UNIVERSITY OF SUSSEX

Unitarity at all Scales:

Towards 2-to-2 scattering of scalars in the Asymptotic Safety scenario

Angelo P. Chiesa, work w/ Jan M. Pawlowski & Manuel Reichert

based on 2603.10168 &
APC, Reichert in prep

“NExT Workshop” @Covener’s House, Abingdon - 24/08/2026

THE STAGE

Asymptotically Safe QG in a nutshell

Perturbative Non-Renormalizable GR

$$S_{EH}[g_{\mu\nu}] = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(2\Lambda - R)$$

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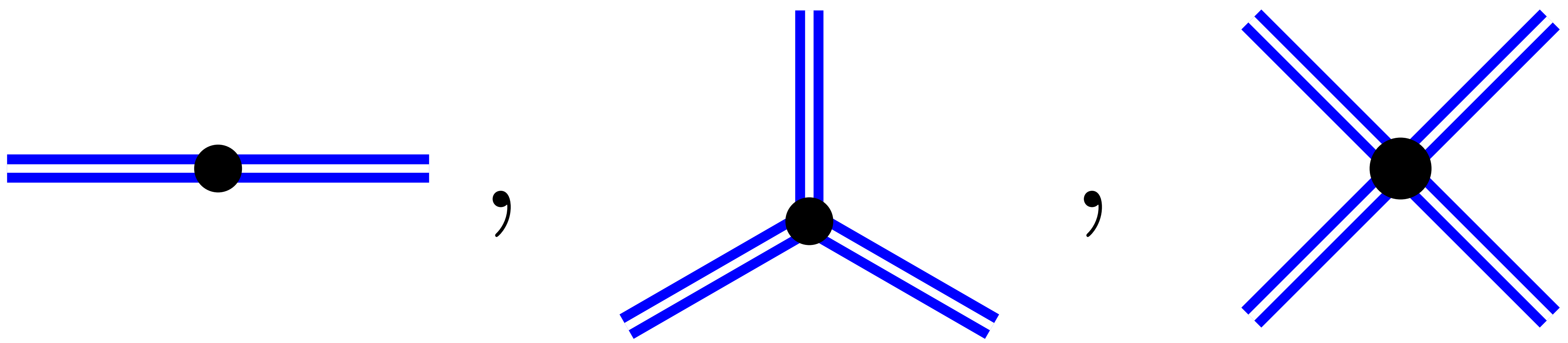
$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu} \Leftarrow \text{ Graviton}$$



$\cdots$

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Quantum Scale Symmetry

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[Weinberg '79] [Reuter '98]

$$\beta_g := \partial_t g = (2 + \eta_g) g$$
$$\beta_\lambda := \partial_t \lambda = (-2 + \eta_\lambda) \lambda$$

$$\lambda = \Lambda k^2$$
$$g = G_N k^{-2}$$

- Enhanced symmetry:

