Holography for Strongly Coupled Systems

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Strongly Coupled Systems

Strong coupling – break down of perturbation theory

$$\alpha = e^2/4\pi \quad > \quad 1$$

is part of the Standard Model – QCD…

We can't compute
without supercomputers

- hadronization
- hadron masses
- finite temperature

Expensive and
time consuming

Lattice QCD calculations can’t study finite density (sign problem)

neutron star interiors
heavy ion collisions

Lattice is a slow way to scan the space of (Nc, Nf, representations)

dynamical EWSBing (higgs as sigma/f0 meson)
composite higgs (higgs as pion)
composite dark matter
dynamical susy breaking

Holography is a tool that has emerged from string theory....

It is a (heavily supported) conjecture

It provides a weakly coupled dual description of strong coupling

It is exactly true for a large class of gauge theories (many behaving like QCD)

It provides a different paradigm for the description of strongly coupled regimes – new phenomenological tool

The plan

Storytelling via ppt…

Calculations at the board…

Lecture 1: strong coupling in QCD
string theory
the AdS/CFT conjecture

Lecture 2: How holography works
Instability in AdS
Dynamical chiral symmetry breaking

Lecture 3: AdS/QCD for meson masses
Confinement and string tension

Lecture 4: NJL interactions
Finite T and μ
Composite higgs models

QCD

An SU(3) gauge theory of quark color

$$\mathcal{L} = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R - m\bar{\psi}_L \psi_R$$

$$D^\mu = \partial^\mu + igC^{a\mu}T^a$$

$$\psi \to e^{ig\theta^a(x)T^a} \psi$$

$$C^{a\mu} \to C^{a\mu} - \partial^\mu \theta^a(x)$$

Quarks come in 3 colours

8 gluons mediate the force

$$G^{\mu\nu} = \partial^\mu C^{a\nu} T^a - \partial^\nu C^{a\mu} T^a + g[C^{a\mu}T^a, C^{b\nu}T^b]$$

The generators

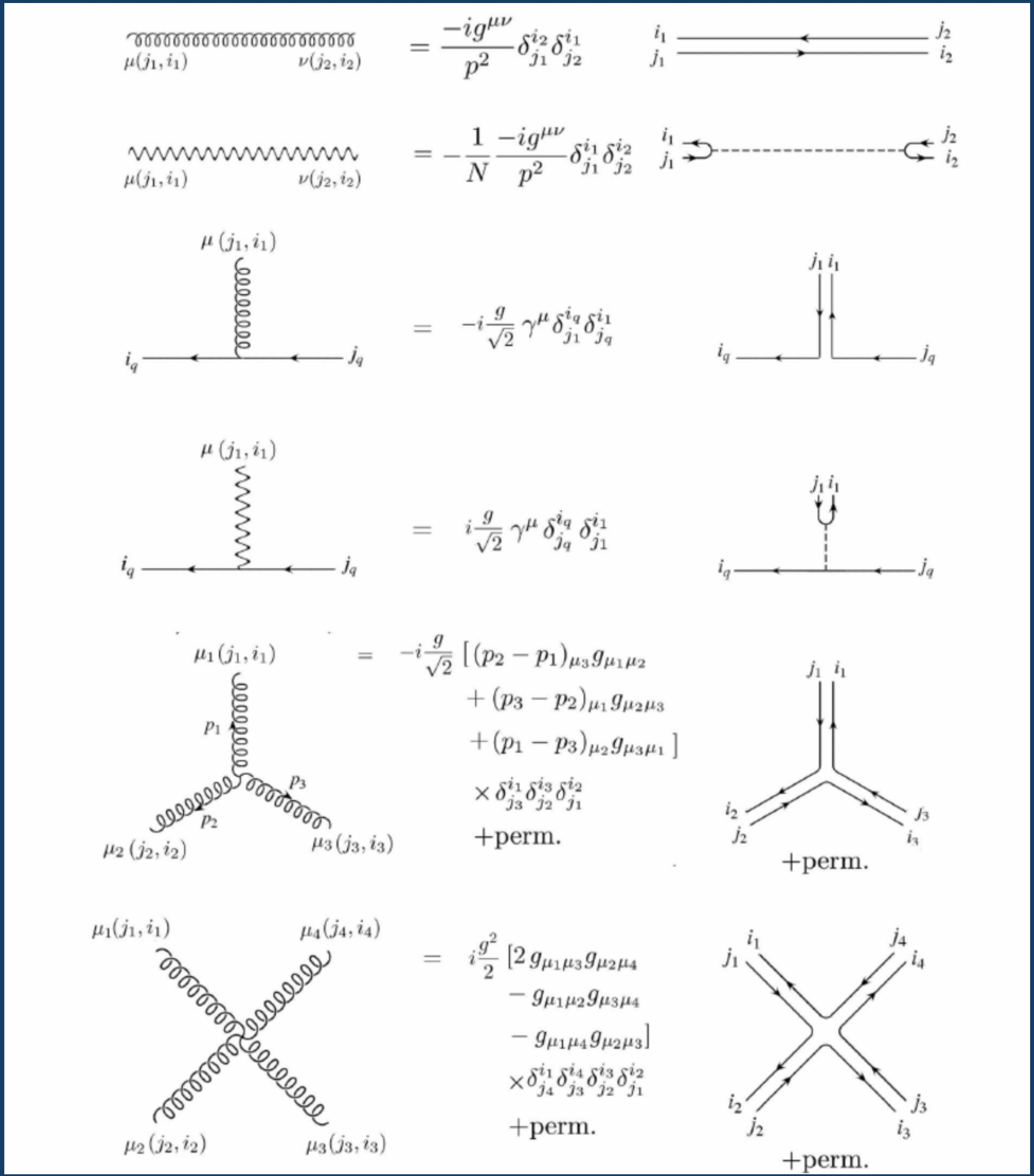
$T^a = ½\, \lambda^a$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

