Holography for Strongly Coupled Systems

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The plan

Storytelling via ppt…

Calculations at the board…

Lecture 1: strong coupling in QCD
string theory
the AdS/CFT conjecture

Lecture 2: How holography works
Instability in AdS
Dynamical chiral symmetry breaking

Lecture 3: AdS/QCD for meson masses
Confinement and string tension

Lecture 4: NJL interactions
Finite T and μ
Composite higgs models

AdS/CFT Conjecture

N=4 SYM U(Nc)*	Supergravity on $AdS_5 \times S^5$
Large Nc/ strongly coupled	Large Radius R
Conformal (gateway drug!)	
SO(2,4) x SO(6)	SO(2,4) x SO(6)

* The difference between Nc^2 and Nc^2-1 is negligible at large Nc

Asserted to be "dual"

Witten's miracle paper 14,250+ cites

hep-th/9802150 [hep-th]

$Tr\phi^2$	20 of SU(4)	dim = 2	ϕ with $M^2 = -4$
$\bar{\lambda}\lambda$	10 of SU(4)	dim = 3	ϕ with $M^2 = -3$
TrF^2	1 of SU(4)	dim = 4	ϕ with $M^2 = 0$

Witten listed all operators in N=4 SYM by symmetries and dimension to predict the states in AdS_5

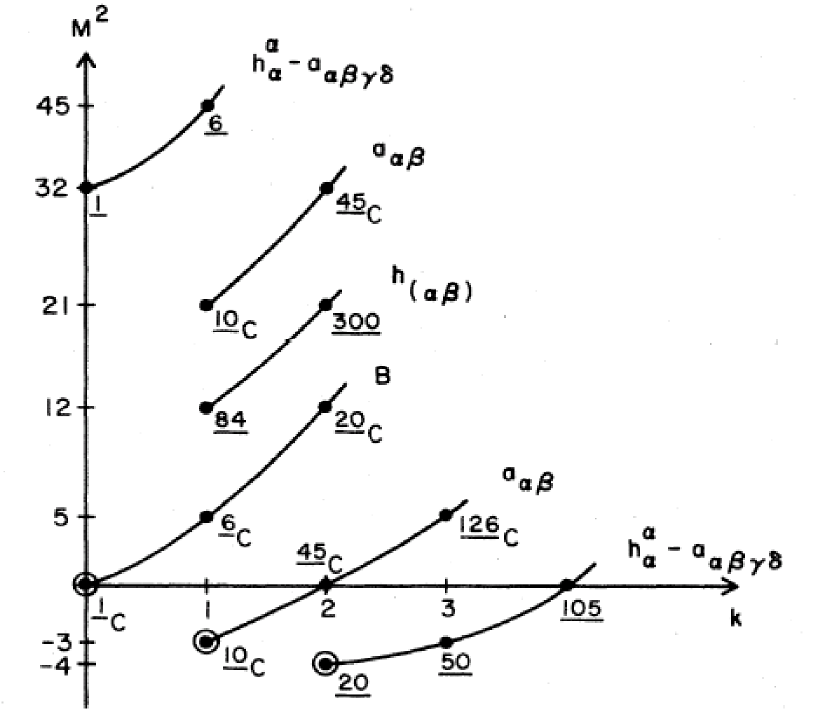


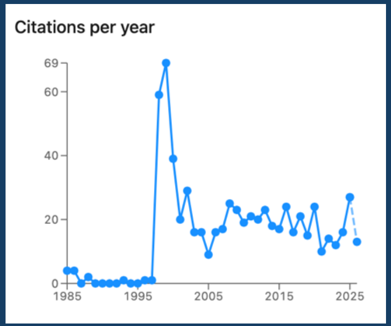
FIG. 2. Mass spectrum of scalars.

Kim, Romans, van Nieuwenhuizen – 1985

Had computed the supergravity spectrum on $AdS_5 x S^5$...

All states match the duality conjecture...

The duality is already clearly true!!!



Conformal Symmetry Breaking in AdS/CFT - Deformation

Since N=4 SYM and $AdS_5 \times S^5$ are dual it must be true that any perturbation of one side must match a perturbation on the other...

So one can look for solutions of the supergravity Einstein equations which look like AdS in some limit...

