Effective Field Theories with applications to collider Phenomenology

Ken Mimasu

University of Southampton

NExT PhD Workshop, 24th August 2026

Science and Technology Facilities Council

University of Southampton

Outline

Part 1: Intro to Effective Field Theory

- What is an EFT? Fermi theory as an example
- Dimensional analysis & renormalisability
- Matching exercise: tree-level and one loop → anomalous dimensions
- Using EFT to search for new physics

Part 2: The Standard Model as an Effective Field Theory

- The effective theory paradigm at the LHC
- Overview of SMEFT

Part 3: Application - SMEFT in the top/Higgs/EW sector

- Impact of SMEFT: new interactions
- Latest results - Global fit and high energy top quark processes
- Tools for SMEFT - state of the art

References

These lectures are partly based on the following:

Reviews

[Burgess; Annu. Rev. Nucl. Part. Sci. 57 (2007) 329, arXiv:hep-th/0701053]

[Manohar; arXiv:1804.05683]

[Skiba; arXiv:1006.2142]

[Cohen; arXiv:1903.03622]

Lectures from previous schools

[Cen Zhang (2015)] https://www.physics.sjtu.edu.cn/madgraphschool/

[Eleni Vryonidou (2018)] https://indico.ihep.ac.cn/event/7822/

[Gauthier Durieux (2018)] https://indico.ihep.ac.cn/event/7822/

Part 1
Introduction to Effective Field Theory

Introduction

Nature contains an abundance of physical scales

- From the Hubble all the way to the Planck scale

In order to make sense of a given physical problem, we must first identify the ***relevant*** *scales*

- Do not need to know the composition of planets to calculate orbital motion
- Or the short distance properties of EW theory for Hydrogen energy levels

Don't need to know everything all of the time

- Depends on required accuracy

In High Energy Physics, we use Quantum Field Theory to compute scattering amplitudes

- Collider cross sections, Dark Matter annihilation/detection, decay rates etc…
- Relevant scales: particle masses, collider energies, momentum transfers,…

Scale separation

Computations can be challenging

- Especially when involving multiple, disparate scales
- **Not all scales may be relevant** at the energy of interest

QFT problems with this **scale separation** can be reorganised into an effective description

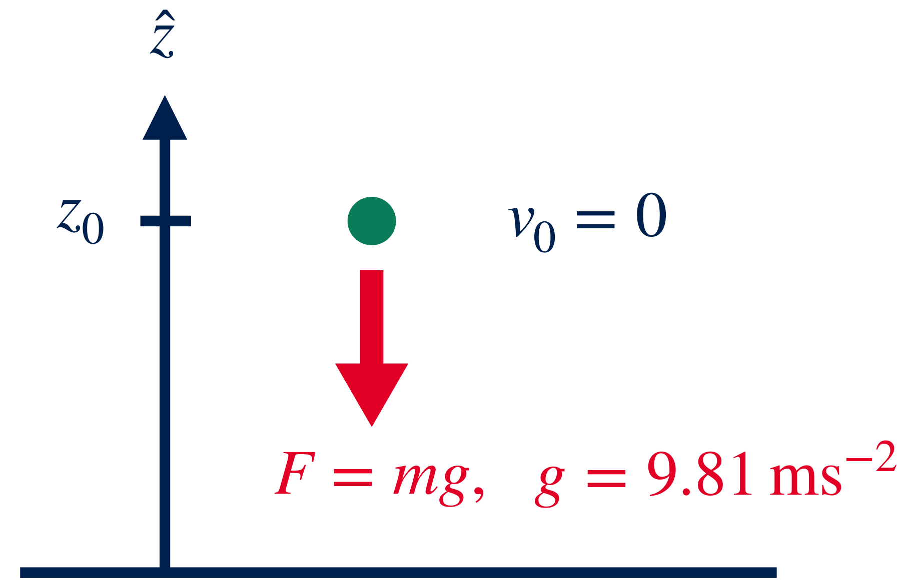
- Controlled by ***ratios*** of scales: E_1/E_2, $\log(E_1/E_2)$

Take a scattering amplitude at CM energy, E, that depends on two mass scales, m and M, with $E^2 \sim m^2 \ll M^2$