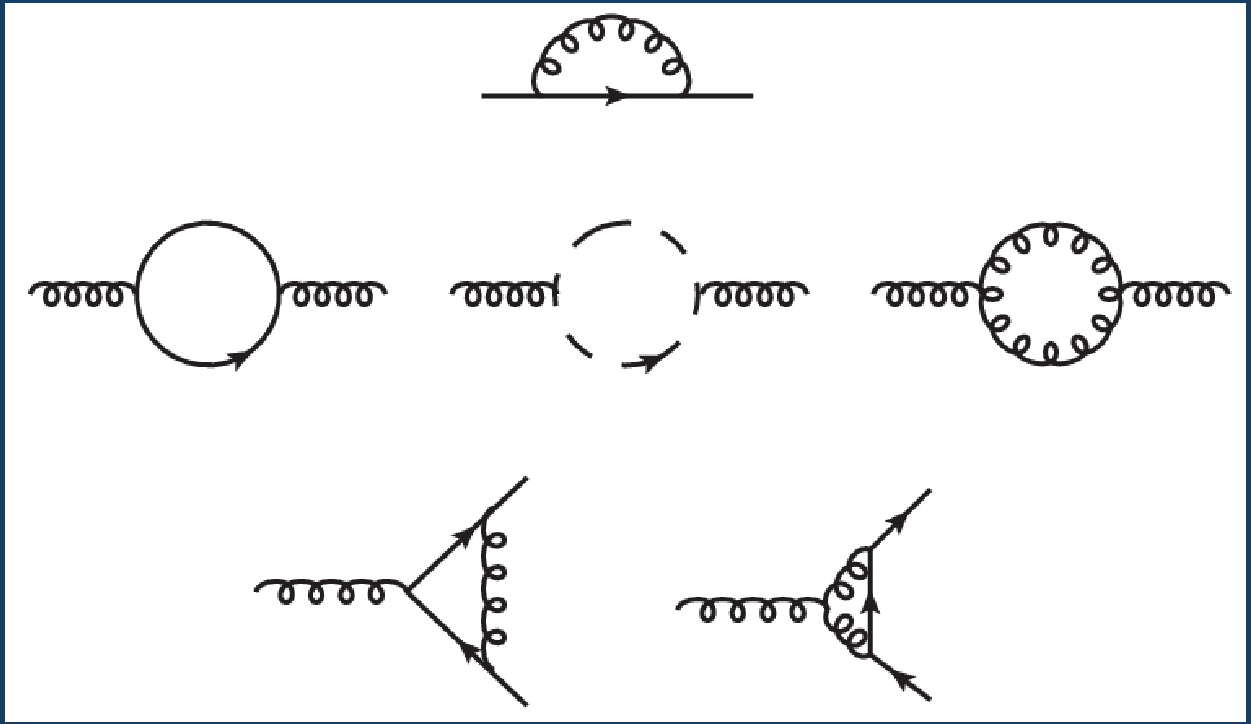
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The Feynman rules



Loop effects



We write an effective tree level vertex coupling that includes these loop effects and can compute

$\beta = \mu\, dg/d\mu$

$$\beta = -\frac{11}{3}N_c + \frac{2C(R)}{3}N_f^{\text{chiral}} \left(+\frac{C(R)}{6}N_{\text{scalar}}^{\text{real}}\right)$$

