HOLOGRAPHY FOR PHENOMENOLOGY

ABINGDON NEXT SCHOOL

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LECTURE 1: CHIRAL SYMMETRY BREAKING

We believe QCD has a dynamical symmetry breaking mechanism....

Massless quarks: $\mathcal{L} = \bar{u}_L \not{\partial} u_L + \bar{u}_R \not{\partial} u_R$
$+ \ldots$

Left & right handed fields separate

$Q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow SU(3)_L \otimes SU(3)_R$ flavour symmetry

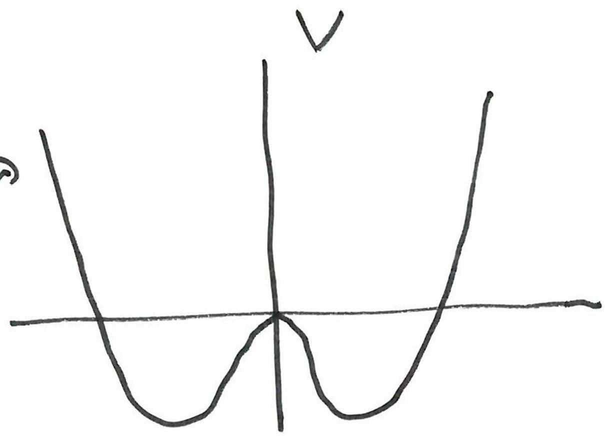
$\longleftrightarrow$
$L \Leftrightarrow R$
parity Z_2

<u>BUT</u> the spectrum does not parity double

Only reps of $SU(3)_V$ [Make L & R together]

We suppose:

generated by QCD at strong coupling



$u_L^\dagger u_R$
$\downarrow$
$\bar{Q}_L Q_R + h.c.$

<u>Pop-science</u>: $\Delta E \Delta t \sim \hbar \rightarrow \bar{q}q$ pairs

strong force $\rightarrow$ huge binding E

$\Rightarrow$ space is full of quarks $\qquad q \rightarrow \bar{q}q \nearrow q \quad \underline{m_q \sim \frac{M_p}{3}}$

Prediction: $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$

Goldstone's theorem: 8 broken generators

$\rightarrow$ 8 Goldstone bosons...

Except $m_u, m_d, m_s \neq 0$ so only approximate.

These states are: π^+, π^-, π^0 $\qquad m_\pi = 140\,\text{MeV}$

$K^+, K^-, K^0, \bar{K}^0$ $\qquad m_K = 490\,\text{MeV}$

η $\qquad m_\eta = 550\,\text{MeV}$

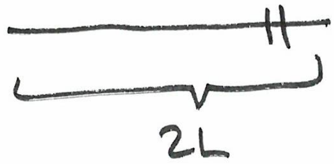
Their masses $< M_\rho$ another $\bar{q}q$ state

$\approx 770\,\text{MeV}$

LECTURE 1: REGGE TRAJECTORIES

A static string of tension T (energy/length)

$$E = T\delta r \;, \quad p = 0$$



Boost to a rotating frame

$$E = \gamma T \delta r \qquad p = \gamma \frac{v}{c} T \delta r$$

with $\gamma = \sqrt{\dfrac{1}{1 - v^2/c^2}}$ & $v = \dfrac{c}{L} r$

ends move at c

$$M = \int_{-L}^{L} \gamma T \, dr$$

$$= T \int_{-L}^{L} \sqrt{\frac{1}{1 - r^2/L^2}} \, dr$$

$$= TL \int_{-\pi/2}^{\pi/2} d\theta = TL\pi$$

$r = L\sin\theta$

$\dfrac{dr}{d\theta} = L\cos\theta$

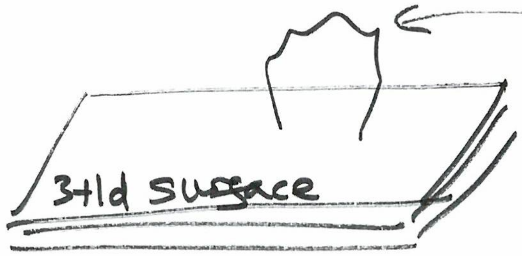
$$J = \int_{-L}^{L} r p \, dr = \int_{-L}^{L} r \gamma \frac{v}{c} T \, dr \overset{\text{same sub}}{=} TL^2 \pi/2$$

$$\Rightarrow \quad \underline{M^2 = 2\pi T J}$$

LECTURE 1 : ADS/CFT CONJECTURE

Surface Theory

stack of N_c D3 branes



10d $U(N_c)$ gauge theory

Open Strings In 10d: $\quad A^M \xleftrightarrow{8\text{ d.o.f}} \lambda$

reduce to 4d world view

$A^\mu + 6$ adjoint real scalars

$+ 4 \times 2\text{ d.o.f.}\ \lambda$

2 d.o.f

"$\mathcal{N}=4$" SYM.