Then try to interpret them on the field theory side...

$$H = \sum_i \frac{R^4}{|u - u_i|^4}$$

Multi-centre Example

Full set of "D3 brane symmetry" geometries are

$$ds^2 = H^{-\frac{1}{2}} dx_{//}^2 + H^{\frac{1}{2}} \sum_{i=1}^{6} du_i^2, \qquad C_4 = H^{-1} dx^0 \wedge .. \wedge dx^3$$

$$H = \frac{1}{u^4}\left(... + a u^4 + b + \frac{c}{u^2} Y_{20} + \frac{d}{u^4} Y_{50} + ... \right)$$

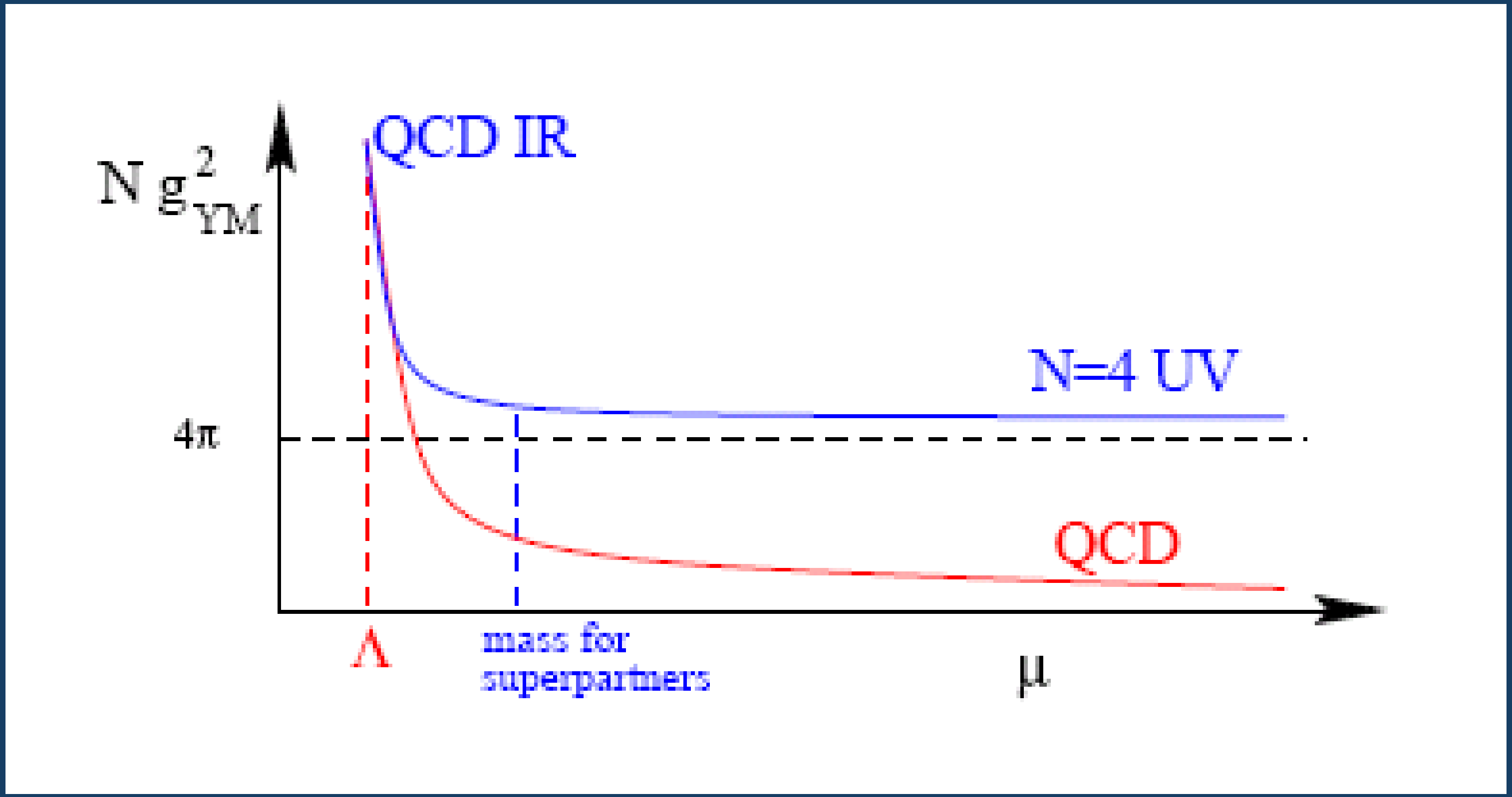
$b \neq 0 \rightarrow$ near horizon - AdS

c has energy dimension 2, is in 20 of SU(4) - $tr\varphi^2$

a has dimension -4 and is singlet - $\mathcal{L} = GtrF^4$

We can introduce relevant and irrelevant operators

We can also in principle put in mass terms for fields we don't like but they decouple at strong coupling…