$g^3/16\,\pi^2$

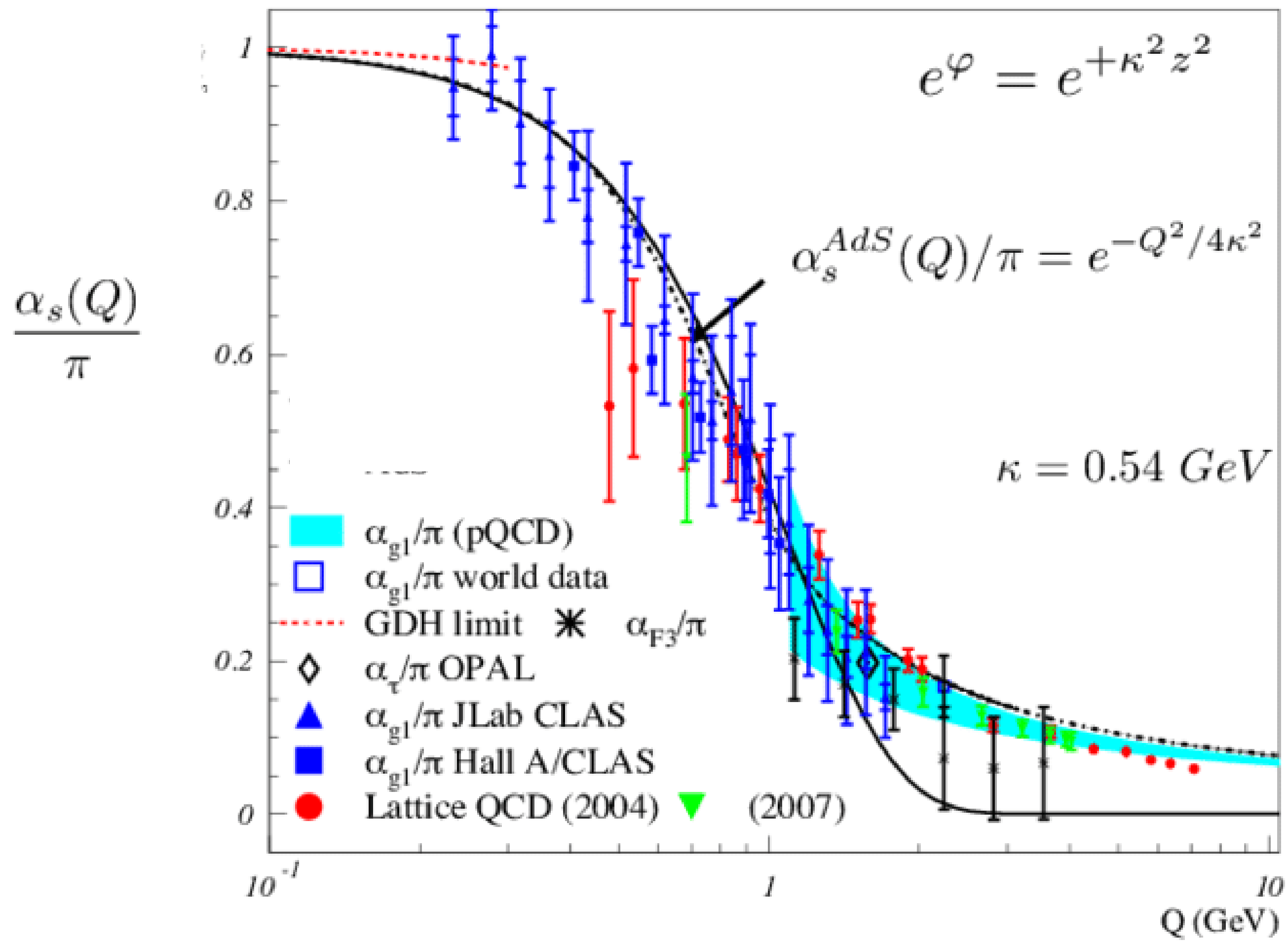
$C(\text{fund}) = (N_c^2-1)/2N_c$
$C(\text{Adj}) = N_c$

Nc=3
Nf=6..3

$\alpha_s = g^2/4\pi$

In the IR perturbation theory breaks down

Pole vs flattening vs gap



Light-front holographic QCD and color confinement. Brodsky et al

Asymptotic freedom – UV complete theory

Set value at some scale eg α_s at M_Z

Strongly Coupled QCD 1
Mass Generation

Technicolour

For a Standard Model fermion doublet Eg (ν,e) the symmetry

$$SU(2)_L \times SU(2)_R \times U(1)_V \longrightarrow SU(2)_V \times U(1)_V$$

by fermion mass formation as a result of the higgs vev…

The gauge groups of the SM: $SU(2)_L \times U(1)_Y$ where $U(1)_Y$ is made of the diagonal $SU(2)_R \times U(1)_V$ symmetry….

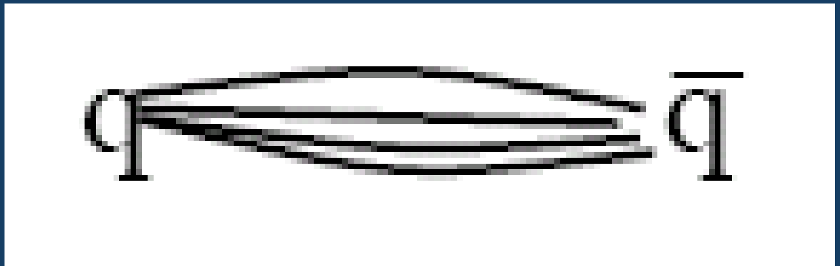
This means you can replace the higgs with a new SCGT that generates a quark condensate at a higher scale…

Phenomenologically now seems unlikely (eg no bound states)