- Heavy physics should not have a big impact on 'low-energy' phenomena
- We can approximate the effects that depend on M
- Describe by an **expansion** in $(m/M)^n$, $(E/M)^n$

Gravity example

Falling body near the earth's surface



- Dropped from height $z_0 \sim 10\text{–}100\,\mathrm{m}$ at t_0
- Hits ground at $t = \sqrt{2z_0/g}$

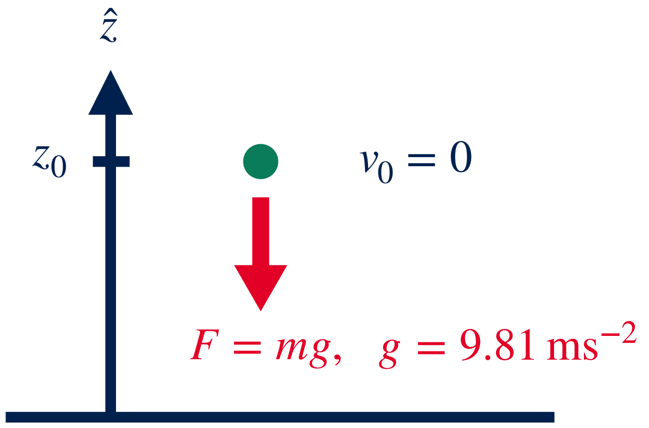
Physical law: Newton

$$F = \frac{GMm}{r^2} = \frac{GM_\oplus m}{(R_\oplus + z_0)^2} \xRightarrow{R_\oplus \gg z_0} m \overset{g}{\boxed{\frac{GM_\oplus}{R_\oplus^2}}} \left(1 - 2\frac{z_0}{R_\oplus} + \cdots\right)$$

$$F = mg\left(1 - 2\frac{z_0}{R} + \cdots\right) = mg\left(1 - 3.1 \times 10^{-7}\, z[m]\right)$$

$F = mg$ is an effective theory of Newtonian gravity on earth

Gravity example



1. Degrees of freedom: point mass, m

2. Symmetries: 2D translations & rotations $(x-y)$

- Different from fundamental theory: $T(3) \times O(3)$

3. Expansion parameter: $z_0/R_\oplus$

- EFT breaks down (not valid) for $z_0 \sim R_\oplus$

Details of fundamental theory encapsulated in $g = \dfrac{GM_\oplus}{R_\oplus}$

General theory for this system:

$$F = F_0\left(1 + c_1\frac{z}{R} + c_2\left(\frac{z}{R}\right)^2 + \cdots\right)$$

- Determine F_0, c_i experimentally
- Try to infer $F(z)$, learn $G, M_\oplus, R_\oplus$

Effective Field Theory

Same principle applied to QFT

1. Degrees of freedom: what fields? $\phi, \psi, A^\mu, \ldots$

2. Symmetries: gauge, global, Lorentz,…

- Determine structure of interactions

3. Expansion parameter: E/M

- "Power counting": crucial ingredient of EFTs

$\Rightarrow$ write down $\mathscr{L}_{\mathrm{EFT}}$ & calculate!

Why does it work?

- To describe physics at some scale p^2, we don't need to know the details of dynamics at some much higher scale, $\Lambda^2 \gg p^2$

Decoupling

Decoupling theorem

[Appelquist & Carrazzone; PRD 11 (1975) 2856]

- Effects of heavy physics with mass M, 'decouple' at low momenta, p
- Result only in shifts of low energy renormalisation constants + $O(p^2/M^2)$

