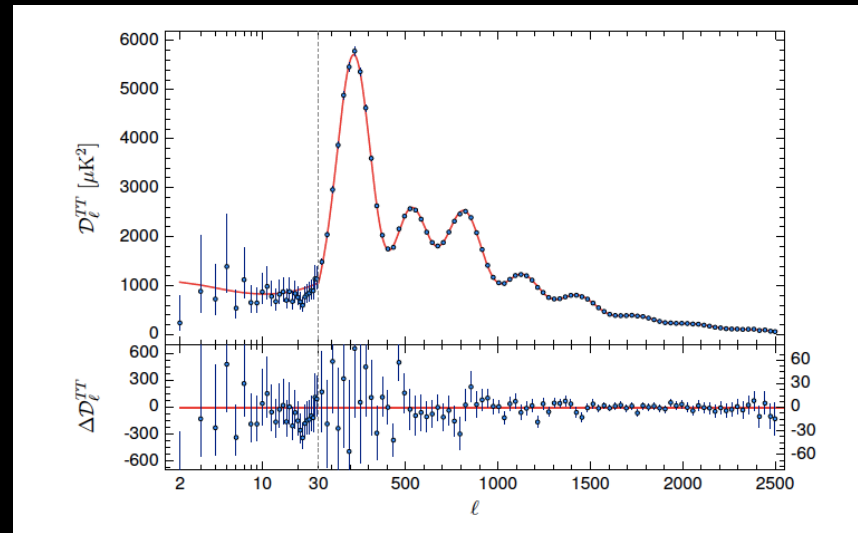
It does not depend on U !

Baryon asymmetry of the universe

(Hu, Dodelson, astro-ph/0110414)



(Planck 2018, 1807.06209)



(CMB+BAO)

$$\Omega_{B0} h^2 = 0.02242 \pm 0.00014$$

$$\eta_{B0} \equiv \frac{n_{B0} - \bar{n}_{B0}}{n_{\gamma 0}} \simeq \frac{n_{B0}}{n_{\gamma 0}} \simeq 273.5 \Omega_{B0} h^2 \times 10^{-10} = (6.12 \pm 0.04) \times 10^{-10} = \eta_{B0}^{CMB}$$

- Consistent with (older) BBN determination but more precise and accurate
- Today the asymmetry coincides with the matter abundance since there is no evidence of primordial antimatter
- Even though all 3 Sakharov conditions are satisfied in the SM, any attempt to reproduce the observed value fails by many orders of magnitude $\Rightarrow$ it requires NEW PHYSICS!

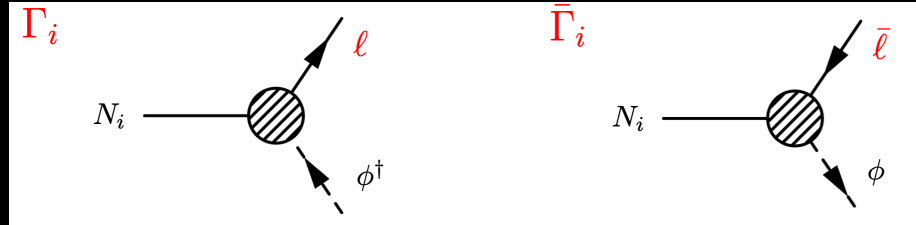
Minimal scenario of leptogenesis

(Fukugita, Yanagida '86)

- Type I seesaw mechanism

- Thermal production of RH neutrinos: $T_{RH} \gtrsim T_{lep} \simeq M_i / (2 \div 10)$

heavy
neutrino
decays



$$\varepsilon_I \equiv -\frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

**total CP
asymmetries**

$$\varepsilon_I \equiv \frac{\Gamma_I - \bar{\Gamma}_I}{\Gamma_I + \bar{\Gamma}_I}$$

$$\Rightarrow N_{B-L}^{fin} = \sum_{I=1,2,3} \varepsilon_I \times K_I^{fin}$$

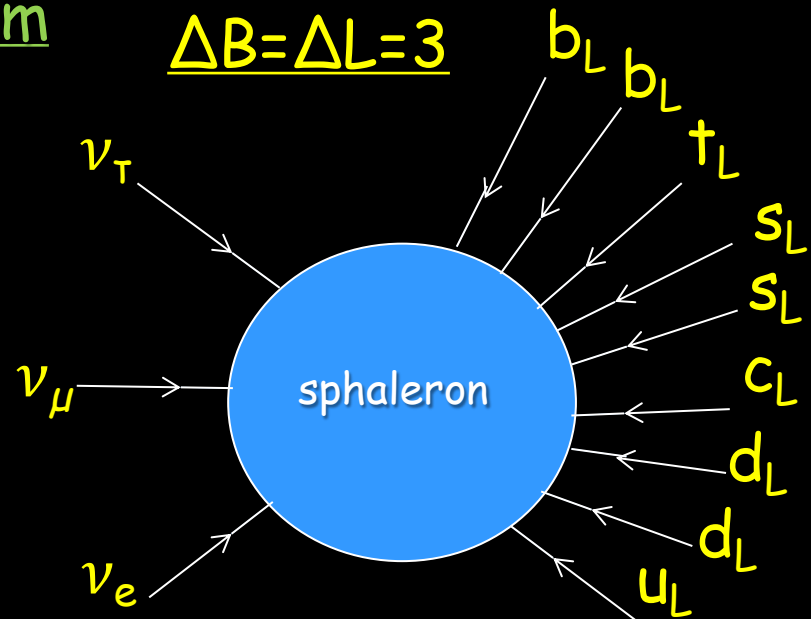
efficiency
factors

- Sphaleron processes in equilibrium

$$\Rightarrow T_{lep} \gtrsim T_{sphalerons}^{off} \simeq 132 \text{ GeV}$$

(Kuzmin, Rubakov, Shaposhnikov '85
D'Onofrio, Rummukainen, Tranberg 1404.3565)

$$\Rightarrow \eta_{B0}^{lep} = \frac{a_{sph} N_{B-L}^{fin}}{N_{\gamma}^{rec}} \simeq 0.01 N_{B-L}^{fin}$$



Seesaw parameter space

Can we test seesaw and leptogenesis combining successful leptogenesis condition $\eta_{B0}^{lep} \simeq \eta_{B0}^{CMB} \simeq 6 \times 10^{-10}$ with low energy neutrino data?

(Casas, Ibarra'01) $m_\nu = -m_D \frac{1}{M} m_D^T \Leftrightarrow \boxed{\Omega^T \Omega = I}$ **Orthogonal parameterisation**

$m_D = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix}$ (in a basis where charged lepton and Majorana mass matrices are diagonal)

light neutrino parameters

heavy neutrino parameters escaping experimental information

❑ Popular solution in the LHC era: *low-scale leptogenesis*, potential direct discovery of RH neutrinos in lab neutrino experiments such as FASER (no signs so far).

❑ *high-scale leptogenesis* is challenging to test but there are a few strategies able to reduce the number of parameters in order to obtain testable predictions on low energy neutrino parameters

Vanilla leptogenesis $\Rightarrow$ upper bound on ν masses

(Buchmüller, PDB, Plümacher '04; Blanchet, PDB '07, Garbrecht et al 2025)

1) Lepton flavor composition is neglected

2) Hierarchical spectrum ($M_2 \gtrsim 2M_1$)

3) Strong lightest RH neutrino wash-out

$$\eta_{B0} \simeq 0.01 N_{B-L}^{final} \simeq 0.01 \varepsilon_1 \kappa_1^{fin}(K_1, m_1)$$

decay parameter: $K_1 \equiv \frac{\Gamma_{N_1}(T=0)}{H(T=M_1)}$

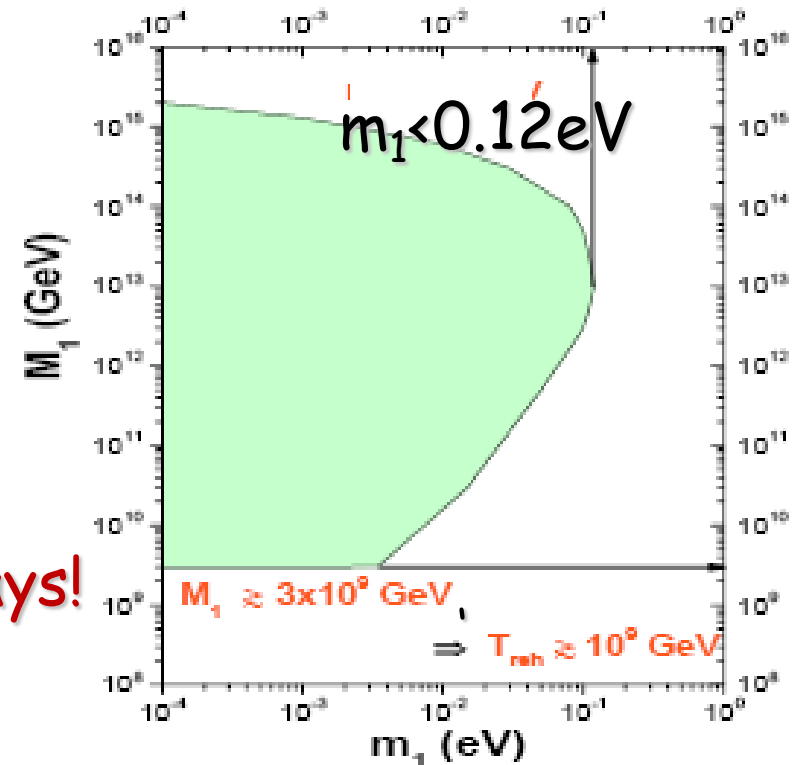
All the asymmetry is generated by the lightest RH neutrino decays!

4) Barring fine-tuned cancellations

(Davidson, Ibarra '02)

$$\varepsilon_1 \leq \varepsilon_1^{\max} \simeq 10^{-6} \left(\frac{M_1}{10^{10} \text{ GeV}} \right) \frac{m_{\text{atm}}}{m_1 + m_3}$$

$$\eta_B^{\max}(m_1, M_1) \geq \eta_B^{\text{CMB}}$$



No dependence on the leptonic mixing matrix U : it cancels out $\Rightarrow$ no sensitivity to mixing parameters.

Independence of the initial conditions (strong thermal leptogenesis)

(Buchmüller, PDB, Plümacher '04)

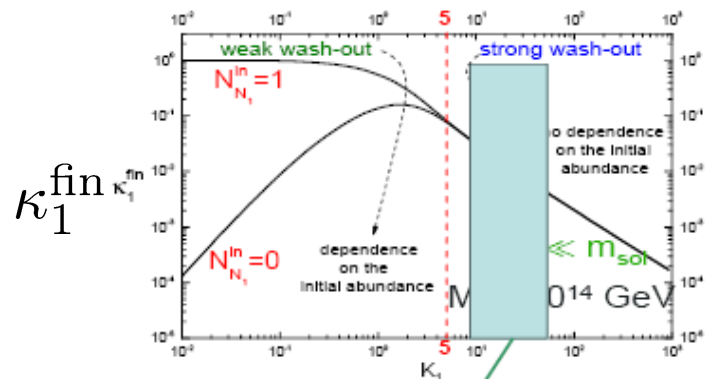
wash-out of a pre-existing asymmetry N_{B-L}^p

$$N_{B-L}^{p, \text{final}} = N_{B-L}^{p, \text{initial}} e^{-\frac{3\pi}{8} K_1} \ll N_{B-L}^{f, N_1}$$

decay parameter: $K_1 \equiv \frac{\Gamma_{N_1}}{H(T = M_1)} \sim \frac{m_{\text{sol, atm}}}{m_* \sim 10^{-3} \text{ eV}} \sim 10 \div 50$ Just a coincidence ?

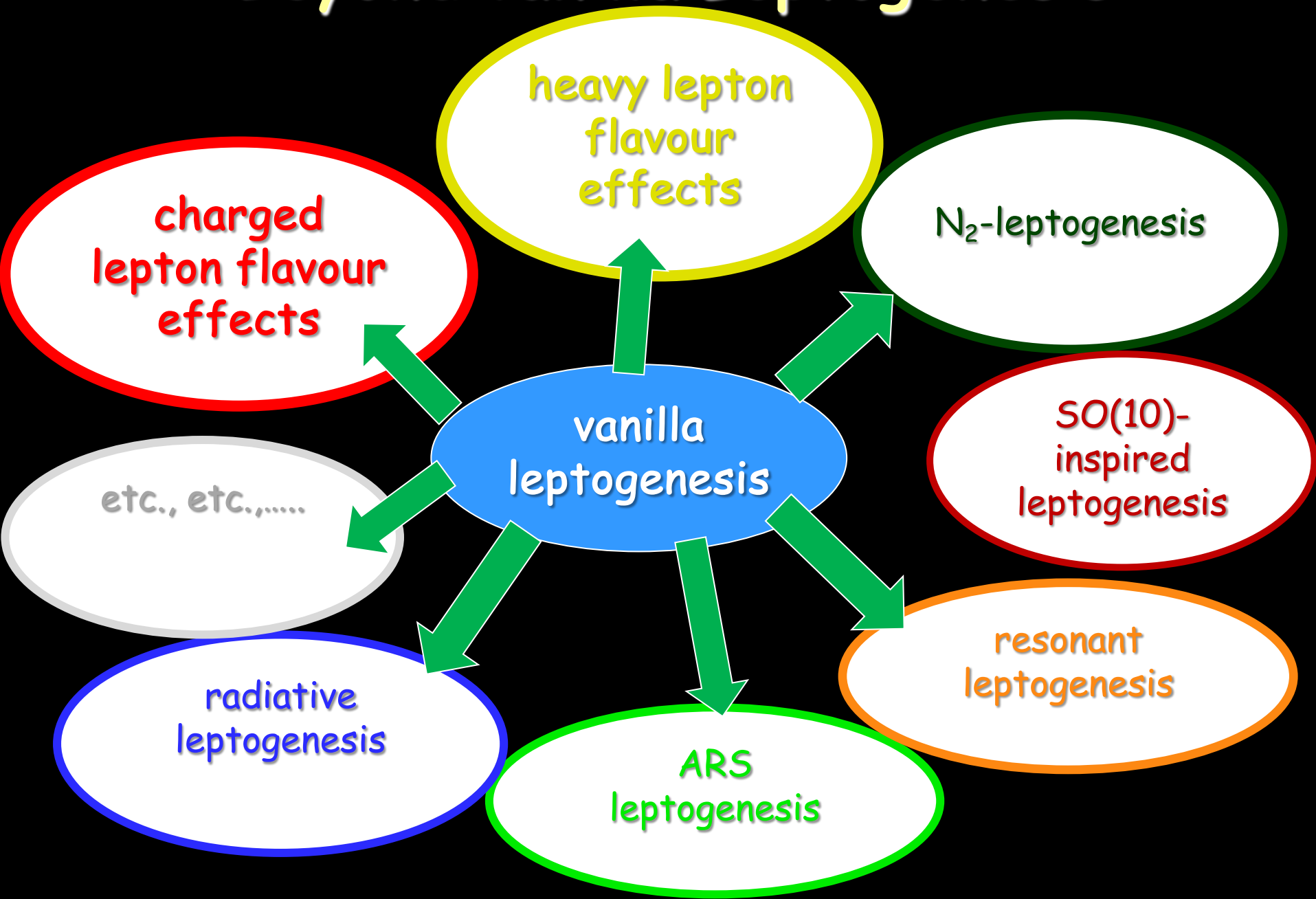
equilibrium neutrino mass: $m_* = \frac{16\pi^{5/2} \sqrt{g_*}}{3\sqrt{5}} \frac{v^2}{M_{\text{Pl}}} \simeq 1.08 \times 10^{-3} \text{ eV}$

Independence of the
initial N_1 abundance



$$K_{\text{sol}} \simeq 9 \lesssim K_1 \lesssim 50 \simeq K_{\text{atm}}$$

Beyond vanilla Leptogenesis



Charged lepton flavour effects

(Barbieri et al '98; Abada et al '06; Nardi et al. '06; Blanchet, PDB, Raffelt '06; Riotto, De Simone '06)

Flavor composition of lepton quantum states matters!

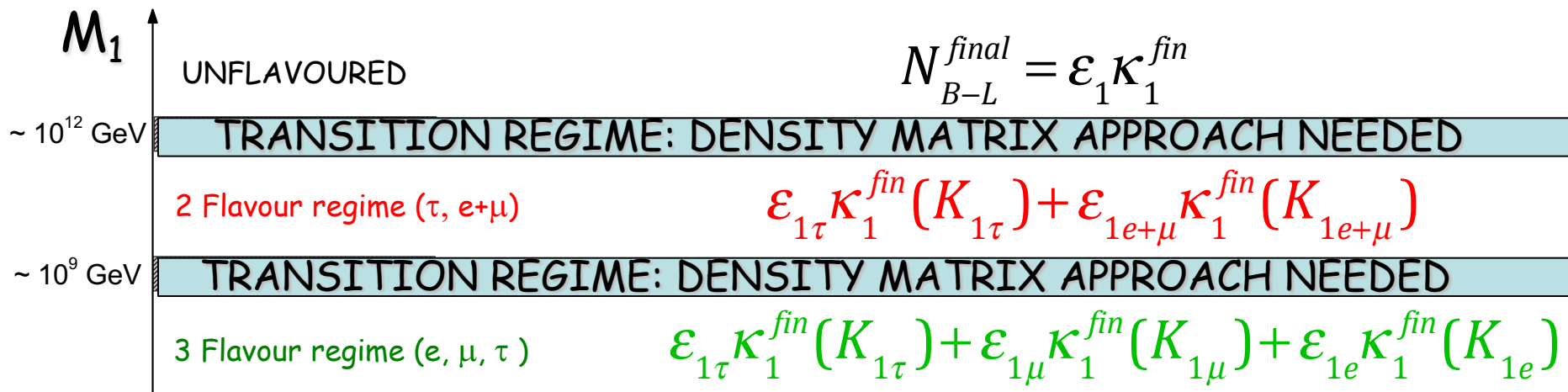
$$|l_1\rangle = \sum_{\alpha} \langle l_{\alpha} | l_1 \rangle |l_{\alpha}\rangle \quad (\alpha = e, \mu, \tau)$$

$$|\bar{l}_1\rangle = \sum_{\alpha} \langle l_{\alpha} | \bar{l}_1 \rangle |\bar{l}_{\alpha}\rangle$$

□ $T \ll 10^{12} \text{ GeV} \Rightarrow \tau$ -Yukawa interactions are fast enough break the coherent evolution of $|l_1\rangle$ and $|\bar{l}_1\rangle$

$\Rightarrow$ incoherent mixture of a τ and of a $\mu+e$ components $\Rightarrow$ 2-flavour regime

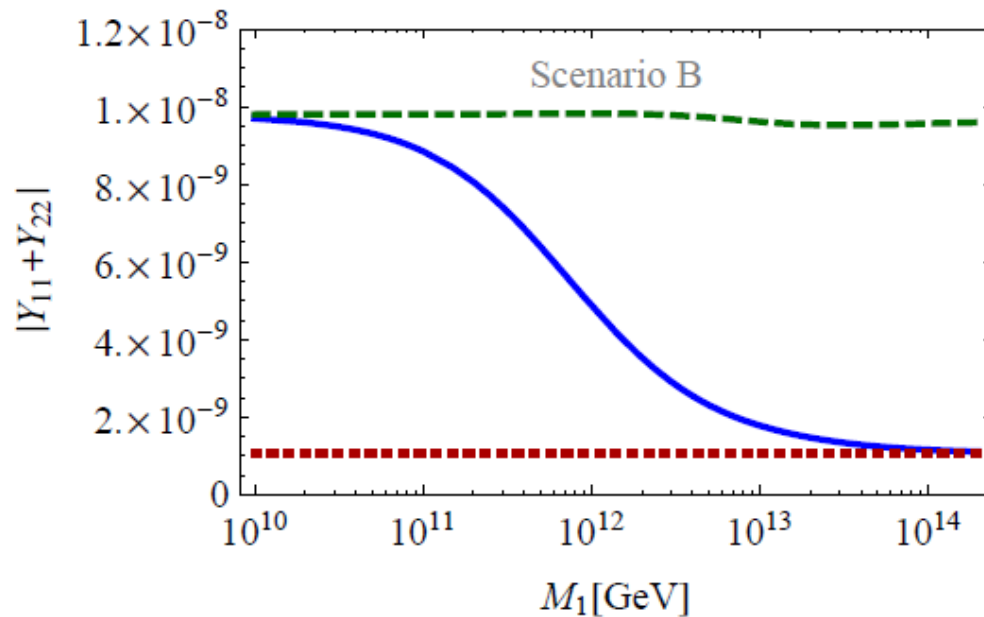
□ $T \ll 10^9 \text{ GeV}$ then also τ -Yukawas in equilibrium $\Rightarrow$ 3-flavour regime



Density matrix and CTP formalism to describe the transition regimes

(De Simone, Riotto '06; Beneke, Gabrecht, Fidler, Herranen, Schwaller '10)

$$\frac{dY_{\alpha\beta}}{dz} = \frac{1}{szH(z)} \left[(\gamma_D + \gamma_{\Delta L=1}) \left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \epsilon_{\alpha\beta} - \frac{1}{2Y_\ell^{\text{eq}}} \left\{ \gamma_D + \gamma_{\Delta L=1}, Y \right\}_{\alpha\beta} \right] - \left[\sigma_2 \text{Re}(\Lambda) + \sigma_1 |\text{Im}(\Lambda)| \right] Y_{\alpha\beta}$$



Fully two-flavoured
regime limit

Unflavoured regime limit

The lower bounds on M_1 and on T_{reh} get relaxed:

(Blanchet, PDB '08)

In the two flavour regime:

$$N_{B-L}^f \simeq 2 \varepsilon_1 \kappa(K_1) + \frac{\Delta p_{1\tau}}{2} \left[\kappa(K_{1\tau_1^\perp}) - \kappa(K_{1\tau}) \right]$$

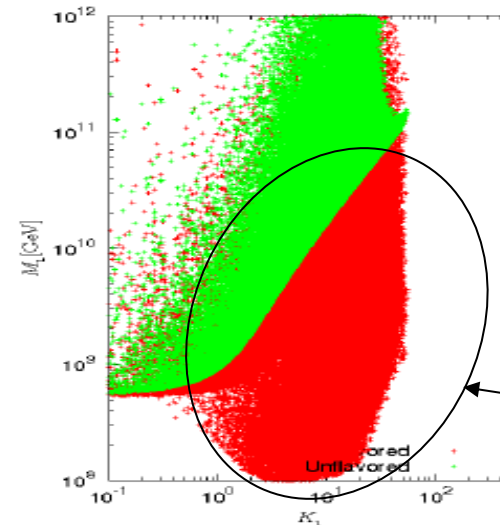
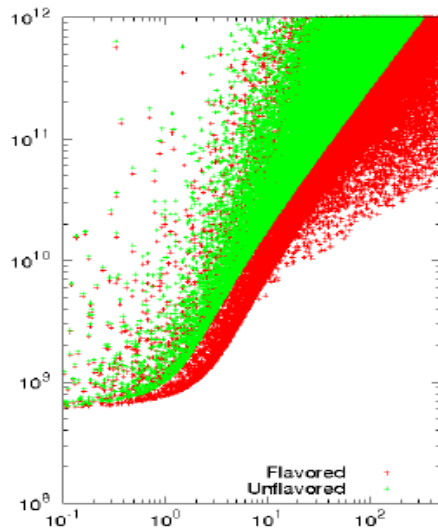
$$\frac{\Delta P_{i\alpha}}{2} \simeq \frac{1}{8\pi (h^\dagger h)_{ii}} \sum_{j \neq i} \left\{ \text{Im} \left[h_{\alpha i}^* h_{\alpha j} \left(\frac{3}{2\sqrt{x_j}} (h^\dagger h)_{ij} + \frac{1}{x_j} (h^\dagger h)_j \right) \right] \right\}$$

$$x_j = \frac{M_j^2}{M_1^2}$$

It dominates for $|\Omega_{ij}| \lesssim 1$ but is upper bounded because of Ω orthogonality:

$$\left| \frac{\Delta P_{1\alpha}}{2} \right| < \bar{\varepsilon}(M_1) \sqrt{P_{1\alpha}^0}$$

