XVI NExT PhD Workshop

# The Impact of Drell-Yan Data on Four-Fermion SMEFT Operators


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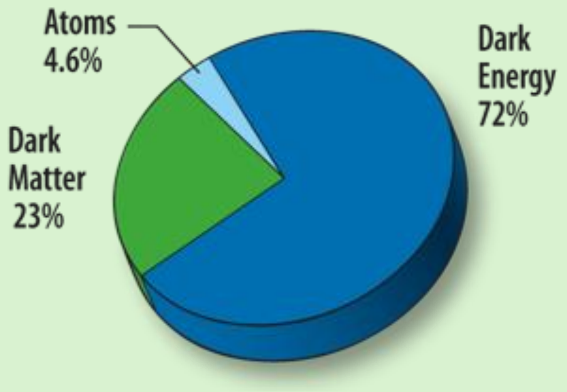
# Standard Model Effective Field Theory (SMEFT)

- SM is the most successful theory in particle physics

- **<u>However</u>**, it cannot explain several observed phenomena
  - Matter-Antimatter asymmetry
  - Neutrino Oscillations
  - Gravity
  - Dark Matter
  - Dark Energy
  - …



# Standard Model Effective Field Theory (SMEFT)

- Many SM extensions predict heavy new particles
- SMEFT extends the SM in a model-independent way

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

- $\Lambda$ is the energy scale of new physics (here fixed at 1 TeV)
- $C_i$ are the so-called ***Wilson coefficients***
- $O_i^{(5)}$ and $O_i^{(7)}$ are not considered because they violate either leptonic or baryonic number

# Standard Model Effective Field Theory (SMEFT)

- We can search for SMEFT effects in precise measurements of SM processes

- Computing the matrix element squared:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\mathrm{SM}}|^2 + 2\,\mathrm{Re}\left(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}^{(6)}_{\mathrm{SMEFT}}\right) + |\mathcal{M}^{(6)}_{\mathrm{SMEFT}}|^2 + \cancel{2\,\mathrm{Re}\left(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}_{\mathrm{SMEFT}}\right)} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Linear / Interference

($c_i$, ...)

($\sim 1/\Lambda^2$)

Squared / 'Pure SMEFT'

($c_i^2$, $c_i c_j$)

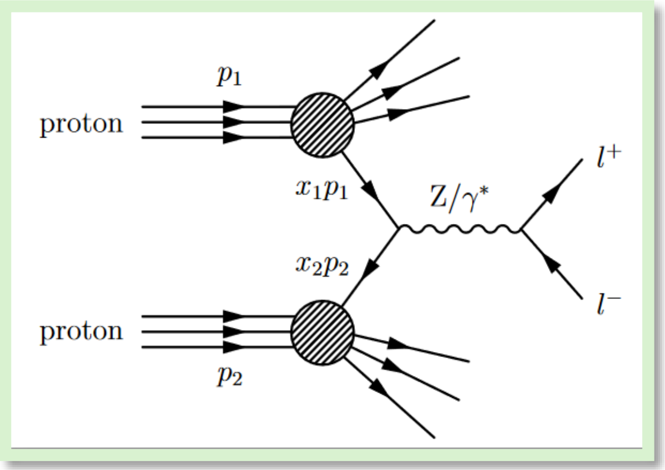
($\sim 1/\Lambda^4$)

Previous works on the topic:
arXiv:2212.10497, 2103.10532, 2204.1312,
2008.12978, 2303.08257, 2502.12250, 2310.19638

- At dimension 6, there are 59 different operators

# The Drell-Yan (DY) Process

- DY has a simple final state
- Many measurements of DY have been performed at colliders
- Only 10 relevant four-fermion operators




*Neutral DY process*

# Four fermion operators in DY

- Only 10 relevant four-fermion operators
- Varying chirality, quark flavour, Lorentz structures
- Many free parameters $\Rightarrow$ include multiple measurements

**Operators** $(LL)(LL)$

$Q_{\ell q}^{(1)} = (\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$

Charged Current DY

$Q_{\ell q}^{(3)} = (\bar{\ell}_p \gamma_\mu \tau^I \ell_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

**Operators** $(RR)(RR)$

$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$

$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$

**Operators** $(LL)(RR)$

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$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$

**Operators** $(LR)(LR)$ **and** $(LR)(RL)$

$Q_{ledq} = (\bar{\ell}_p^j e_r)(\bar{d}_s q_t^j)$

$Q_{lequ}^{(1)} = (\bar{\ell}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$

$Q_{lequ}^{(3)} = (\bar{\ell}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Excluded in the linear

[+] *Dimension-Six Terms in the Standard Model Lagrangian - arXiv:1008.4884v3*

$$\frac{d^3\sigma}{dm_{\ell\ell}\,dy\,d\cos\theta^*}$$

- All four-fermion operators grow with mll
- Up-type operators and down-quark operators have different y dependency
- Right and left operators have different cosθ* dependency

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# Input Observables and Measurements

| Exp. | $\sqrt{s}$ (TeV) | Type | Observable | Mass range (GeV) | L ($fb^{-1}$) | Ref. |
|---|---|---|---|---|---|---|
| ATLAS | 8 | NC | $d^2\sigma / (dm_{ll}\, dy)$ | 116–1500 | 20.3 | 1606.01736 |
| ATLAS | 8 | NC | $d^3\sigma / (dm_{ll}\, dy\, d\cos\theta^*)$ | 46–200 | 20.2 | 1710.05167 |
| ATLAS | 13 | CC | $d^2\sigma / (dm_T\, dy)$ | 200-5000 | 140 | 2502.21088 |
| CMS | 13 | NC | $A_0$, $A_{FB}$ / $dm_{ll}$ | 170–13000 | 140 | 2202.12327 |
| CMS | 13 | NC | $d\sigma / dm_{ll}$ | 122-6020 | 140 | 2103.02708 |
| LEP, CHARM, VENUS, PDG and more | Low energy | Compilation of neutrino scattering ratio, atomic parity violation, quark pair production… | | 0-209 | / | 1706.03783 |

**Considering only the electron channel and assuming flavour-universality**

# Fitting strategy

- For the fit, we use the *signal strength* $\mu$:
- And minimize the $\chi^2$:

$$\boxed{\mu \equiv \frac{\sigma_{\text{exp}}}{\sigma_{\text{SM}}}}$$

$$\boxed{\chi^2(\vec{C}) = \left(\vec{\mu}_{\text{data}} - \vec{\mu}_{\text{th}}(\vec{C})\right)^T \mathbf{V}^{-1} \left(\vec{\mu}_{\text{data}} - \vec{\mu}_{\text{th}}(\vec{C})\right)}$$

- Where:
  - C is a vector of the Wilson coefficients
  - V is the full covariance matrix
  - $\mu_{th}$ is defined as:

$$\mu_{\text{th}}(\vec{C}) = 1 + \sum_i C_i\, \mu_i^{\text{INT}} + \sum_{i \le j} C_i C_j\, \mu_{ij}^{\text{SQR}}$$