Flavour Symmetry: $SO(6) \equiv SU(4)$ $\qquad$ λ are a $\underline{4}$

ϕ are a $\underline{6}$

β-function : $\quad \beta_{1\,loop} = -\dfrac{11 N_c}{3} + \dfrac{2}{3} \underbrace{C_{Adj}}_{N_c} \underbrace{N_f^{chiral}}_{4} + \dfrac{\overbrace{C_{Adj}}^{N_c} \overbrace{N_{scalar}}^{6}}{6}$

$= 0$ $\quad$ true at all loop level

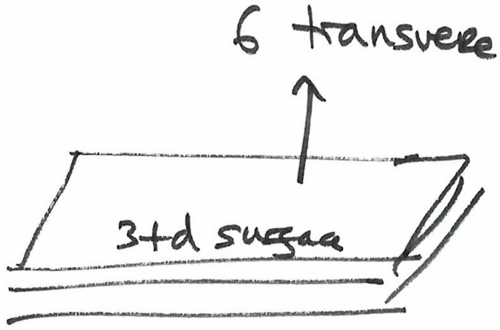
"CONFORMAL" : no scale $\rightarrow$ no bound states

$\rightarrow$ vacuum empty

$SO(1,3) \longrightarrow SO(2,4)$ $\quad$ more in LT2.

Bulk Theory

$N_c \to \infty$



They generate a black hole like geometry

Near horizon region is $r \times S_5$

$$ds^2 = \frac{r^2}{R^2} dx_{0..3}^2 + \frac{R^2}{r^2} \underbrace{dx_{4..9}^2}_{dr^2 + r^2 d\Omega_5^2}$$

$$ds^2 = \underbrace{\frac{r^2}{R^2} dx_{0..3}^2 + \frac{R^2}{r^2} dr^2}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S_5}$$

$R \sim N_c$

Large N_c is small curvature so neglect strings $\to$ supergravity

The QFT becomes strongly coupled ~ $g^2 N_c$ effect

Symmetries:

$SO(2,4)$ — AdS_5 is a sub-surface in a 2,4 space...

$SO(6)$ — rotations

MALDACENA: Matching global symmetries $\to$ two copies of the same physics....

<u>Dilatations</u> one of the ~~extra~~ bits in $SO(2,4)$

eg QFT $\int d^4x \; \frac{1}{2}(\partial^\mu \phi)^2 + i \bar{\psi} \not{\partial} \psi$

$\phi \to \# \phi \qquad \#^{-4} \quad \#^2 \; \#^2 \qquad \psi \to \psi \#^{3/2}$

$x \to x/\#$

This is how determine energy dimensions of fields. (& sources)

$$\int d^4x \; M^2 \phi^2 \implies M \text{ dim } 1.$$

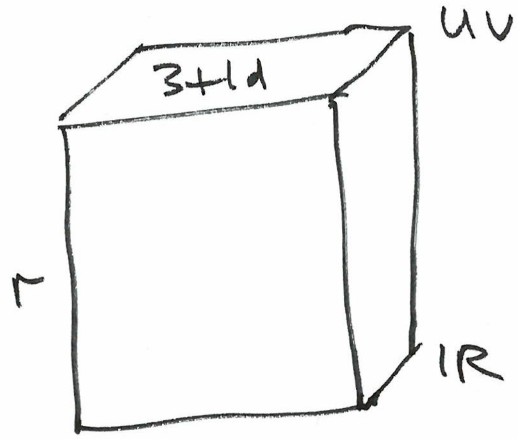
$M \to \# M$ is a "spurious" symmetry

<u>AdS_5</u> - here this symmetry is a <u>metric</u> symmetry

~~[illegible] dropped R^2~~

$$ds^2 = R^2 \frac{dr^2}{r^2} + \frac{r^2}{R^2} dx^2_{3+1} \qquad \eta^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

$x \to x/\# \qquad r \to \# r$



r is naturally a continuous energy $\to$ RG scale

You can see $r^2 dx^2_{3+1}$ rescales space

Dual Quantities — if the theories are the same they must have the same observables

ie operators $\mathcal{O}$ & sources J

$S_{QFT} = \int d^4x \, \mathcal{O} J$ eg $\int m \bar{\Psi}\Psi \, d^4x$

Here scalars that can change with RG μ / r so...

Scalars in AdS_5

$$S_{AdS} = \int d^4x \, dr \sqrt{-g} \left[\tfrac{1}{2} (\partial^M \phi)^2 - \tfrac{1}{2} M^2 \phi^2 \right]$$

d^5x is not an invariant under coord redefs.

$\sqrt{-g} \, d^5x$ is.... $(\sqrt{-\det g})$

Let's consider a vacuum config where $\phi(r)$ only (not x^μ or break Lorentz sym.)

$$\sqrt{-g} = r^3 / R^3$$

$$\begin{pmatrix} R^2/r^2 & & \\ & r^2/R^2 & \\ & & \ddots \end{pmatrix} \Big\} 1,\ \Big\} 4$$

$$S_{AdS} = \frac{1}{R^3} \int d^4x \, dr \, r^3 \left[\tfrac{1}{2} \underbrace{r^2/R^2}_{g^{rr}} (\partial_r \phi)^2 - \tfrac{1}{2} M^2 \phi^2 \right]$$

check dim : $\int d^4x \, dr \, r^3$ dim 0

$r^2 \partial_r^2$ "   "