It is usually neglected but since it is not upper bounded by orthogonality, for $|\Omega_{ij}| \gg 1$ it can be important



$$|\Omega_{ij}^2| \lesssim 10$$

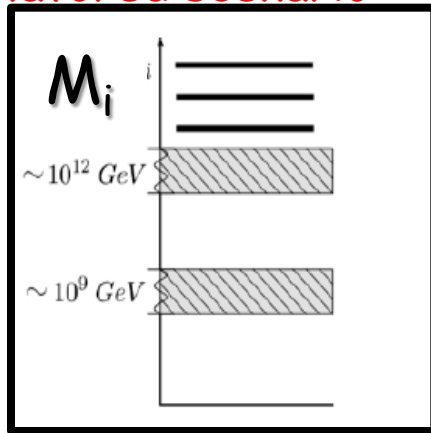
The lower bound gets relaxed

Heavy neutrino lepton flavour effects: 10 scenarios

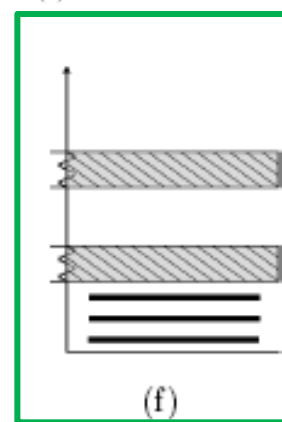
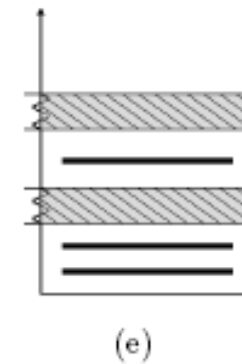
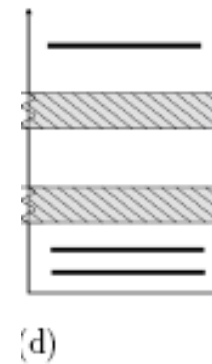
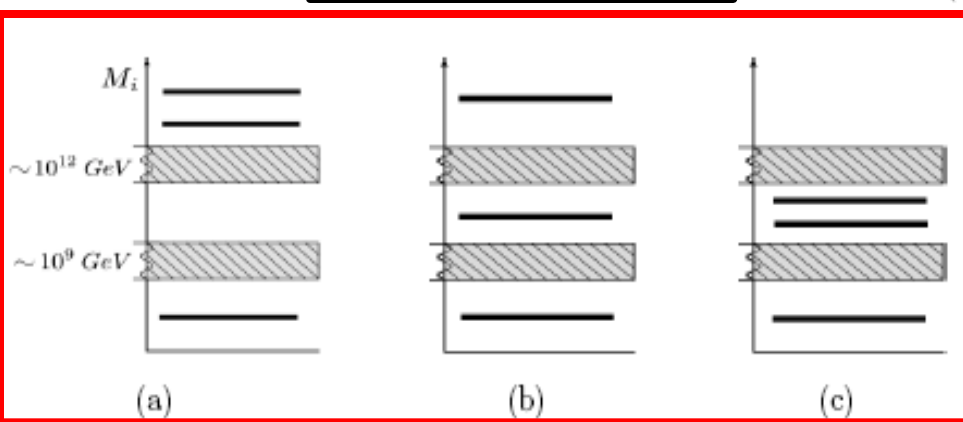
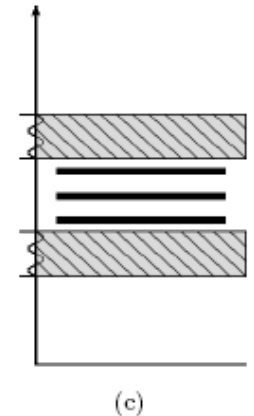
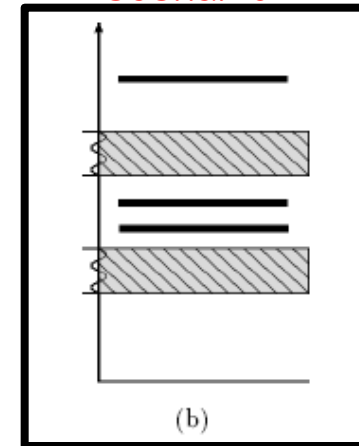
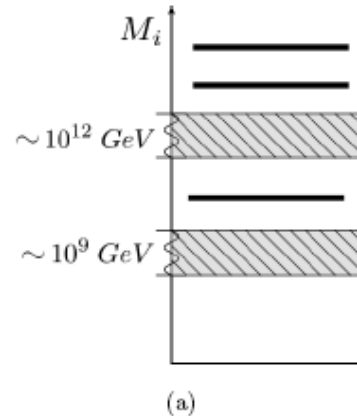
(PDB hep-ph/0502082; Bertuzzo, PDB, Feruglio, Nardi 2009; Bertuzzo, PDB, Marzola 2009)

Heavy neutrino flavored scenario

Typically rising in discrete flavour symmetry Models: if $T_{RH} < M_2$ then vanilla leptog. is recovered



2 RH neutrino scenario



N_2 -dominated scenario:

- N_1 produces negligible asymmetry;
- (b) is the only that can realise strong thermal leptogenesis...
- ...and emerges within $SO(10)$ -inspired models.

Low scale leptogenesis

Examples: resonant, ARS, radiative leptogenesis

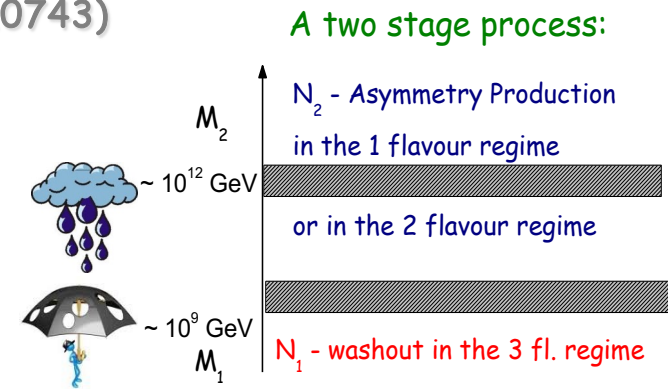
N₂-leptogenesis

(PDB hep-ph/0502082, Vives hep-ph/0512160; Blanchet, PDB 0807.0743)

- **Unflavoured case:** asymmetry produced from N₂ - RH neutrinos is typically washed-out

$$\eta_{B0}^{lep(N_2)} \simeq 0.01 \cdot \varepsilon_2 \cdot \kappa^{fin}(K_2) \cdot e^{-\frac{3\pi}{8}K_1} \ll \eta_{B0}^{CMB}$$

- **Adding flavour effects:** lightest RH neutrino wash-out acts on individual flavour $\Rightarrow$ much weaker: $K_{1e} + K_{1\mu} + K_{1\tau} = K_1$



$$N_{B-L}^f(N_2) = P_{2e}^0 \varepsilon_2 \kappa(K_2) e^{-\frac{3\pi}{8} K_{1e}} + P_{2\mu}^0 \varepsilon_2 \kappa(K_2) e^{-\frac{3\pi}{8} K_{1\mu}} + P_{2\tau}^0 \varepsilon_2 \kappa(K_2) e^{-\frac{3\pi}{8} K_{1\tau}}$$

- With flavor effects the domain of successful N₂ dominated leptogenesis greatly enlarges: the probability that $K_1 < 1$ is less than 0.1% but the probability that either K_{1e} or $K_{1\mu}$ or $K_{1\tau}$ is less than 1 is ~50% (taking into account experimental data)

(PDB, Michele Re Fiorentin, Rome Samanta 1812.07720)

- Existence of the heaviest RH neutrino N₃ is necessary for the ε_{2a} 's not to be negligible
- It is the only hierarchical scenario that can realise strong thermal leptogenesis (independence of the initial conditions) if the asymmetry is **tauon-dominated** and if $m_{ee} \gtrsim 10 \text{ meV} \Rightarrow m_1 \gtrsim 10 \text{ meV}$ (corresponding to $\sum_i m_i \gtrsim 80 \text{ meV}$).

(PDB, Michele Re Fiorentin, Sophie King arXiv 1401.6185)

This is an important motivation for $0\nu\beta\beta$ experiments

- **N₂-leptogenesis rescues SO(10)-inspired leptogenesis**

SO(10)-inspired models

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03; PDB, Riotto '08; PDB, Re Fiorentin, Marzola 1411.5478; PDB, Re Fiorentin 1705.01935)

If one imposes SO(10)-inspired conditions:

$$m_{D1} = \alpha_1 m_{up}; \quad m_{D2} = \alpha_2 m_{charm}; \quad m_{D3} = \alpha_3 m_{top}; \quad \alpha_i = O(1)$$

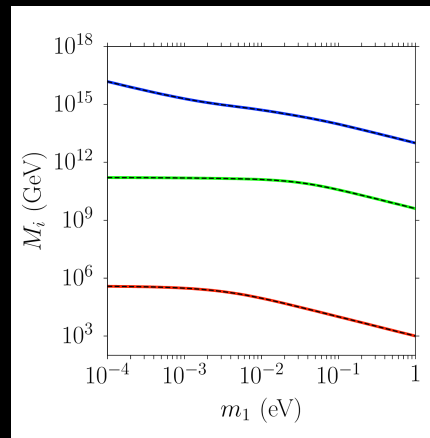
Barring very fine-tuned solutions, one obtains a very hierarchical RH neutrino mass spectrum

light neutrino mass matrix in the Yukawa basis: $m_\nu \rightarrow \tilde{m}_\nu = V_L m_\nu V_L^T$

RH neutrino masses
$$M_1 \simeq \frac{\alpha_1^2 m_u^2}{|(\tilde{m}_\nu)_{11}|}, \quad M_2 \simeq \frac{\alpha_2^2 m_c^2}{m_1 m_2 m_3} \frac{|(\tilde{m}_\nu)_{11}|}{|(\tilde{m}_\nu^{-1})_{33}|}, \quad M_3 \simeq \alpha_3^2 m_t^2 |(\tilde{m}_\nu^{-1})_{33}|$$

typical RH neutrino mass spectrum

the lightest RH neutrino is lighter than 10^9 GeV



This very hierarchical RH neutrino mass spectrum is obtained for any choice of V_L barring very fine-tuned crossing level solutions (PDB, Xubin Hu 2507.06144 and in preparation)

Full analytical solution (V_L arbitrary): RH neutrino mass spectrum and mixing matrix

(PDB, Marzola, Re Fiorentin 1411.5478; PDB, Re Fiorentin 1705.01935)

light neutrino mass
matrix in the Yukawa
basis

$$m_\nu \rightarrow \tilde{m}_\nu = V_L m_\nu V_L^T$$

RH neutrino masses

$$M_1 \simeq \frac{\alpha_1^2 m_u^2}{|(\tilde{m}_\nu)_{11}|}, \quad M_2 \simeq \frac{\alpha_2^2 m_c^2}{m_1 m_2 m_3} \frac{|(\tilde{m}_\nu)_{11}|}{|(\tilde{m}_\nu^{-1})_{33}|}, \quad M_3 \simeq \alpha_3^2 m_t^2 |(\tilde{m}_\nu^{-1})_{33}|$$

RH neutrino phases

$$\Phi_1 \simeq -\text{Arg}[-(\tilde{m}_\nu)_{11}^*], \quad \Phi_2 \simeq \text{Arg}\left[\frac{(\tilde{m}_\nu)_{11}}{(\tilde{m}_\nu^{-1})_{33}}\right] - 2(\rho + \sigma) - 2(\rho_L + \sigma_L), \quad \Phi_3 \simeq \text{Arg}[(\tilde{m}_\nu^{-1})_{33}]$$

RH neutrino
mixing matrix

$$U_R \simeq \begin{pmatrix} 1 & -\frac{m_{D1}}{m_{D2}} \frac{(\tilde{m}_\nu)_{12}^*}{(\tilde{m}_\nu)_{11}^*} & \frac{m_{D1}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{13}^*}{(\tilde{m}_\nu^{-1})_{33}^*} \\ \frac{m_{D1}}{m_{D2}} \frac{(\tilde{m}_\nu)_{12}}{(\tilde{m}_\nu)_{11}} & 1 & \frac{m_{D2}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{23}^*}{(\tilde{m}_\nu^{-1})_{33}^*} \\ \frac{m_{D1}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{13}}{(\tilde{m}_\nu^{-1})_{33}} & -\frac{m_{D2}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{23}}{(\tilde{m}_\nu^{-1})_{33}} & 1 \end{pmatrix} D_\Phi, \quad D_\Phi \equiv \begin{pmatrix} e^{-i\frac{\Phi_1}{2}}, e^{-i\frac{\Phi_2}{2}}, e^{-i\frac{\Phi_3}{2}} \end{pmatrix}$$

General analytical solution for the asymmetry

Flavoured decay
parameters

$$K_{I\alpha} = \frac{\sum_{k,l} m_{Dk} m_{Dl} V_{Lk\alpha} V_{Ll\alpha}^* U_{Rkl}^* U_{Rli}}{M_I m_*}$$

Flavoured CP
asymmetries

$$\epsilon_{2\alpha} = \frac{3}{16\pi v^2} \frac{|(\tilde{m}_\nu)_{11}|}{m_1 m_2 m_3} \frac{\sum_{k,l} m_{Dk} m_{Dl} \text{Im}[V_{Lk\alpha} V_{Ll\alpha}^* U_{Rk2}^* U_{Rl3} U_{R32}^* U_{R33}]}{|(\tilde{m}_\nu^{-1})_{33}|^2 + |(\tilde{m}_\nu^{-1})_{23}|^2}$$

Final B-L
asymmetry

$$N_{B-L}^{\text{lep,f}} = \epsilon_{2e} \kappa(K_{2e} + K_{2\mu}) e^{-\frac{3\pi}{8} K_{1e}} + \epsilon_{2\mu} \kappa(K_{2e} + K_{2\mu}) e^{-\frac{3\pi}{8} K_{1\mu}} + \epsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8} K_{1\tau}}$$

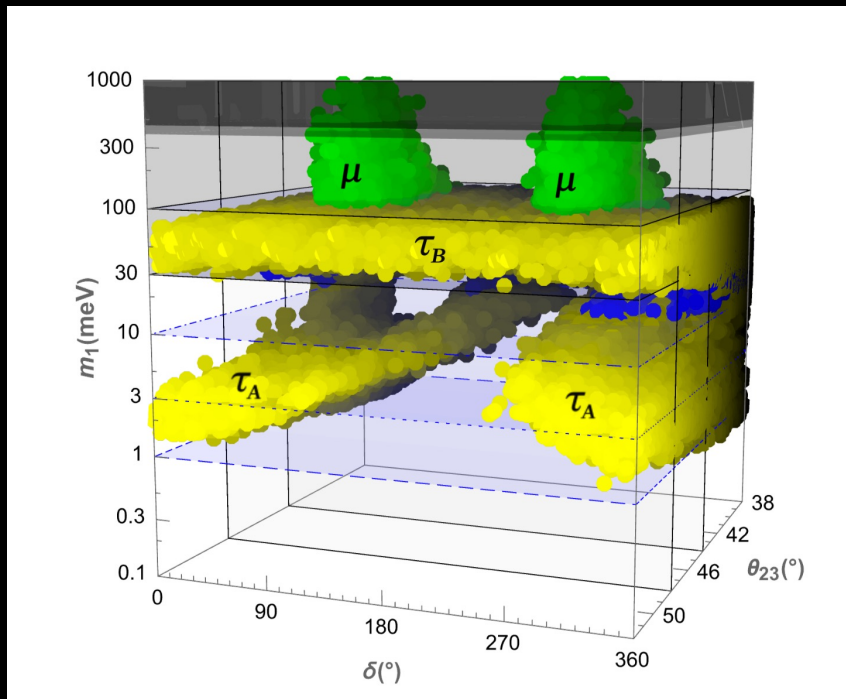
Successful SO(10)-inspired leptogenesis condition:

$$\eta_B^{SO10lep}(m_1, m_{sol}, m_{atm}, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \rho, \sigma; \alpha_2, V_L) = \eta_B^{\text{obs}}$$

- There is no dependence on α_1 and α_3 determining M_1 and M_3
- SO(10) inspired conditions also impose $V_L \sim V_{CKM}$
- In the approximation $V_L = I$ this would identify a hypersurface in the space of low energy neutrino parameters yielding interesting constraints and predictions.
- Turning small angles in V_L at the level of the angles in V_{CKM} gives some thickness to the hypersurface but the constraints remain.

3-dim projection of the allowed region (NO)

$\alpha_2=5$ NORMAL ORDERING $I \leq V_L \leq V_{CKM}$



$\sim 2 \times 10^6$ points
out of
 $\sim 2 \times 10^9$ trials
(success rate is
 $\sim 0.1\%$)

● tauonic

● muonic

● electronic

● Strong-thermal
leptogenesis (subset of
tauonic)

(PDB, R. Samanta 2005.03057; PDB, Xubin Hu 2507.06144)

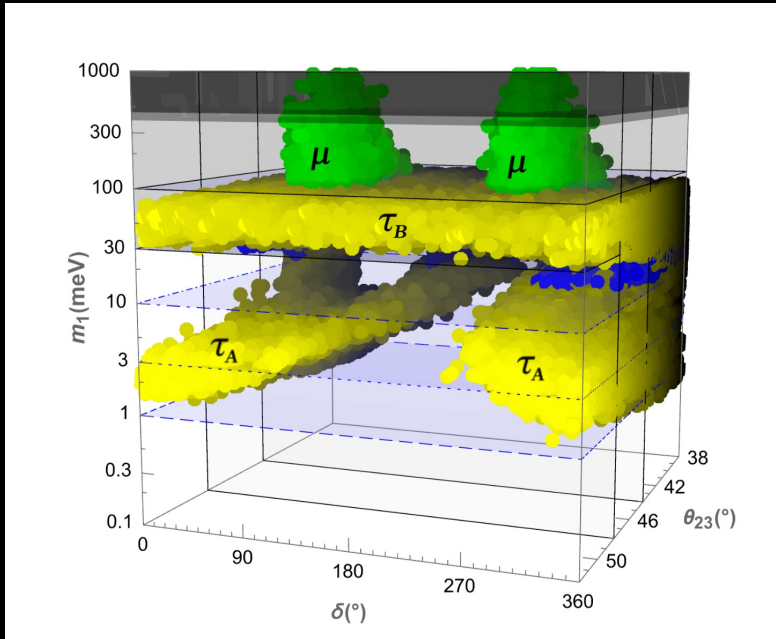
➤ N_2 -leptogenesis (with flavour effects) does rescue
SO(10)-inspired models! It works only for NO

(PDB, Riotto 0809.2285 and 1012.2343;0810.1104)

3-dim projection of the allowed region (NO)

$\alpha_2=5$ NORMAL ORDERING $I \leq V_L \leq V_{CKM}$

without flavour coupling



(PDB, Xubin Hu 2507.06144)

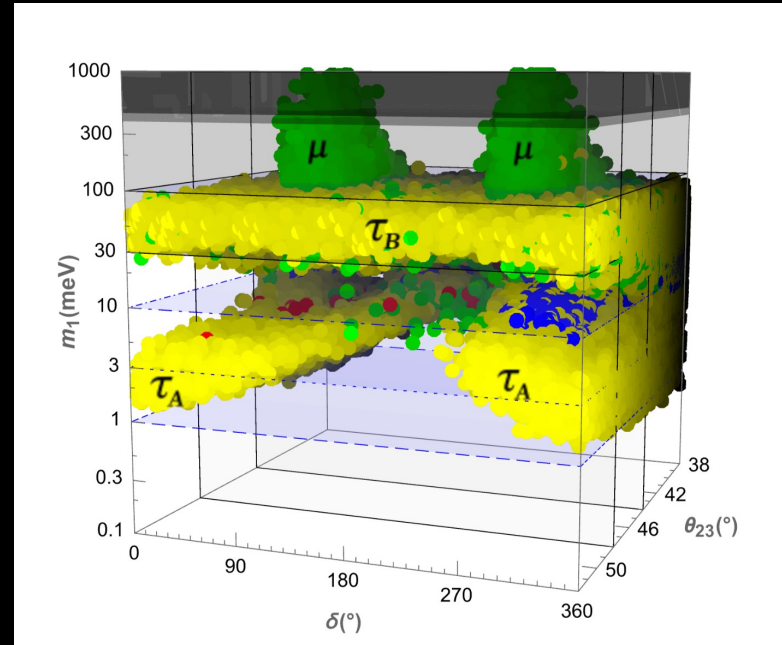
● tauonic

● muonic

● electronic

● Strong-thermal
leptogenesis (subset of
tauonic)

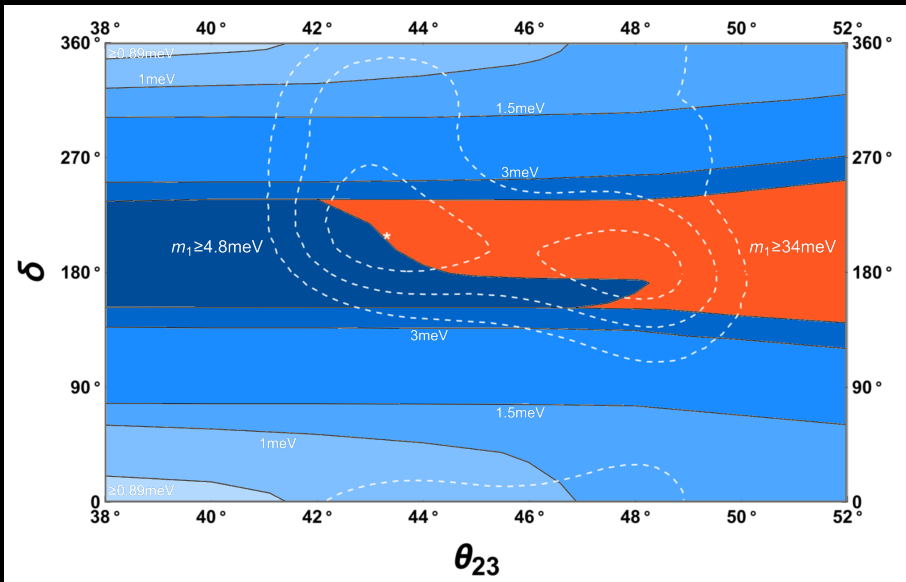
with flavour coupling (originating from
spectator processes, mainly Higgs asymmetry)



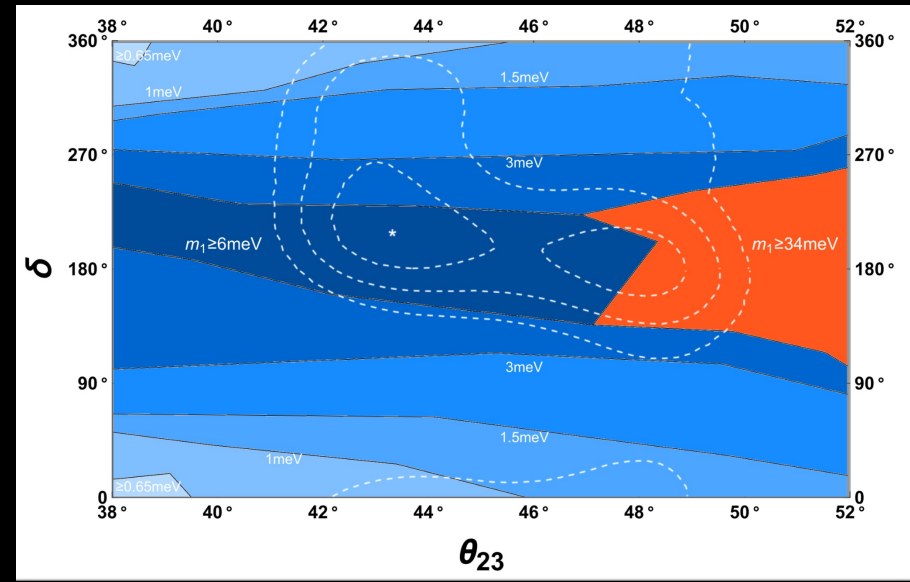
Including flavour coupling, new muonic solutions appear and even some very marginal electronic solutions (red points)