$$\beta_g = \beta_\lambda = 0 \text{ at } (g^*, \lambda^*)$$

Fixed Point

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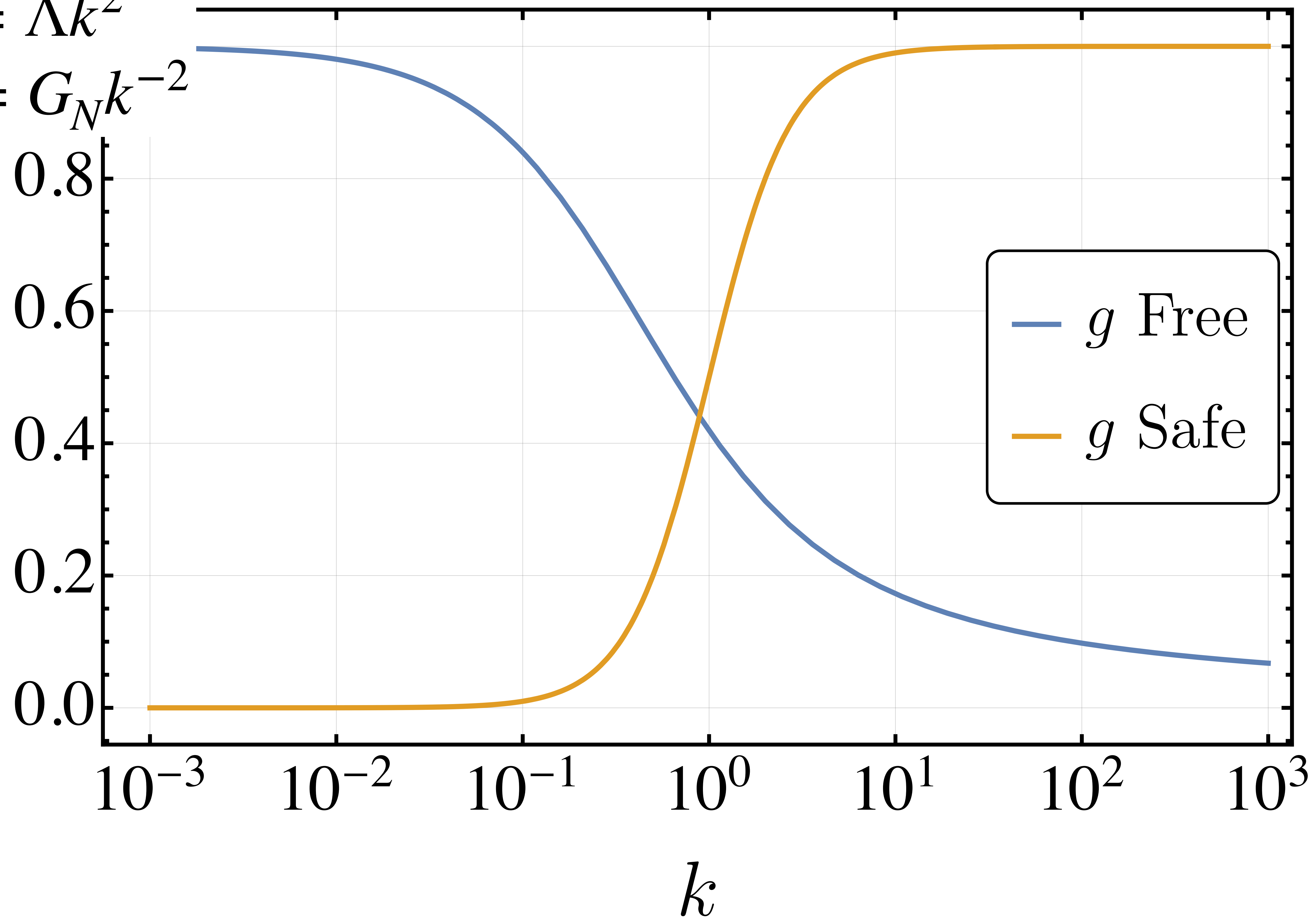

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- $d = 4$ **UV Non-perturbative**

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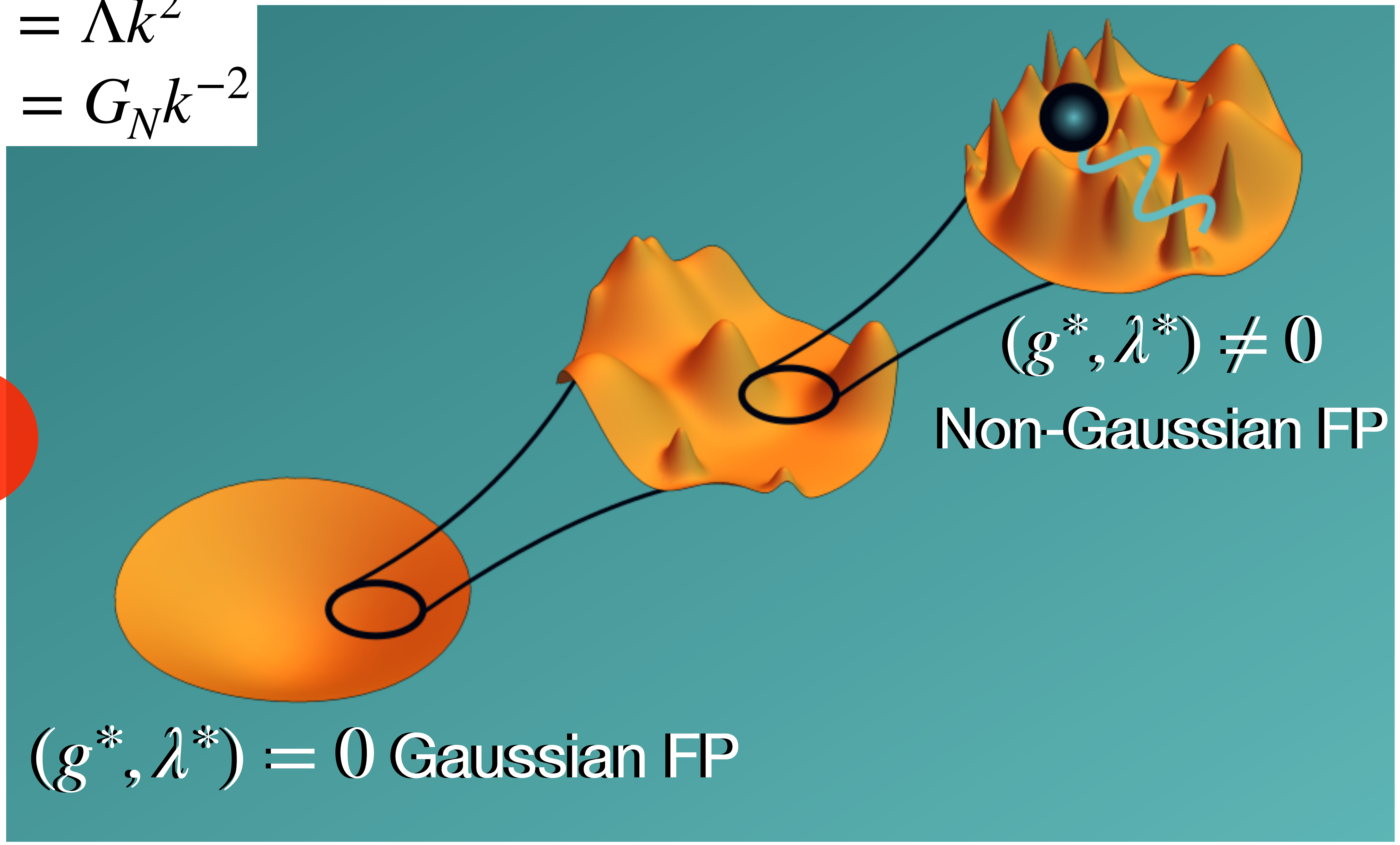