Note: M^2 & ϕ^2 are dimensionless under field theory dilatations

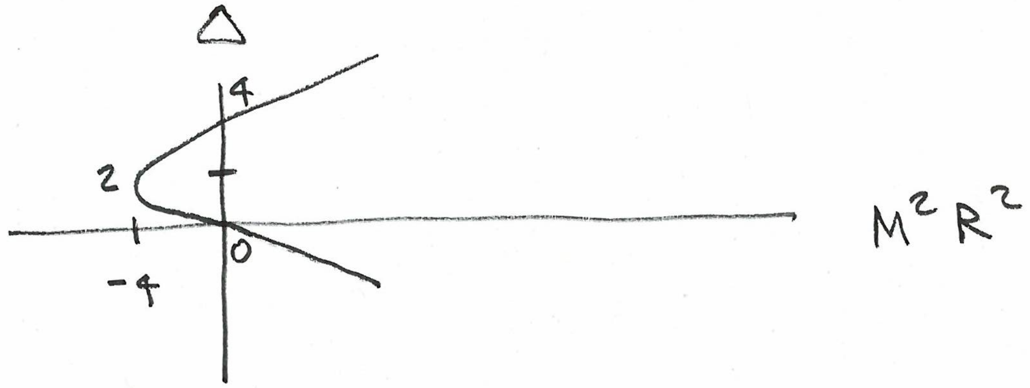
E-L eq'n: $\partial_r \left(r^5 \partial_r \phi \right) + r^3 M^2 R^2 \phi = 0$

Trial soln $\phi \sim 1/r^\Delta$

$$\partial_r \left(r^5 . -\Delta . r^{-\Delta-1} \right) + r^{3-\Delta} M^2 R^2 = 0$$

$$-\Delta . (-\Delta+4) . r^{-\Delta+3} + r^{3-\Delta} M^2 R^2 = 0$$

$$\boxed{M^2 R^2 = \Delta (4-\Delta)}$$



Our solution for a given $M^2 R$ is

$$\phi \sim a/r^\Delta + b/r^{4-\Delta}$$

dim = 0 $\Rightarrow$ a has dim Δ

b " " $4-\Delta$

Since $\Delta + 4 - \Delta = 4$ they can play the role of $\mathcal{O}$ & J that we've seen in the field theory.

Every $\mathcal{O}$, J pair $\Rightarrow$ bulk field....

[Witten's paper]

Breitenlohner - Freedman Bound

The stable solutions for $M^2 < 0$ is a result of the space-time curvature. Here the stability bound is

$$M^2 < -4/R^2 \qquad \underline{\text{BF bound}}$$

below this are only complex solutions ... this is the equivalent of $M^2 < 0$ in flat space.

$\underline{M^2 > 0}$ correspond to $\Delta > 4$ operators $\rightarrow$ HDOs.

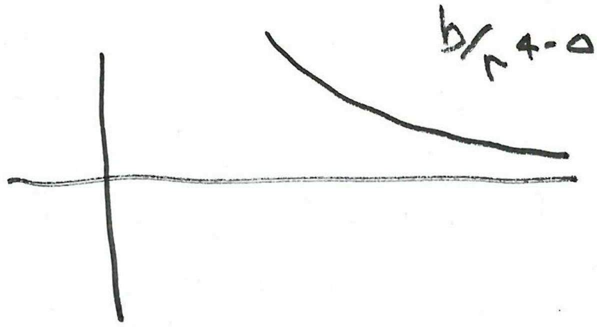
$$\text{eg} \quad \underbrace{\frac{g^2}{\Lambda^2}}_{\text{dim } -2} \underbrace{\bar{\Psi}\Psi\bar{\Psi}\Psi}_{\text{dim } 6} \qquad M^2 = 12$$

In UV soln grows as r^2 $\left(1/r^{\Delta} \quad \Delta = -2\right)$

$$r^3 M^2 \phi \sim r^5 \quad \text{but} \quad r^3 \phi^3 \sim r^9$$

so higher order terms in action would be important....

$\underline{N=4}$ there are no dimensional sources....
ie $a = 0$



$b=0$ is regular solution

$\Rightarrow$ no vevs & vacuum is empty.

(conformal symmetry breaking)

Dynamical AdS/QCD $N_f = 2$ SU(3) QCD

The vacuum operator is $\bar{u}_L u_R + \bar{d}_L d_R + h.c.$

$\rightarrow$ holographic field L

Here I choose it to have conformal dim 1 but could write as $L = r\phi$

$$S = \int \underbrace{dr\, d^4x}_{\text{dim } -3} \left(r^3 \underbrace{(\partial_r L)^2}_{\text{dim } 0} + r \underset{?\text{dim } 0}{\Delta m^2} L^2 \right)$$

If integrate by parts get $M^2_\phi = -3$.