Lower bound on the absolute neutrino mass scale

Without flavour coupling

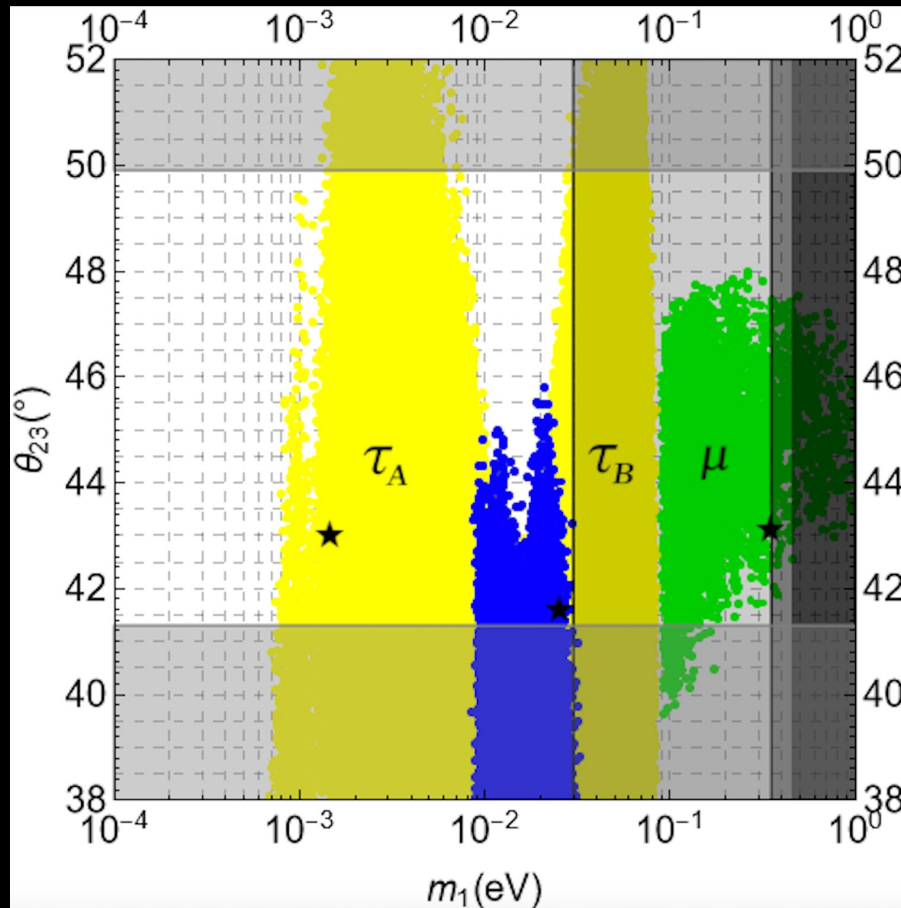


With flavour coupling



(PDB, Xubin Hu 2507.06144)

Upper bound on the atmospheric mixing angle



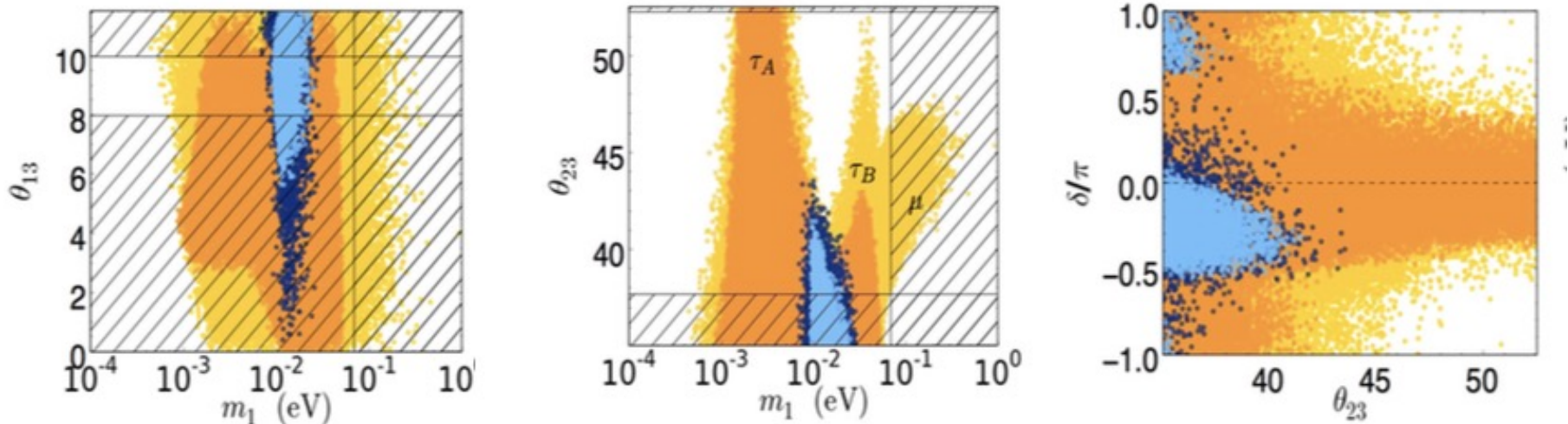
For $10 \text{ meV} \lesssim m_1 \lesssim 30 \text{ meV}$ the atmospheric mixing angle has to be in the first octant

Strong thermal $SO(10)$ -inspired leptogenesis

(PDB, Marzola 09/2011, DESY workshop and 1308.1107; PDB, Re Fiorentin, Marzola 1411.5478)

- **Strong thermal leptogenesis** condition can be satisfied for a subset of the solutions only for NORMAL ORDERING

$\alpha_2=5$ □ blue regions: $N_{B-L}^{pre-ex} = 10^{-3}$ ($I \leq V_L \leq V_{CKM}$)



- Absolute neutrino mass scale: $8 \lesssim m_1/\text{meV} \lesssim 30 \Leftrightarrow 70 \lesssim \sum_i m_i/\text{meV} \lesssim 120$
- **Non-vanishing Θ_{13}** (first results presented before Daya Bay discovery)
- Θ_{23} preferably in the first octant;

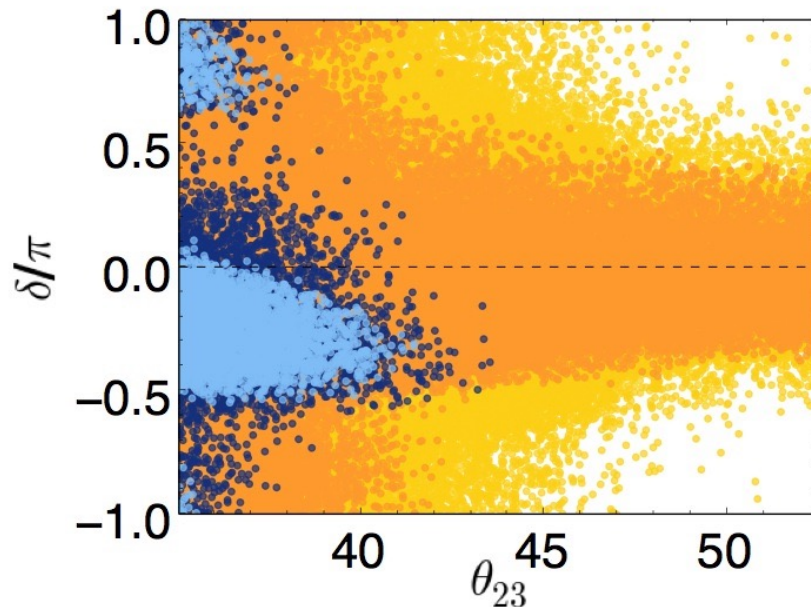
Why do we live in a matter (and not antimatter) dominated universe?

(PDB, Marzola, Re Fiorentin, 1411.5478)

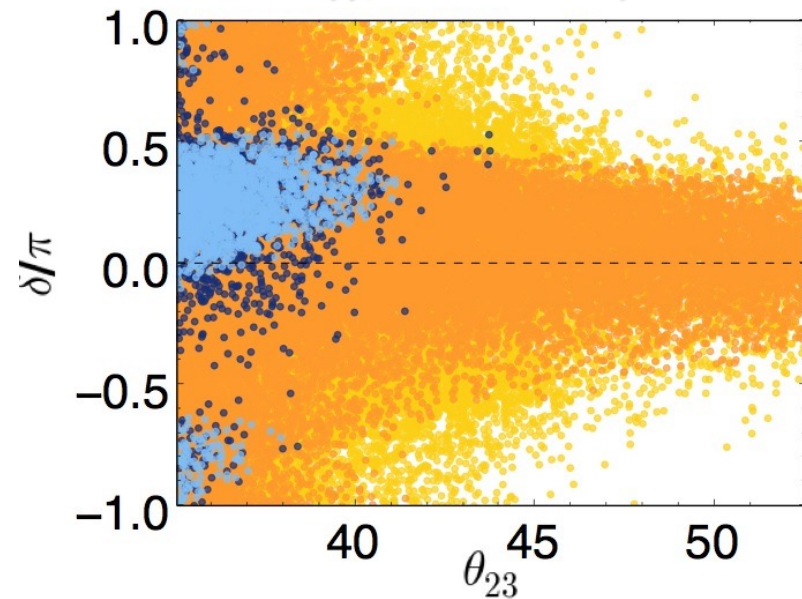
$$\alpha_2=5$$

□ blue regions: $N_{B-L}^{pre-ex} = 10^{-3}$ ($I \leq V_L \leq V_{CKM}$; $V_L = I$)

Matter dominated universe
($\eta_B \sim +6 \times 10^{-10}$)



Antimatter dominated universe
($\eta_B \sim -6 \times 10^{-10}$)



For sufficiently large θ_{23} one has $\text{sign}(\eta_B) = -\text{sign}(\sin \delta)$

$\Rightarrow$ We would live in a matter dominated universe because $\sin \delta < 0$

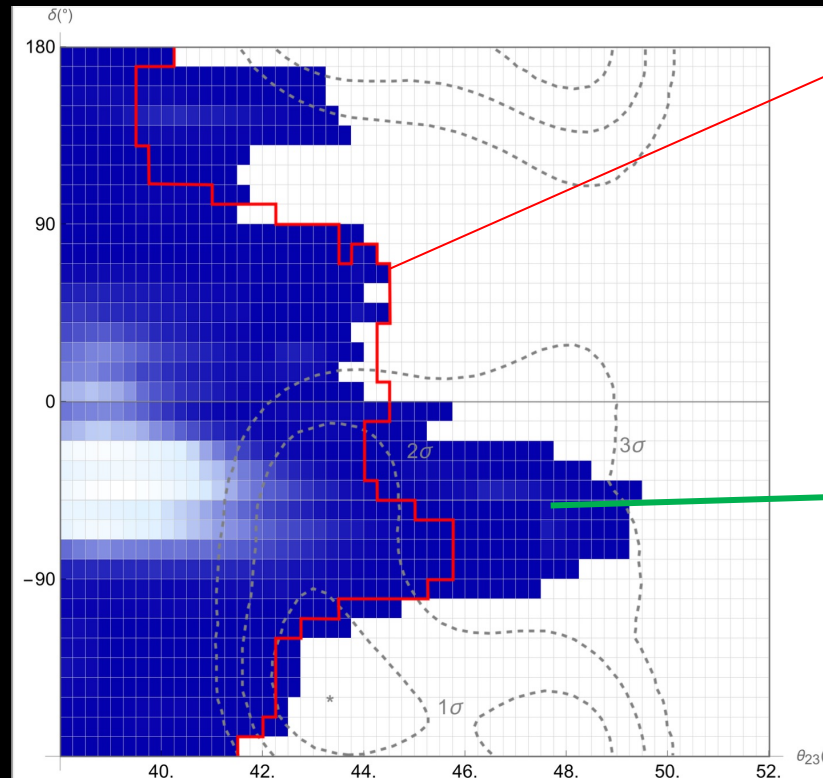
New atmospheric neutrino remove the tension

(PDB, Xubin Hu, 2507.06144)

The new SK atmospheric data seem to favour first octant when combined in global analysis (ν fit September 2024) and moreover $\Delta\chi^2(\text{IO-NO})=6.1$: there is a potential interesting overlap now

Pre-existing initial asymmetry: $N_{B-L}^{p,i} = 10^{-3}$

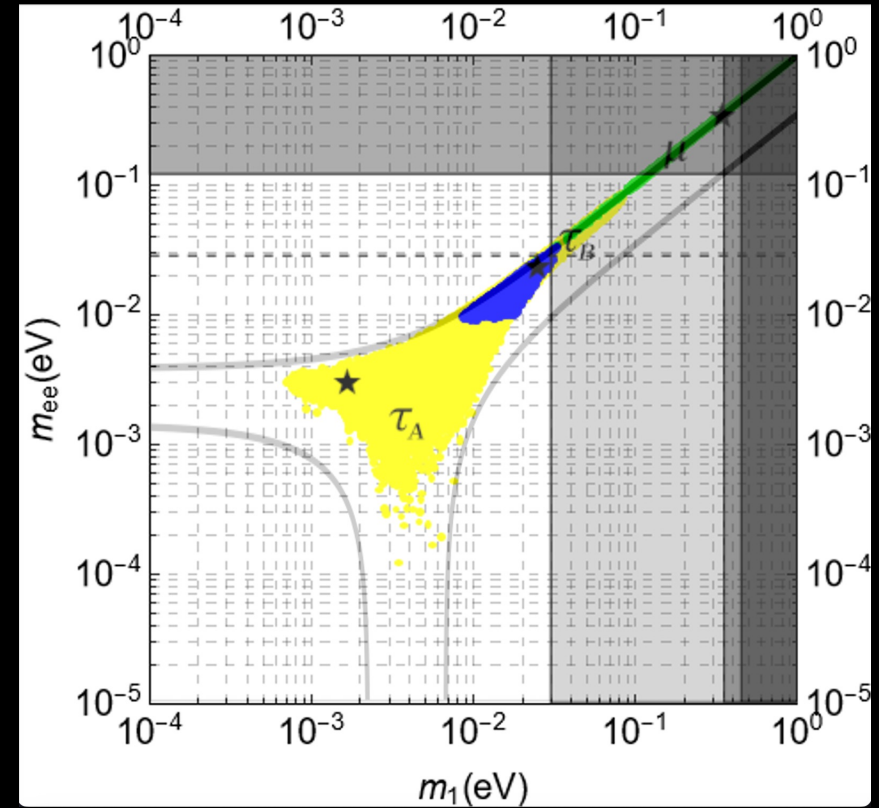
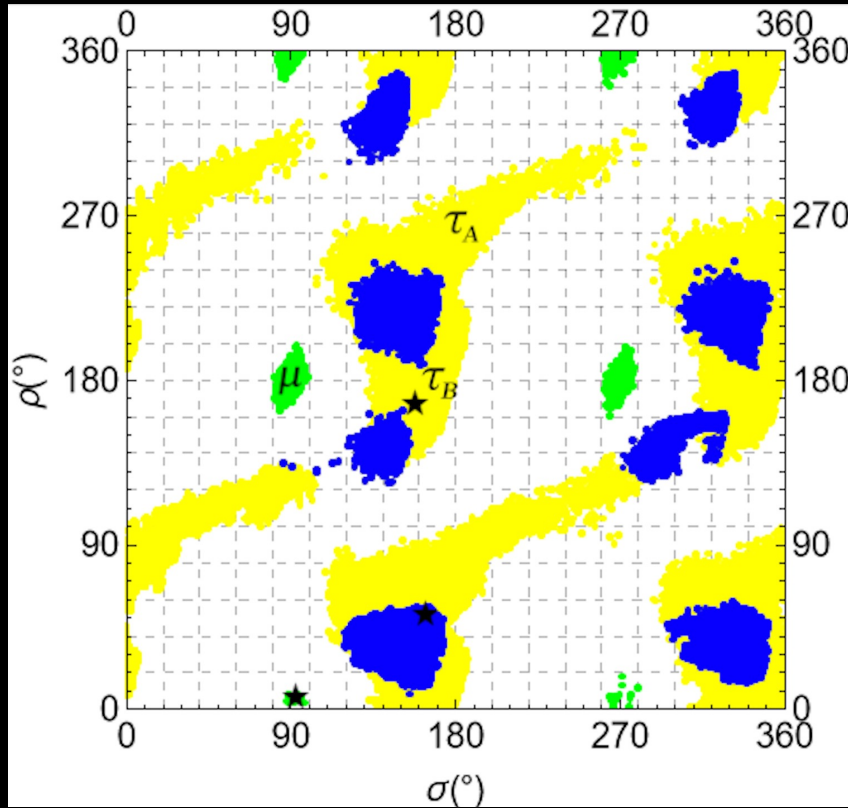
$$\alpha_2 = m_{D2} / m_{\text{charm}} = 5$$



"old" solutions confirmed (no flavour coupling effects)

new solutions found including flavour coupling

Majorana phases and $0\nu\beta\beta$ effective neutrino mass



There are strong constraints on the Majorana phases and this yields a lower bound on m_{ee}

KamLAND2-Zen, LEGEND1000, CUPID aims at improving the upper bound to $m_{ee} < 20$ meV in next years (talks by Shimizu at Neutrino 2024 and by BORRA and IMBERT at this workshop)

Leptogenesis in SO(10) with a minimal Yukawa sector

(Babu, Bajc, Saad, 1612.04329; K Babu, PDB, C.S. Fong, S. Saad 2409.03840)

- The most attractive models realizing SO(10)-inspired leptogenesis are.....
.....SO(10) models !
- SO(10) unifies all fermions of a generation into a single spinorial 16-dimensional representation that, in addition to SM fermions, contains 1 RH neutrino: it naturally predicts 3 RH neutrino species
- Masses of fermions arise from Yukawa interactions of two 16s with vevs of suitable Higgs fields. Since:

$$16 \otimes 16 = 10_S \oplus \overline{126}_S \oplus 120_A$$

⇒ The Higgs fields of renormalizable SO(10) models can belong to 10-, 126-, 120-dim representations yielding Yukawa part of the Lagrangian

$$\mathcal{L}_Y = 16 (Y_{10} 10_H + Y_{126} \overline{126}_H + Y_{120} 120_H) 16.$$

- After SSB of the fermions at $M_{GUT}=2 \times 10^{16}$ GeV one obtains the masses:

up-quark mass matrix

down-quark mass matrix

neutrino mass matrix

charged lepton mass matrix

RH neutrino mass matrix

$$\begin{aligned} M_u &= v_{10}^u Y_{10} + v_{126}^u Y_{126} + v_{120}^u Y_{120}, \\ M_d &= v_{10}^d Y_{10} + v_{126}^d Y_{126} + v_{120}^d Y_{120}, \\ M_D &= v_{10}^u Y_{10} - 3v_{126}^u Y_{126} + v_{120}^D Y_{120}, \\ M_l &= v_{10}^d Y_{10} - 3v_{126}^d Y_{126} + v_{120}^l Y_{120}, \\ M_R &= v_{126}^R Y_{126}, \end{aligned}$$

Simplest case but clearly non-realistic: it predicts no mixing at all (both in quark and lepton Sectors). For realistic models one has to add at least the 126 contribution

A recent realistic fit

(K Babu, PDB, C.S. Fong, S. Saad 2409.03840)

Observables (Δm_{ij}^2 in eV^2)	Values at M_Z scale		
	Input	Benchmark Fit: NO	Benchmark Fit: IO
$y_u/10^{-6}$	6.65 ± 2.25	7.30	10.0
$y_c/10^{-3}$	3.60 ± 0.11	3.59	3.57
y_t	0.986 ± 0.0086	0.986	0.986
$y_d/10^{-5}$	1.645 ± 0.165	1.636	1.635
$y_s/10^{-4}$	3.125 ± 0.165	3.122	3.148
$y_b/10^{-2}$	1.639 ± 0.015	1.639	1.637
$y_e/10^{-6}$	2.7947 ± 0.02794	2.7945	2.7906
$y_\mu/10^{-4}$	5.8998 ± 0.05899	5.9011	5.9080
$y_\tau/10^{-2}$	1.0029 ± 0.01002	1.0022	1.0023
$\theta_{12}^{\text{CKM}}/10^{-2}$	22.735 ± 0.072	22.729 ($\theta_{12}^{\text{CKM}} = 13.023^\circ$)	22.730 ($\theta_{12}^{\text{CKM}} = 13.023^\circ$)
$\theta_{23}^{\text{CKM}}/10^{-2}$	4.208 ± 0.064	4.206 ($\theta_{23}^{\text{CKM}} = 2.401^\circ$)	4.204 ($\theta_{23}^{\text{CKM}} = 2.408^\circ$)
$\theta_{13}^{\text{CKM}}/10^{-3}$	3.64 ± 0.13	3.64 ($\theta_{13}^{\text{CKM}} = 0.208^\circ$)	3.64 ($\theta_{13}^{\text{CKM}} = 0.208^\circ$)
δ_{CKM}	1.208 ± 0.054	1.209 ($\delta_{\text{CKM}} = 69.322^\circ$)	1.212 ($\delta_{\text{CKM}} = 69.457^\circ$)
$\Delta m_{21}^2/10^{-5}$	7.425 ± 0.205	7.413	7.506
$\Delta m_{31}^2/10^{-3}$ (NO)	2.515 ± 0.028	2.514	-
$\Delta m_{32}^2/10^{-3}$ (IO)	-2.498 ± 0.028	-	-2.499
$\sin^2 \theta_{12}$	0.3045 ± 0.0125	0.3041 ($\theta_{12} = 33.46^\circ$)	0.3067 ($\theta_{12} = 33.63^\circ$)
$\sin^2 \theta_{23}$ (NO)*	0.5705 ± 0.0205	0.4473 ($\theta_{23} = 41.98^\circ$)	-
$\sin^2 \theta_{23}$ (IO)*	0.576 ± 0.019	-	0.5784 ($\theta_{23} = 49.51^\circ$)
$\sin^2 \theta_{13}$ (NO)	0.02223 ± 0.00065	0.02223 ($\theta_{13} = 8.57^\circ$)	-
$\sin^2 \theta_{13}$ (IO)	0.02239 ± 0.00063	-	0.02238 ($\theta_{13} = 8.60^\circ$)
δ_{CP}° (NO)	207.5 ± 38.5	240.49	-
δ_{CP}° (IO)	284.5 ± 29.5	-	263.49
$\eta_B/10^{-10}$	$6.12 \pm 0.04^\dagger$	7.6 (7.6)	9.6 (51)
χ^2	-	1.45	5.76 [†]

For NO:

light neutrino masses

$$m_1 = 0.038 \text{ meV}$$

$$m_2 = 8.6 \text{ meV}$$

$$m_3 = 50.1 \text{ meV}$$

$$m_{ee} = 3.7 \text{ meV}$$

heavy neutrino masses

$$M_1 = 6.6 \times 10^4 \text{ GeV}$$

$$M_2 = 2.1 \times 10^{12} \text{ GeV}$$

$$M_3 = 8.1 \times 10^{14} \text{ GeV}$$

Why is the lower bound on m_1 violated? Because $\theta_{23}^L \simeq 45^\circ \Rightarrow$ extended SO(10)-insp. lep

Resonant leptogenesis

- leptogenesis with a degenerate heavy neutrino spectrum:

$$\frac{M_2 - M_1}{M_1} \ll 1 \Rightarrow \text{(under some conditions)} \quad \varepsilon_1 \rightarrow \xi_\varepsilon \varepsilon_1, \quad \xi_\varepsilon \gg 1$$

$$\Rightarrow M_1^{\min} \rightarrow \frac{M_1^{\min}}{\xi_\varepsilon}$$

the bounds can be evaded but one has to justify such small degeneracies

- Resonant leptogenesis can be realized with RH neutrino masses as low as 1 TeV: testable at colliders? (Pilaftsis, Underwood 0309342)
- In a minimal picture (just type I seesaw extension) it is highly fine-tuned (strong compensation between asymmetry produced and washed-out)
- Introducing a $U_{B-L}(1)$ gauge symmetry one can produce the RH neutrinos from Z' decays (Blanchet Chakco Granor Mohapatra 0904.2174)
- Recently, a novel mechanism accounting for two-loop thermal effects is claimed to produce the asymmetry at low scale without requiring strong mass degeneracies (Li, Pilaftsis 2604.06493)

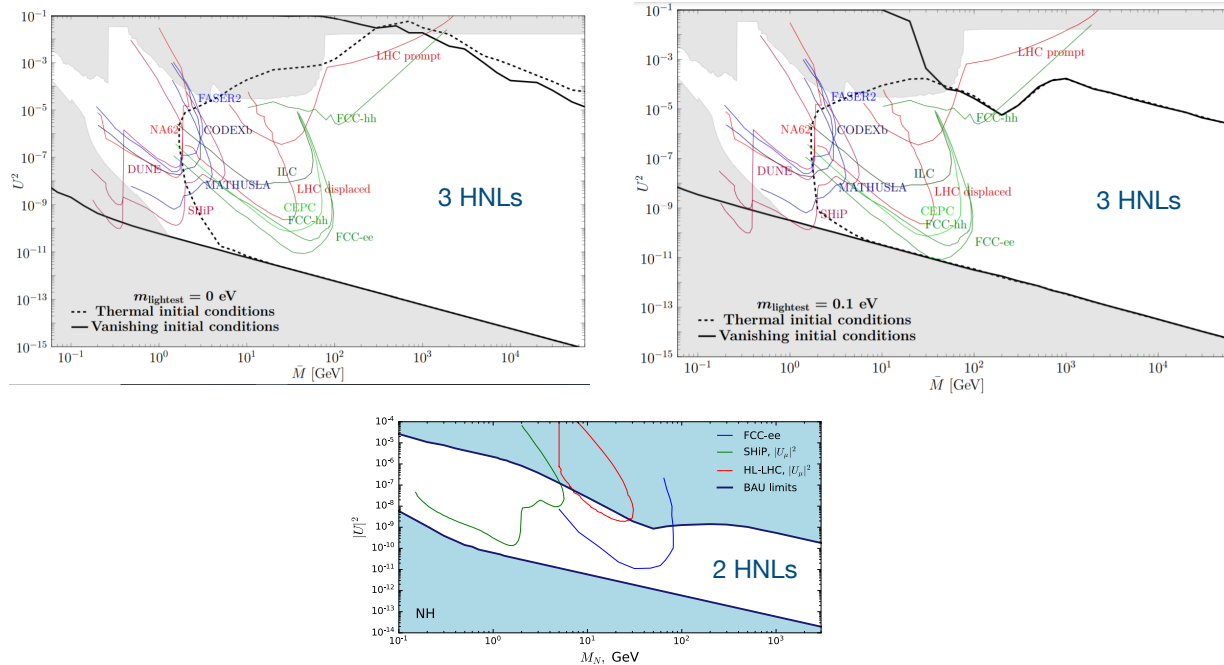
Leptogenesis via RH neutrino mixing (ARS leptogenesis)