- For the **linear terms**, we can minimize the $\chi^2$ *analytically*

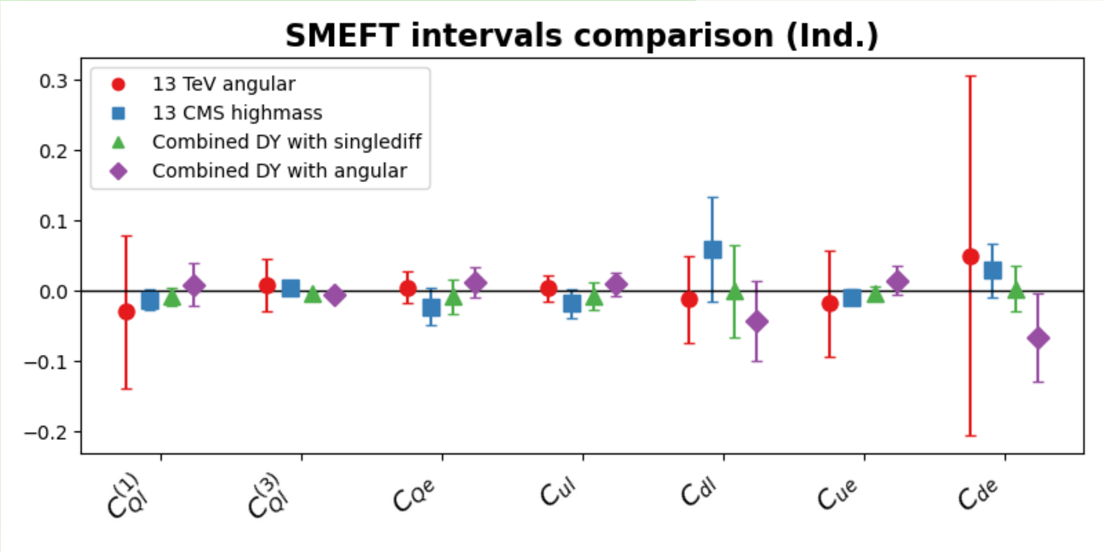
# Input Observables and Measurements

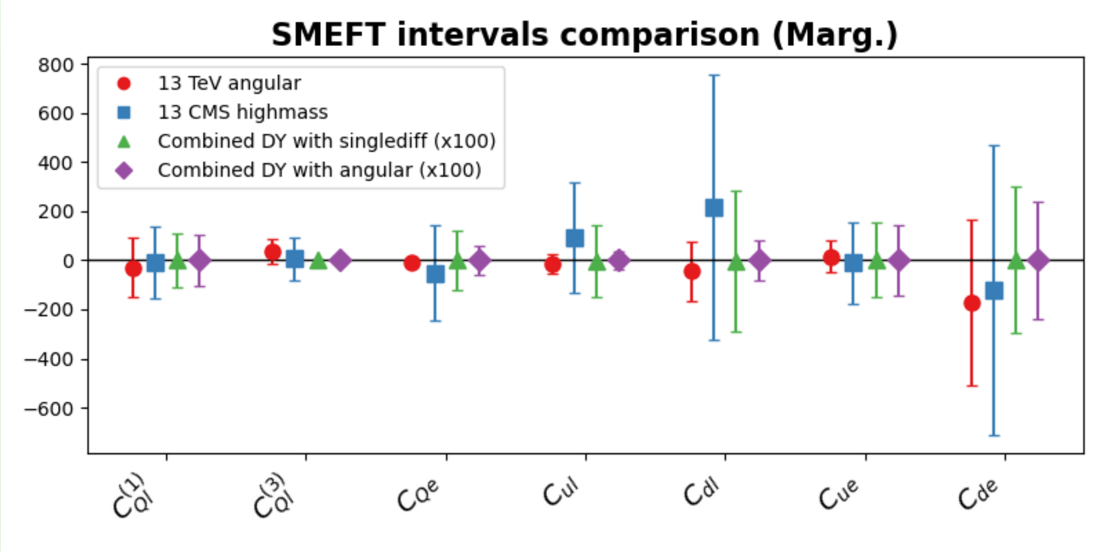
- Is it worth it to include lower energy datasets, now that we have 13 TeV measurements?

- What's the optimal way to include 13 TeV data?

  (Single differential vs angular coefficients)

# Results at *linear* level



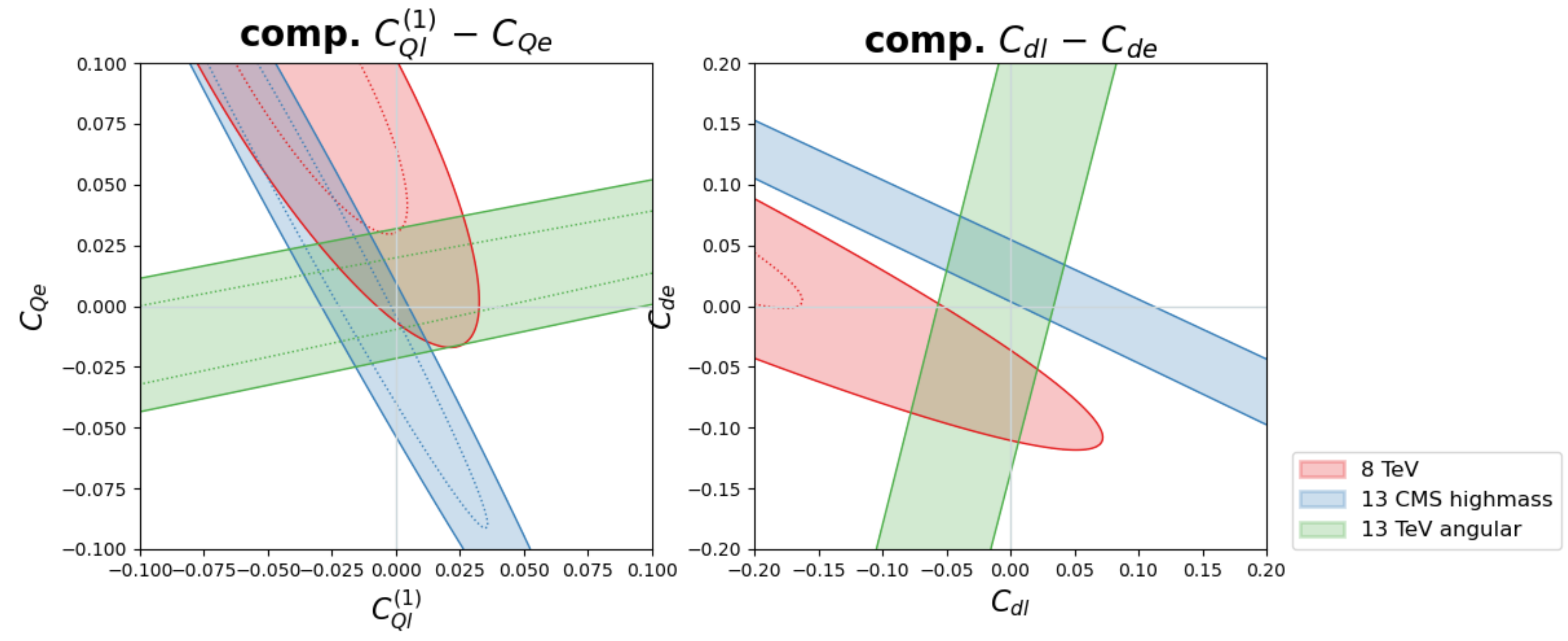

- For both **individual** (one parameter at a time) and **marginalized** (all parameters are allowed to vary) fits, 13 TeV datasets alone provide worst constraints

- The single differential dataset gives best constraints on the individual bounds, the angular dataset on the marginalized ones

# Results at *linear* level

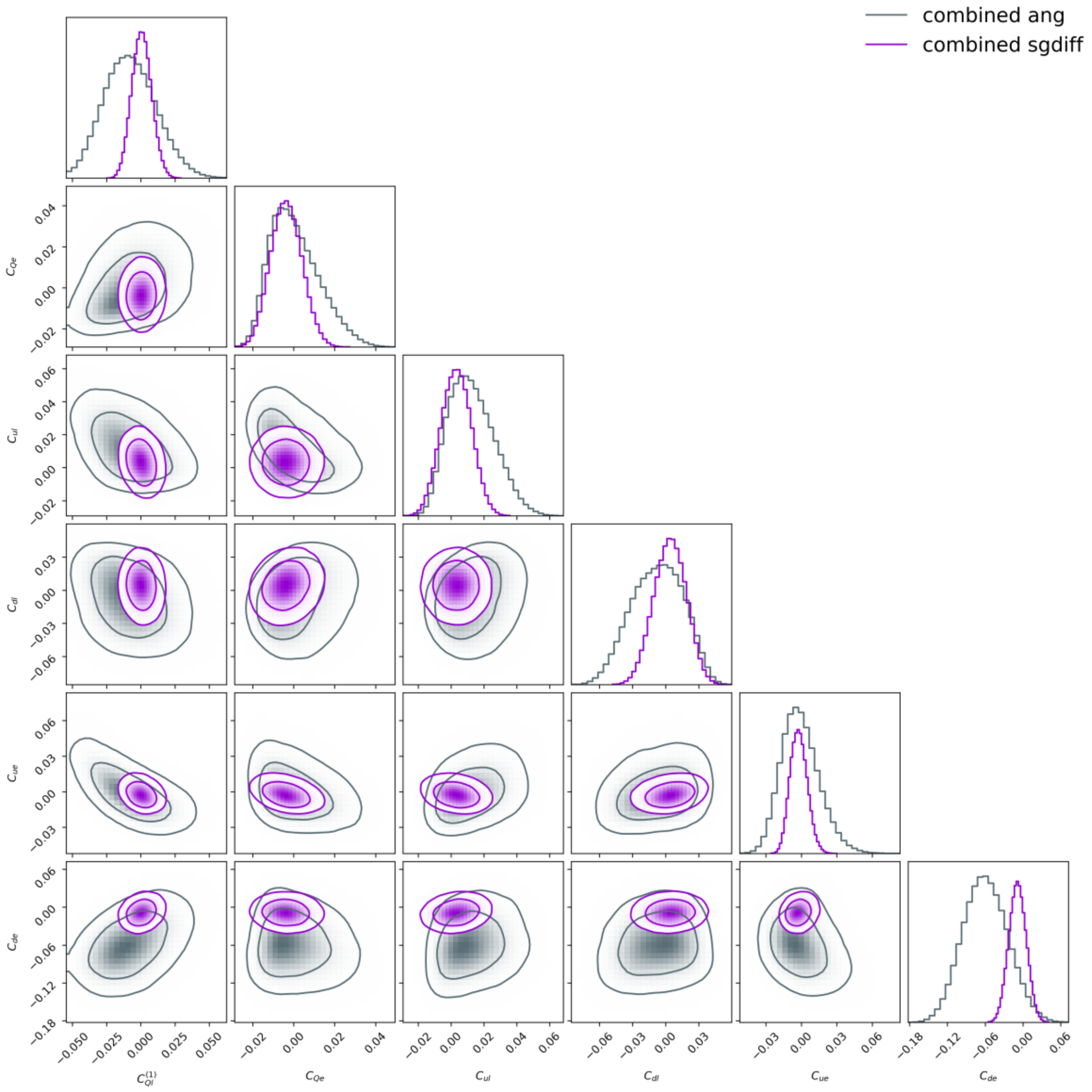
- The single differential dataset provides constraints in the *same* direction as the 8 TeV data