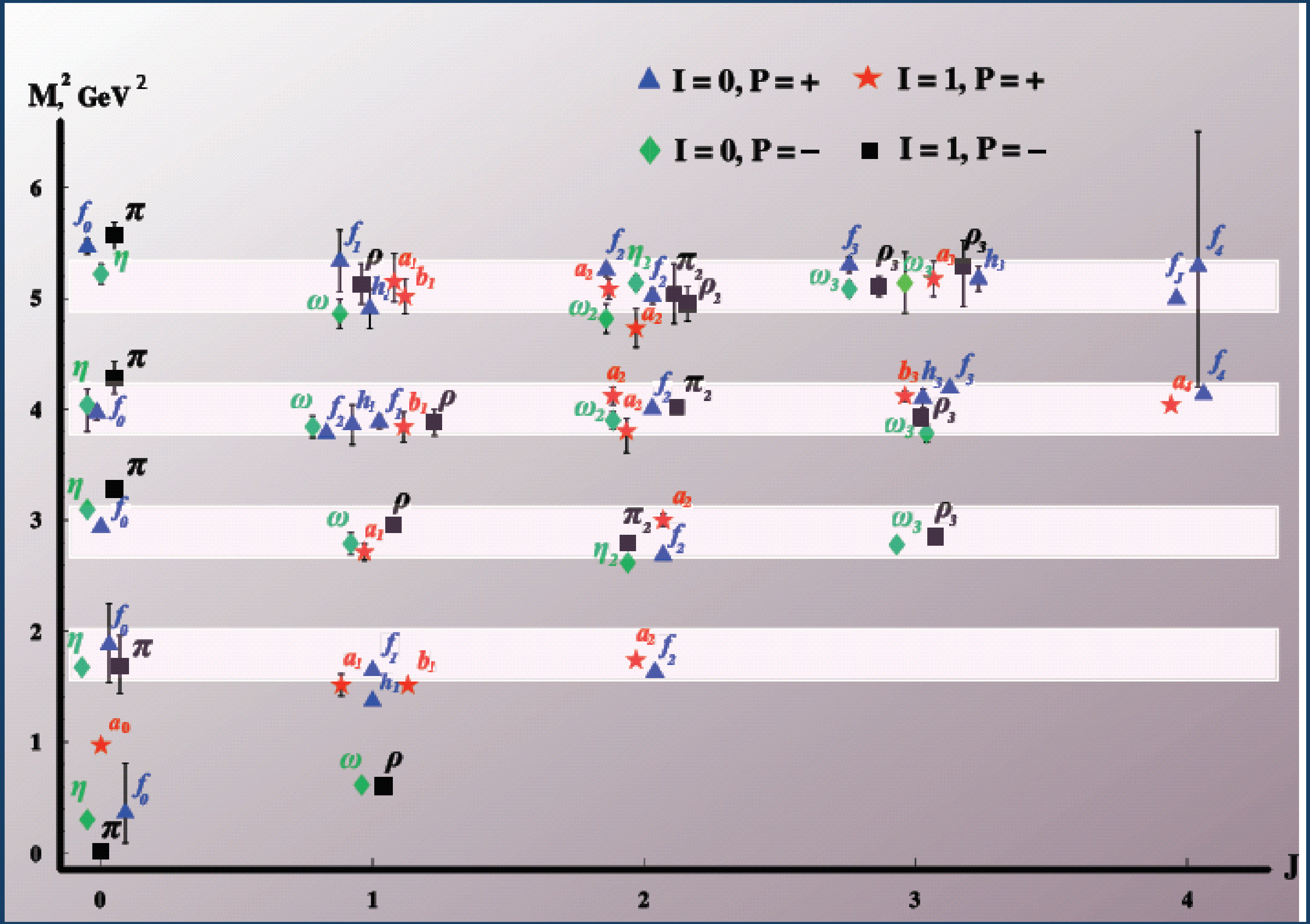
Strongly Coupled QCD 2
Confinement

Fundamentally no one has ever seen a free quark outside a hadron...

A naïve picture is of a chromo-electric field "flux tube"