(Akhmedov, Rubakov, Smirnov 1999)



Parameter space for 3 HNLs

Drewes, Georis, Klaric: much more space is available



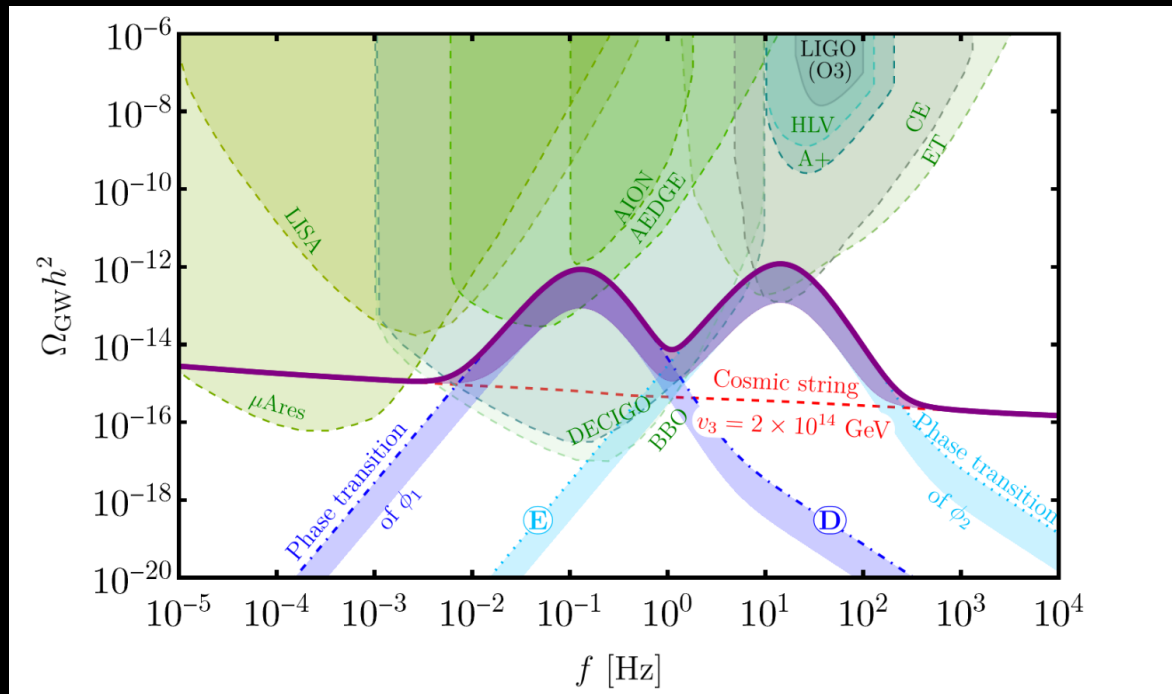
(M. Shaposhnikov at Neutrino 2024)

- The asymmetry is generated when seesaw neutrinos are ultrarelativistic and it is never washed-out (freeze-out leptogenesis)
- For this reason there is a strong dependence on the initial conditions and the observed value is not predicted but fitted.
- However, it can be tested in LLP searches experiments and it complies 'naturalness' idea
- (no destabilizing scale of new physics above the EW scale)

Can we test leptogenesis with gravitational waves?

There is a flourish of activities:

- From FOPTs associated to Majorana mass generation (PDB, D.Marfatia, Y.Zhou 2106.00025)
- From cosmic strings produced during symmetry breakings of GUTs (J.A.Dror, T. Hiramatsu, K.Kohri, H. Murayama, G.White 1908.03227
B.Fu, S.F. King, Marsili, Pascoli, Turner, Zhou 2209.00021)
-
- Combination of cosmic string and FOPTs production:



$$M_1 \sim 100 \text{ GeV}$$

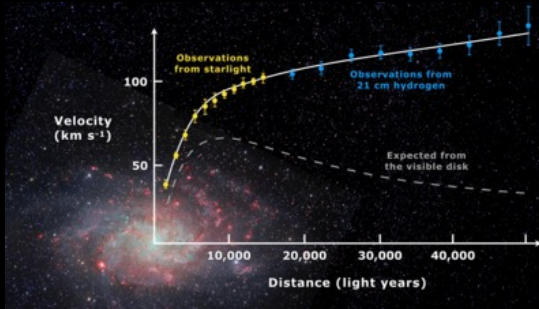
$$M_2 \sim 10^4 \text{ GeV}$$

(PDB, S.F. King, M. Rahat 2306.04680)

Dark Matter

At the present time dark matter acts as a cosmic glue keeping together

stars in galaxies and



galaxies in clusters of galaxies

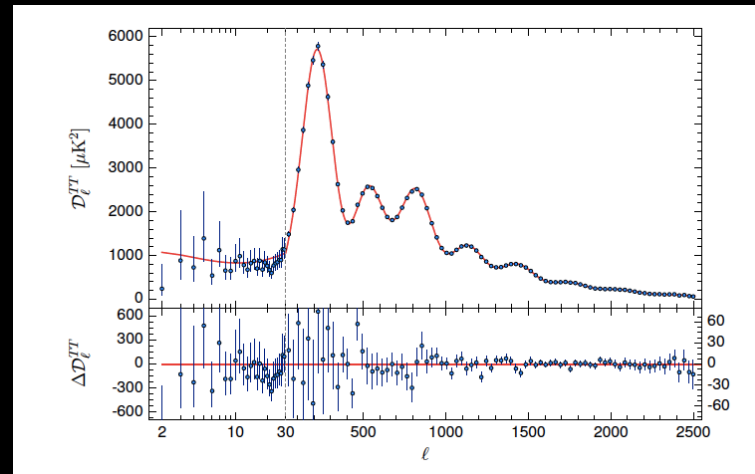
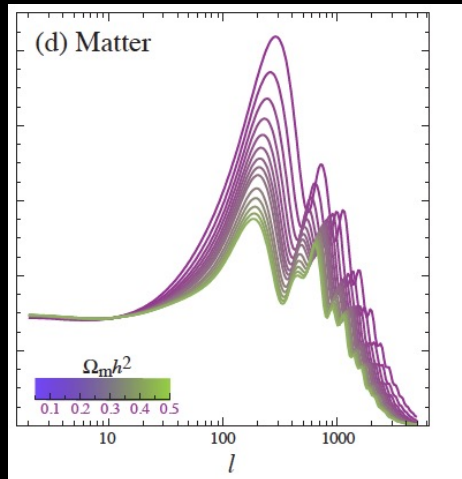


Coma cluster



bullet cluster

...but it also needs to be primordial to understand structure formation and CMB anisotropies



CMB +
BAO

(Hu, Dodelson, astro-ph/0110414)

(Planck 2018, 1807.06209)

$$\Omega_{CDM,0} h^2 = 0.11933 \pm 0.0009 \sim 5 \Omega_{B,0} h^2$$

Dark matter from active-sterile neutrino mixing

(Dodelson Widrow '94; Shi, Fuller '99; Dolgov and Hansen '00; Asaka, Blanchet, Shaposhnikov '05)

- LH-RH
(active-sterile)
neutrino mixing

$$\nu_{1L} \simeq U_{1\alpha}^\dagger \left(\nu_{L\alpha} - \frac{m_{D\alpha 1}}{M_1} \nu_{R1}^c \right)$$

$$N_{1R} \simeq \nu_{1R} + \frac{m_{D\alpha 1}}{M_1} \nu_{L\alpha}^c \longrightarrow \text{lightest RH neutrino}$$

- Solving Boltzmann equations an abundance is produced at $T \sim 100$ MeV:

$$\Omega_{N_1} h^2 \sim 0.1 \frac{\theta^2}{10^{-8}} \left(\frac{M_1}{\text{keV}} \right)^2 \sim \Omega_{DM,0} h^2 \quad \theta^2 \equiv \frac{\sum |m_{D\alpha 1}|^2}{M_1^2}$$

- For $M_1 \ll m_e \Rightarrow \tau_1 = 5 \times 10^{26} \text{ s} \left(\frac{M_1}{\text{keV}} \right)^{-5} \left(\frac{10^{-8}}{\theta^2} \right) \gg t_0$

- The lightest neutrino mass $m_1 \lesssim 10^{-5} \text{ eV} \Rightarrow \text{hierarchical neutrino masses}$
- The N_1 's also radiatively decay and this produces constraints from X-rays (or opportunities to observe it).
- Considering also structure formation constraints, one is forced to consider a resonant production induced by a large lepton asymmetry
- $L \sim 10^{-4}$: 3.5 keV line? (Horiuchi et al. '14; Bulbul et al. '14; Abazajian '14)
- Not confirmed by a recent reanalysis (2309.03254) and new XRISM satellite data (2510.24560)
- See <https://arxiv.org/pdf/2507.20659> for a revamp of the solution with much larger values $L_\alpha \sim 0.1$

Heavy RH neutrino as dark matter ?

(Anisimov,PDB '08)

What production mechanism? For high masses just a tiny abundance is needed:

$$N_{DM} \simeq 10^{-9} (\Omega_{DM,0} h^2) N_\gamma(t_{prod}) \frac{\text{TeV}}{M_{DM}}$$

Suppose there is a RH neutrino with tiny Yukawa couplings (e.g., proportional to a small symmetry breaking parameter) referred to as **dark neutrino** N_D :

$$m_D \simeq \begin{pmatrix} \varepsilon_{e1} & m_{De2} & m_{De3} \\ \varepsilon_{\mu1} & m_{D\mu2} & m_{D\mu3} \\ \varepsilon_{\tau1} & m_{D\tau2} & m_{D\tau3} \end{pmatrix} \text{ or } m_D \simeq \begin{pmatrix} m_{De1} & \varepsilon_{e2} & m_{De3} \\ m_{D\mu1} & \varepsilon_{\mu2} & m_{D\mu3} \\ m_{D\tau1} & \varepsilon_{\tau2} & m_{D\tau3} \end{pmatrix} \text{ or } m_D \simeq \begin{pmatrix} m_{De1} & m_{De2} & \varepsilon_{e3} \\ m_{D\mu1} & m_{D\mu2} & \varepsilon_{\mu3} \\ m_{D\tau1} & m_{D\tau2} & \varepsilon_{\tau3} \end{pmatrix}$$

$$m_D = V_L^\dagger D_{m_D} U_R \quad D_{m_D} \equiv v \text{diag}(h_A, h_B, h_C) \text{ with } h_A \leq h_B \leq h_C$$

$$\tau_{DM} = \frac{4\pi}{h_A^2 M_{DM}} = 0.87 h_A^2 10^{-26} \frac{\text{TeV}}{M_{DM}} s$$

$\Rightarrow$

$$\tau_{DM} > \tau_{DM}^{\min} \simeq 10^{28} s \Rightarrow h_A < 10^{-27} \sqrt{\frac{\text{TeV}}{M_{DM}} \times \frac{10^{28} s}{\tau_{DM}^{\min}}}$$

Too small to reproduce the correct abundance with any production mechanism within a minimal type-I seesaw extension

5-dimensional Higgs portal-like operators as a way out

(Anisimov hep-ph/0612024, Bezrukov, Gorbunov, Shaposhnikov 0812.3622 Anisimov,PDB 0812.5085)

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Y+M}^{\nu} + \mathcal{L}_A$$

Type-I
seesaw
Lagrangian

$$-\mathcal{L}_{Y+M}^{\nu} = \bar{L}_{\alpha} h_{\alpha I} N_I \tilde{\phi} + \frac{1}{2} \overline{N_I^c} M_I N_I + h.c.$$

Anisimov
operator(s)

$$\mathcal{L}_A = \sum_{I,J} \frac{\lambda_{IJ}}{\Lambda} \phi^{\dagger} \phi \overline{N_I^c} N_J + h.c.$$

$$= \frac{\lambda_{DS}}{\Lambda} \phi^{\dagger} \phi \overline{N_D^c} N_S + \frac{\lambda_{SS}}{\Lambda} \phi^{\dagger} \phi \overline{N_S^c} N_S + \frac{\lambda_{DD}}{\Lambda} \phi^{\dagger} \phi \overline{N_D^c} N_D + h.c. \quad (N_D = N_3; N_S = N_2)$$

Remarks: $\longrightarrow$ RH-RH (sterile-sterile) Higgs-induced neutrino mixing (RHINO)

- from SMEFT to vSMEFT
- They are Weinberg-like operators but a further step up
- They extend Higgs portal renormalizable operator (Patt, Wilczek hep-ph/0605188)

RHINO dark matter

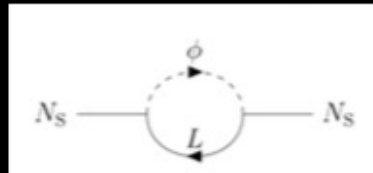
(Anisimov '06, Anisimov,PDB '08)

Focus on the RH-RH Higgs-induced neutrino mixing (RHINO) operator:

$$\mathcal{L}_A = \frac{\lambda_{DS}}{\Lambda} \phi^\dagger \phi \overline{N_D^c} N_S \quad \tilde{\Lambda}_{DS} = \Lambda / \lambda_{DS}$$

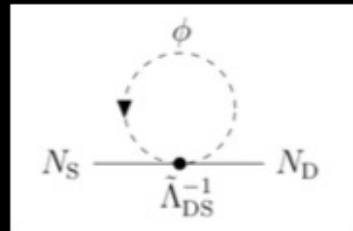
In general, $\lambda_{DS} \neq 0$ generates a dark-source RH neutrino mixing. The Yukawa and Anisimov interactions both generate effective potentials from self-energies:

From Yukawa interactions



$$\Rightarrow V_S^Y = \frac{T^2}{8p} h_S^2$$

From mixing



$$\Rightarrow V_{DS}^\Lambda = \frac{T^2}{12\Lambda} \lambda_{DS}$$

Effective mixing Hamiltonian :

$$\Delta H \simeq \begin{pmatrix} -\frac{\Delta M^2}{4p} - \frac{T^2}{16p} h_S^2 & \frac{T^2}{12\tilde{\Lambda}_{DS}} \\ \frac{T^2}{12\tilde{\Lambda}_{DS}} & \frac{\Delta M^2}{4p} + \frac{T^2}{16p} h_S^2 \end{pmatrix}$$

mixing term

$$\Delta M^2 \equiv M_S^2 - M_D^2$$

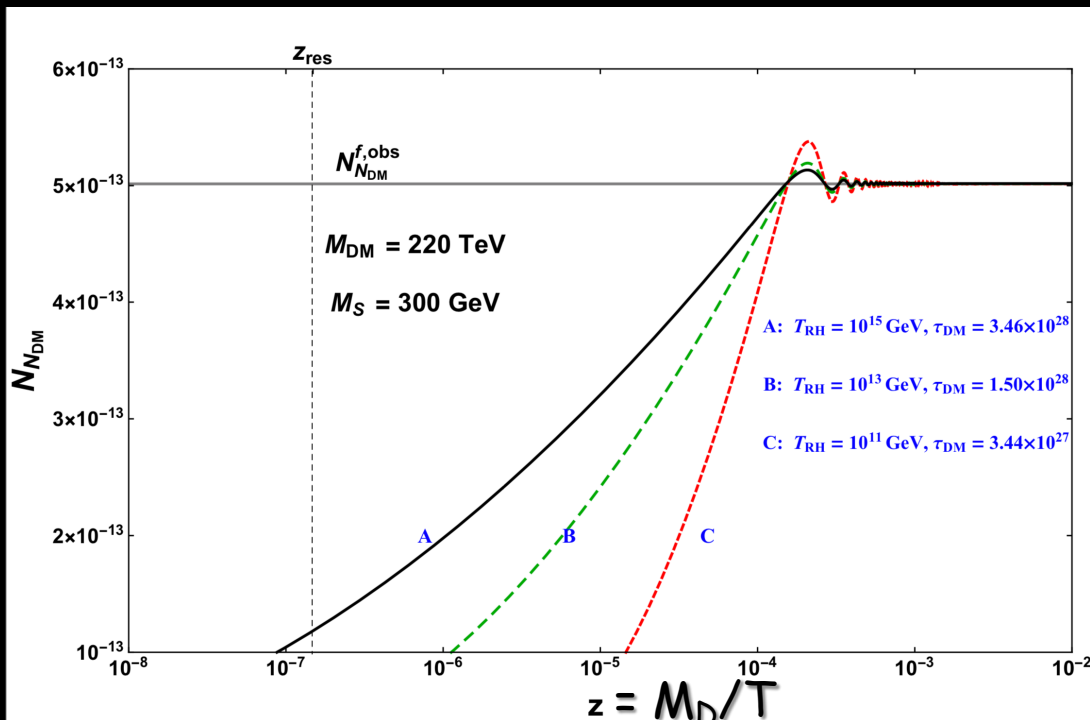
Density matrix calculation of the relic abundance

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

Density matrix equation for the dark-bright mixed RH neutrinos (using a monochromatic approximation $p \sim 3T$)

$$\frac{dN_{IJ}}{dt} = -i[\Delta H, N]_{IJ} - \begin{pmatrix} 0 & \frac{1}{2}(\Gamma_D + \Gamma_S)N_{DS} \\ \frac{1}{2}(\Gamma_D + \Gamma_S)N_{SD} & (\Gamma_D + \Gamma_S)(N_{N_S} - N_{N_S}^{eq}) \end{pmatrix}$$

Assuming an initial thermal N_S -abundance

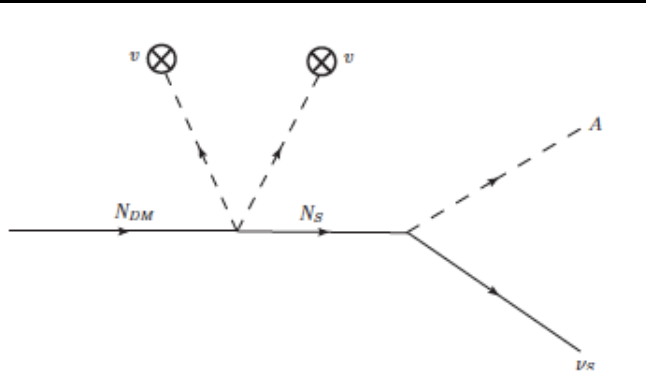


Dark neutrinos are necessarily unstable

(Anisimov,PDB '08; Anisimov,PDB'10; P.Ludl,PDB,S.Palomarez-Ruiz'16)

2 body decays ($M_S > M_W$)

Dark neutrinos unavoidably decay today into $A + \text{leptons}$ ($A = H, Z, W$) through the same mixing that produced them in the very early Universe



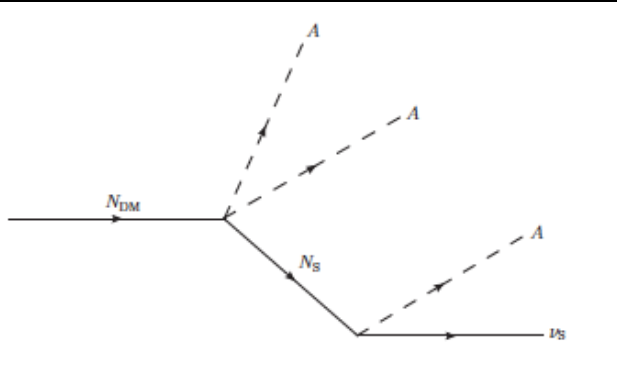
$$\theta_{\Lambda 0} = \frac{2 v^2 / \tilde{\Lambda}_{DS}}{M_D (1 - M_S / M_D)}$$

mixing angle today
(for $\theta_{\Lambda 0} \ll 1$)

$$\Gamma_{D \rightarrow A + \ell_S} = \frac{h_S^2}{\pi} \left(\frac{v^2}{\tilde{\Lambda}} \right)^2 \frac{M_D}{(M_D - M_S)^2}.$$

$\Rightarrow$ Lower bound on M_D

4 body decays



$$N_{DM} \rightarrow 2 A + N_S \rightarrow 3 A + \nu_S \quad (A = W^\pm, Z, H).$$

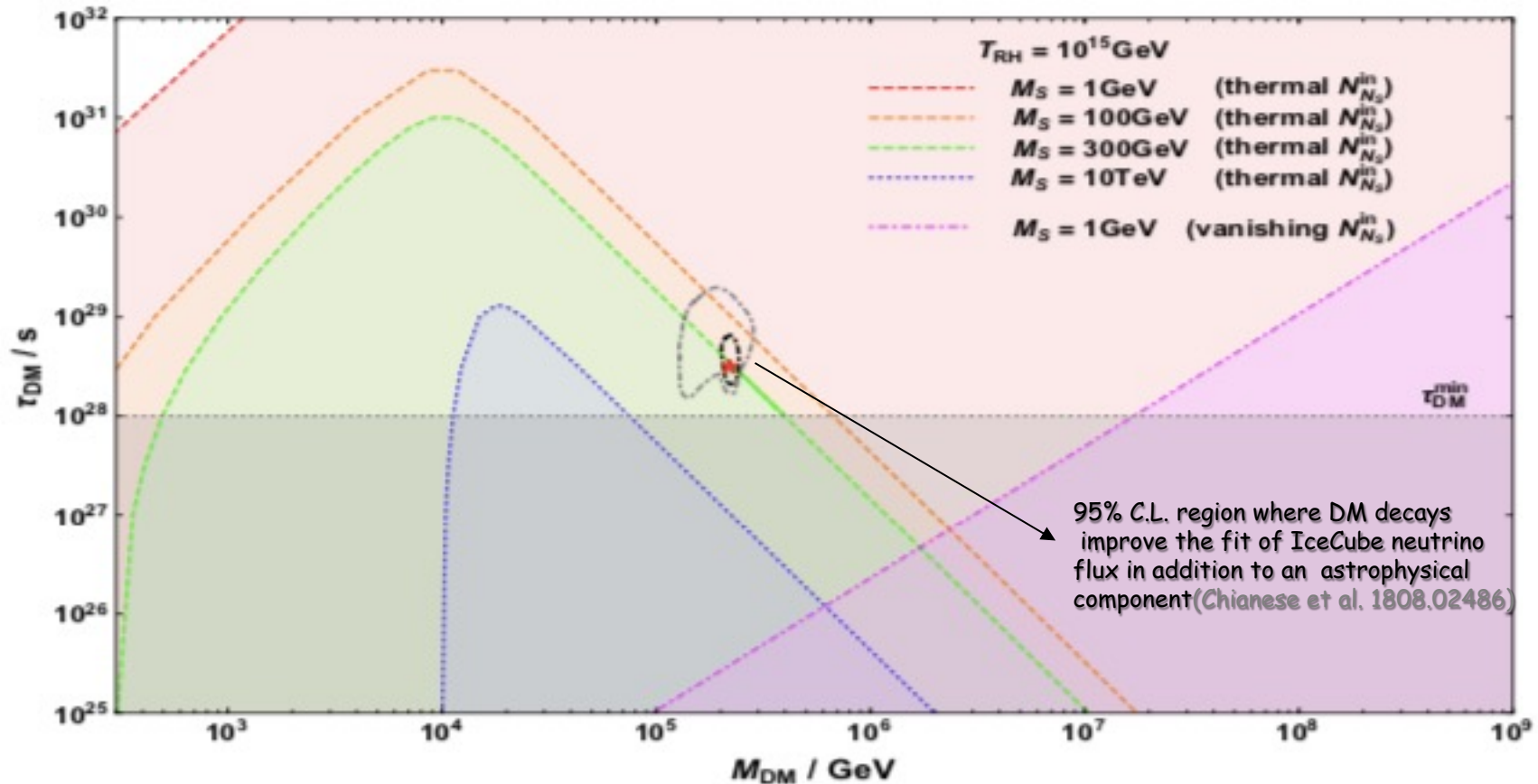
$$\Gamma_{D \rightarrow 3A + \ell_S} = \frac{\Gamma_S}{15 \cdot 2^{11} \cdot \pi^4} \frac{M_D}{M_S} \left(\frac{M_D}{\tilde{\Lambda}_{DS}} \right)^2$$