This is the basis of the viability of **Effective Field Theories**

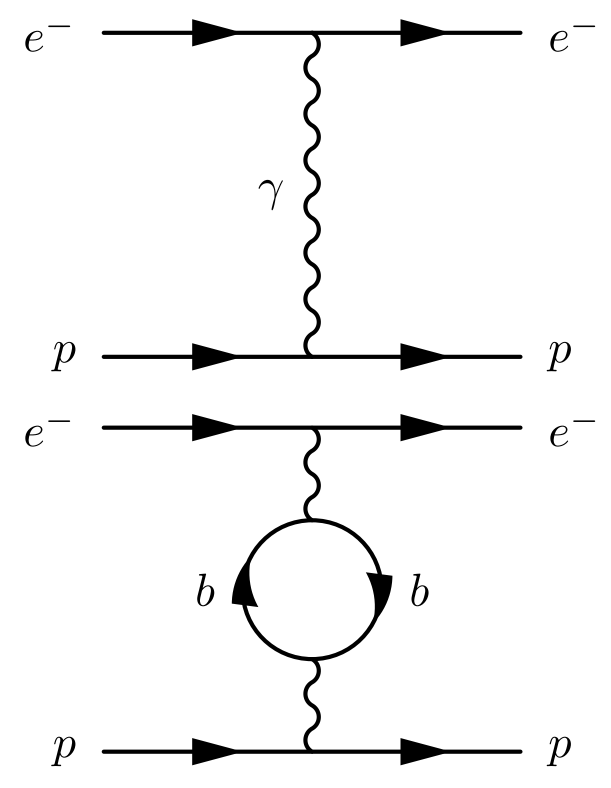
- When applied at the right scale, EFTs can predict with arbitrary precision
- The world can be divided into successive EFT 'slices', each describing relevant physics at that scale

For QFT this means only including particles that are heavy enough to be produced at the energy of interest

- All other fields are 'integrated out' and appear indirectly via $(E/M)^n$ corrections

$$\mathcal{L}_{\mathrm{full}}\left(\phi_{\mathrm{light}}, \phi_{\mathrm{heavy}}\right) \longrightarrow \mathcal{L}_{\mathrm{full}}\left(\phi_{\mathrm{light}}\right) + \mathcal{L}_{\mathrm{eff.}}\left(\phi_{\mathrm{light}}\right)$$

Hydrogen



Low energy theory: QED

- Energy scales: $m_e, m_e\alpha, m_e\alpha^2$, coupling: α

Full theory: Standard Model

- Many more scales $m_b, m_t, m_Z, m_h, \ldots$

Ground state

$$E = E_0\left(1 + \mathcal{O}(m_e^2/m_b^2) + \cdots\right) \qquad E_0 = E_0(\alpha)$$

$\alpha_0 = 1/137$

$\alpha(m_Z) = 1/128$

Presence of heavy particles renormalizes α

- Value of α_0 depends on $m_b, m_t, m_Z, m_h, \ldots$ via threshold corrections

Contribute power corrections to energy levels

Different EFTs

Two main reasons for using EFT

A. If 'full theory' is **known**: greatly simplify calculations — 'Top Down'

B. If 'full theory' is **unknown**: universally parametrise UV effects — 'Bottom up'

Every EFT has a…

- **Power counting** expansion: appropriately small ratio of scales
- **Range of validity**: scales at which it reliably approximates full theory

Examples in HEP

A. Heavy quark effective theory, Soft collinear effective theory, Weak effective theory, non-relativistic QCD,…

B. Fermi theory, Chiral perturbation theory, EW chiral Lagrangian, **Standard model effective field theory**,…

A tower of EFTs

UV E

Full/UV theory — Unknown EFT? $\frac{M_{NP}}{M'_{NP}}$

$M_{\rm NP}$

Standard model EFT

EW chiral Lagrangian

$\frac{p}{M_{\rm NP}}$ $\frac{M_{\rm EW}}{M_{\rm NP}}$

LHC

LEP

M_{EW}

Fermi Theory $\frac{m_\mu}{M_{\rm EW}}$ — Weak effective theory $\frac{p}{M_{\rm EW}}$