Regge Trajectories

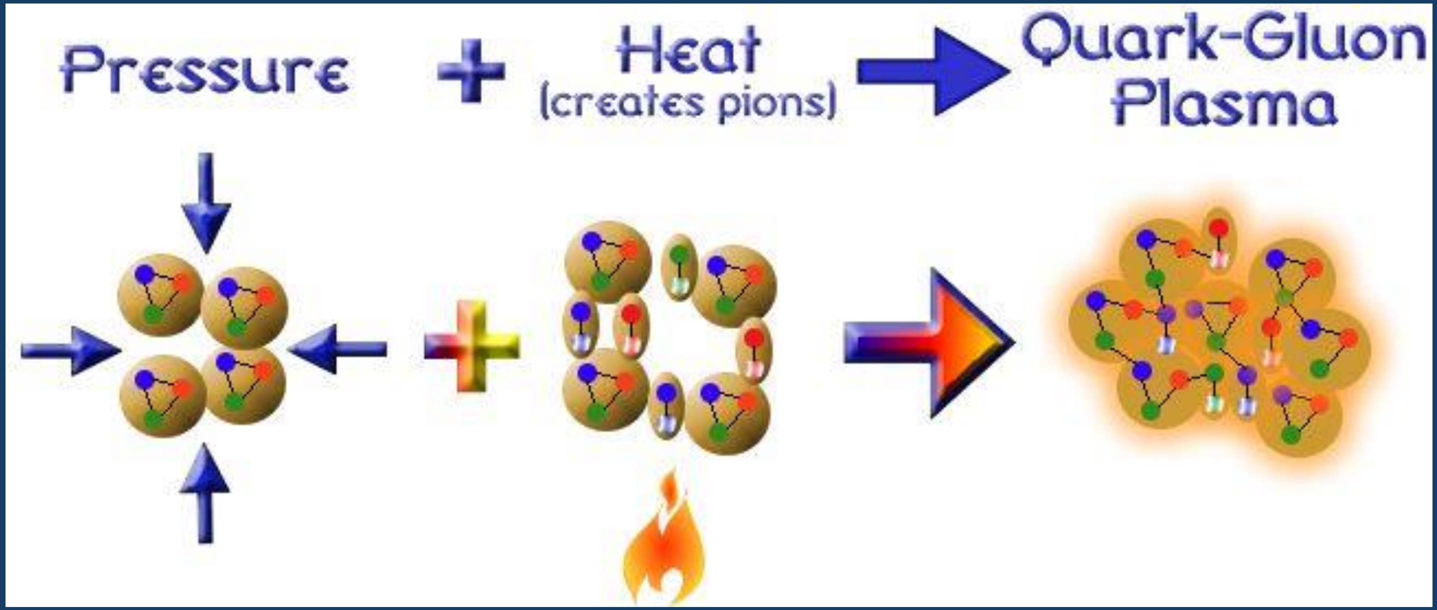


$$M^2 \propto J$$

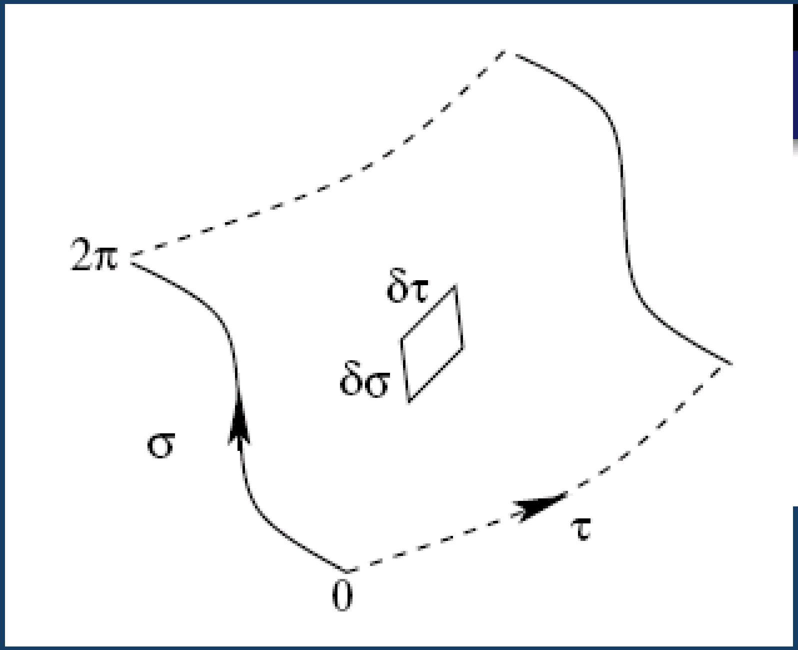
Deconfinement at finite T or density

Temperature fills the space with quark anti-quarks pairs. When the density becomes so great that all quarks are within $1/\Lambda$ of each other asymptotic freedom makes the plasma weakly coupled...

Density fills the space with quarks.. same out come...



String Theory 101



Action = area of the world sheet

$$= T \int d\sigma^2 \sqrt{\det\left(G_{\mu\nu}\frac{dX^\mu}{d\sigma^a}\frac{dX^\mu}{d\sigma^b}\right)}$$

$$dA = xy\sin\theta$$
$$= \sqrt{x^2y^2 - x^2y^2\cos^2\theta}$$
$$= \delta\sigma\delta\tau\sqrt{\left(\frac{dX^\mu}{d\sigma}\right)^2\left(\frac{dX^\nu}{d\tau}\right)^2 - \left(\frac{dX^\mu}{d\sigma}\frac{dX_\mu}{d\tau}\right)^2}$$

Nambu-Goto action

Only parameter is

$T = 1/2\pi\alpha'$ is string tension

$$\begin{aligned}\delta S &= \int d^2\sigma \left(\frac{\partial L}{\partial \dot{X}^\mu}\delta \dot{X}^\mu + \frac{\partial L}{\partial X'^\mu}\delta X'^\mu\right) \\ &= \int d^2\sigma \left(-\frac{\partial}{\partial\tau}\left(\frac{\partial L}{\partial \dot{X}^\mu}\right) - \frac{\partial}{\partial\sigma}\left(\frac{\partial L}{\partial X'^\mu}\right)\delta X^\mu\right) \\ &\quad + \int d\tau \left(\frac{\partial L}{\partial X'^\mu}\delta X^\mu\right)\Big|_{\sigma=0}^{2\pi}\end{aligned}$$

$$\begin{aligned}dA &= xy\sin\theta \\ &= \sqrt{x^2y^2 - x^2y^2\cos^2\theta} \\ &= \delta\sigma\delta\tau\sqrt{\left(\frac{dX^\mu}{d\sigma}\right)^2\left(\frac{dX^\nu}{d\tau}\right)^2 - \left(\frac{dX^\mu}{d\sigma}\frac{dX_\mu}{d\tau}\right)^2}\end{aligned}$$