$\Rightarrow$ Upper bound on M_D

3 body decays and annihilations can also occur but yield weaker constraints

DM lifetime vs. mass plane: allowed regions

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)



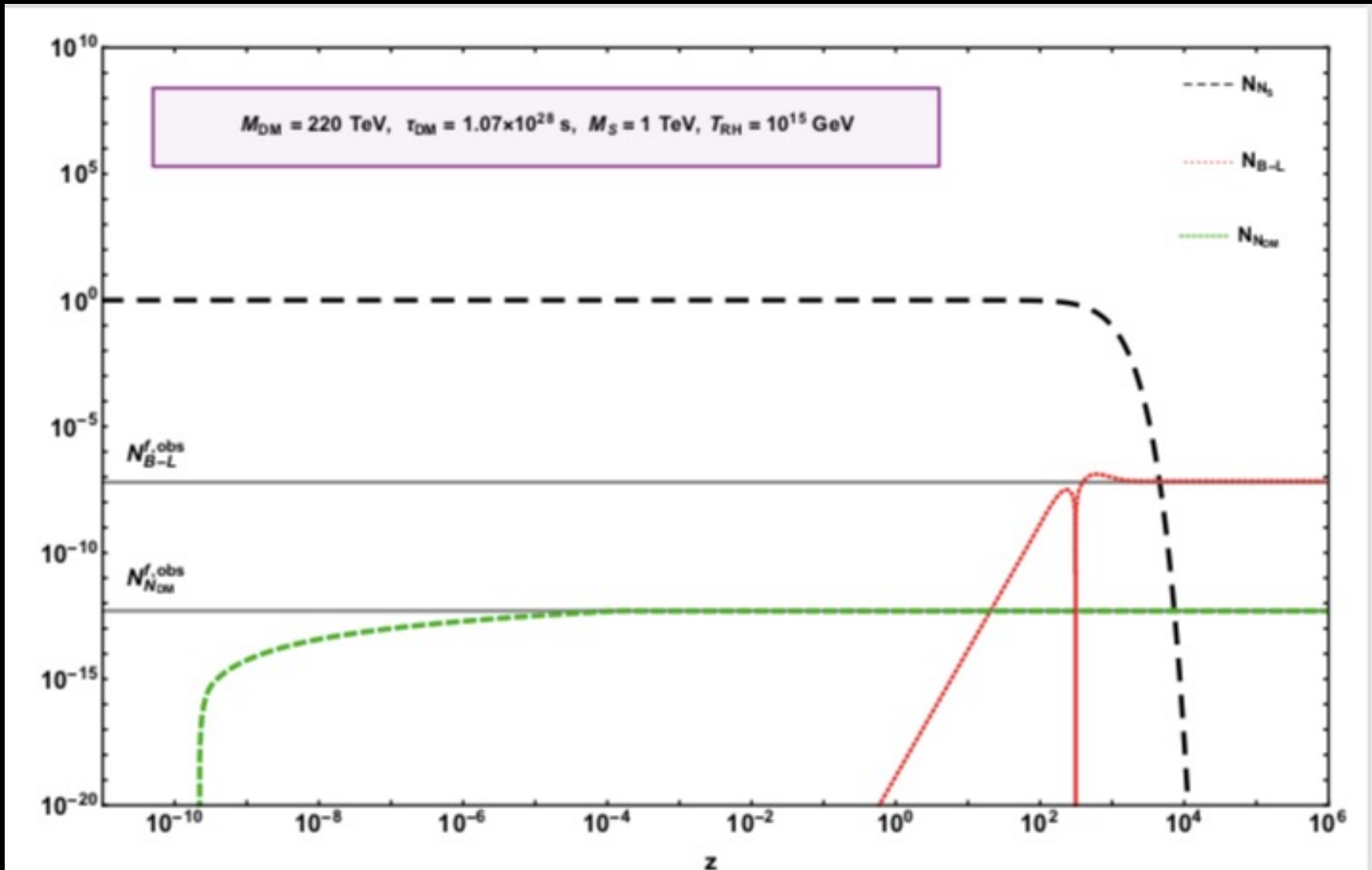
It works only for **initial thermal N_S abundance**, unless $M_S \sim 1 \text{ GeV}$ and $M_D \gtrsim 10^7 \text{ GeV}$

Can one think of processes able to thermalize the N_S abundance prior to the oscillations?

Unifying Leptogenesis and Dark Matter

(PDB, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

A solution for initial thermal N_S abundance:



Including Higgs portal interactions for N_S

(PDB, A. Murphy, arXiv 2210.10801)

$$\mathcal{L}_A = \frac{\lambda_{DS}}{\Lambda} \phi^\dagger \phi \overline{N_{DM}^c} N_S + \frac{\lambda_{SS}}{\Lambda} \phi^\dagger \phi \overline{N_S^c} N_S$$

effective scales

$$\tilde{\Lambda}_{DS} \equiv \Lambda / \lambda_{DS}$$

$$\tilde{\Lambda}_{SS} \equiv \Lambda / \lambda_{SS}$$

Can these interactions thermalise the source neutrinos prior to oscillations?
Let us modify the kinetic equations including these processes:

$$\frac{dN_{IJ}}{dt} = -i[\Delta H, N]_{IJ} - \begin{pmatrix} 0 & \frac{1}{2}(\Gamma_D + \Gamma_S)N_{DS} \\ \frac{1}{2}(\Gamma_D + \Gamma_S)N_{SD} & (\Gamma_D + \Gamma_S)(N_{N_S} - N_{N_S}^{eq}) + \frac{\langle \sigma_{\phi\phi \rightarrow N_S N_S}^{\nu} \rangle}{R^3} (N_{N_S}^2 - N_{N_S}^{eq2}) \end{pmatrix}$$

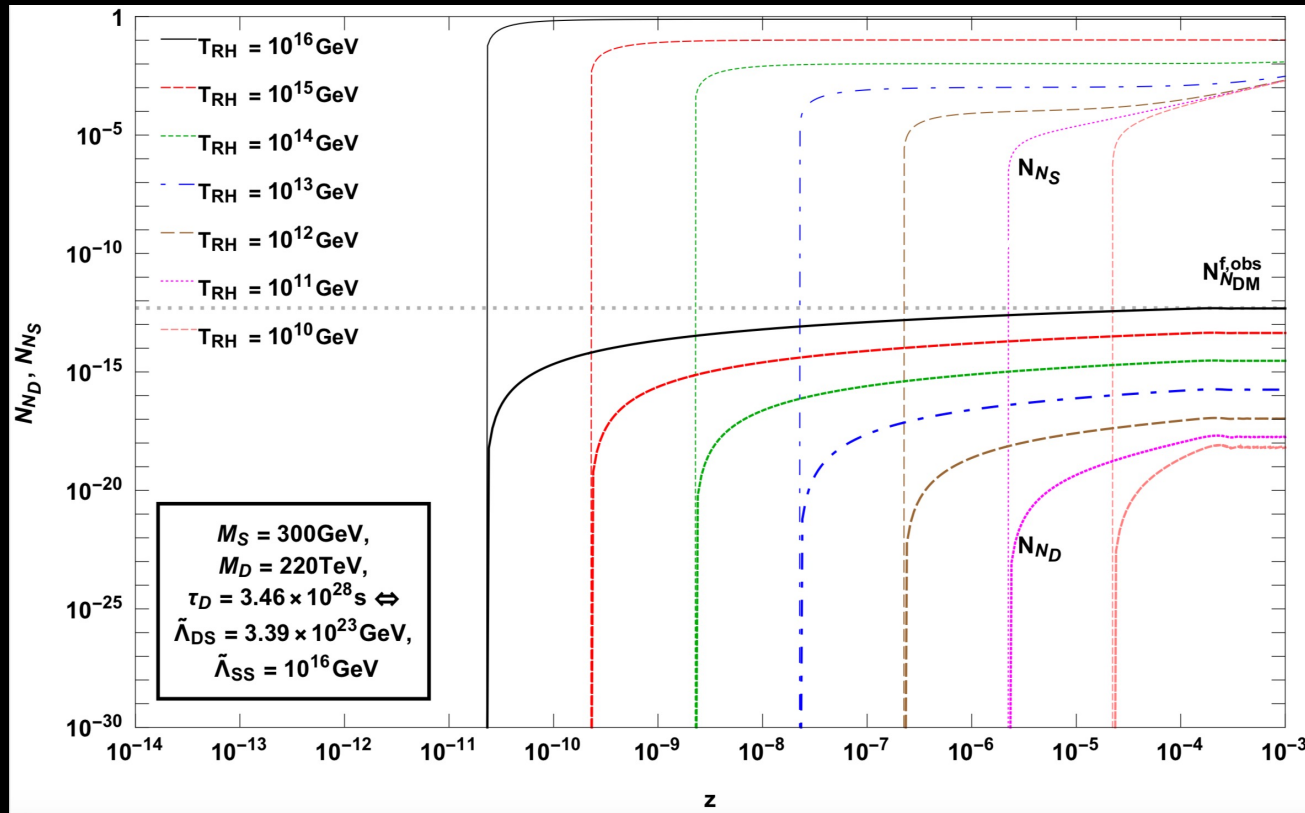
$$A(z) \equiv \frac{\langle \sigma_{\phi\phi \rightarrow N_S N_S}^{\nu} \rangle}{R^3 H z} = \frac{A(z=1)}{z^2}; \quad \langle \sigma_{\phi\phi \rightarrow N_S N_S}^{\nu} \rangle_{T \gg M_S} \simeq \frac{1}{\sim^2 4\pi \Lambda_{SS}} \quad (\text{Kolb, Long, 1708.04293})$$

$$\Rightarrow A(z=1) \simeq g_N \frac{3}{16} \frac{\xi(3)}{\pi^3} \sqrt{\frac{90}{8\pi^3 g_R}} \frac{M_D M_{Pl}}{\sim^2 \Lambda_{SS}}$$

Condition for the thermalisation of the N_S abundance

(PDB, A. Murphy, arXiv 2210.10801)

$$\Rightarrow N_{N_S}(z_{in} \ll z \ll 1) - N_{N_S}(z_{in}) \simeq \frac{A_1}{z_{in}} \simeq 1.0 \times \left(\frac{T_{in}}{10^{16} \text{ GeV}} \right) \left(\frac{10^{16} \text{ GeV}}{\tilde{\Lambda}_{SS}} \right)^2 \simeq 1$$



$$\tilde{\Lambda}_{DS} \sim 10^{23} \text{ GeV}$$

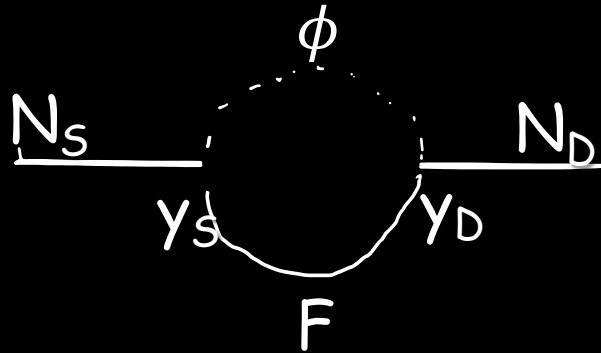
$$\tilde{\Lambda}_{SS} \sim 10^{16} \text{ GeV}$$

δ
 FUT
 $\text{scad}!!$

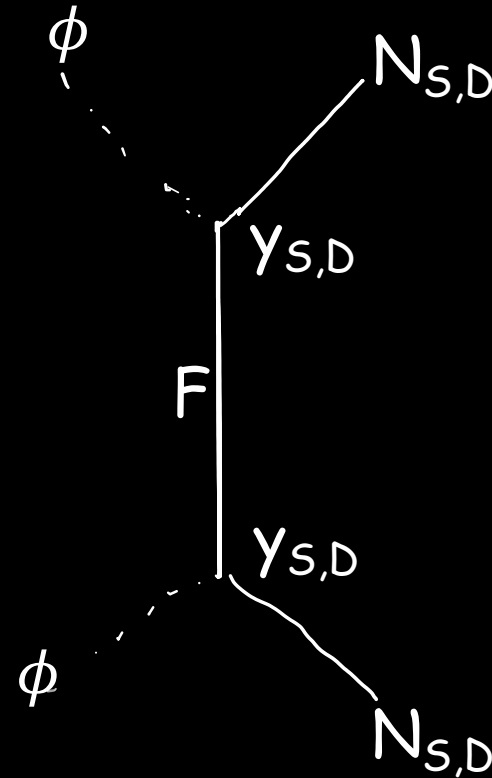
A possible GUT origin ? Heavy fermion F as mediator

(Anisimov,PDB, 2008; PDM, A. Murphy 2210.10801)

$$\mathcal{L}_F = \bar{F} (i \not{\partial} - M_F) F - \sum_I y_I (\bar{F} \phi N_I + \bar{N}_I \phi^\dagger F)$$



$$-\mathcal{L}_F^{\text{eff}} = \sum_{I,J} \frac{y_I y_J}{M_F} \bar{N}_I N_J \phi^\dagger \phi, \implies \Lambda = M_F \text{ and } \lambda'_{IJ} = y_I y_J.$$



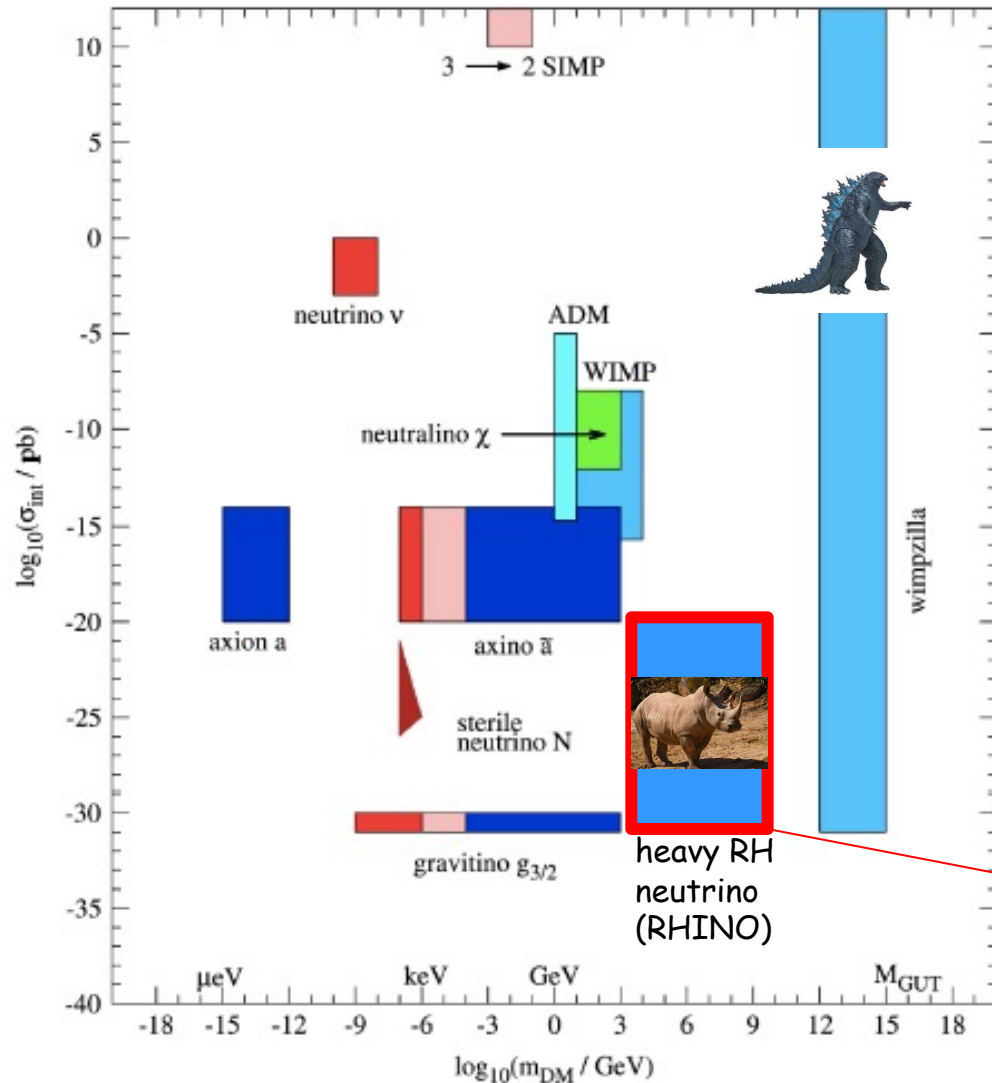
This time one can have one scale $\Lambda = M_F \sim M_{\text{GUT}}$ and for $y_S \sim 1$ and $y_D \sim 10^{-7}$:

$$\tilde{\Lambda}_{DS} = \frac{\Lambda}{y_D y_S} \sim 10^{23} \text{ GeV} \quad \tilde{\Lambda}_{SS} = \frac{\Lambda}{y_S y_S} \sim \Lambda \sim 10^{16} \text{ GeV} \quad \tilde{\Lambda}_{DD} = \frac{\Lambda}{y_D y_D} \sim 10^{30} \text{ GeV}$$

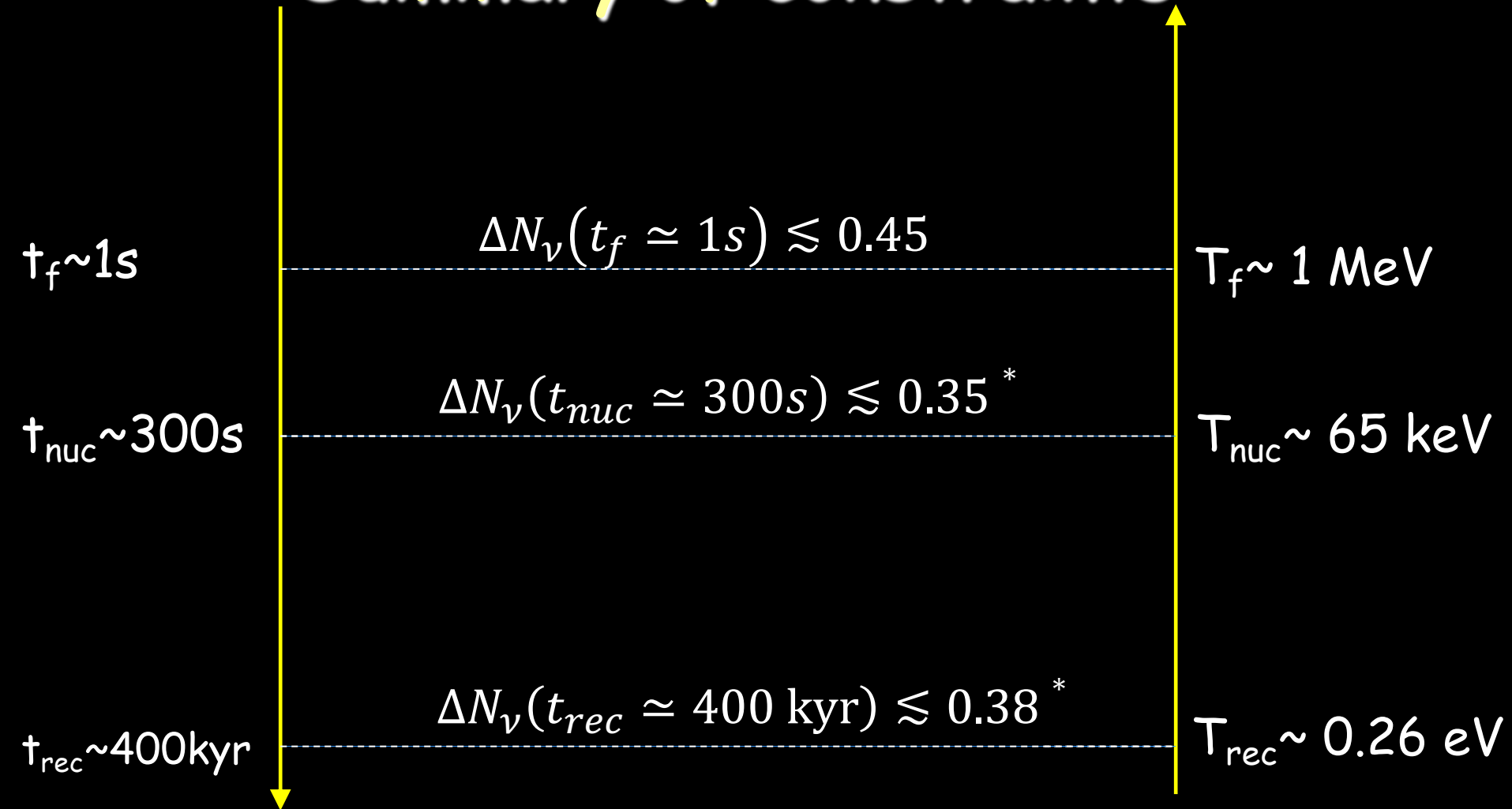
$y_D \sim 10^{-7}$ can be understood as a small symmetry (e.g. Z_2) breaking parameter

Beyond the WIMP paradigm: the DM particle zoo

(from Baer et al.1407.0017)



Summary of constraints



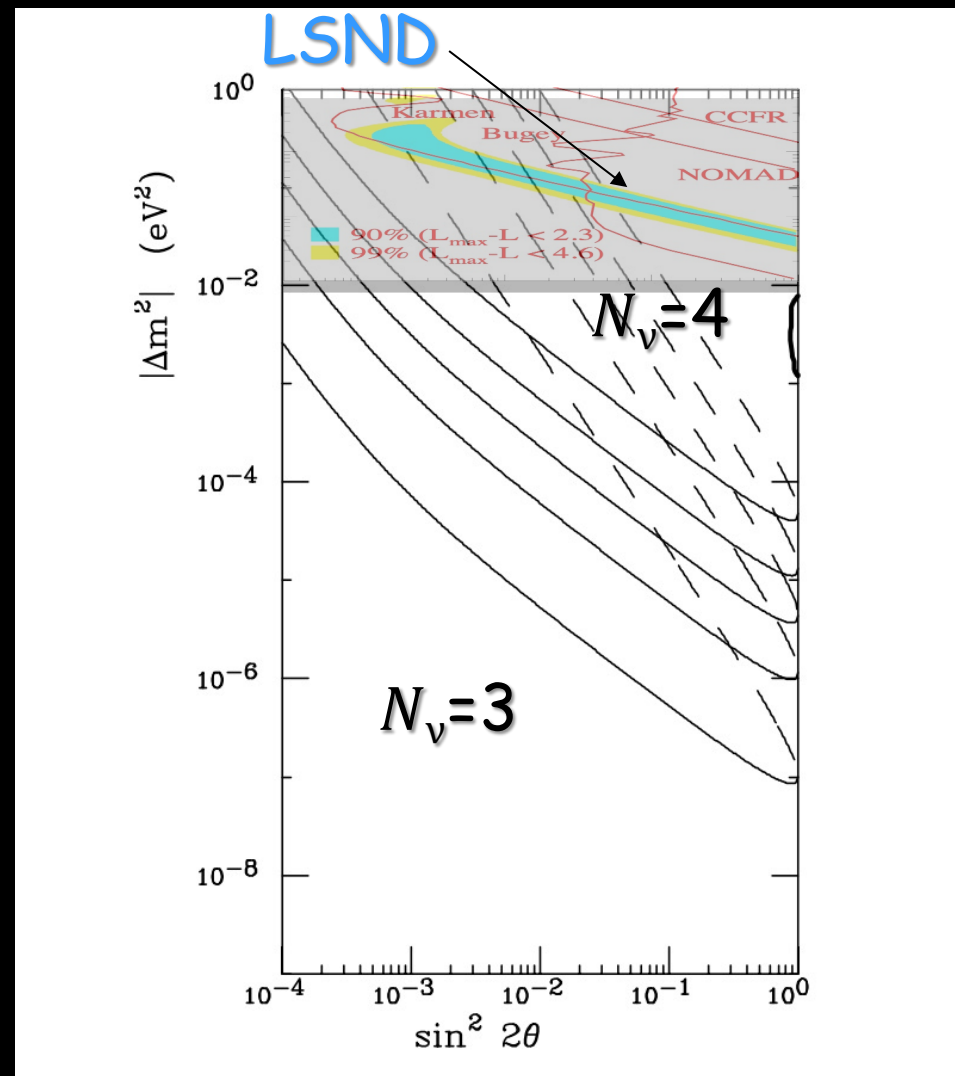
- A typical case corresponds to an injection (or freeze-out) at some high $T_* \gg T_f$: in that case $\Delta N_\nu(t_i) = \text{const}$
- **Example:** SM is augmented by 1 massless species X decoupling (in u.r. eq.) at $T_{X,f} = T_* \gg 100 \text{ GeV} \Rightarrow \Delta N_\nu \sim g_{\rho X^*} (4/7) (43/427)^{4/3} \sim 0.054 g_{\rho X^*}$

Split majoron model
addressing
cosmological tensions
and anomalies

pre-Planck Hubble tension: active-sterile neutrino oscillations in the early universe

(Dolgov '81; Enqvist, Kainulainen '90; Barbieri Dolgov '90; Cline '92)

- An explanation of the LSND anomaly implies that the sterile neutrino gets fully thermalized: $N_\nu=4$ (barring large lepton asymmetry)
- Combining with BBN data $\Rightarrow$ case for a sterile neutrino "friendly" cosmology (Hamann et al. 1006.5276)
- WMAP9 data were still compatible with $\Delta N_\nu \sim 1$ ($N_\nu = 3.84 \pm 0.40$)
- though BBN less friendly, still caveats justifying $\Delta N_\nu \sim 1$
- *Planck* results rule out a fully thermalised sterile neutrino but....