m_b, m_c

Heavy quark EFT $\frac{\Lambda_{\rm QCD}}{m_{b,c}}$

LHCb
Belle
BaBar

$\Lambda_{\rm QCD}$

Chiral perturbation theory $\frac{p}{\Lambda_{\rm QCD}}$

IR

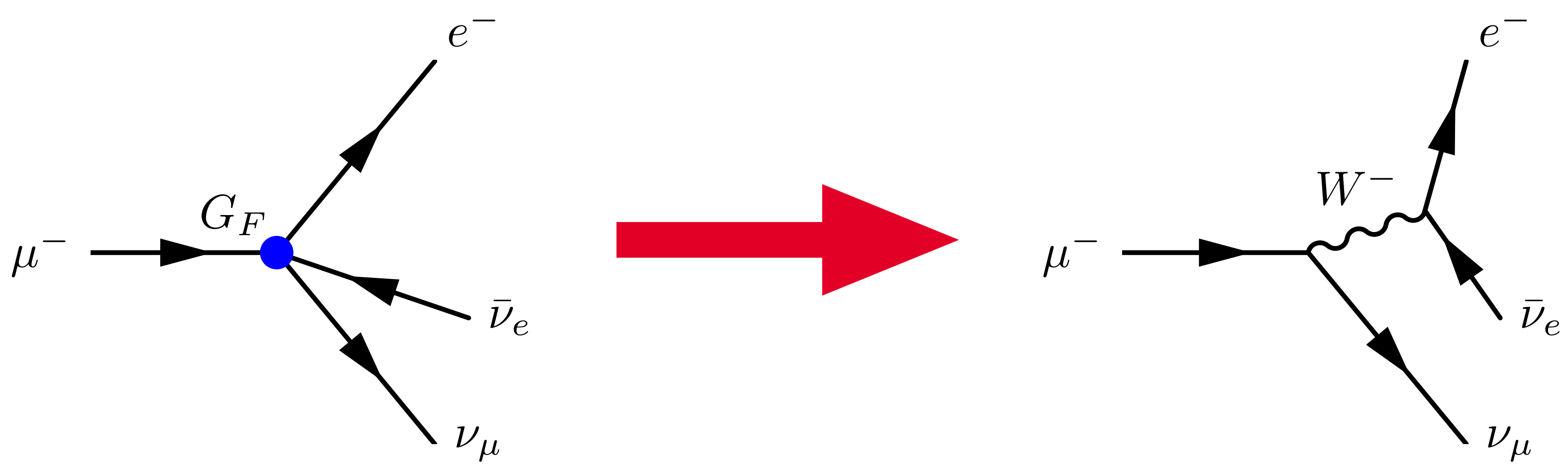
The most famous EFT

Fermi theory of weak interactions

- 30 years before the development of EW theory
- Describe beta decay, muon decay via **4-fermion contact interaction**

$$\mathcal{L}_{\rm Fermi} = -\frac{G_F}{\sqrt{2}}\,[\bar{\nu}_\mu\,\gamma^\mu(1-\gamma_5)\,\mu]\,[\bar{e}\,\gamma_\mu(1-\gamma_5)\,\nu_e]$$

$$G_F = 1.167\times 10^{-5}\ \text{GeV}^{-2} \qquad\qquad [G_F] = -2$$



We know now that it is mediated by the W boson

- Scale separation: $(m_\mu/m_W)^2$

The most famous EFT

We can **match** EW theory to the Fermi Lagrangian

- Demand that the two descriptions give the same result at a given scale

Full theory:

$$\mathcal{M}_{EW} = \frac{ig^2}{8m_W^2}\left(\frac{m_W^2 g_{\mu\nu} - k_\mu k_\nu}{k^2 - m_W^2}\right)\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma^\nu(1-\gamma^5)v(q_3)\right)$$

$$k^2 \ll m_W^2 \to -\frac{ig^2}{8m_W^2}\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma_\mu(1-\gamma^5)v(q_3)\right)$$

EFT:

$$\mathcal{M}_{\text{Fermi}} = -\frac{iG_F}{\sqrt{2}}\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma_\mu(1-\gamma^5)v(q_3)\right)$$

Tree-level matching: $\mathcal{M}_{EW} = \mathcal{M}_{\text{Fermi}} \longrightarrow G_F = \dfrac{g^2}{4\sqrt{2}m_W^2} = \dfrac{1}{\sqrt{2}v^2}$

Both theories predict the same IR ($q^2 \ll m_W^2$) physics but differ in the UV ($q^2 \gtrsim m_W^2$), where on-shell W can occur

Dimensional analysis

Fermi interaction is a higher dimensional operator

4D QFT path integral: $Z = \int \mathcal{D}\phi\, e^{iS[\phi]}, \quad S = \int d^4x\, \mathcal{L}[\phi(x)]$