- The angular dataset, on the other hand, provides information in the *orthogonal* direction



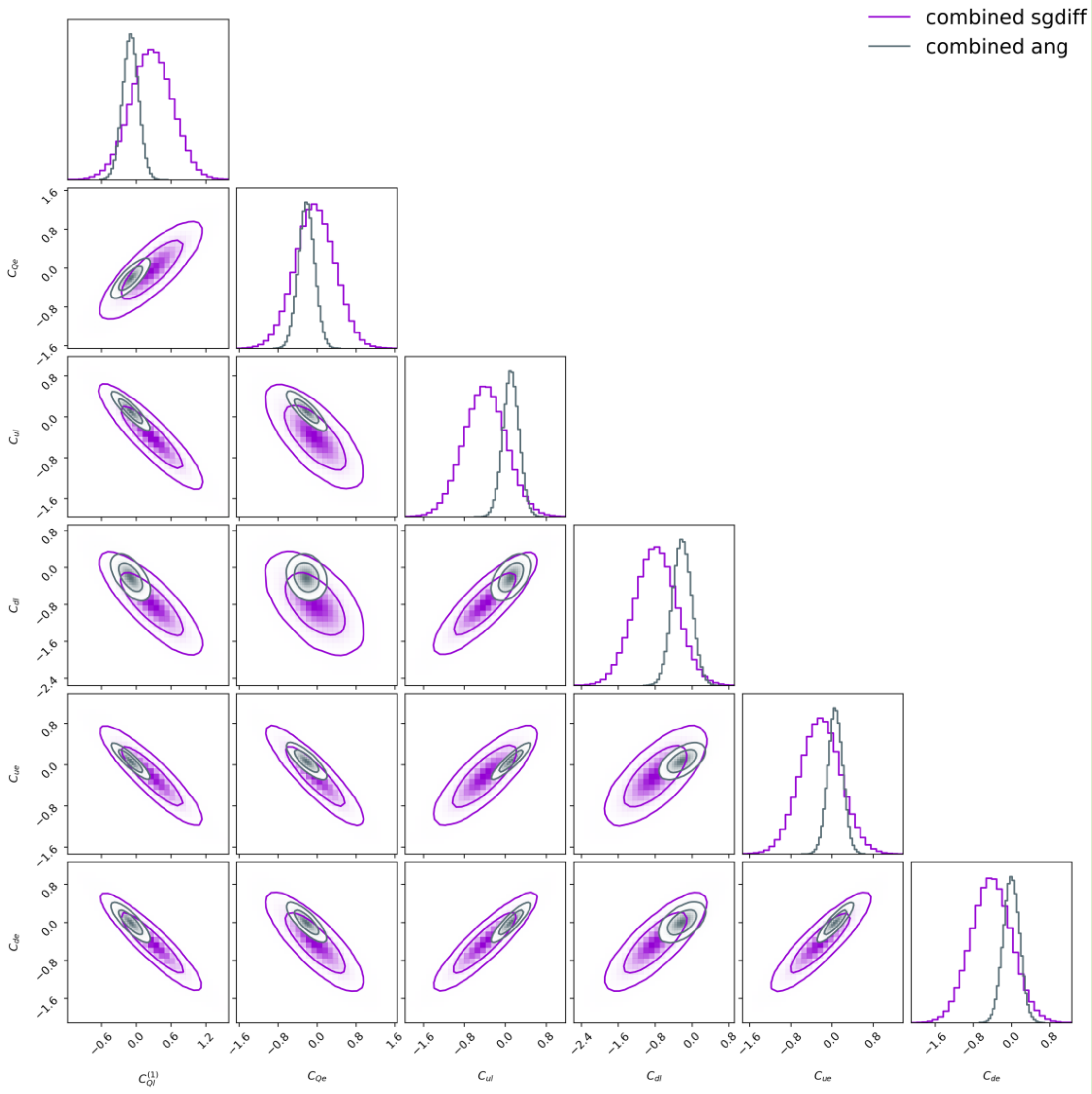
# How do things change at *quadratic* level?

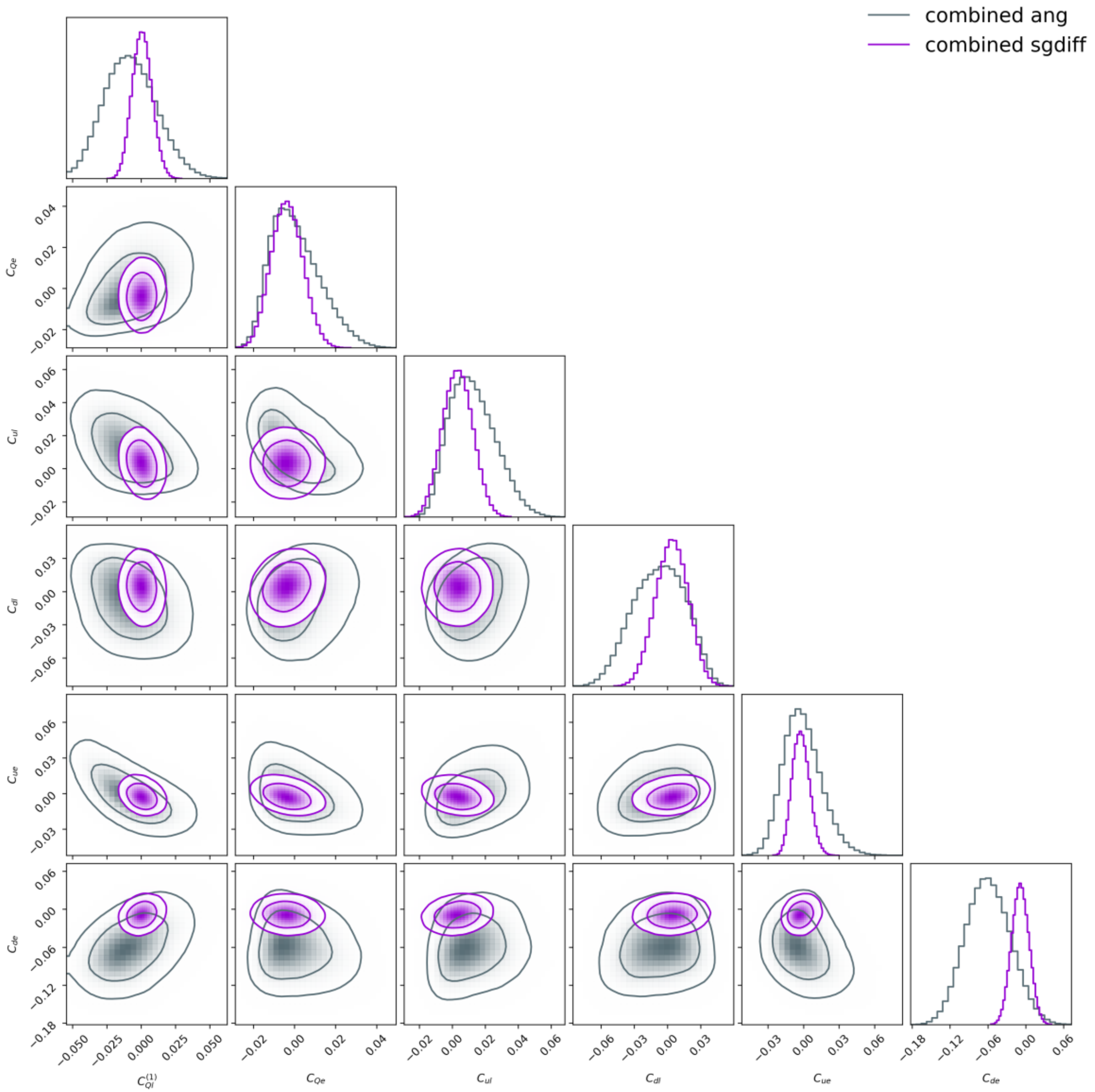
- At quadratic level, comparing the lower energy data with single-differential cross section measurement at 13 TeV gives the best constraints
- Combining the same data with the angular measurements results less powerful
- This is probably due to the size of quadratic effects in the distribution tail