(from PDB et al. hep-ph/9907548)

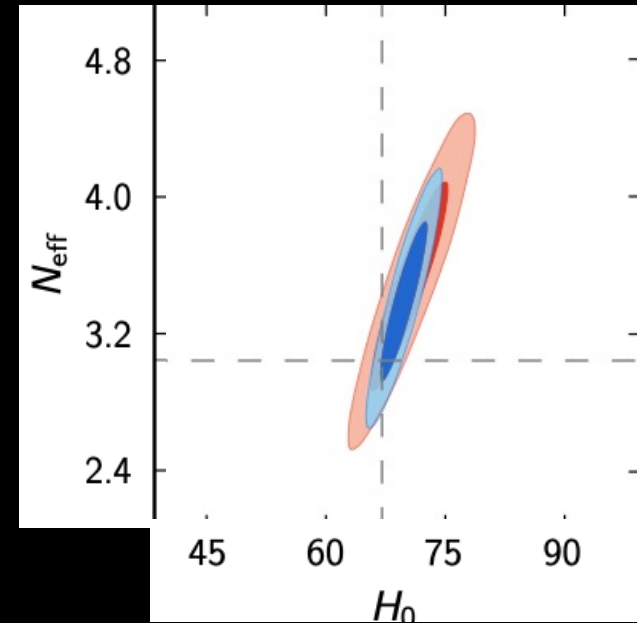
Rising of the Hubble tension; a fractional N_ν ?

$$H_0^{(Planck13)} = 67.3 \pm 1.2 \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$N_\nu^{(Planck13)} = 3.36 \pm 0.34$$

$$H_0^{(SNe)} = 73.8 \pm 2.4 \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$N_\nu^{(Planck13+SNe)} = 3.62 \pm 0.25$$



Many proposed models for $\Delta N_\nu(T_{rec}) \sim 0.5$:

(from Planck2013 1303.5076)

- **Long-lived particle decays** (PDB, S.F. King, A. Merle 1303.6267)

In this case dark radiation can be dominantly produced after 1s but before nucleosynthesis so **predicted deuterium abundance** would be non-standard but Helium-4 abundance would be standard. This is interesting because there is indeed a claim that the **observed deuterium abundance** is non-standard and there is evidence for $\Delta N_\nu(t_{nuc}) \sim 0.3$ (Pitrou, Coc, Uzan, Vangioni 2011.11320)

- **Axionic dark radiation** (J.Conlon, M.C. David Marsh, 1304.1804)

- **Decoupled Goldstone boson** (S. Weinberg 1305.1971):

$$T_* \sim 100 \text{ MeV} \Rightarrow \Delta N_\nu \sim (4/7)(43/57)^{1/3} \sim 0.39$$

Cosmological tensions: beyond a fractional N_ν

The situation today is more articulated (more observations, more accurate):

- Hubble tension:

$$H_0^{(P18)} = 67.66 \pm 0.42 \text{ km s}^{-1}\text{Mpc}^{-1} \xleftrightarrow{\sim 5\sigma \text{ tension}} H_0^{(SH0ES)} = 73.30 \pm 1.04 \text{ km s}^{-1}\text{Mpc}^{-1}$$

- Growth tension
- Cosmic dipoles
- CMB anisotropy anomaly

A model should improve the Λ CDM baseline model rather than solve one tension in isolation, without spoiling the success of the Λ CDM. A mere injection of extra-radiation would solve the Hubble tension but spoil the nice fit of the acoustic peaks.

A **majoron model** is quite well motivated in particle physics: it can explain the generation of the neutrino Majorana masses in the seesaw model, as the Higgs mechanism explains the Dirac masses.....and it would not just produce dark radiation

Majorana mass generation in the Majoron model

(Y. Chikashige, R. Mohapatra, R. Peccei 1981)

$$-\mathcal{L}_{N_I+\sigma} = \overline{L}_\alpha h_{\alpha I} N_I \tilde{\Phi} + \frac{\lambda_I}{2} \sigma \overline{N_I^c} N_I + V_0(\sigma) + h.c. \quad (\text{respecting } U_L(1) \text{ symmetry})$$

$$\sigma = \frac{1}{\sqrt{2}}(\sigma_1 + i\sigma_2), \quad \langle \sigma \rangle = \frac{v_I}{\sqrt{2}}$$

At the end of the σ -phase transition, after SSB, L is violated and

$$\sigma = \frac{e^{i\theta}}{\sqrt{2}}(v_0 + S + iJ) \quad M_I = \lambda_I \frac{v_0}{\sqrt{2}} \sim M \quad (\text{seesaw scale})$$

Typically, one assumes that the σ -phase transition occurs before EWSB

Dirac neutrino mass matrix $m_D = v_{ew} h / \sqrt{2}$ generated after EWSB

$$\text{After both symmetry breakings: } m_\nu = -\frac{v_{ew}^2}{2} \frac{h_{\alpha I} h_{\beta I}}{M_I}$$

S is a massive boson while J is a massless (Goldstone) boson referred to as **majoron**

DARK SECTOR $\equiv N_I$'s + J + S

VISIBLE SECTOR $\equiv$ SM particles

Majoron model at low energies and neutrino rethermalisation

(Chacko,Hall,Okui,Oliver hep-ph 0312267; PDB, Rahat 2307.03184)

- Let us now assume that the temperature of the σ -phase transition T_* occurs not only below EWSB but even after neutrino decoupling ($T_* \lesssim 1$ MeV)
- This very low energy phase transition generates Majorana masses for N' light RH neutrinos (minimal case $N' = 1$).
- At these temperatures ordinary neutrinos interact with the majoron and some additional (real) scalar η . This can (or cannot) be regarded as a second majoron (see PDB, King, Rahat 2306.04680 for a multiple Majoron model) that was produced during a high energy phase transition:

$$-\mathcal{L}_{\nu\text{-dark}} = \frac{i}{2} \sum_{i=2,3} \lambda_i \bar{\nu}_i \gamma^5 \nu_i \eta + \frac{i}{2} \lambda_1 \bar{\nu}_1 \gamma^5 \nu_1 J + \text{h.c.},$$

- These interactions couple neutrinos to majorons, so that the **dark sector** thermalises prior to the phase transition to a common (ν -D) temperature T_D :

$$r_{\nu-D} \equiv \frac{T_D}{T_\nu} = \left(\frac{3.044}{3.044 + N' + \frac{12}{7} + 4 \Delta g/7} \right)^{\frac{1}{4}} < 1$$

contribution from η and σ

- Minimal case: $N' = 1$ and $\Delta g=0 \Rightarrow r_{\nu-D}=0.815$
- Notice that T_ν here denotes the standard neutrino temperature
- The model predict a **CvB** temperature lower than in the standard case

Constraint from BBN + Deuterium abundance

(PDB, Rahat 2307.03184)

$$g_\rho(T) = g_\rho^{\gamma+e^\pm+3\nu}(T) + \frac{7}{4}\Delta N_\nu(T) \left(\frac{T_\nu}{T}\right)^4$$

- Prior to neutrino rethermalisation, above neutrino decoupling, ΔN_ν is negligible
- After the phase transition and the decay of N_h massive particles (S + N' right-handed neutrinos):

$$\Delta N_\nu \simeq 3.044 \left[\left(\frac{3.044 + N' + 12/7 + 4\Delta g/7}{3.044 + N' + 12/7 + 4\Delta g/7 - N_h} \right)^{\frac{1}{3}} - 1 \right]$$

- For $N'=1$ and $\Delta g = 0, 1, 2, 3 \Rightarrow \Delta N_\nu = 0.46, 0.41, 0.37, 0.33$

For $T_* > T_{\text{nuc}} \simeq 65$ keV one has to confront BBN+D/H constraint.

There are 2 different results:

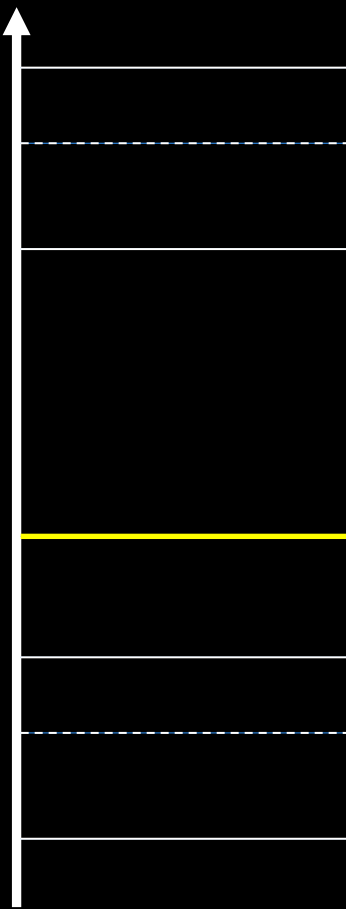
- $\Delta N_\nu(T_{\text{nuc}}) = -0.05 \pm 0.22 \Rightarrow \Delta N_\nu(t_{\text{nuc}}) \lesssim 0.4$ (95% C.L.) (Pisante et al. 2011.11537)
- $\Delta N_\nu(T_{\text{nuc}}) = 0.3 \pm 0.15$ (Pitrou et al. 2011.11320)

The model can nicely address this potential *deuterium problem*

Split majoron model

(PDB, Marfatia, Zhou 2106.00025; PDB, Rahat 2307.03184)

canonical
seesaw
scale

$$\eta, \nu'_0, T'_* \left\{ \begin{array}{l} M_N \\ M_1 \end{array} \right.$$


The diagram shows a vertical axis with an upward-pointing arrow. It features several horizontal lines representing energy scales. From top to bottom, there are two solid lines, a dashed line, and another solid line. A large curly brace on the left groups the top two solid lines, with M_N above it and M_1 below it. Further down, a single solid line is highlighted in yellow, with the label $\sim 100 \text{ MeV}$ to its left. Below this yellow line, there is another dashed line, and at the bottom, a solid line. A second curly brace on the left groups these bottom two lines, with $M_{N'}$ above it and $M_{1'}$ below it.

mini-seesaw scale
or dark sector
low scale

$$\sigma, \nu_0, T_* \left\{ \begin{array}{l} M_{N'} \\ M_{1'} \end{array} \right.$$

The lightest RH neutrino(s) could be responsible for the lightest neutrino mass (as in the ν MSM model)

Confronting the cosmological tensions

(M.Escudero, S. Whitte 1909.04044)

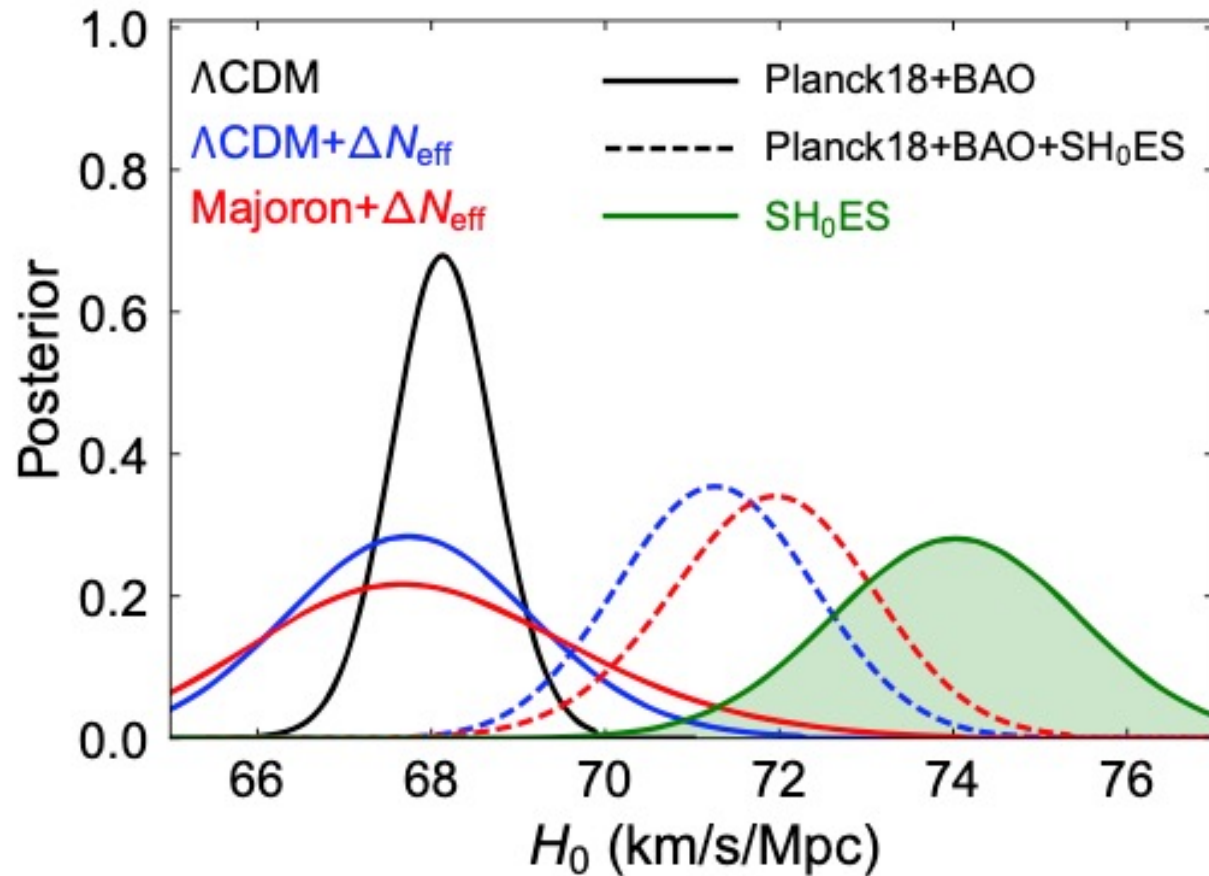
In addition to extra radiation, it also couples the majoron background to neutrinos reducing r_s and allowing for larger H_0 (so that peak positions are unchanged)

Parameter	Λ CDM	Λ CDM + ΔN_{eff}	Majoron + ΔN_{eff}
ΔN_{eff}	—	$0.43 (0.358) \pm 0.18$	$0.52 (0.545) \pm 0.19$
m_ϕ/eV	—	—	(0.33)
Γ_{eff}	—	—	(8.1)
$100 \Omega_b h^2$	$2.252 (2.2563) \pm 0.016$	$2.270 (2.2676) \pm 0.017$	$2.280 (2.2765) \pm 0.02$
$\Omega_{\text{cdm}} h^2$	$0.1176 (0.11769) \pm 0.0012$	$0.125 (0.1243) \pm 0.003$	$0.127 (0.1279) \pm 0.004$
$100 \theta_s$	$1.0421 (1.04223) \pm 0.0003$	$1.0411 (1.04125) \pm 0.0005$	$1.0410 (1.04102) \pm 0.0005$
$\ln(10^{10} A_s)$	$3.09 (3.1102) \pm 0.03$	$3.10 (3.072) \pm 0.03$	$3.11 (3.116) \pm 0.03$
n_s	$0.971 (0.9690) \pm 0.004$	$0.981 (0.9780) \pm 0.006$	$0.990 (0.99354) \pm 0.010$
τ_{reio}	$0.051 (0.0500) \pm 0.008$	$0.052 (0.0537) \pm 0.008$	$0.052 (0.0576) \pm 0.008$
H_0	$68.98 (69.04) \pm 0.57$	$71.27 (70.60) \pm 1.1$	$71.92 (71.53) \pm 1.2$
$(R - 1)_{\text{min}}$	0.009	0.009	0.03
$\chi^2_{\text{min}} \text{ high-}\ell$	2341.56	2345.39	2338.84
$\chi^2_{\text{min}} \text{ lowl}$	22.45	21.56	20.81
$\chi^2_{\text{min}} \text{ lowE}$	395.72	395.89	396.40
$\chi^2_{\text{min}} \text{ lensing}$	9.91	9.21	10.69
$\chi^2_{\text{min}} \text{ BAO}$	4.74	4.5	4.69
$\chi^2_{\text{min}} \text{ SH}_0\text{ES}$	12.34	5.82	3.10
$\chi^2_{\text{min}} \text{ CMB}$	2769.6	2772.1	2766.7
$\chi^2_{\text{min}} \text{ TOT}$	2786.7	2782.4	2774.5
$\chi^2_{\text{min}} - \chi^2_{\text{min}} _{\Lambda\text{CDM}}$	0	-4.3	-12.2

$L_{\phi} \sim 10^{-13} - 10^{-12}$

Significant improvement compared to the Λ CDM model but new calculations neutrino-majoron interaction rate seems to reduce the statistical significance (S. Sandner, M.Escudero, S. Whitte 2305.01692)

Ameliorating the Hubble tension



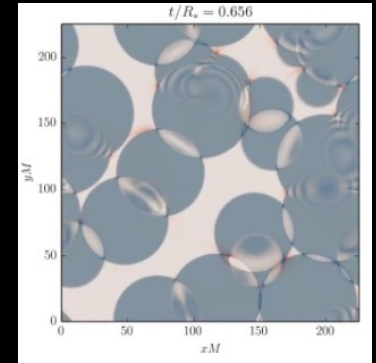
(M.Escudero, S. Witte 1909.04044)

Gravitational waves from first order phase transitions

(Hindmarsh et al. 1704.05871; D. Weir 1705.01783; PDB, Rahat 2307.03184)

GW spectrum

$$h^2 \Omega_{\text{GW}0}(f) = \frac{1}{\rho_{\text{c}0} h^{-2}} \frac{d\rho_{\text{GW}0}}{d \ln f}$$



3 (known) contributions: bubble wall collision, sound waves and turbulence but in the case of a PTA in the dark sector the sound wave contribution is the dominant one, approximately:

$$h^2 \Omega_{\text{sw}0}(f) = 1.845 \times 10^{-6} \frac{\tilde{\Omega}_{\text{gw}}}{10^{-2}} \frac{v_w(\alpha)}{\beta/H_\star} \left[\frac{\kappa(\alpha_D) \alpha}{1 + \alpha} \right]^2 \left(\frac{15.5}{g'_{s\star}} \right)^{4/3} \left(\frac{g'_{\rho\star}}{15.5} \right) S_{\text{sw}}(f) \Upsilon(\alpha, \alpha_D, \beta/H_\star).$$

Peak frequency

$$f_{\text{sw}} = 8.9 \mu\text{Hz} \frac{1}{v_w} \frac{\beta}{H_\star} \frac{T_\star}{100 \text{ GeV}} \left(\frac{g_\rho^\star}{106.75} \right)^{1/6}$$

Spectral shape

$$S_{\text{sw}}(f) = (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}$$

Bubble wall
velocity
(detonation)

$$v_w = \frac{\sqrt{1/3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha} \geq c_s$$

Efficiency
factor

$$\kappa(\alpha_D) \simeq \frac{\alpha_D}{0.73 + 0.083\sqrt{\alpha_D} + \alpha_D}$$

Approximately the GW spectrum depends on $\beta/H_\star$, α and α_D . They can be derived from the effective potential for a given choice of the parameters of the model

A barrier at zero temperature

(PDB, Marfatia,Zhou 2106.00025; PDB, Rahat 2307.03184)

$$V_0(\eta, \sigma) = V_0(\sigma) + V_{\eta\sigma}(\eta, \sigma) + V_\eta(\eta)$$

The most important term is contained in

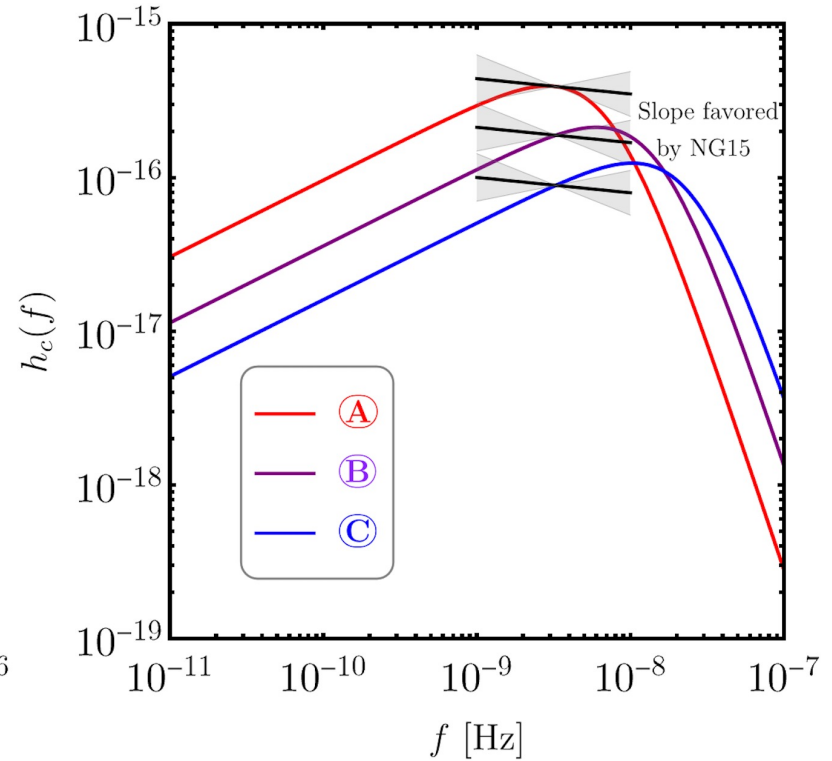
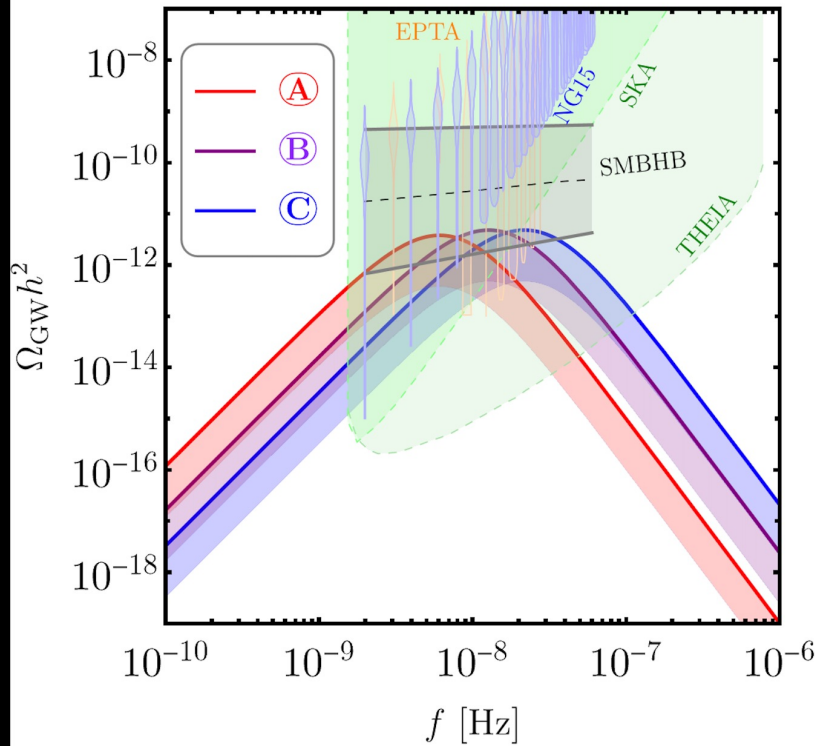
$$V_{\eta\sigma}(\eta, \sigma) = \frac{\delta_1}{2} |\sigma|^2 \eta + \frac{\delta_2}{2} |\sigma|^2 \eta^2$$

The scalar field η can be regarded as a Majoron from a high energy phase transition that generated usual seesaw Majorana masses (see PDB, King, Rahat 2306.04680 for a **multiple Majoron model**)

$$\Rightarrow V_{eff}^T(\sigma_1) = \frac{1}{2} \tilde{M}_T^2 \sigma_1^2 - (A T + \tilde{\mu}) \sigma_1^3 + \frac{1}{4} \lambda_T \sigma_1^4 \quad \text{with } \tilde{\mu} \propto \delta_2$$