To get to QCD need a weakly coupled UV which is OK in principle but would involve strong gravity (small R)

Example: N=2*

Pilch & Warner, Polchinski, Peet, Buchel

N=4 SYM + mass for 2 gauginos + equal mass for 4 scalars

+ lightest 2 scalars can have vev

$$\rho = \exp(\alpha)$$

scalar mass and vev – cf multi-centre solutions

$$A_{(2)} = e^{i\phi}\left(a_1(r,\theta)\,d\theta\wedge\sigma_1 + a_2(r,\theta)\,\sigma_2\wedge\sigma_3 + a_3(r,\theta)\,\sigma_1\wedge d\phi + a_4(r,\theta)\,d\theta\wedge d\phi\right)$$

$$\begin{aligned}
a_1(r,\theta) &= -i\,\frac{4}{g^2}\,\tanh(2\chi)\,\cos\theta\,,\\
a_2(r,\theta) &= i\,\frac{4}{g^2}\,\frac{\rho^6\sinh(2\chi)}{X_1}\,\sin\theta\cos^2\theta\,,\\
a_3(r,\theta) &= \frac{4}{g^2}\,\frac{\sinh(2\chi)}{X_2}\,\sin\theta\cos^2\theta\,.
\end{aligned}$$

χ is fermion mass

Example: N=2*

Pilch & Warner, Polchinski, Peet, Buchel

$$ds^2_{10} = \Omega^2 ds^2_{1,4} + ds^2_5 \ ,$$

$$ds^2_{1,4} = e^{2A(r)}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + dr^2 \ ,$$

$$ds^2_5 = L^2 \frac{\Omega^2}{\rho^2}\left[\frac{d\theta^2}{c} + \rho^6 \cos^2\theta \left(\frac{\sigma_1^2}{cX_2} + \frac{\sigma_2^2 + \sigma_3^2}{X_1}\right) + \sin^2\theta \frac{d\phi^2}{X_2}\right]$$

$$\begin{aligned} \Omega^2 &= \frac{(cX_1X_2)^{1/4}}{\rho} \\ X_1 &= \cos^2\theta + \rho^6 \cosh(2\chi)\sin^2\theta \\ X_2 &= \cosh(2\chi)\cos^2\theta + \rho^6 \sin^2\theta \ . \end{aligned}$$

$$\lambda = i\left(\frac{1-B}{1+B}\right) \qquad B = \left[\frac{b^{1/4} - b^{-1/4}}{b^{1/} + b^{-1/4}}\right] e^{2i\phi} \ , \quad \text{where } b \equiv c\frac{X_1}{X_2}$$

$$C_{(4)} = e^{4A}\frac{X_1}{g_s\rho^2} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3$$

$$\begin{aligned} \frac{d\alpha}{dr} &= \frac{g}{6}\left(\frac{1}{\rho^2} - \rho^4\cosh(2\chi)\right) \\ \frac{d\chi}{dr} &= -\frac{g}{4}\rho^4 \sinh(2\chi) \\ \frac{dA}{dr} &= \frac{g}{3}\left(\frac{1}{\rho^2} + \frac{1}{2}\rho^4\cosh(2\chi)\right) \end{aligned}$$

Numerical solutions have a blow up in the dilaton (enhancon) that can be matched onto solutions for gauge theory coupling....

Example: YangMills*

Babington, Crooks, NE

N=4 SYM + mass for 4 gauginos + equal mass for 6 scalars

$$ds^2_{10} = (\xi_+\xi_-)^{\frac{1}{2}} ds^2_{1,4} + (\xi_+\xi_-)^{-\frac{3}{2}} ds^2_5$$

$$ds^2_5 = \xi_- \cos^2\alpha \; d\Omega^2_+ + \xi_+ \sin^2\alpha \; d\Omega^2_- + \xi_+\xi_- d\alpha^2$$

$$ds^2_{(1,4)} = e^{2A(r)} dx^\mu dx_\mu + dr^2$$

$$\xi_\pm = c^2 \pm s^2 \cos 2\alpha, \quad c = \cosh\lambda, \quad s = \sinh\lambda$$

$$A_{(2)} = iA_+ \cos^3\alpha \; \cos\theta_+ d\theta_+ \wedge d\phi_+ - A_- \sin^3\alpha \; \cos\theta_- d\theta_- \wedge d\phi_-$$

$$A_\pm = \sinh 2\lambda / \xi_\pm$$

$$F_{(4)} = F + \star F, \quad F = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\omega$$

$$\omega(r) = e^{4A(r)} A'(r)$$

$$B = \frac{\sinh^2\lambda \cos 2\alpha}{\cosh^2\lambda + (\xi_+\xi_-)^{1/2}}$$

$$C + ie^{-\Phi} = i\frac{(1-B)}{(1+B)}$$

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial\lambda}, \qquad -3A'' - 6A'^2 = \lambda'^2 + 2V$$