Preliminary

# At linear level




*Preliminary*

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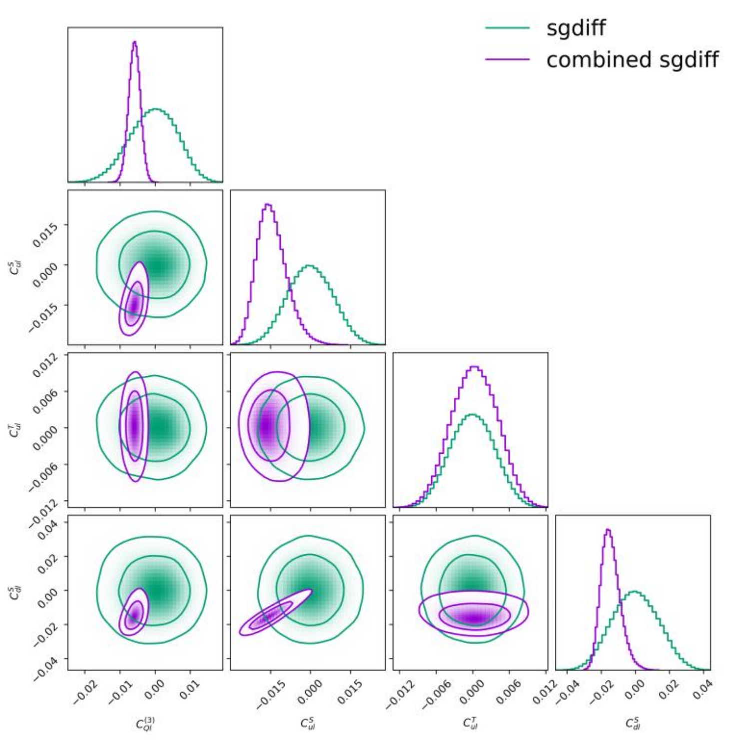
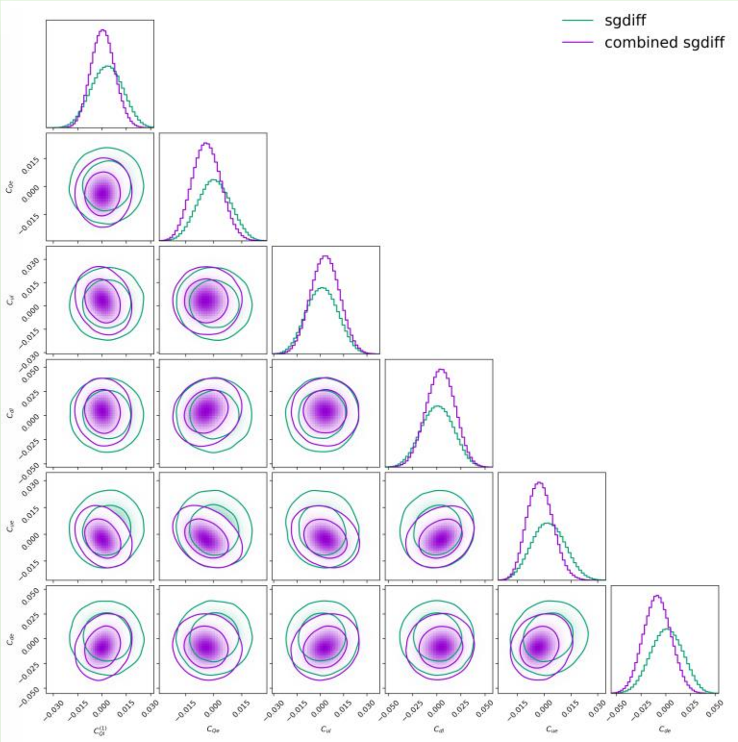



*Preliminary*

- Lower energy data and 13 TeV charged DY constraint certain coefficients particularly well (cQl$^{(3)}$, cult, culs, cdls)
- This slightly improve the constraint on the other coefficients as well
- It *is* worth it to include complementary datasets

# Conclusion & Next Steps

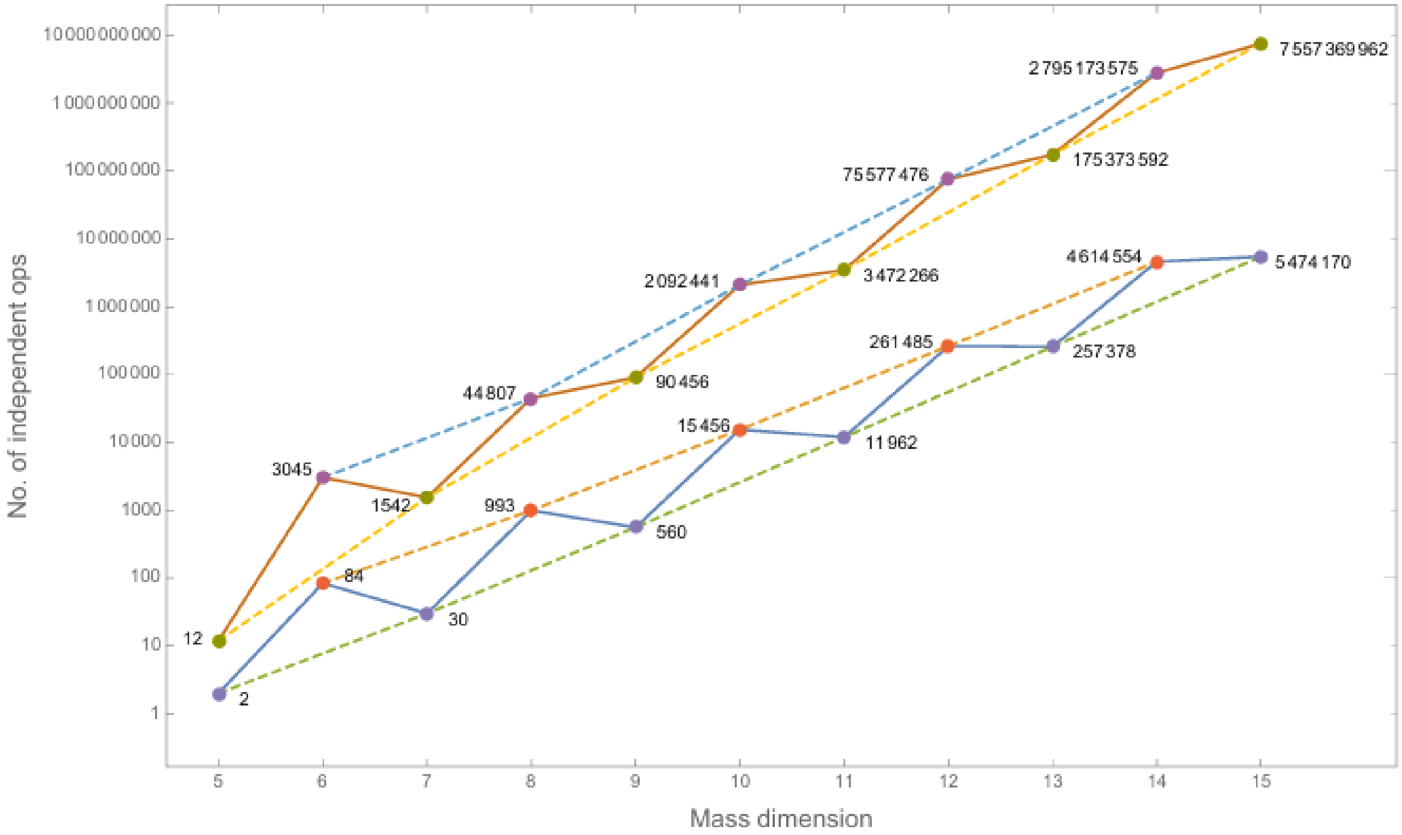
- I have presented results from a SMEFT fit to **8 TeV, 13 TeV Drell–Yan data and other low energy measurements**
- Highlighted the importance of complementary measurements, even when high energy data are available
- Found that although measurements of angular coefficients constraint Wilson coefficients well at linear level, at quadratic level single differential cross section is more powerful