The split majoron model confronts the NANOGrav signal

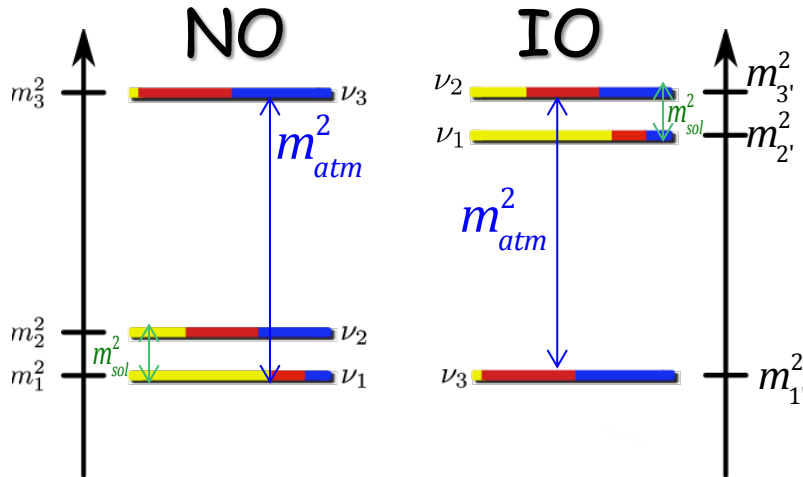
(PDB, Rahat 2307.03184)



B.P.	N'	λ_1	v_1 [keV]	M [keV]	C [keV]	α	α_D	κ_D	β / H_*	T_* [keV]
A	1	0.001	71.0	20.0	0.75	0.52	2.40	0.74	424.0	240.58
B	1	0.001	83.0	23.0	1.70	0.60	2.62	0.75	399.73	515.11
C	1	0.001	144.0	40.0	3.0	0.59	2.56	0.75	393.63	888.35

Neutrino mass tension

Neutrino masses ($m_1 < m_2 < m_3$)



$$NO: m_2 = \sqrt{m_1^2 + m_{sol}^2}, \quad m_3 = \sqrt{m_1^2 + m_{atm}^2}$$

$$IO: m_2 = \sqrt{m_1^2 + m_{atm}^2 - m_{sol}^2}, \quad m_3 = \sqrt{m_1^2 + m_{atm}^2}$$

$$m_{sol} = (8.6 \pm 0.1) \text{ meV}$$

$$m_{atm} = (50.0 \pm 0.3) \text{ meV}$$

Lower bound on neutrino masses (NO)

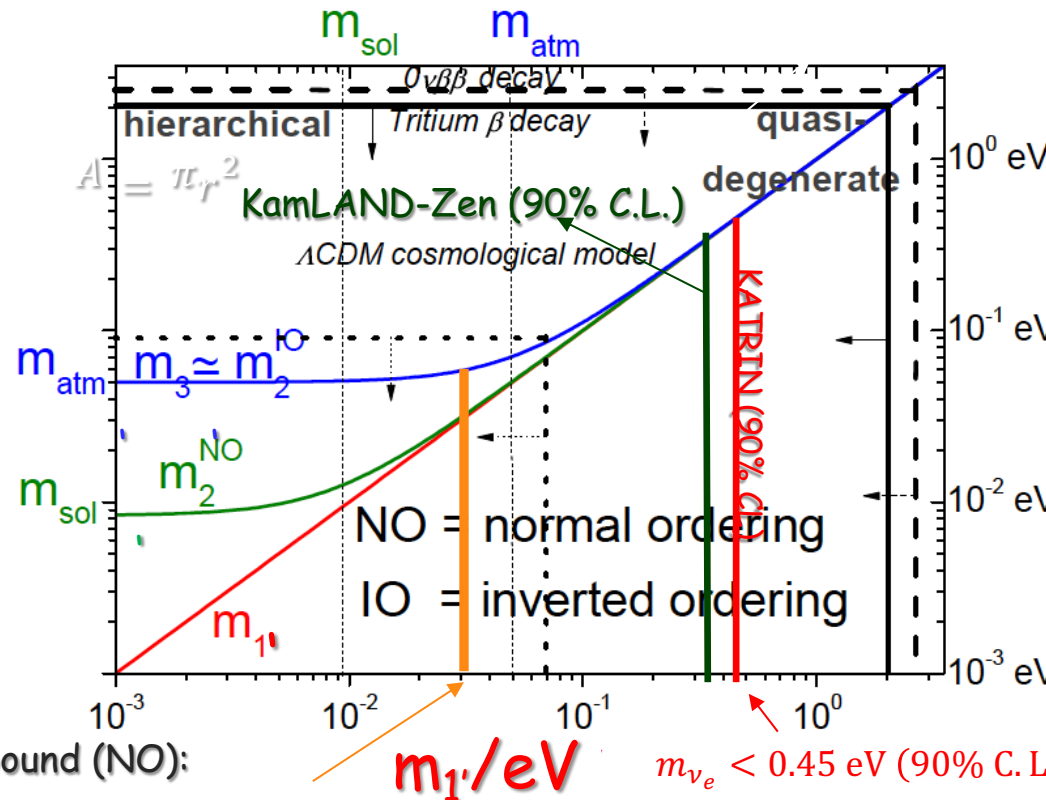
$$\Rightarrow \sum m_i \gtrsim 58 \text{ meV (95\%CL)}$$

For IO it would be much more stringent:

$$\sum m_i \gtrsim 98 \text{ meV (95\%CL)}$$

Cosmological upper bound (NO):

$$m_1 < 30 \text{ meV (95\% C.L.)}$$



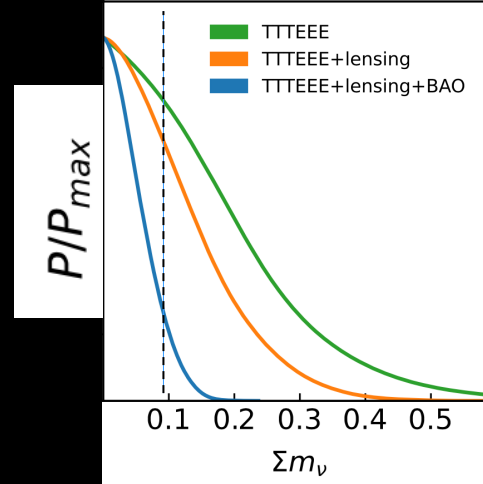
...but also some failure:

Cosmological upper bound on neutrino masses

- Neutrino masses do not seem to play any role in **structure formation**,
- In fact, **neutrino masses are even detrimental** contributing to unwanted **hot dark matter** and for this reason from cosmology (combining CMB + BAO-SDSS) one obtains an **upper bound on the sum of neutrino masses**:

$$\text{NO}_{\text{BAO}} \quad \sum_i m_i \leq 0.26 \text{ eV (95\% C. L.)} \quad \text{WITH}_{\text{BAO}} \quad \sum_i m_i \leq 0.11 \text{ eV (95\% C. L.)} \quad \text{Planck 2023 (2309.10034)}$$

- most stringent upper bound on the absolute neutrino mass scale



Inverted ordering is just marginally allowed when BAO-SDSS data are added

Planck 2023
(2309.10034)

$$\Omega_{\text{stars},0} / 3 \leq \Omega_{\nu 0} \simeq \frac{\sum_i m_i}{45 \text{ eV}} \leq \Omega_{\text{stars},0} \simeq 0.004$$

Neutrino contribution to matter today seems to be comparable to that of stars

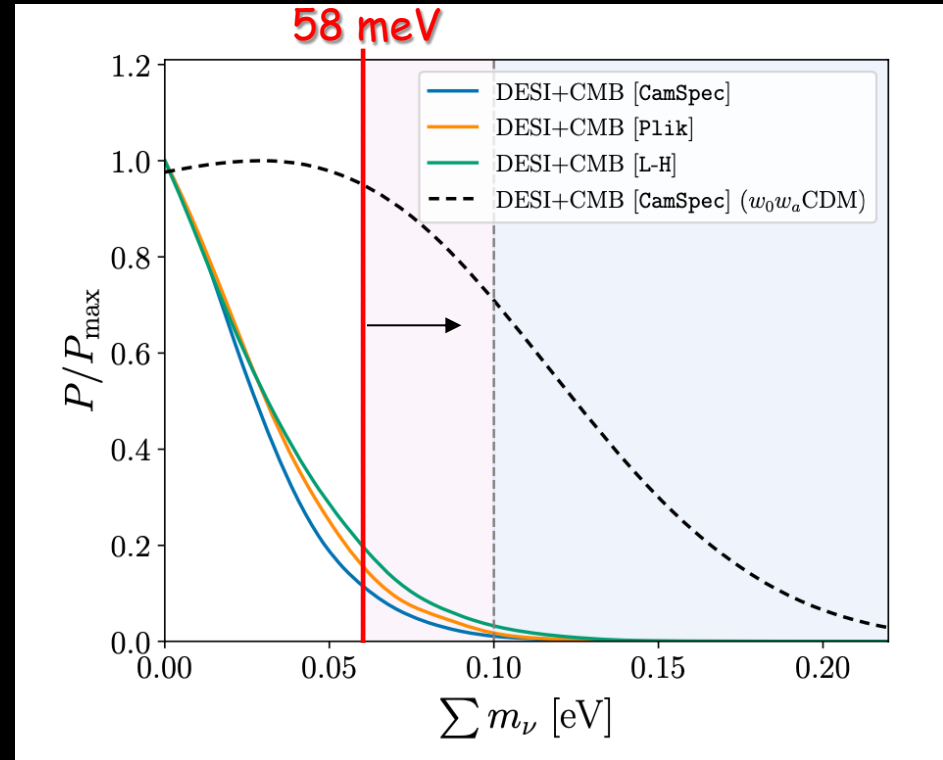
DESI DR2 2025: a neutrino mass tension?

(DESI collaboration 2503.14738)

The best fit is for $\sum_i m_{\nu i} = 0$
(imposing a prior $\sum_i m_{\nu i} \geq 0$)

$$\Rightarrow \sum m_i \lesssim 64 \text{ meV (95\%CL)} \quad (\text{assuming } \Lambda\text{CDM})$$

Clear tension with the
lower bound from
neutrino oscillations



Cosmological solution: going beyond ΛCDM

+Pantheon SNe assuming $w_0 w_a \text{CDM}$ $\Rightarrow \sum m_i \lesssim 117 \text{ meV (95\%CL)}$ (DESI collaboration 2503.14738)

assuming suppressed matter
perturbation growth rate $\Rightarrow \sum m_i \lesssim 134 \text{ meV (95\%CL)}$ (Giare et al. 2507.01848)

...but there is also a simple particle physics solution

No ν 's good news?

(J.F. Beacom, N. Bell, S. Dodelson astro-ph/0404085; P. Serpico astro-ph/0701699; M. Escudero, J. Lopez-Pavon, N. Rius, S. Sandner 2007.04994; N. Craig, D. Green, J. Meyers, S. Rajendran, 2405.00836)

- This tension can be solved if neutrinos decay with a lifetime lower (at least) than about 1 order of magnitude than the the age of the universe but longer than $\sim 10^9 \text{s} (\sum_i m_{\nu_i} / 50 \text{meV})^3$ not to clash with CMB anisotropy observations (free streaming neutrinos at recombination is necessary to explain the peaks, as we have seen).
- Radiative decays are strongly constrained by the upper bound on CMB spectral distortions $\Rightarrow$ **they need to decay invisibly**;

- Example:

$$\mathcal{L}_\phi \supset \frac{\lambda_{ij}}{2} \bar{\nu}_i \nu_j \phi + \frac{\tilde{\lambda}_{ij}}{2} \bar{\nu}_i \gamma_5 \nu_j \phi + \text{h.c.}$$

(N. Craig, D. Green, J. Meyers, S. Rajendran, 2405.00836)

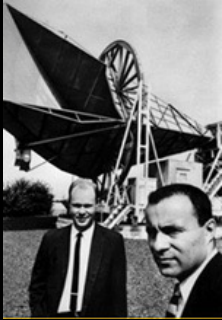
- This results into:

$$\tau(\nu_i \rightarrow \nu_j \phi) \simeq 7 \times 10^{17} \text{ s} \times \left(\frac{0.05 \text{ eV}}{m_{\nu_i}} \right) \left(\frac{10^{-15}}{\tilde{\lambda}_{ij}^2} \right)^2$$

A low scale dark sector destabilizing neutrinos?

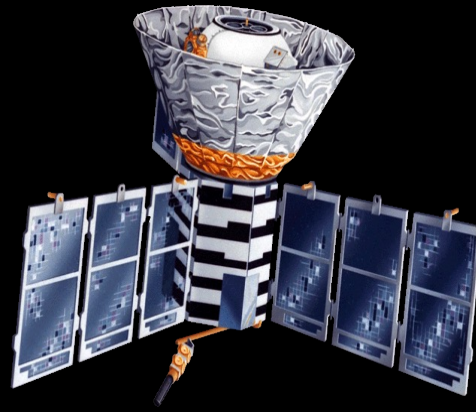
The excess radio
background mystery

The CMB spectrum: most perfect Planckian in nature



Penzias and Wilson (1965)

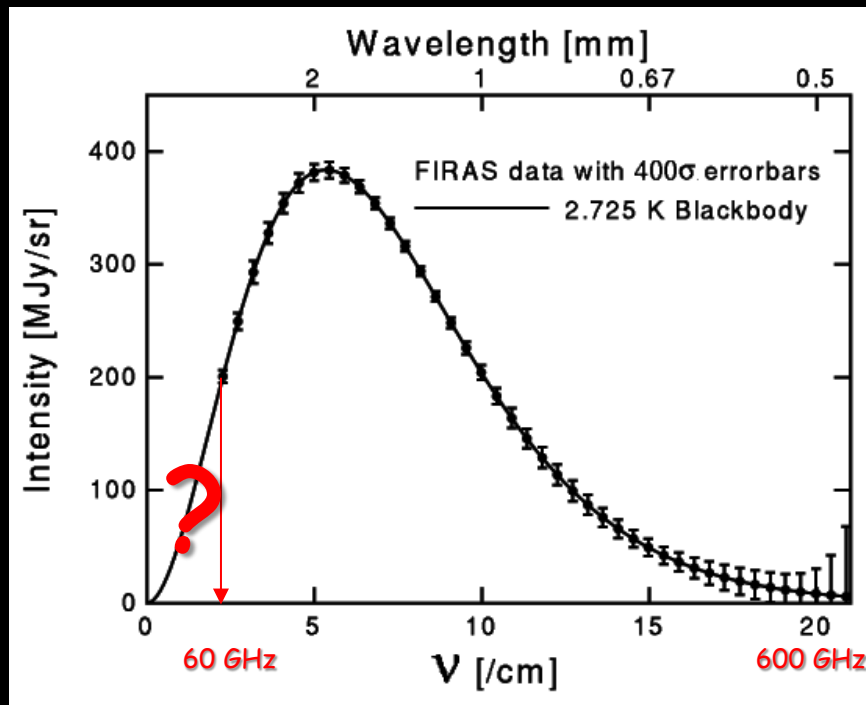
$$T_{\nu 0} = (3.5 \pm 1) \text{ }^{\circ}\text{K}$$



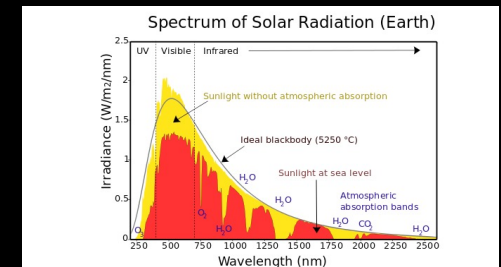
COBE satellite

FIRAS instrument of COBE (1990)

$$T_{\nu 0} = (2.725 \pm 0.001) \text{ }^{\circ}\text{K} \quad (\text{Fixsen and Mather 2002})$$



for a comparison

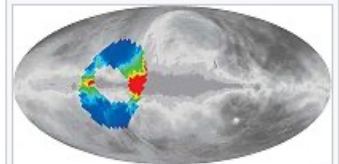
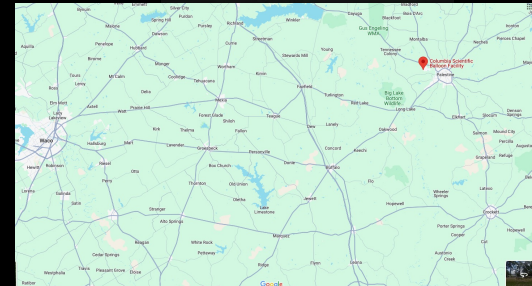


ARCADE 2: The 2006 FLIGHT

(Fixsen, Kogut, Levin, Limon, Lubin, Mirel, Seiffert, Singal, Wollack, Villela, Wuensche 0901.0555)



- Launched from Palestine TX (Columbia scientific balloon facility) on a 29 MCF balloon on 22 July 2006 at 1:15 UT
- It reached a float altitude of 37 km at 4:41 UT
- The entire gondola with the instrument was rotated so that 8.4% of the entire sky was observed
- The most useful observations were from 5:35 to 7:40 UT: **with only two hours of balloon flight observations, ARCADE 2 approaches the absolute accuracy of long-duration space missions**
- The uncertainty in the sky temperature is dominated by thermal gradients in the calibrator
- One main advantage of balloon flight is that at 37km the instrument is above about 99.7% of the atmosphere and an even larger fraction of water vapour and, second, it is well above the nearest source of any radio transmitters



ARCADE viewed about 7% of the sky. The observed region is colored on this all-sky radio map. The plane of our galaxy, the Milky Way, runs across the center.



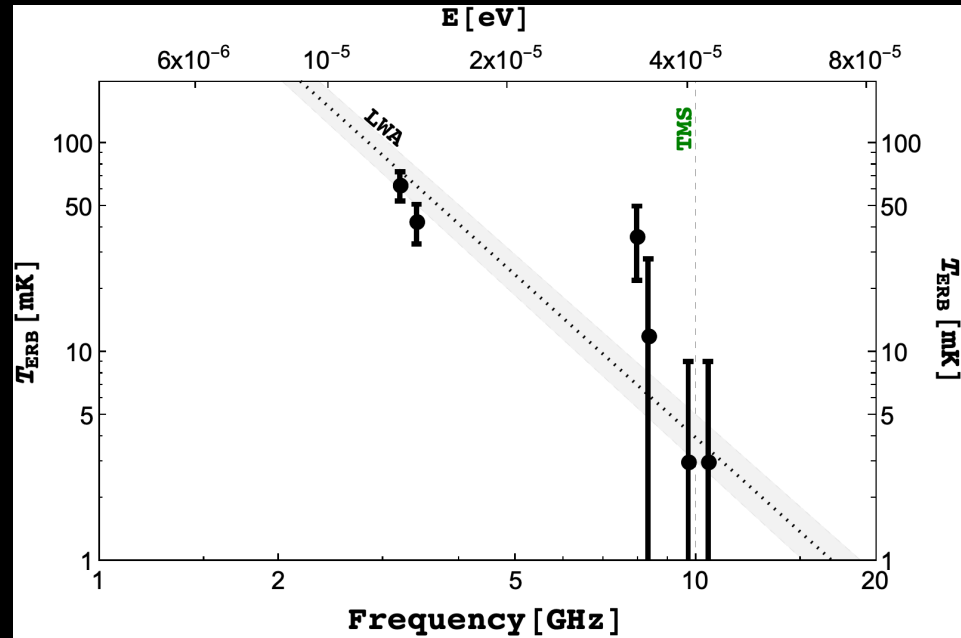
Photograph of the instrument upon landing, 2005. The electronics box and magnetometers which are mounted on the dewar are visible.

ARCADE 2: Results

(Fixsen, Kogut, Levin, Limon, Lubin, Mirel, Seiffert, Singal, Wollack, Villela, Wuensche 0901.0555)

$$T_{\text{ERB}} \equiv T_{\gamma 0}(\nu) - T_{\text{CMB},0}$$

↓
effective temperature



(Figure courtesy of Rishav Roshan)

- The ARCADE 2 measurement of the CMB temperature is in excellent agreement with the FIRAS instrument above 10 GHz
- Below 10 GHz, the detected radio background is brighter than expected ($\sim 5\sigma$ deviation)
- The Long Wavelength Array (LWA) measured the diffuse radio background in the 40-80 MHz, also finding an excess. In combination with ARCADE 2 data, they find a good fit in terms of a power-law:

$$T_{\text{ERB}} = (30.4 \pm 2.6) \left(\frac{\nu}{310 \text{ MHz}} \right)^{-2.58 \pm 0.05} \text{ K.}$$

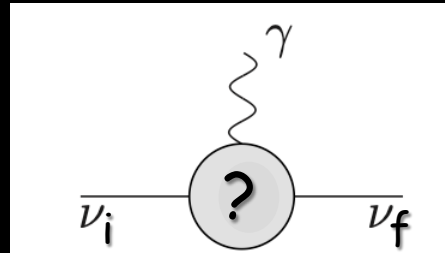
The excess radio background is a mystery

- Part of the excess is due to galactic synchrotron radiation but this galactic contamination is significantly below the measured excess. (N.Fornengo, R.A. Lineros, M. Regis, M. Taoso 1402.2218)
- The excess cannot be explained by known population of sources: they give a contribution to the effective temperature that is 3-10 times smaller than the measured one. (J. Singal et al. 1711.09979)
- Low-redshift populations of discrete extra-galactic radio sources have been also excluded by cross correlating data of the diffuse radio sky with matter tracers at different redshifts provided by galaxy catalogs and CMB lensing. (E.Todarello, et al. 2311.17641)
- Exotic astrophysical explanations have been proposed (supermassive black holes and star forming galaxies) but typically they tend to produce also other (unobserved) signals and more importantly the Australian Telescope Compact Array (ATCA) constrains the contribution to the excess of extended source with angular size below 2 arcmin: **the ERB is extremely smooth**. (T. Vernstrom et al 1408.4160)
- In order to satisfy this constraint, the source of the excess should be active only at redshifts $z \gtrsim 5$ (G.P. Holder 1207.0856)
- Even explanations in terms of new physics encounter similar difficulty: e.g., excess radiation from dark matter decays and/or annihilations would also produce anisotropies that have not been observed.
- "The radio synchrotron level reported by ARCADE 2 is spatially uniform enough to be considered a BACKGROUND. Thus, it would join the astrophysical backgrounds known in all other regions of the EM spectrum. ...**the origin of the radio background would be one of the mysteries of contemporary astrophysics**". (Singal et al., *The second radio synchrotron background workshop*, 2211.16547)
- "Highly accurate measurements.....The nature of the background is still unknown" (R. Sunayev 2408.01858)

Relic neutrino radiative decays

(Chianese, Farrag, PDB, Samanta 2018; Dev, PDB, Martinez-Soler, Roshan 2312.03082)

An intriguing possibility is that the source of non-thermal radiation is relic neutrinos decaying radiatively due to the existence of some new physics:



The decay active-to-active neutrino cannot explain the necessary small $\Delta m_i \ll 2.5 \times 10^{-4} \text{ eV}$
Then necessarily the final neutrino needs to be a sterile neutrino:

$$\nu_f = \nu_{\text{sterile}}$$

Assume that decaying and final sterile neutrino are quasi-degenerate: $\Delta m_i \equiv m_i - m_f \ll m_i$

Moreover, assume that the ordinary neutrinos decay non-relativistically:
with these assumptions the **final photon is monochromatic at the decay**.