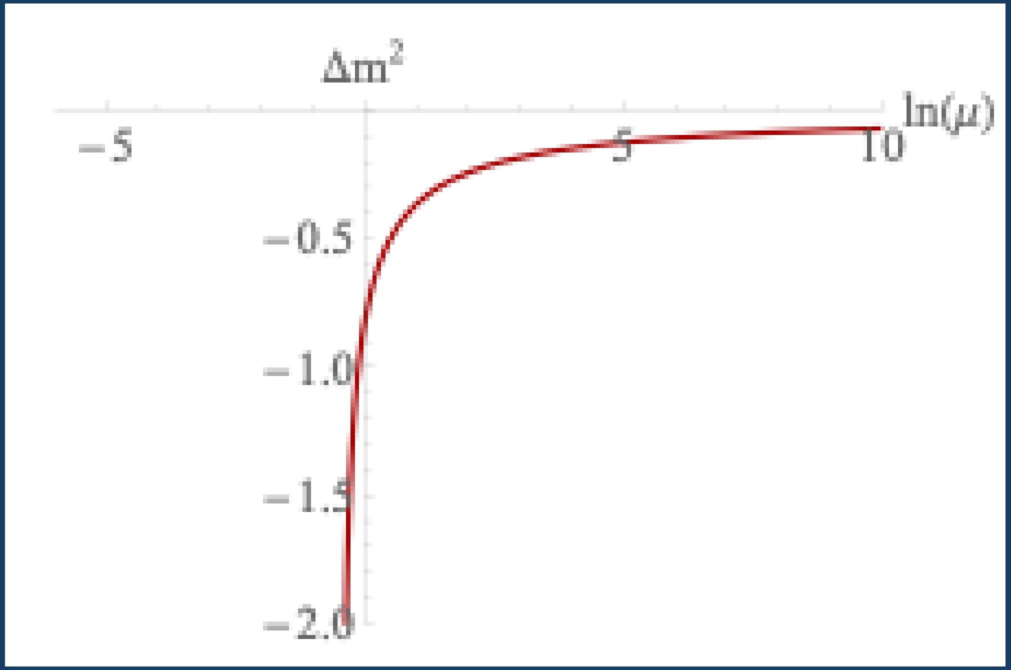
$$V = -\tfrac{3}{2}\left[1 + \cosh^2\lambda\right]$$

Now the IR singularities are too hard to decipher…

You are now a career supergravity constructor and you’re unlikely to return to phenomenology!

Can we learn lessons and model build?