**Outlook**

- Broaden the analysis including other operators relevant for DY
- Quantify the impact of NLO SMEFT terms
- Address flat directions further by including more LHC measurements (ATLAS)

# Backup Slides

# How many SMEFT operators?




Growth of the number of independent operators in the SM EFT up to mass dimension 15. Points joined by the lower solid line are for one fermion generation; those joined by the upper solid line are for three generations. Dashed lines are to guide the eye to the growth of the even and odd mass dimension operators in both cases.

*Ref: arXiv:1512.03433v2*

# Helicity Amplitudes of DY Subprocesses

$\bar{u}u \to e^+e^-$

$$\{\bar{R}L\bar{L}L\},\ \{\bar{R}L\bar{R}R\},\ \{\bar{L}L\bar{L}R\},\ \{\bar{R}R\bar{R}L\},\quad 0$$

$$\{\bar{L}R\bar{R}R\},\ \{\bar{L}R\bar{L}L\},\ \{\bar{R}R\bar{L}R\},\ \{\bar{L}L\bar{R}L\},\quad 0$$

$$\{\bar{R}L\bar{R}L\},\ \{\bar{L}R\bar{L}R\},\quad 0$$

$$\{\bar{R}L\bar{L}R\}\quad -\frac{4ic_{ulS}^{1111}\,s\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$$\{\bar{L}L\bar{L}L\}\quad \frac{is(c_{Ql1x}^{1111}-c_{Ql3x}^{1111})(1+\cos\theta)\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}+\frac{ie^2\,(1+\cos\theta)\,\delta_{\mathrm{Col1,Col2}}(3c_w^4 s+4c_w^2 s_w^2(s-2M_Z^2)+ss_w^4)}{12c_w^2 s_w^2(M_Z^2-s)}$$

$$\{\bar{L}L\bar{R}R\}\quad \frac{ie^2\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}(c_w^2(4M_Z^2-s)-ss_w^2)}{6c_w^2(M_Z^2-s)}-\frac{2ic_{Qe}^{1111}s\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$$\{\bar{R}R\bar{L}L\}\quad \frac{ie^2\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}(c_w^2(2M_Z^2-s)-ss_w^2)}{3c_w^2(M_Z^2-s)}-\frac{ic_{ul}^{1111}s\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$$\{\bar{R}R\bar{R}R\}\quad \frac{2ie^2\,(1+\cos\theta)\,\delta_{\mathrm{Col1,Col2}}(c_w^2(M_Z^2-s)-ss_w^2)}{3c_w^2(M_Z^2-s)}-\frac{ic_{ue}^{1111}s\,(1+\cos\theta)\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$$\{\bar{L}R\bar{R}L\}\quad \frac{4ic_{ulS}^{1111}\,s\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$\bar{d}d \to e^+e^-$

$$\{\bar{R}L\bar{L}L\},\ \{\bar{R}L\bar{R}R\},\ \{\bar{L}L\bar{L}R\},\ \{\bar{R}R\bar{R}L\},\quad 0$$

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$$\{\bar{R}L\bar{L}R\},\ \{\bar{L}R\bar{R}L\},\quad 0$$

$$\{\bar{R}L\bar{R}L\}\quad \frac{-4ic_{dls}^{1111}s\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

$$\{\bar{L}L\bar{L}L\}\quad \frac{is(c_{Ql1x}^{1111}+c_{Ql3x}^{1111})(\cos\theta+1)\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}-\frac{ie^2\left(\frac{1+\cos\theta}{2}\right)\delta_{\mathrm{Col1,Col2}}\left(3c_w^4 s+2c_w^2 s_w^2(s-2M_Z^2)-ss_w^4\right)}{6c_w^2 s_w^2(M_Z^2-s)}$$

$$\{\bar{L}L\bar{R}R\}\quad \frac{ic_{Qe}^{1111}s(\cos\theta-1)\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}+\frac{ie^2(\cos\theta-1)\delta_{\mathrm{Col1,Col2}}\left(c_w^2(2M_Z^2+s)+ss_w^2\right)}{6c_w^2(M_Z^2-s)}$$

$$\{\bar{R}R\bar{L}L\}\quad \frac{i\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}\,c_{dl}^{1111}s}{\Lambda^2}-\frac{i\,(1-\cos\theta)\,\delta_{\mathrm{Col1,Col2}}\,EL^2\left(cw^2(s-2MZ^2)+s\,sw^2\right)}{6\,cw^2(MZ^2-s)}$$

$$\{\bar{R}R\bar{R}R\}\quad \frac{2}{3}ie^2\left(\frac{1+\cos\theta}{2}\right)\delta_{\mathrm{Col1,Col2}}\left(\frac{ss_w^2}{c_w^2(M_Z^2-s)}-1\right)-\frac{2ic_{de}^{1111}s\left(\frac{1+\cos\theta}{2}\right)\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

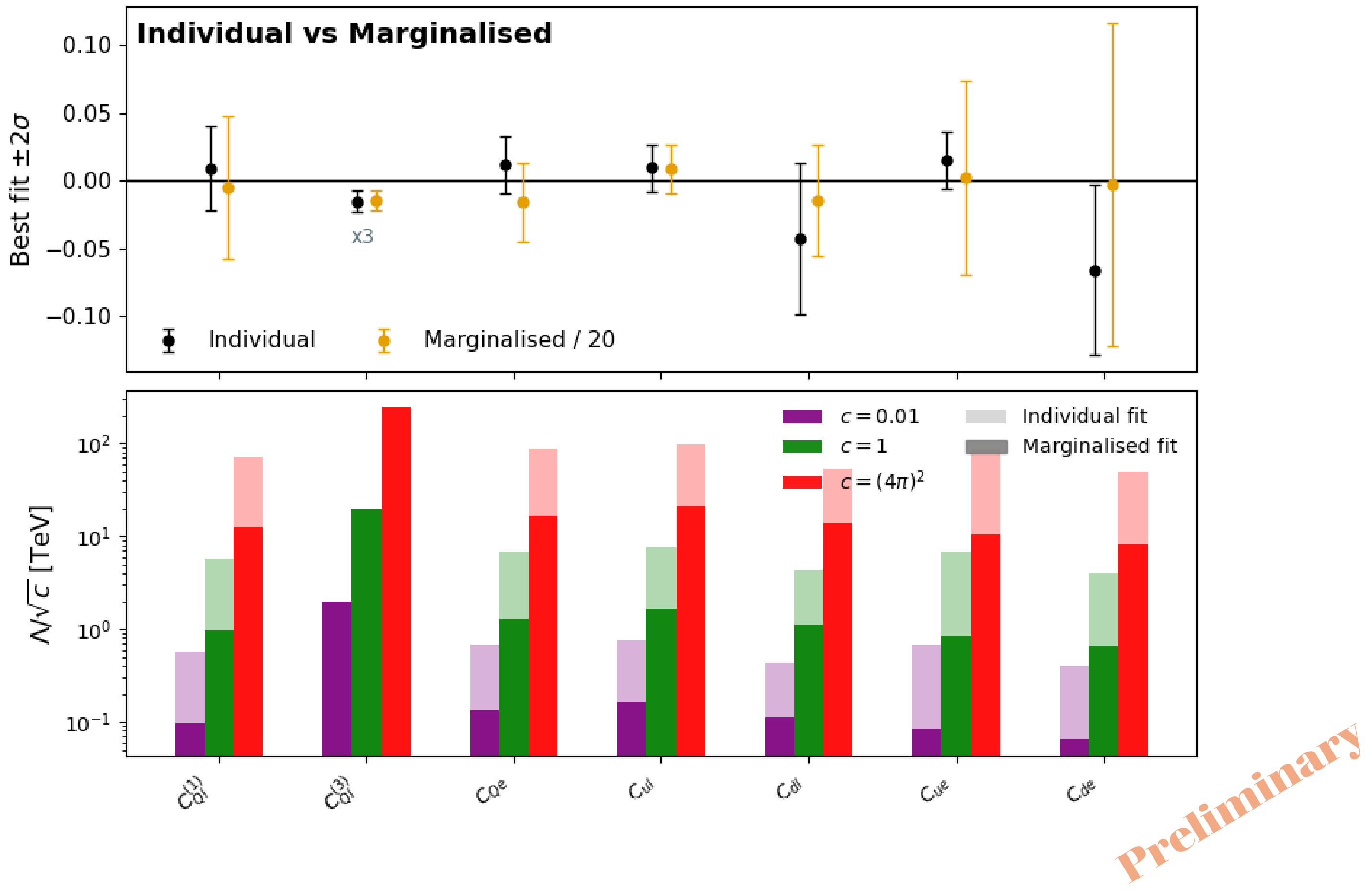
$$\{\bar{L}R\bar{L}R\}\quad \frac{4ic_{dls}^{1111}s\,\delta_{\mathrm{Col1,Col2}}}{\Lambda^2}$$