At redshift z_D :

$$E_\gamma(z_D) = \Delta m_i$$

...but not at the detection since cosmological expansion will redshift energies:

$$E_{\gamma 0} = \frac{\Delta m_i}{1 + z_D} \leq \Delta m_i$$

For definiteness, we can assume that the lightest neutrino decays $\Rightarrow i = 1$

Specific intensity of the radiation from relic neutrino decays

(Masso, Toldra 1999; Chianese, Farrag, PDB, Samanta 2018; Dev, PDB, Martinez-Soler, Roshan 2312.03082)

$$E = \Delta m_1 \frac{1+z}{1+z_D} \leq \Delta m_1 \Rightarrow a_D \equiv \frac{1}{1+z_D} = \frac{E}{\Delta m_1(1+z)} \quad \text{scale factor at the decay}$$

$$\text{energy density} \quad \varepsilon_{\gamma_{nth}}(z) = \frac{\Delta m_1}{\tau_1} n_{\nu_1}^{\infty}(z) \int_0^a da_D \frac{e^{-\frac{t(a_D)}{\tau_1}}}{H(a_D)a}$$

specific intensity

effective temperature

$$I_{\gamma_{nth}}(E, z) = \frac{1}{4\pi} \frac{d\varepsilon_{\gamma_{nth}}}{dE} \Big|_z = \frac{n_{\nu_1}^{\infty}(z)}{4\pi} \frac{e^{-\frac{t(a_D)}{\tau_1}}}{H(a_D)\tau_1} \Rightarrow T_{\gamma_{nth}}(E, z) \simeq \frac{4\pi^3}{E^2} I_{\gamma_{nth}}(E, z)$$

$$\text{expansion rate at the decay} \quad H(a_D) = H_0 \sqrt{\Omega_{M0} a_D^{-3} + \Omega_{\Lambda 0}} = H_0 \sqrt{\Omega_{M0}} a_D^{-\frac{3}{2}} \left(1 + \frac{a_D^3}{a_{eq}^{M\Lambda^3}} \right)^{\frac{1}{2}}$$

$$\text{age of the universe at the decay} \quad t(a_D) = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda 0}}} \ln \left[\sqrt{\left(\frac{a_D}{a_{eq}^{M\Lambda}} \right)^3} + \sqrt{1 + \left(\frac{a_D}{a_{eq}^{M\Lambda}} \right)^3} \right]$$

relic neutrino number density at the detection (at redshift z)

$$n_{\nu_1}^{\infty}(z) = \frac{6}{11} \frac{\xi(3)}{\pi^2} T^3(z)$$

Fitting the ARCADE 2 excess radio background

(Dev, PDB, Martinez-Soler, Roshan 2312.03082)

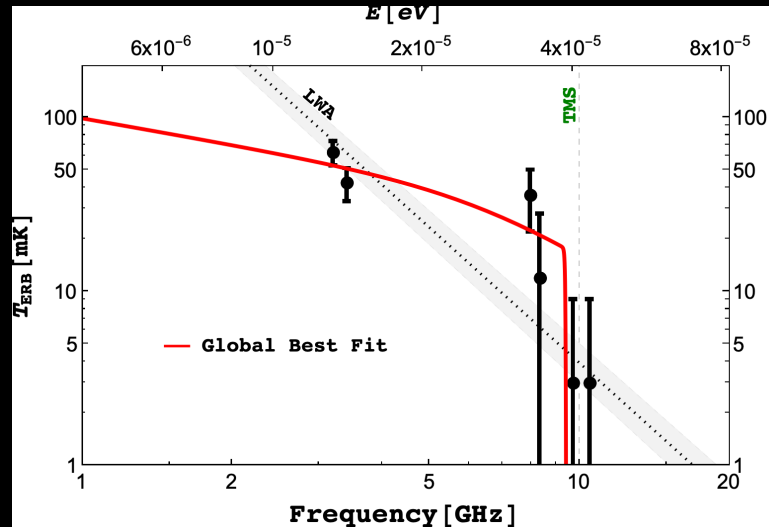
The general expression for the specific intensity gets now specialized into ($z=0$):

$$I_{\gamma_{\text{nth}}}(E, 0) = \frac{n_{\nu_1}^{\infty}(0)}{4\pi} \frac{e^{-\frac{t(a_D)}{\tau_1}}}{H(a_D) \tau_1}$$

Moreover, we can consider solutions with $\tau_1 \gg t_0$ so that we can neglect the exponential:

$$T_{\gamma_{\text{nth}}}(E, 0) \simeq \frac{6\zeta(3)}{11\sqrt{\Omega_{M0}}} \frac{T_0^3}{E^{1/2} \Delta m_1^{3/2}} \frac{t_0}{\tau_1} \left(1 + \frac{a_D^3}{a_{\text{eq}}^3}\right)^{-\frac{1}{2}}$$

Fitting the 6 values of the effective temperature measured by ARCADE 2:



For our best fit we find $\chi^2/4\text{d.of.} = 0.96$, to be compared with $\chi^2/4\text{d.of.} = 2.5$ for the power law

A clash with the upper bounds on the effective magnetic moment

(Dev, PDB, Martinez-Soler, Roshan 2509.03441)

$$\Gamma_{\nu_j \rightarrow \nu_i + \gamma} = \frac{\mu_{\text{eff},ij}^2}{8\pi} \left(\frac{m_j^2 - m_i^2}{m_j} \right)^3$$

Upper bound on μ_{eff} from neutron-electron elastic scattering

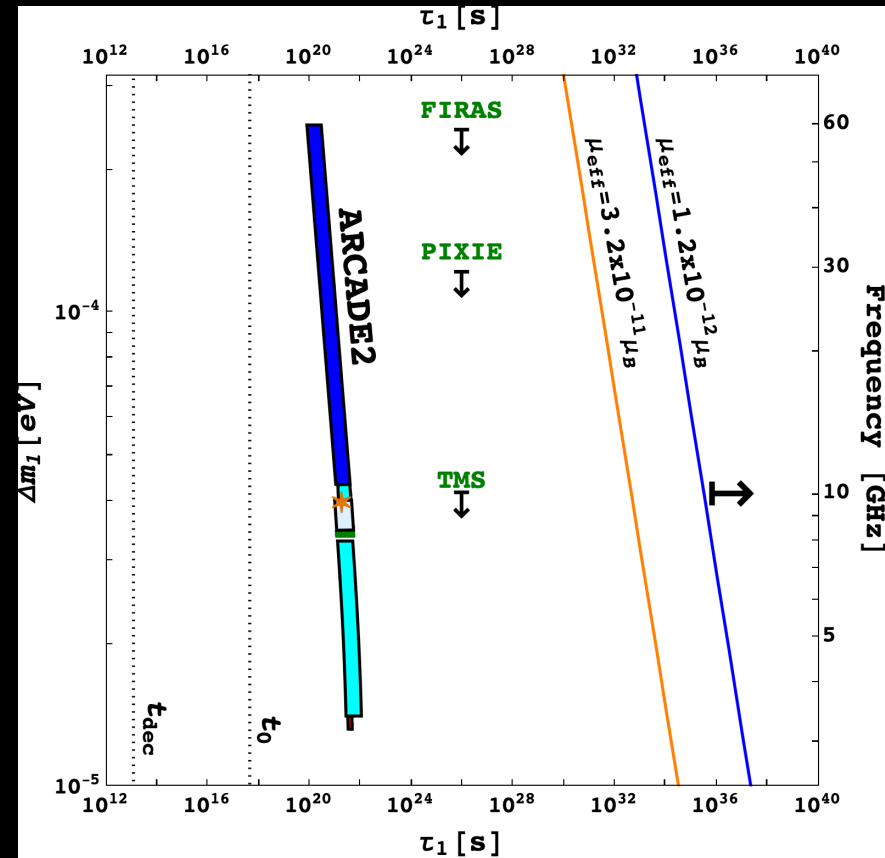
$$\mu_{\text{eff}} \lesssim 3.2 \times 10^{-11} \mu_B$$

(GEMMA collaboration, 1005.2736)

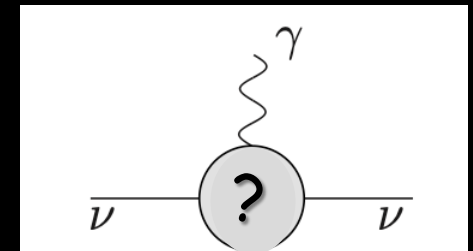
...and from plasmon decays in globular cluster stars:

$$\mu_{\text{eff}} \lesssim 1.2 \times 10^{-12} \mu_B.$$

Raffelt, PRL64, 2856 (1990)



Very challenging clash: how to solve it?



Boomerang mechanism

(Dev, PDB, Martinez-Soler, Roshan 2509.03441)

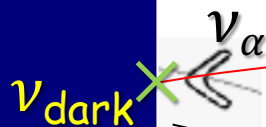
visible sector

active-dark neutrino mixing

$$\nu_1 = \cos \theta_0 \nu_\alpha + \sin \theta_0 \nu_{dark}$$

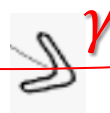
dark sector

$$\nu_{dark} \rightarrow \psi' + \gamma'$$



$$T_\nu^{res}(L_i) \sim 100 \text{ keV}$$

(PDB, Foot hep-ph/0103192)

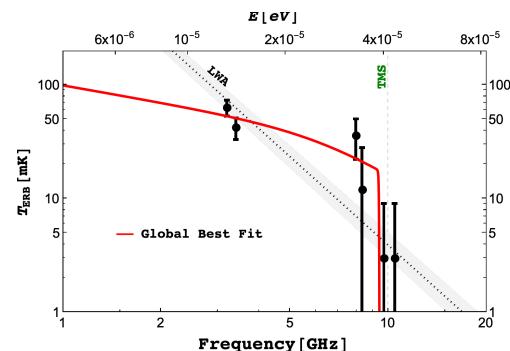


Kinetic mixing

$$\gamma' = \cos \chi_0 \gamma_{dark} + \sin \chi_0 \gamma$$

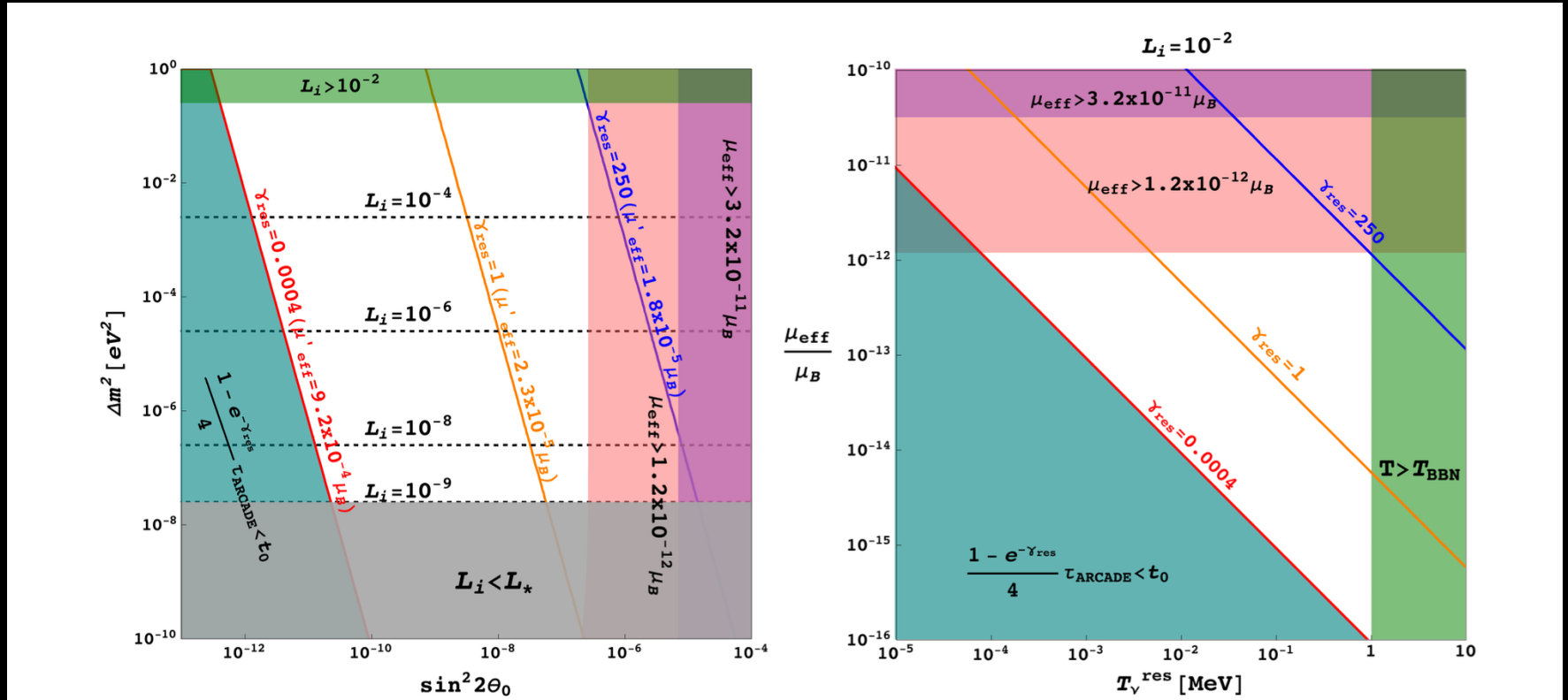


$$\mu_{eff} = \sin^2 \theta_0 \mu'_{eff}$$



Constraints and allowed region

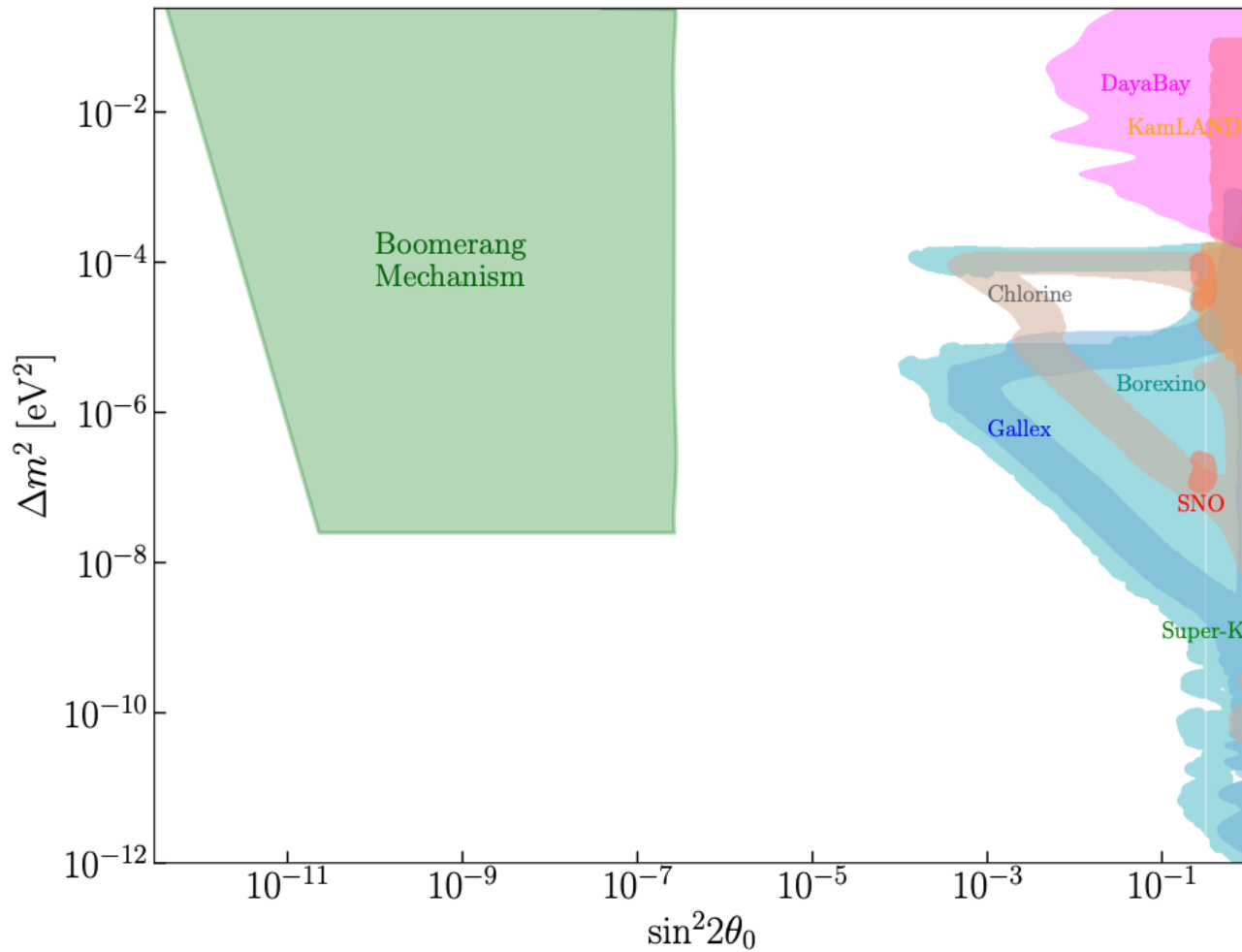
(Dev, PDB, Martinez-Soler, Roshan 2509.03441)



A non-adiabatic mixing description (talk by Shreya in March) is useful both for active-sterile neutrino mixing and for photon-dark photon kinetic mixing

Boomerang mechanism versus neutrino oscillation constraints

(Dev, PDB, Martinez-Soler, Roshan 2509.03441)



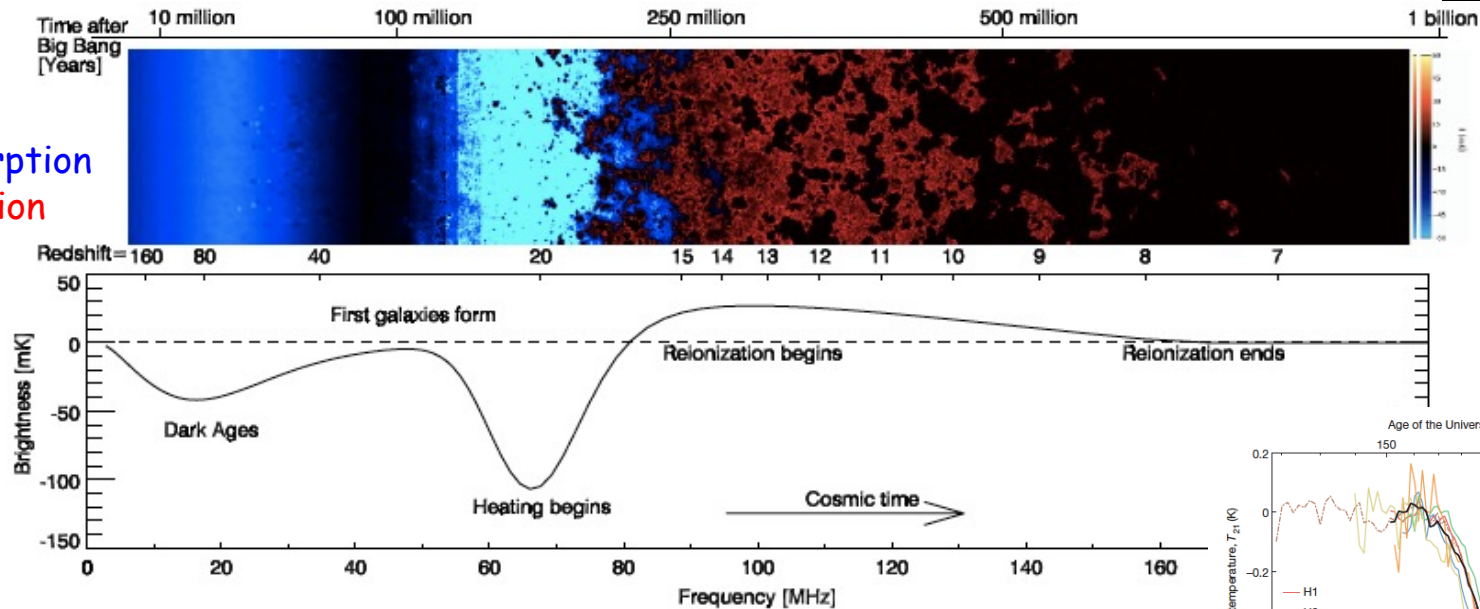
21cm cosmology:
shedding light on dark ages

21 cm cosmology (global signal)

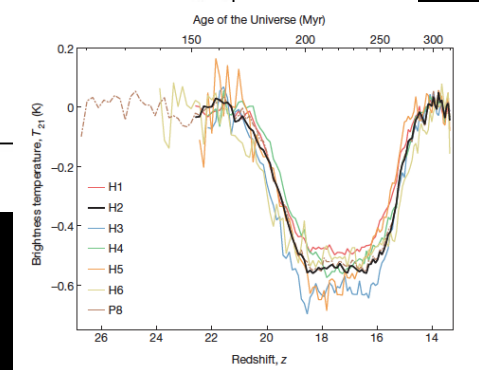
- 21 cm line (emission or absorption) is produced by hyperfine transitions between the two energy levels of 1s ground state of Hydrogen atoms. The energy splitting between the two level is $E_{21}=5.87\mu\text{eV}$
- The **21cm brightness temperature** parametrises the brightness contrast :

$$T_{21}(z) \simeq 23 \text{ mK} (1 + \delta_B) x_{H_I}(z) \left(\frac{\Omega_B h^2}{0.02} \right) \left[\left(\frac{0.15}{\Omega_m h^2} \right) \left(\frac{1+z}{10} \right) \right]^{1/2} \left[1 - \frac{T_\gamma(z)}{T_S(z)} \right]$$

spin
temperature



EDGES (anomalous) signal →



EDGES anomaly and relic neutrino decays

(Chianese, PDB, Farrag, Samanta, arXiv 1805.11717; Dev, PDB, Martinez-Soler, Roshan 2312.03082)

A solution of the EDGES anomaly requires an additional non-thermal photon component with:

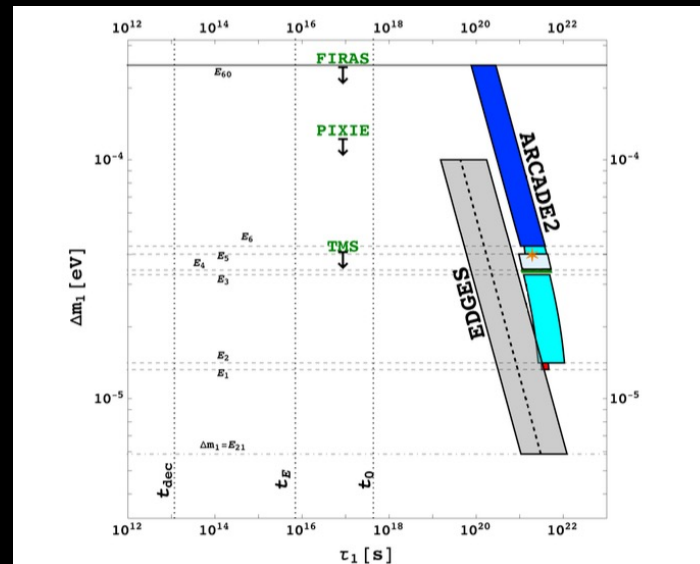
$$T_{\gamma_{\text{nth}}}^{\text{EDGES}}(\bar{z}_E) = (60_{-41}^{+101}) \text{ K}$$

$$\bar{z}_E \simeq 17$$

If we want to reproduce this value with relic neutrino decays we have to use:

$$T_{\gamma_{\text{nth}}}(E_{21}, z_E) \simeq \frac{6 \zeta(3)}{11 \sqrt{\Omega_{\text{M}0}}} \frac{T_0^3 (1 + z_E)^{3/2}}{E_{21}^{1/2} \Delta m_1^{3/2}} \frac{t_0}{\tau_1}$$

And the result is:



The solution of the excess radio background would imply a much smaller non-thermal photon temperature

$$T_{\gamma-\text{nth}}(z_E) \sim 6 \text{ K}$$

However, the EDGES result is controversial and many groups think it might be contaminated by some foreground contribution (ionosphere? Ground inhomogeneities? SARAS3 experiment has rebutted EDGES, so we need to wait for more results. 3 lunar-based experiments (to avoid ionosphere and ground reflection) are planned in a close future

21cm global signal: lunar-based experiments

➤ Lunar Surface Electromagnetic Experiment (LuSEE)

- 4 monopole antennas mounted on a rotating platform;
- 0.1-50 MHz band
- far side of the Moon
- scheduled to land in early 2026;



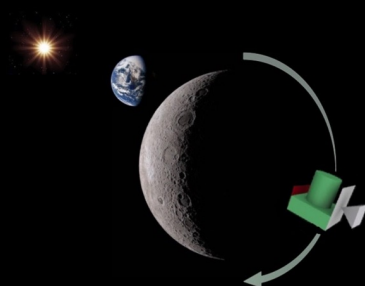
➤ Discovering Sky at the Longest wavelength (DSL)

- 1 mother satellite + 8 daughter satellites;
- 2h lunar orbit period;
- high precision in 30-120 MHz band
- mission launch in 2026;



➤ Probing ReionizATIion of the Universe using Signal from Hydrogen (PRATUSH)

- Moon-orbiting experiment;
- 40-200 MHz band
- Pre-project funded by the Indian Space Research Organisation



Conclusions

- ❑ Neutrino cosmology is a traditional field but still very lively and with many exciting open problems potentially leading to new physics.
- ❑ There are potential hints pointing to an unstable cosmic neutrino background
- ❑ The existence of low energy dark sector might destabilize cosmic neutrinos and this might solve some puzzles such as the excess radio background and give a 21 cm cosmological signal
- ❑ The matter-antimatter asymmetry puzzle might be related to an explanation of neutrino masses, this seems today the most attractive scenario. Discovery of $0\nu\beta\beta$ would be a strong evidence
- ❑ The dark matter puzzle has different solutions, some of which could also be related to neutrino physics.
- ❑ Primordial background of GWs is a new exciting tool to probe physics of the early universe and also neutrino cosmology