The two loop running of α gives the resulting Δm^2



$\alpha(1) = 0.65$

BF bound violation…

$$\partial_\rho(\rho^3 \partial_\rho L) - \rho\, \Delta m^2 L = 0\,.$$

$$L(\rho) = m + c/\rho^2$$

In the UV where $\Delta m^2 \to 0$

$$\partial_\rho(\rho^3\partial_\rho L) - \rho\,\Delta m^2 L = 0\,.$$

We must pick boundary conditions

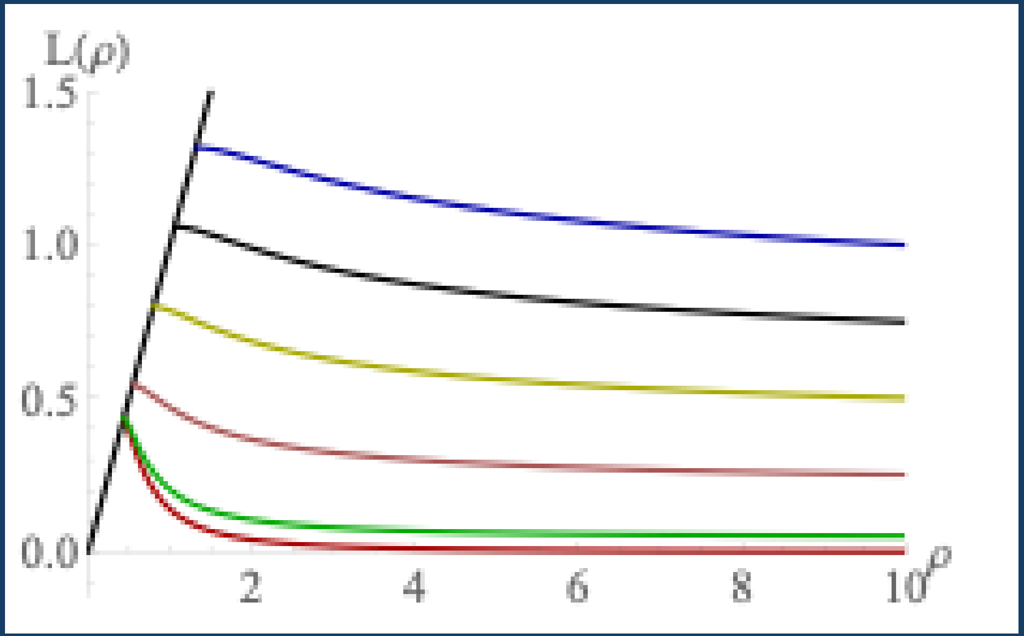
$$L(\rho)|_{\rho=\rho_{IR}} = \rho_{\mathrm{IR}}\,,$$

On mass shell condition

$$\partial_\rho L(\rho)|_{\rho=\rho_{IR}} = 0\,.$$

IR regularity

Now solve numerically (shooting) using Mathematicas NDSolve

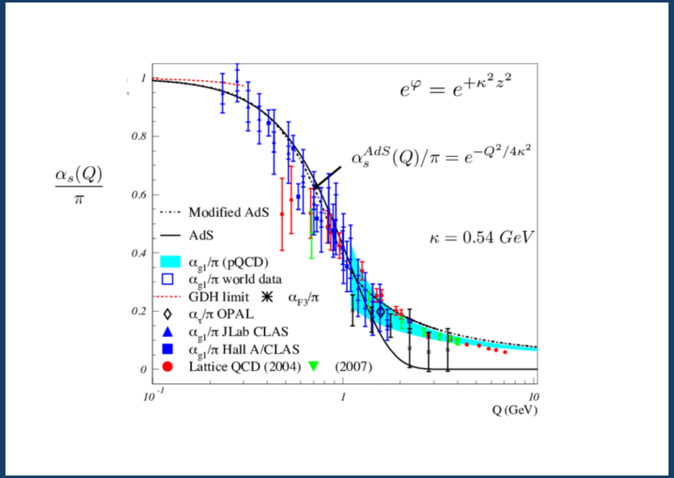


Read off mass in UV..

Consider m=0 case

$$L(\rho) = m + c/\rho^2$$

Can treat plot as running mass...



$\alpha(1)=0.65$ $\qquad$ $m_{IR} = 0.43$

$\alpha(700\text{ MeV})=0.65$ $\qquad$ $m_{IR} = 300$ MeV

One third the proton mass!

Light-front holographic QCD and color confinement. Brodsky et al

AdS/QCD extensions

- Axial mesons – add an axial gauge field in the bulk – it has a covariant derivative to L since $\bar{q}q$ has axial charge – it is higgsed by the L vev…

- See Son et al. hep-ph/0501128 [hep-ph]

- Baryons – use the Dirac equation in the bulk with a mass to describe a dimension 9/2 operator (qqq) – more involved GR

$$\left(\partial_\rho^2 + \mathcal{P}_1\partial_\rho + \frac{M_B^2}{r^4} + \mathcal{P}_2\frac{1}{r^4} - \frac{m^2}{r^2} - \mathcal{P}_3\frac{m}{r^3}\,\gamma^\rho\right)\psi = 0\,,$$

$$m = 5/2$$

$$\begin{aligned}
\mathcal{P}_1 &= \frac{6}{r^2}\left(\rho + L_0\,\partial_\rho L_0\right)\,,\\
\mathcal{P}_2 &= 2\left((\rho^2 + L_0^2)L\partial_\rho^2 L_0 + (\rho^2 + 3L_0^2)(\partial_\rho L_0)^2 + 4\rho L_0\partial_\rho L_0 + 3\rho^2 + L_0^2\right)\,,\\
\mathcal{P}_3 &= \left(\rho + L_0\,\partial_\rho L_0\right)\,.
\end{aligned}$$