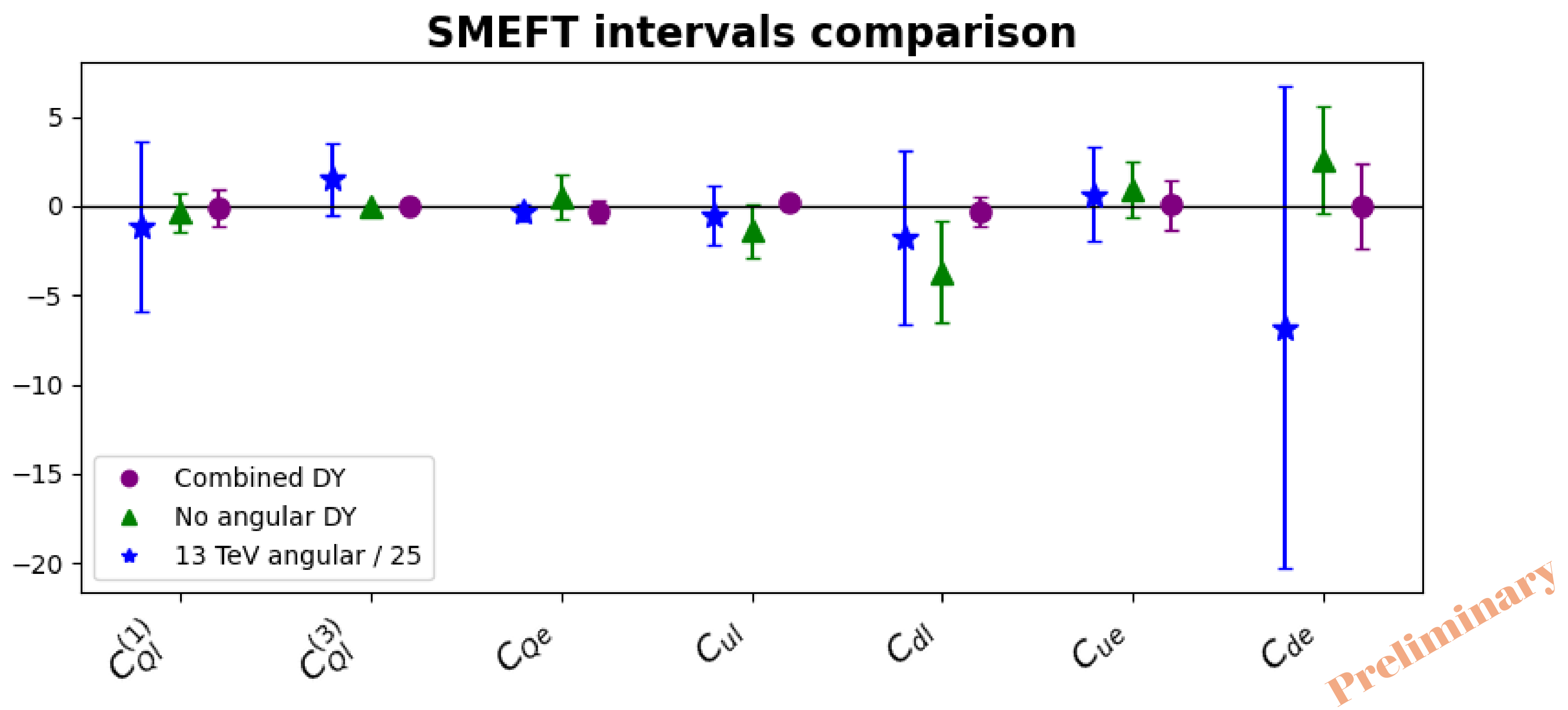
These are the subprocesses helicity amplitudes at order *~1/Λ²*
(SM is marked in *black*, interference terms are in *red*)

# Results

- <u>Complete</u> **linear** fit:
    - Individual vs marginalised

- New physics scale:
    - Individual vs marginalised
    - Different values of the coupling






# Results

- We can compare the fit results with and without the dataset 'angular 13 TeV'
- The 13 TeV angular data improves the marginalized fit substantially