Natural units, $(\hbar = c = 1)$: $[\text{Mass}] = [\text{Length}]^{-1} \quad \Rightarrow \quad [\mathcal{L}] = 4$

kinetic term: $[\partial_\mu \phi \partial^\mu \phi] = 4,\ [\partial_\mu] = 1 \Rightarrow [\phi] = 1$

$$\Rightarrow \quad [\psi] = \frac{3}{2},\ [D_\mu] = 1,\ [A_\mu] = 1,\ [g] = 0$$

canonical (mass) dimension

Renormalisable interactions have couplings $[c] \geq 0$

$$\mathcal{L}_{\text{int.}} = c\,\mathcal{O}, \quad [\mathcal{O}] \leq 4$$ ‘operator’

- Renormalisable: need a **finite number** of counter-terms (CT) to absorb divergences in loop computations to **all orders** in perturbation theory

$[\mathcal{O}] < 4,\ [c] > 0$	$[\mathcal{O}] = 4,\ [c] = 0$	$[\mathcal{O}] > 4,\ [c] < 0$
‘Relevant’	‘Marginal’	‘Irrelevant’

Renormalisability

Fermi interaction: ψ^4 = dimension-6

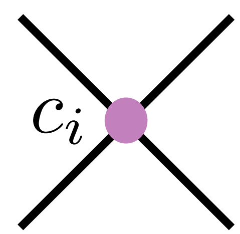
c = 'Wilson coefficient'

$$G_F \bar{\psi}\psi\bar{\psi}\psi \to \frac{c}{\Lambda_F^2}\bar{\psi}\psi\bar{\psi}\psi, \quad [c] = 0$$

Λ_F = 'cutoff' (nothing to do with loop integration)

Inserting an operator once into a 2→2 amplitude:

$$\mathcal{L}_{\text{eff.}} = \sum_i \frac{c_i \mathcal{O}_i^d}{\Lambda^{4-d}}$$



$$[\mathcal{A}] = 0 \to \mathcal{A} \sim c_i \left(\frac{p}{\Lambda}\right)^{d-4}$$

Expect a power-like dependence on the external momenta

- **At most** equal to the power of Λ in the denominator
- Holds beyond tree-level: only physical (IR) scales result from loop integration
- Easily seen using dimensional regularisation, which discards power-like dependence on unphysical scales (unlike, e.g., cut-off regulator)

$$I_{\text{DR}} \sim f\left(\boxed{p^2,\, m^2}, \log\frac{\boxed{\mu^2}}{\boxed{p^2}}, \log\frac{\boxed{\mu^2}}{\boxed{m^2}}, \boxed{\frac{1}{\epsilon}}, \ldots\right)$$

physical

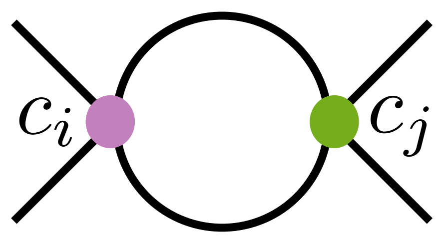
renormalisation

poles

Non-renormalisability?

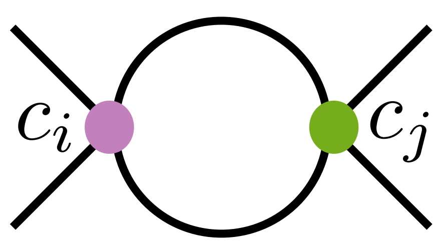
Consequence:

- Higher order corrections involving further d>4 operators lead to **higher power** momentum dependence

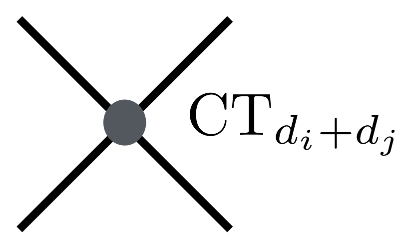


$$\longrightarrow \quad \mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n_i+n_j}$$

- Renormalisation requires counter-terms from higher dimensional operator (d_i+d_j) to cancel divergent piece