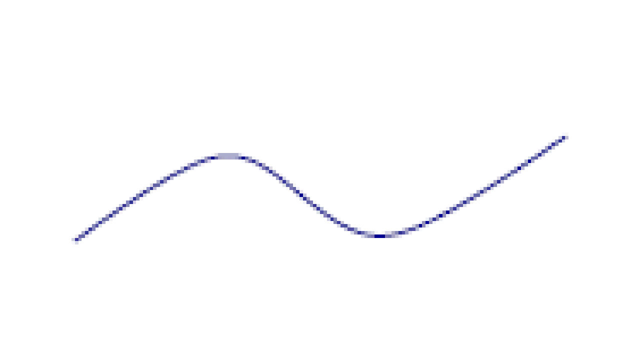
The boundary terms in time are dropped.. those in space not...

Natural solutions are

$$\frac{\partial p^\mu_\tau}{\partial\tau} + \frac{\partial p^\mu_\sigma}{\partial\sigma} = 0$$

$$p^\mu_\sigma = \frac{\partial L}{\partial X'^\mu} = 0$$

$$\sigma = 0, 2\pi$$



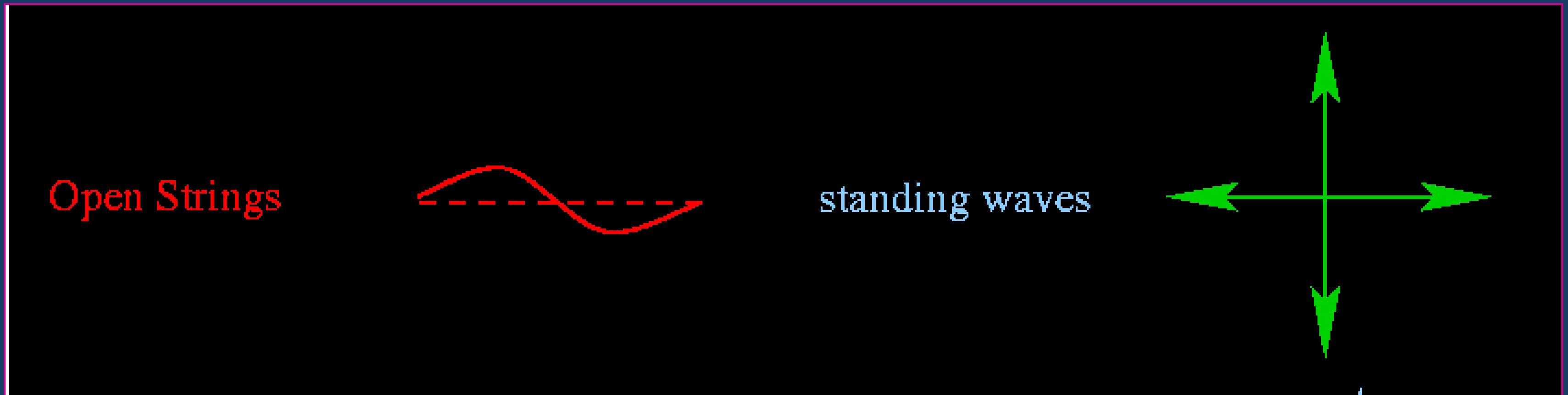
Waves with no energy flux off the end of a string... energy rises in steps with frequency...

Quantum String

There's nothing surprising – the centre of mass is a quantum particle... each wave frequency can be excited with quanta...

The energy rises with each quanta by $h\nu$...

The lowest energy first excited state has the Lorentz group structure of a photon.... But isn't massless... so add a negative worldsheet cosmological constant!



Now the ground state is a tachyon...

Supersymmetry to the rescue

Supersymmetric theories have positive definite potentials (no tachyons)

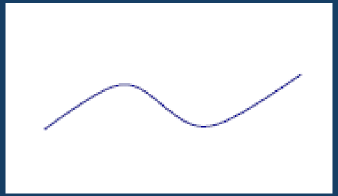
Enforcing susy on strings is messy

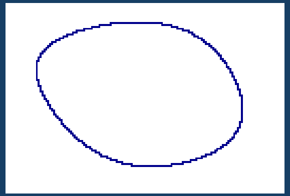
- add world-sheet super partners for X^μ
– self consistently eject some states (GSO projection)