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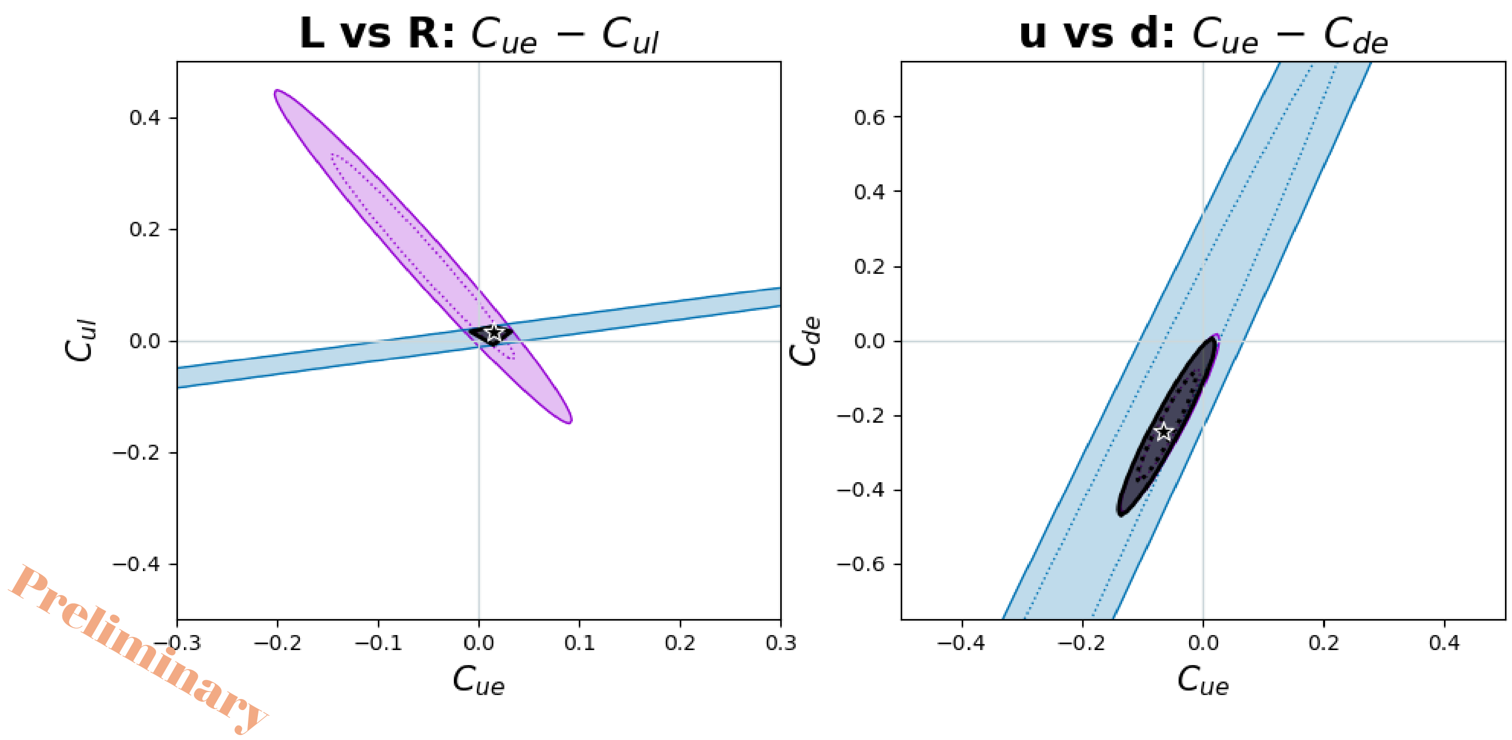
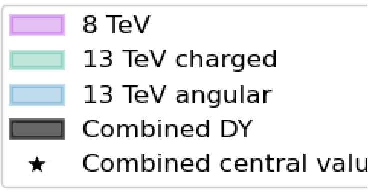
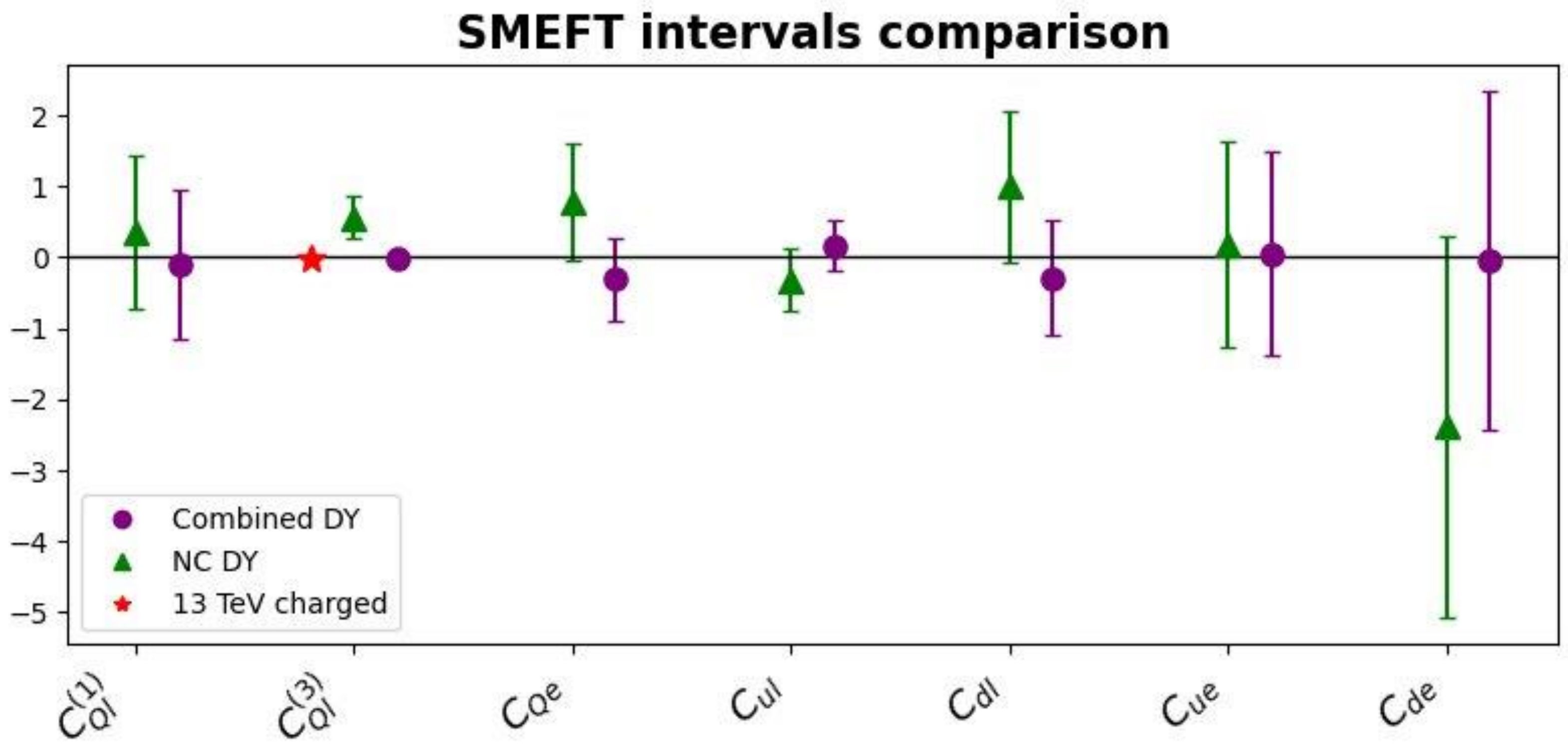



- Using differential observables helps to solve flat directions
- This improves constraints significantly
- 13 TeV angular is not differential in $y_{ll}$



# Impact of 13 TeV charged on the marginalized fit



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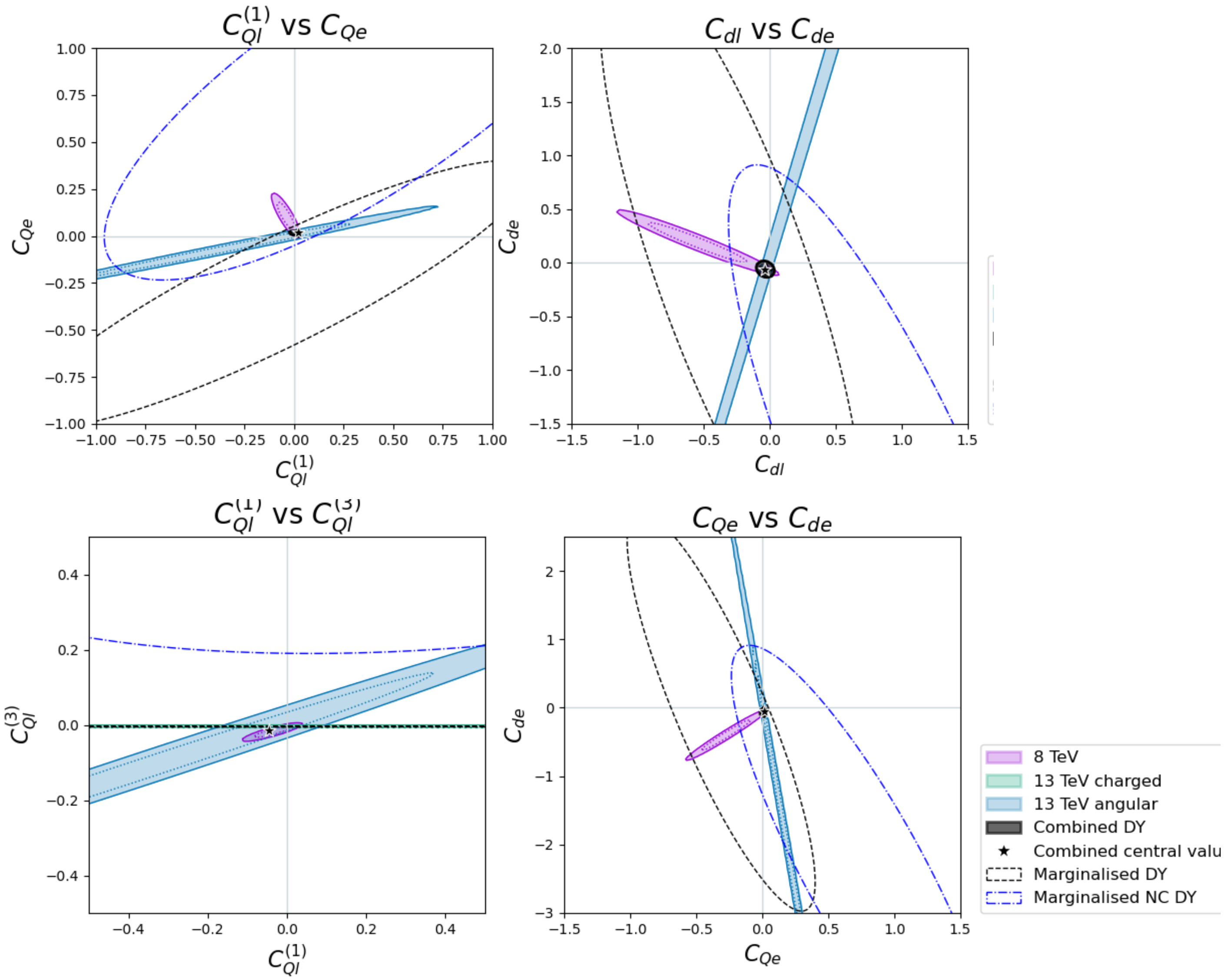