$+$



$=$ finite

A. EFTs require an infinite number of CTs to cancel poles to all orders: formally **non-renormalisable**

B. Poles can be cancelled (renormalisable) **order-by-order** in Λ

Toy model

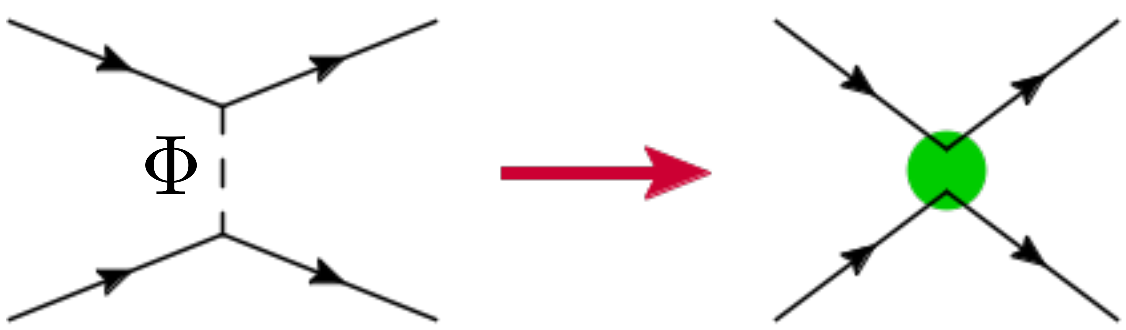
Let's work with a toy model to see this in action

- Yukawa theory of massless fermions, ψ, and a heavy scalar, ϕ

$$\mathcal{L}_{\rm Full} = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\left(D_\mu\Phi\right)^2 - \frac{1}{2}\underbrace{\boxed{M^2}}_{\text{Heavy mass scale}}\Phi^2 - \underbrace{\boxed{\lambda\Phi\bar{\psi}\psi}}_{\text{Yukawa interaction}}$$

Find the EFT that describes the physics below the scale M

- Like Fermi theory, integrating out scalar leads to 4-fermion interaction



$$\mathcal{L}_{\rm EFT} = i\bar{\psi}\partial\!\!\!/\psi + \frac{c_S}{M^2}\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

- Let's perform the matching

Tree-level matching

Full

EFT

Diagrammatic method: compare amplitude $\psi\psi \to \psi\psi$

Full:

$$\mathcal{M} = \bar{u}(p_3)(-i\lambda)u(p_1)\bar{u}(p_4)(-i\lambda)u(p_2)\left[\frac{i}{(p_1-p_3)^2 - M^2}\right] - (3 \leftrightarrow 4)$$

$$= \mathcal{U}_S \frac{i\lambda^2}{M^2}\left[1 + \mathcal{O}(q^2/M^2) + \cdots\right]$$

define: $\mathcal{U}_S = \bar{u}(p_3)u(p_1)\bar{u}(p_4)u(p_2) - (3 \leftrightarrow 4)$

EFT: $\dfrac{c_S}{M^2}\dfrac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi) \quad \longrightarrow \quad \mathcal{M} = \mathcal{U}_S \dfrac{ic_S}{M^2}$

Matching condition
$c_S = \lambda^2$

- After matching, dimension-6 EFT below M is:

$$\mathcal{L}_{\mathrm{EFT}} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

Improved matching

Full

EFT

Can be improved by including higher dimension operators

- Keep more terms in the propagator expansion

Full

$$\mathcal{M} = \mathcal{U}'_S \frac{i\lambda^2}{M^2}\left[1 + \frac{(p_1 - p_3)^2}{M^2} + \cdots\right] - (3 \leftrightarrow 4)$$

$$\mathcal{L}_{\rm EFT} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) + \boxed{\frac{c^{(8)}}{M^4}\left(\partial_\mu\bar{\psi}\partial^\mu\psi\right)\left(\bar{\psi}\psi\right)}$$

EFT

$$\mathcal{M} = \mathcal{U}'_S \frac{ic_S}{M^2} + \mathcal{U}'_S\left(\frac{-ic^{(8)}}{M^4}\right)(p_1 \cdot p_3 + p_2 \cdot p_4) - (3 \leftrightarrow 4)$$

Matching condition
$$\boxed{c^{(8)} = \lambda^2}$$

- EFTs can be systematically improved by higher dimension operators and higher order calculations