I've dropped Rs because we will only look at ratios in which it cancels.

$$\rightarrow \quad \partial_r (r^3 \partial_r L) - r \Delta m^2 L = 0$$

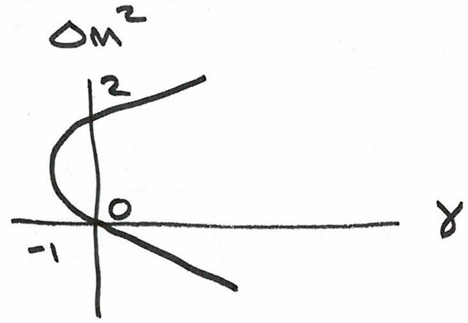
$$\Delta m^2 = 0 \quad \rightarrow \quad \underset{\text{dim } 1}{L} = \underset{\text{dim } 1}{m} + \underset{\text{dim } 3}{c/r^2} \qquad \text{(by inspection)}$$

$$\Delta m^2 \neq 0 \quad \rightarrow \quad \underset{?\text{dim } 1}{L} = \underset{?\text{dim } 1+\gamma}{m\, r^{-\gamma}} + \underset{?\text{dim } 3-\gamma}{c\, r^{\gamma - 2}}$$

$$\Delta m^2 = \gamma(\gamma - 2)$$

BF bound at $\Delta m^2 = 1$

ie $\gamma = 1$



"$\gamma = 1$ instability criteria"

Anomalous Dimensions in QCD

We can write J/θ pairs

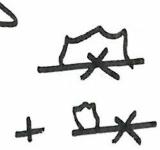
$$J = \tilde{J}\,\mu^{d} \qquad \theta = \tilde{\theta}\,\mu^{4-d}$$

dimless holds dim

$$d = \frac{1}{J}\,\mu\,\frac{dJ}{d\mu}$$

At 1-loop

eg mass



$$J = Z_0 J$$

$$\frac{1}{J}\,\mu\frac{dJ}{d\mu} = d + \underbrace{\frac{1}{Z_0}\,\mu\,\frac{dZ_0}{d\mu}}_{\gamma}$$

"anomalous dimension"

In QCD $\qquad \gamma_{n\,legs} = n\,\frac{\alpha(\mu)}{\pi}$

$$\gamma_{\bar{q}q} = \frac{2\alpha(\mu)}{\pi}.$$

So the dim of m rises... and that of $\bar{q}q$ falls...

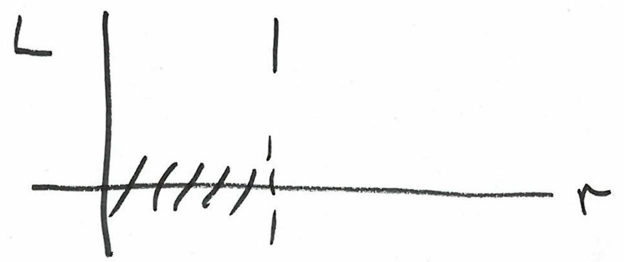
In the holographic model $\qquad \Delta m^2 = \gamma(\gamma - 2)$

Perturbative regime $\qquad \Delta m^2 \simeq -2\gamma$

insert $\frac{2\alpha(\mu)}{\pi}$

Naively one associates $\mu \Leftrightarrow r$

however this leads to an instability at low r for all values of L . . .



A solution is backreaction

$$r^2 \to r^2 + |L|^2$$

Now the instability is only at low L, r

fully stable solutions are possible.

Top-down string models do this naturally (eg D3/probe D7) ... here it's just dimensionally & symmetry allowed "back reaction"

LECTURE 3 : BOUND STATES

Mesons are the quanta associated to wave solutions of operators on top of the vacuum

eg $\quad \sigma / f_0 \qquad \bar{q} q$

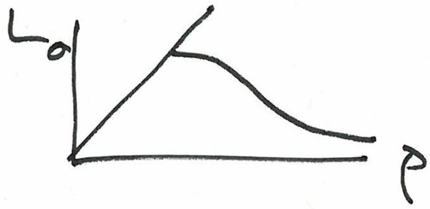
$\qquad \rho \qquad \bar{q} \gamma^\mu q$

$\qquad \pi \qquad \bar{q} \gamma_5 q$

$$\text{fluc} \sim e^{-ip \cdot x} \qquad p^2 = -M^2 \qquad (-1, +1, +1, +1)$$

AdS/QCD Last time we used a holographic field L to describe the vacuum with $\langle \bar{q} q \rangle \neq 0$.

$$\partial_\rho (\rho^3 \partial_\rho L_0) - \rho \Delta m^2 L_0 = 0$$



so now set $L = L_0 + S(\rho, x)$ & linearize

$$\partial_\rho (\rho^3 \partial_\rho S) - \rho \Delta m^2(L_0) S - \rho L_0 S \left.\frac{\partial \Delta m^2}{\partial L}\right|_{L_0}$$

$$+ \rho^3 \frac{\partial_\mu \partial_\nu S}{\rho^4} \sim \frac{g^{xx}}{g^{\rho\rho}}$$

Back reaction: more complex string models allow

$$\rho^2 \to r^2 = \rho^2 + L_0^2$$

It's dimensional correct & corresponds to "living on the L_0 surface".

It communicates the gap to the excitations.

NB/ This substitution is harder at the $\mathcal{L}$ level than EoM!