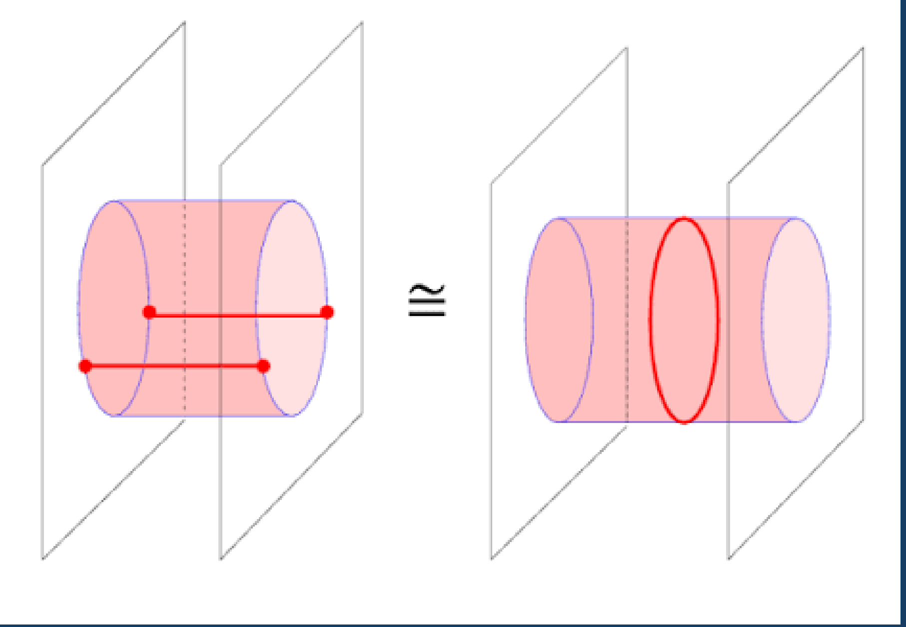
But.. After the event we are left with a theory of massless photons & gauginos...

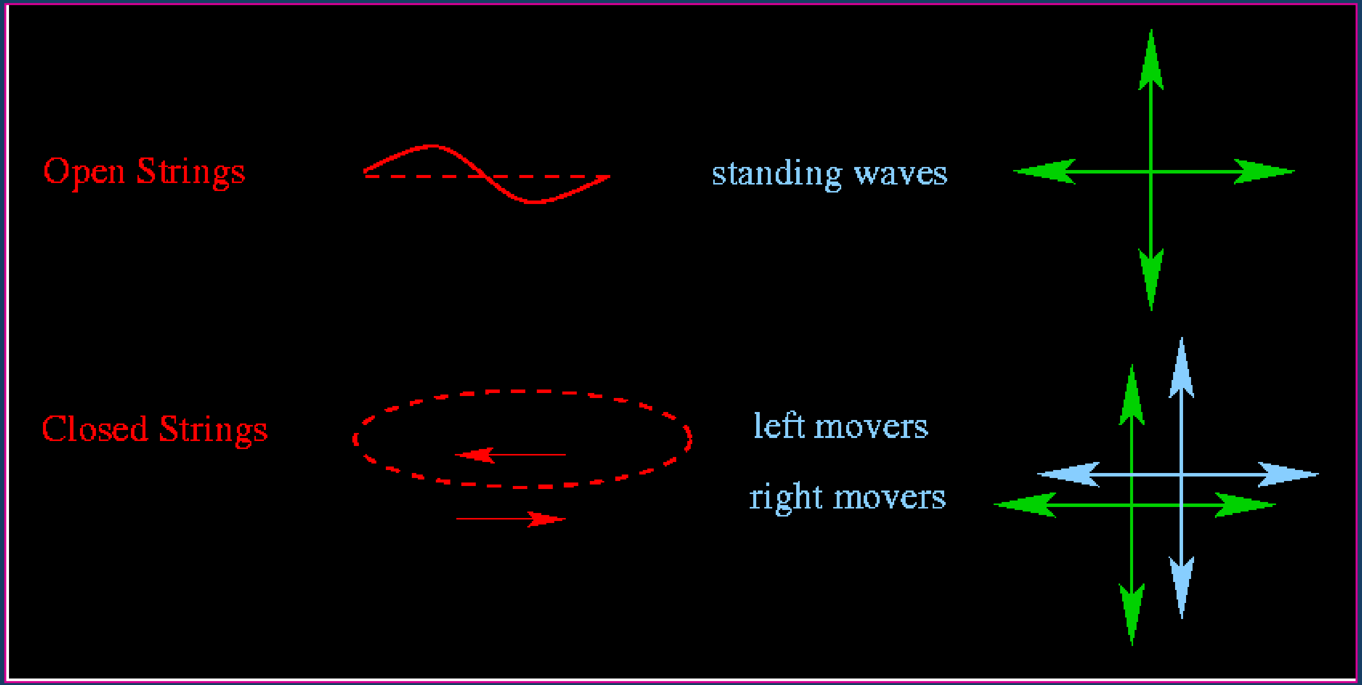
Closed strings too

An open (ended) string quantum loop is a closed string…









There are left moving and right moving waves… the first excitation has the polarizations of two superimposed photons…

It’s a graviton… $G^{\mu\nu}$

Force its mass = 0

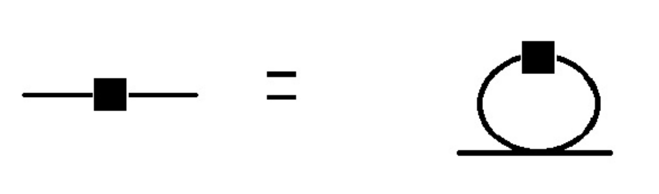
Supergravity in 10 dimensions

We have described *a* string... how many create a spacetime is unclear (!)... but there is a remarkable self consistency calculation....

$$S = T\int d^2\sigma \sqrt{\det\left(G_{\mu\nu}\frac{dX^\mu}{d\sigma^a}\frac{dX^\nu}{d\sigma^b}\right)}$$

$$G^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{3}R_{\mu\nu\lambda\kappa}X^\lambda X^\kappa + \dots$$

These loop calculations are inconsistent with the world sheet symmetries unless...