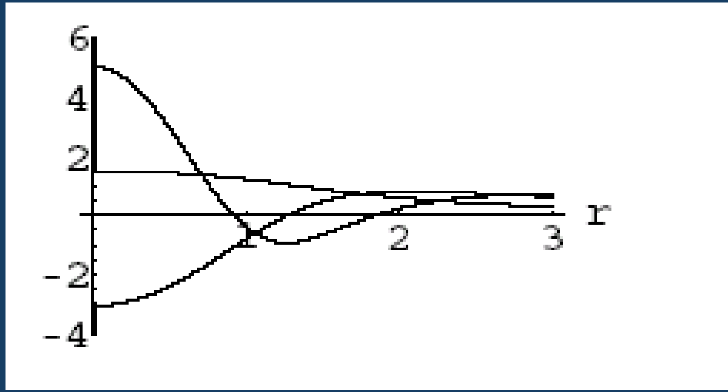
First done by Brodsky and de Teramond hep-ph/0602252 [hep-ph] – works well in baryon sector…

To match with meson sector need running m/γ   NE 2304.10816 [hep-ph]

- Recovering the full meson action hep-ph/0501128 [hep-ph]

eg ρ

$$S_{boson} = \int d^5x \, \rho^3 \left(F_{L,MN} F_L^{MN} + (L \leftrightarrow R) \right)$$



Substitute the solutions for the radial wave functions for mesons and sources… integrate over r

Normalize the rho kinetic term….
Normalize the source solution to match UV QCD…

Cross terms give you F_ρ^2

- Non-abelian flavour hep-ph/0501128 [hep-ph], NE 2304.09190 [hep-th]
  On the board we did a single operator and its phase as a pion but the flavour structure can be richer…. Eg 2 flavour QCD use flavour matrix…

$$U = \begin{pmatrix} \bar{u}_L u_R & \bar{d}_L u_R \\ \bar{u}_L d_R & \bar{d}_L d_R \end{pmatrix}$$

It transforms under flavour as $U \to L^\dagger U R$

So write all flavour symmetry invariants and then go in AdS…

$$S = \int d^5x \, Tr \, \partial U^\dagger \partial U + ..$$

eg you could now split the u and d quark masses… (would need to include that in Δm^2s)…

- ## Hard walls, Soft walls, Excited state scaling

The original Son AdS/QCD paper didn't determine L(r) dynamically but simply used c/r^2 ending at a hard IR wall….

The model predicts for excited states that $M^2 \sim n^2$

Shifman argued this is wrong…

$$\text{string tension} \sim \Lambda^2 \qquad M \sim \Lambda^2 \,.\, \text{length}$$

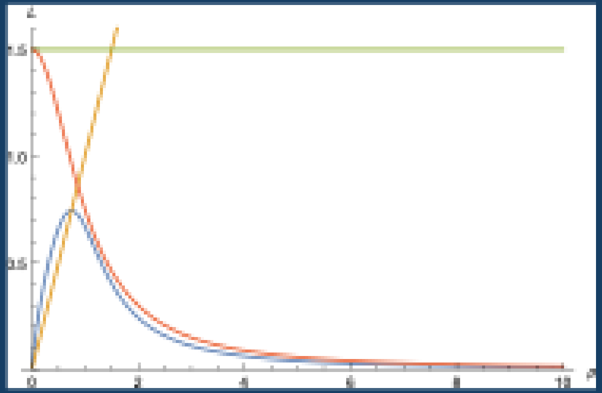
Bohr type quantization condition. $p.L \sim n$ $\quad M^2 \sim n$

In hep-ph/0602229 [hep-ph] Son et al changed the IR geometry away from AdS (soft wall) to correct to $M^2 \sim n$

But shouldn't they have included strings?

This is a large Nc problem – in AdS/CFT the tension is infinite and all bound states are very small…

In the model we've seen you can change L(r) to get soft wall behaviour… can't justify below mass shell condition…
1508.06540 [hep-ph]



Observables (MeV)	QCD	AdS/SU(3) 2 F 2 $\bar{F}$	Deviation
M_ρ	775	775*	fitted
M_A	1230	1183	- 4%
M_S	500/990	973	+64%/-2%
M_B	938	1451	+43%
f_ρ	345	321	- 7%
f_A	433	368	-16%
$M_{\rho,n=1}$	1465	1678	+14%
$M_{A,n=1}$	1655	1922	+19%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%
$M_{B,n=1}$	1440	2406	+50%

Table 1: The predictions for masses and decay constants (in MeV) for $N_f = 2$ massless QCD. The ρ-meson mass has been used to set the scale (indicated by the *).