We seek solutions

$$S = f(\rho)\, e^{-ip.x} \qquad p^2 = -M^2$$

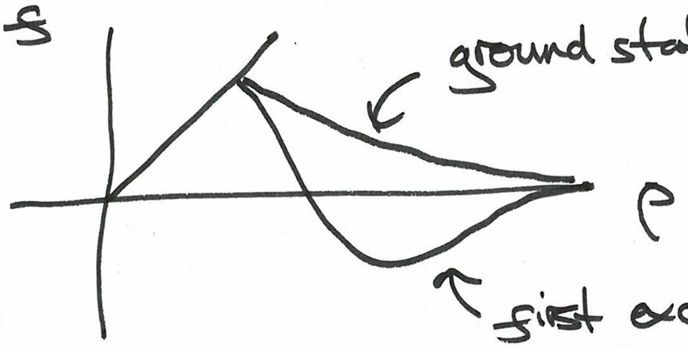
Boundary conditions on f

UV : $f \sim M + C/\rho^2$ — so want $f \to 0$ (no M fluctuation)

IR : $f'(L_{IR}) = 0$ — regularity

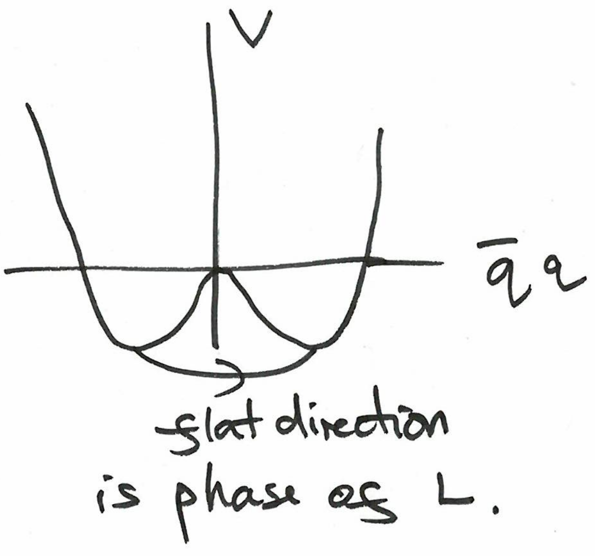
Solutions:

For linearized field IR b.c. fixes solution. Vary M^2 to match UV b.c.



→ predicts masses of ~~f₀~~ f_0 bound states.

PIONS The pions are the Goldstone bosons of chiral symmetry breaking



$$L \to L_0 e^{i\pi}$$

$$S = \int d^4x \, dr \quad r^3 \left[(\partial_r L_0 e^{i\pi})^2 + \frac{1}{r^4} (\partial_x L_0 e^{i\pi})^2 \right]$$

note π not in Δm^2 - symmetry so depends on L^+L.

$$\rightarrow \quad \partial_r (r^3 L_0^3 \partial_r \pi) + \frac{M_\pi^2}{r^4} \rho^3 L_0^2 \pi = 0 \qquad *$$

Is there a massless solution?

when $M_\pi^2 = 0$ $\quad \pi = $ const is a solution

ie $L = L_0 e^{i\pi} = L_0 + i L_0 \pi$

linearization scaling

The Im piece is a massless fluctuation provided

$L_0 \xrightarrow{r \to \infty} 0$ $\quad$ ie only for the massless theory. ✓

For $M_q \neq 0$ solve $*$ & $M_\pi^2 \neq 0$.

RHO MESONS $\qquad \bar{q} \gamma^\mu q \rightarrow A^\mu$ in the bulk.

must be massless gauge field

In 4d A^μ $\quad 4 - 2 = 2$ d.o.f

gauge fixed to 0.

In 5d ~~set~~ $A^r = 0$, $A^t = 0$

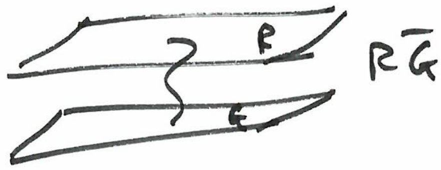
$$S = \int d^4x \, d\rho \; \rho^3 \; F^{\mu\nu} F_{\mu\nu}$$

eg $A_z(r, x)$ $\quad S = \int d^4x \, d\rho \; \rho^3 (\partial_\rho A_z)^2 + \frac{\rho^3}{\rho^4} (d_x A_z)^2$

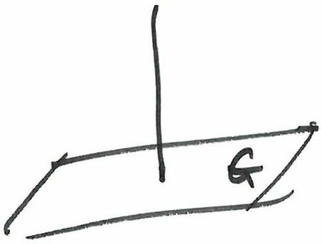
$$\rightarrow \quad \partial_\rho (\rho^3 \partial_\rho A_z) + \frac{M_\rho^2}{\rho} A_z^2 = 0.$$

LECTURE 3: CONFINEMENT

In AdS/CFT adjoint fields are made of strings with ends on the D3

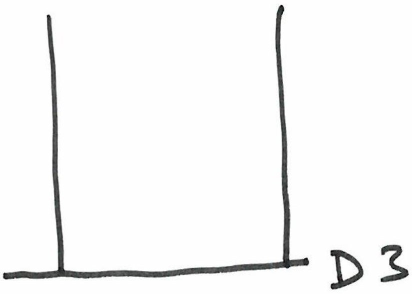


If we "detach" one end & take it to ∞



The resulting ∞ massive object is in the fundamental rep.

What is the interaction between two such "quarks"?