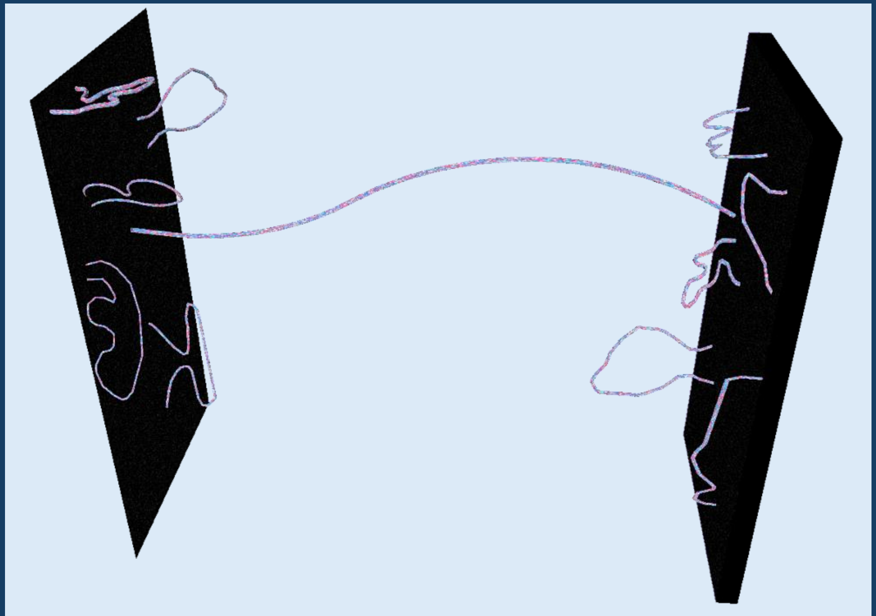


D=10

The background field equations are those of supergravity

Branes

$$\begin{aligned}\delta S &= \int d^2\sigma \left(\frac{\partial L}{\partial \dot{X}^\mu}\delta \dot{X}^\mu + \frac{\partial L}{\partial X'^\mu}\delta X'^\mu\right)\\ &= \int d^2\sigma \left(-\frac{\partial}{\partial \tau}\left(\frac{\partial L}{\partial \dot{X}^\mu}\right) - \frac{\partial}{\partial \sigma}\left(\frac{\partial L}{\partial X'^\mu}\right)\delta X^\mu\right)\\ &\quad + \int d\tau \left(\frac{\partial L}{\partial X'^\mu}\delta X^\mu\right)\Big|_{\sigma=0}^{2\pi}\end{aligned}$$



We picked the momenta to vanish at the end points.. But you can just enforce $dX^\mu=0$...

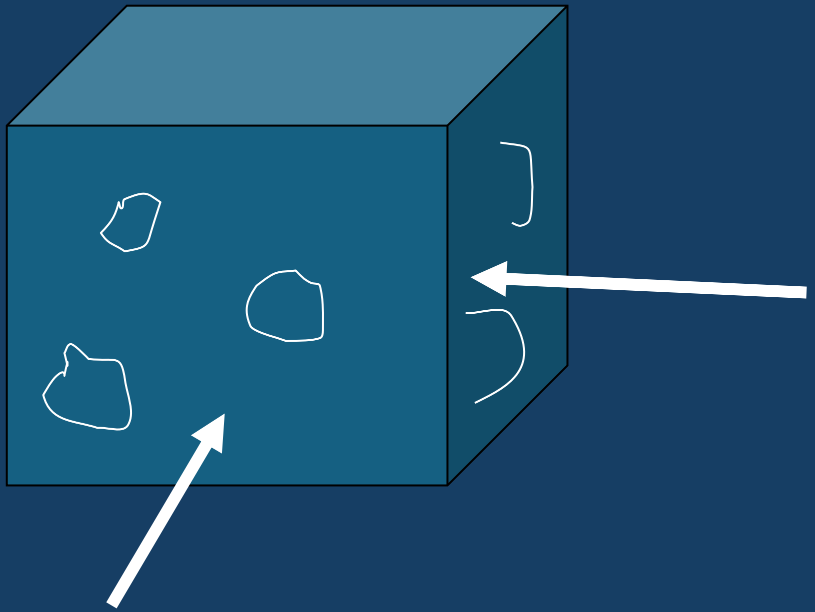
The brane is stuck at one place... in any number of directions you want...

Dirichlet p branes have p+1d world volumes... eg D3

Including multiple D-branes in the space provides flavour indices for the string ends points.... U(1) -> U(N) gauge symmetries

D-branes

Only open strings are restricted… not closed strings…



Electromagnetic particles live on a sub-space or "brane"

Gravitons live in a higher dimension "bulk"

$$A^M \to A^\mu + \phi$$

The extra scalar's vev measures the position of a dynamical brane in the bulk….

Lecture 2: How holography works

Instability in AdS

Dynamical chiral symmetry breaking

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Lecture 4: NJL interactions

Finite T and μ

Composite higgs models