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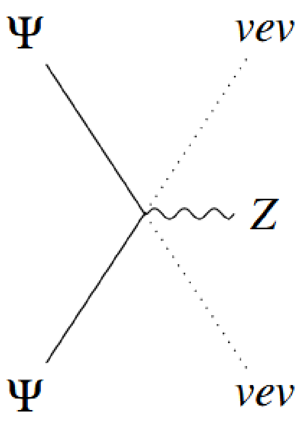
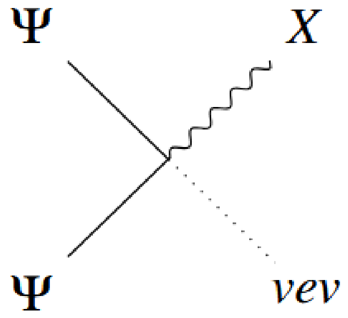
# Dim-6 SMEFT Operators in DY

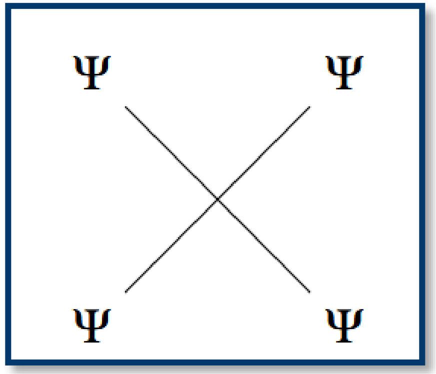
There are three classes of dim-6 operators for DY:




- $\Psi^2\phi^2 D$: these operators include a single derivative, a Higgs doublet and two fermions
  - They shift the SM coupling of the fermions to gauge bosons
  - *They have already been constrained at LEP*




- $\Psi^2 X\phi$: two fermions coupled to a gauge boson ($X = B_{\mu\nu}$, $G_{\mu\nu}$, $W_{\mu\nu}$) and a Higgs.
  - *They don't interfere with the SM*




- **$\Psi^4$: four-fermion operators**
  - **10 different operators (7 interfere with the SM)**
  - **They grow with $E^2/\Lambda^2$**

# Lam-Tung Relation

$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1\sin(2\theta)\cos\phi + \frac{A_2}{2}\sin^2\theta\cos(2\phi)$$
$$+ A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big]$$

- At LO, for the Drell-Yan process:

$$\boxed{A_0 = A_2 = 0}$$

- The only non-zero SMEFT contributions are:

**Operators** $(LR)(LR)$ **and** $(LR)(RL)$

$$Q_{ledq} = (\bar{\ell}_p^j e_r)(\bar{d}_s q_t^j)$$

$$Q_{lequ}^{(1)} = (\bar{\ell}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$$

$$Q_{lequ}^{(3)} = (\bar{\ell}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$$

# Analytical Fit

$$\mu(c) = \mu_{\mathrm{SM}} + H \cdot c$$

$$\boxed{\chi^2(c) = \chi^2_{\mathrm{SM}} - 2\, c^\top \cdot \omega + c^\top \cdot F \cdot c}$$

$$\omega = H^\top \cdot V^{-1} \cdot (y - \mu_{\mathrm{SM}}) \qquad F = H^\top \cdot V^{-1} \cdot H$$

$$\nabla_c \chi^2(c) = -2\,\omega + 2\,F \cdot c = 0$$

$$\boxed{\Rightarrow \quad \hat{c} = F^{-1} \cdot \omega = \left(H^\top \cdot V^{-1} \cdot H\right)^{-1} \cdot H^\top \cdot V^{-1} \cdot (y - \mu_{\mathrm{SM}})}$$

# Analytical Fit

$$\mu(c) = \mu_{\text{SM}} + H \cdot c$$

$$\chi^2(c) = \chi^2_{\min} + (c - \hat{c})^\top \cdot \hat{w} + (c - \hat{c})^\top \cdot F \cdot (c - \hat{c})$$

$$\hat{w} = \nabla_c \chi^2(c)\Big|_{c=\hat{c}} = 0$$

$$F = \frac{1}{2}(\nabla_c \otimes \nabla_c)\,\chi^2(c) = H^\top \cdot \tilde{V} \cdot H$$

$$\chi^2_{\min} = \chi^2(\hat{c}) = \chi^2_{\text{SM}} - 2\,\hat{c}^\top \cdot \omega + \hat{c}^\top \cdot F \cdot \hat{c} = \chi^2_{\text{SM}} - \hat{c}^\top \cdot F \cdot \hat{c}$$

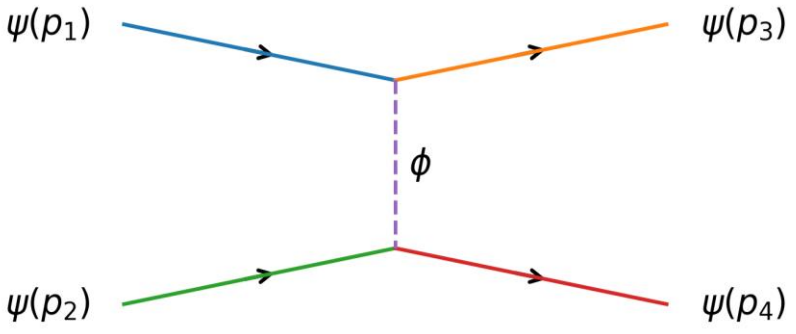
$$F^{-1} \equiv U$$

# A Toy Effective Field Theory+

Let's consider this toy model:

$$S = \int d^4x \left\{ -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} M^2 \phi^2 + \bar{\Psi}\,(i\gamma^\mu \partial_\mu - m)\Psi + iy\,\phi\,\bar{\Psi}\gamma^5 \Psi - \frac{1}{4!}\,\lambda\phi^4 \right\}$$

- Ψ is a fermionic field with small mass
- Φ is a scalar field with high mass m << M
- *y*, λ << 1 so that we can work in perturbation theory




The matrix element for an elastic scattering of two fermions

$$i\mathcal{M} = (-y)^2 \left\{ \bar{u}_{\lambda_3,p_3}\gamma^5\, u_{\lambda_1,p_1} \frac{-i}{(p_3-p_1)^2 + M^2}\, \bar{u}_{\lambda_4,p_4}\gamma^5\, u_{\lambda_2,p_2} - (3 \leftrightarrow 4) \right\}$$

+*An Introduction to Effective Field Theories - arXiv:2006.16285v1*

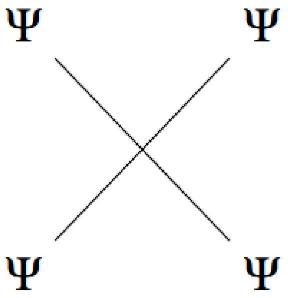
# A Toy Effective Field Theory

The scalar propagator can be expanded in powers of momenta (considering $m^2 \ll p^2 \ll M^2$).
The first term is:

$$i\mathcal{M} \simeq -\frac{i\,y^2}{M^2}\left[\bar{u}_{\lambda_3,p_3}\gamma^5 u_{\lambda_1,p_1}\,\bar{u}_{\lambda_4,p_4}\gamma^5 u_{\lambda_2,p_2} - (3 \leftrightarrow 4)\right] + \mathcal{O}\!\left(\frac{E^2}{M^2}\right)$$

We would have obtained the same result with the effective theory:

$$S_{\text{eff}} = \int d^4x \left\{ \bar{\Psi}(i\,\partial\!\!\!/ - m)\Psi \;-\; \frac{y^2}{2M^2}\left(\bar{\Psi}\gamma^5\Psi\right)^2 \right\}$$




- At low energy ($E \ll M$) the two theories are *indistinguishable*
- The effective theory does **not** postulate a scalar boson