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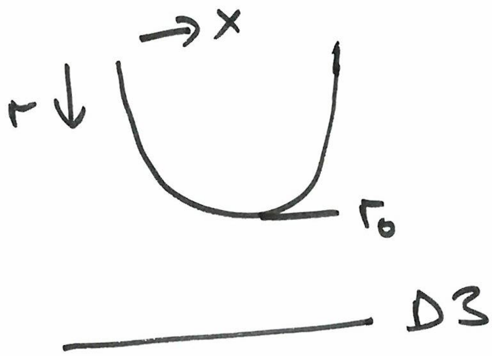


We must find the minimum energy config

$$S_{\substack{\text{Nambu}\\ \text{Goto}}} = T \int d\tau\, d\sigma \sqrt{-\det G_{ab}}$$

$$G_{ab} = G^{MN} \frac{dX_M}{d\xi^a} \frac{dX_N}{d\xi^b}$$

AdS metric

$$ds^2 = \frac{dr^2}{r^2} + r^2 dx^2_{3+1}$$

$$G_{ab} = \begin{pmatrix} -r^2 & \\ & r^2 + \frac{1}{r^2}(\partial_x r)^2 \end{pmatrix}$$

$t = \tau$
$\sigma = x$.

$$S = T \int d\sigma \sqrt{r^4 + (\partial_x r)^2}$$

There is no explicit x dependence so there's a conserved quantity ("Hamiltonian")

$$H = (\partial_x r) \cdot \frac{\partial_x r}{\sqrt{r^4 + (\partial_x r)^2}} - \frac{r^4 + (\partial_x r)^2}{\sqrt{r^4 + (\partial_x r)^2}}$$

$$p \cdot \frac{\partial L}{\partial p} - L$$

$$= \frac{r^4}{\sqrt{r^4 + (\partial_x r)^2}} = \text{const} = r_0^2$$

minimum r when $\partial_x r = 0$

square: $$r^8 = r_0^4 \left(r^4 + (\partial_x r)^2\right)$$

$$\frac{\partial r}{\partial x} = \sqrt{r^8/r_0^4 - r^4}$$

$$\int_0^{L/2} dx = \int_{r_0}^{\infty} \frac{dr}{\sqrt{r^8/r_0^4 - r^4}}$$

$y = r/r_0$ $$L/2 = \frac{1}{r_0} \int_1^{\infty} dy / y^2 \sqrt{y^4 - 1}$$ $L \propto 1/r_0$

The more separated the quarks the more the string dips into AdS.

Integrating the action over σ/x gives the energy

$$E = T \int d\cancel{x} \sqrt{\cancel{r^4} + r^8/r_0^4 - \cancel{r^4}}$$

$$\sim \frac{dr}{\sqrt{r^8/r_0^4 - r^4}}$$

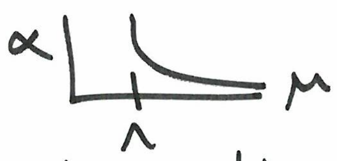
$$= T r_0 \int_1^\infty dy \; y^4 \Big/ \sqrt{y^8 - y^4} \qquad y = r/r_0$$

$$\propto 1/L .$$

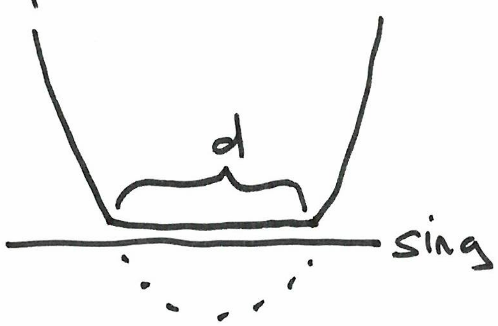
This is all it could be because there is no scale but L... the string in AdS represents the $1/L$ Coulomb law energy ~~as~~ the $N=4$ interactions.

In models with running couplings that become strong in ~~the~~ IR – supergravity deformations ⇒ singularity

– $\alpha(\mu)$



If a repulsive singularity develops then widely separated quark strings:



Reduces central action at lowest r...

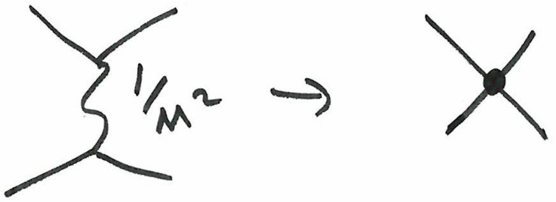
Now $E = T d$

Recovers confining behaviour.

LECTURE 4:

NJL INTERACTIONS

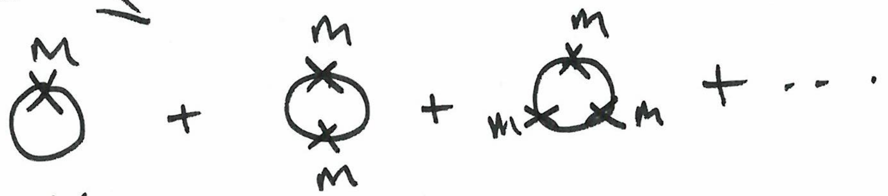
A common idea in BSM physics is heavy gauge fields that are strongly coupled



$$\frac{g^2}{\Lambda_{uv}^2}\,\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L + h.c.$$

In field theory these can generate a fermion mass....

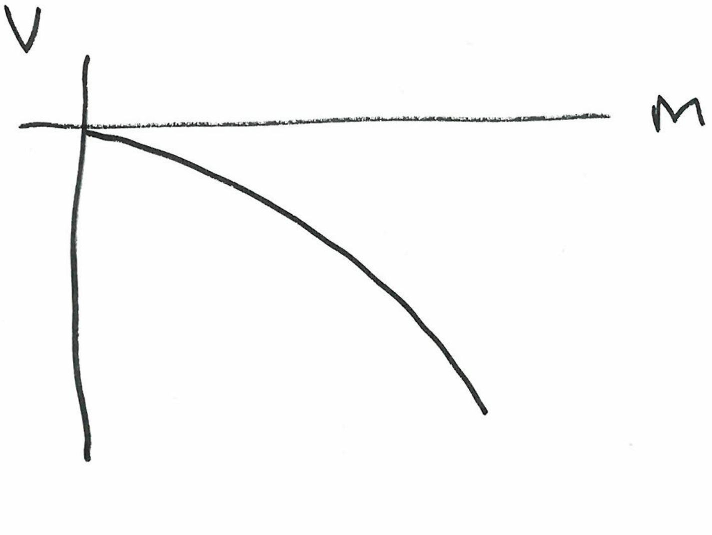
Vacuum energy of a fermion due to Coleman & Weinberg



$$Tr\,\frac{m\not{k}}{k^2} \to 0 \qquad \text{odd insertions vanish}$$

Even insertions sum to

$$\Delta V_{eff} = -\int_0^{\Lambda_{uv}} \frac{d^4k}{(2\pi)^4}\, Tr \ln\left(1 + m^2/k^2\right)$$



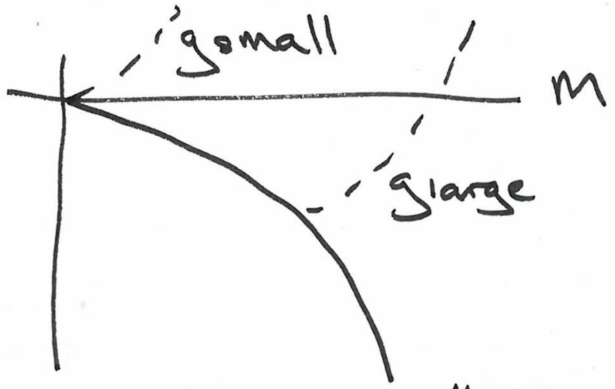
Normally think of m as fixed so not a run away...

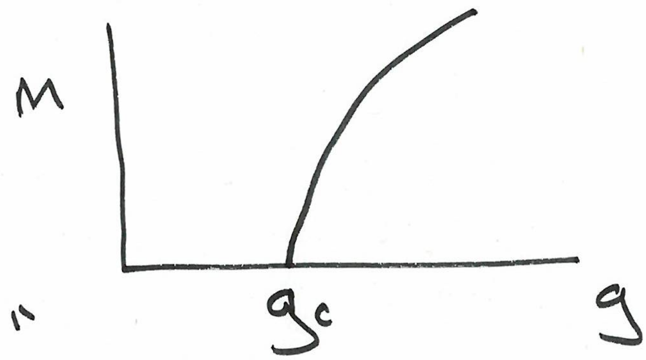
Now add $\frac{g^2}{\Lambda_{UV}^2} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \rightarrow m = \frac{g^2}{\Lambda_{UV}^2} \langle \bar{\psi}_L \psi_R \rangle$

assume condenses

$K_{\text{term}} \rightarrow \frac{m^2 \Lambda_{UV}^2}{g^2}$

Now include this term in potential & treat m as a determined quantity





"NJL model"

<u>Holographic Version</u> : Witten's "multi-trace prescription"

we had solutions ~~for~~ $\bar{q}q$ vacuum

$L(r) = m + c/r^2$ in the UV.

Now reinterpret as m & $c = \bar{q}q$ due to NJL interaction

ie $m = \frac{g^2}{\Lambda^2} \langle \bar{\psi}_L \psi_R \rangle$
$\uparrow c$

We can read ~~off~~ m

→ ppt examples

FINITE TEMPERATURE

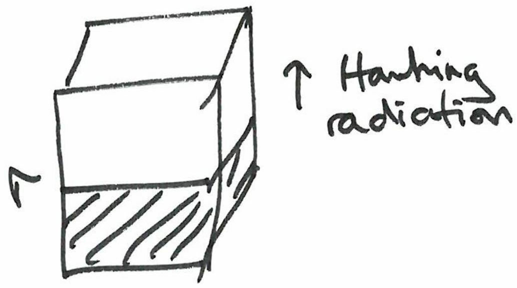
Witten immediately linked field theory T to an AdS - Black Hole geometry

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)\,dt^2 + dx_3^2\right) + \frac{R^2}{r^2 f(r)}\,dr^2$$

$$f(r) = 1 - r_0^4/r^4$$

The horizon is at r_0 – plays the role of IR cut off at $E = kT$.

[does the interior mean anything?]



<u>More formally</u>: in field theory T is included by $t \to it$ & then making compact with circumf. $\beta = 1/kT$

$$Z = e^{i\int dt\, d^3x\, \mathcal{L}} \rightarrow e^{-\int_0^{\beta} dt \underbrace{\int d^3x\, V}_{E}}$$

$$\beta = 1/kT$$

$$\rightarrow e^{-E/kT}.$$

If we look at geometry near horizon

$$f = 1 - (1 + \delta r/r_0)^{-4} \simeq +4\,\delta r/r_0$$

$$ds^2_{Euc} = \frac{r_0^2}{R^2}\frac{4\delta r}{r_0}\,dt_E^2 + \frac{R^2 r_0}{r_0^2\, 4\delta r}\,d\delta r^2 + \ldots$$

Set $\rho^2 = \frac{R^2 \delta r}{r_0}$ & $\theta = \frac{2r_0}{R} t_E$

$$\rightarrow ds^2_{EUC} = d\rho^2 + \rho^2 d\theta^2$$

θ must go $0 \rightarrow 2\pi$ (else a cone with singular tip)

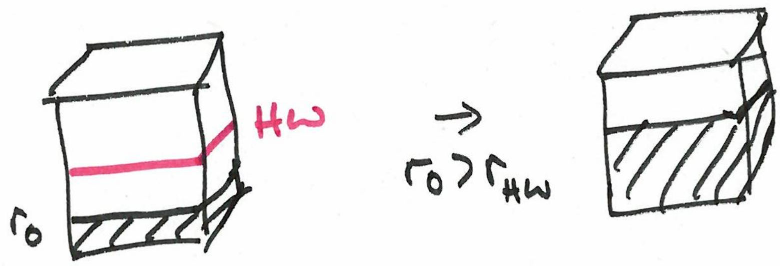
$$\rightarrow t \text{ from } 0 \rightarrow \frac{2\pi R^2}{2r_0} = 1/kT = \beta$$

$$1/T \sim 1/r_0 \quad \checkmark$$

First Order Transitions

Computing space-time energy (Hawking Page) shows in N=4 transition at T>0.

Hard Wall



This sharp transition is unlike QCD... but expected at large N_c.

Meson Melting

- in a hard wall geometry we had a discrete spectrum of mesons...

Consider again the near horizon geometry

$$ds^2 = -\frac{4 r_0 \delta r}{R^2} dt^2 + \frac{R^2}{4 r_0 \delta r} d\delta r^2 + \frac{r_0^2}{R^2} dx_3^2$$

Let's do a massless scalar in this space

$$S = \int dr\, d^4x \, \frac{r_0^3}{R^3} \left[\frac{4 r_0 \delta r}{R^2} (\partial_r \phi)^2 + \frac{R^2}{4 r_0 \delta r} (\partial_t \phi)^2 \right]$$

Trial static meson solution $\phi(r)\, e^{-i\omega t}$

$$\partial_r \left(\frac{4 r_0 \delta r}{R^2} \partial_r \phi \right) + \frac{\omega^2 R^2}{4 r_0 \delta r} \phi = 0$$

$$\phi'' + \frac{\phi'}{\delta r} + \frac{\omega^2 R^2}{4 r_0 \delta r} \frac{R^2}{4 r_0 \delta r} \phi = 0$$

$\delta r \to 0$

$$\phi'' + \frac{\tilde{\omega}^2}{16\, \delta r^2} \phi = 0$$

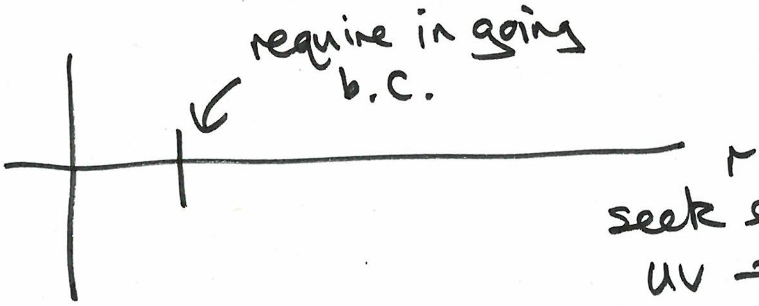
$$\phi = a\, \delta r^{i\tilde{\omega}/4} + b\, \delta r^{-i\tilde{\omega}/4}$$

$\sim$ re-exponentiate $e^{-i\omega(t + \# r)}$

out coming wave

$\sim e^{-i\omega(t - \# r)}$

in going wave



seek solutions uv $\to 0$

Complex ω solutions $\to$ decay width.

DENSITY & EXTERNALLY APPLIED $\vec{E}$ & $\vec{B}$

All of these correspond to aspects of a $U(1)_B$ gauge field in the bulk...

$$\begin{pmatrix} 0 & \mu & & & \\ \mu & 0 & E & E & E \\ & -E & 0 & & \\ & -E & & 0 & B_s \\ & -E & & B_s & 0 \end{pmatrix}$$

density $= \bar{\Psi} \gamma^0 \Psi$

$$\rightarrow S = \int d^4x \, \mu \, \bar{\Psi} \gamma^0 \Psi$$

chemical potential

$$= \int d^4x \, \langle A_0 \rangle \bar{\Psi} \gamma^0 \Psi$$

To influence $L(r)$ need terms

$$S = \int d^5x \quad F^2 (\partial L)^2 + F^2 L^2$$

$\vec{E}$ & μ discourage χSBing...

$\vec{B}$ encourages...