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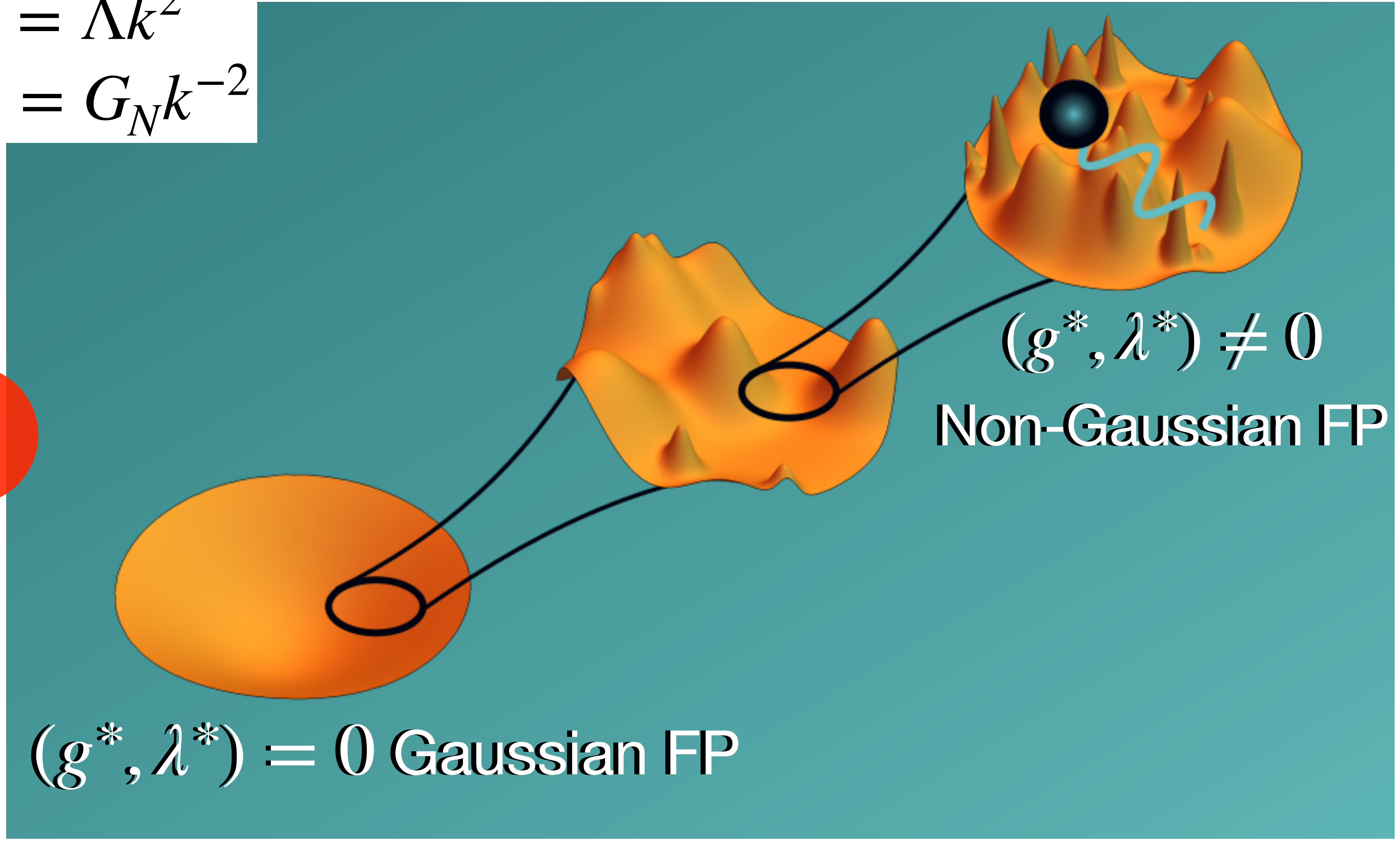

Image from Asymptotic Safety Seminars

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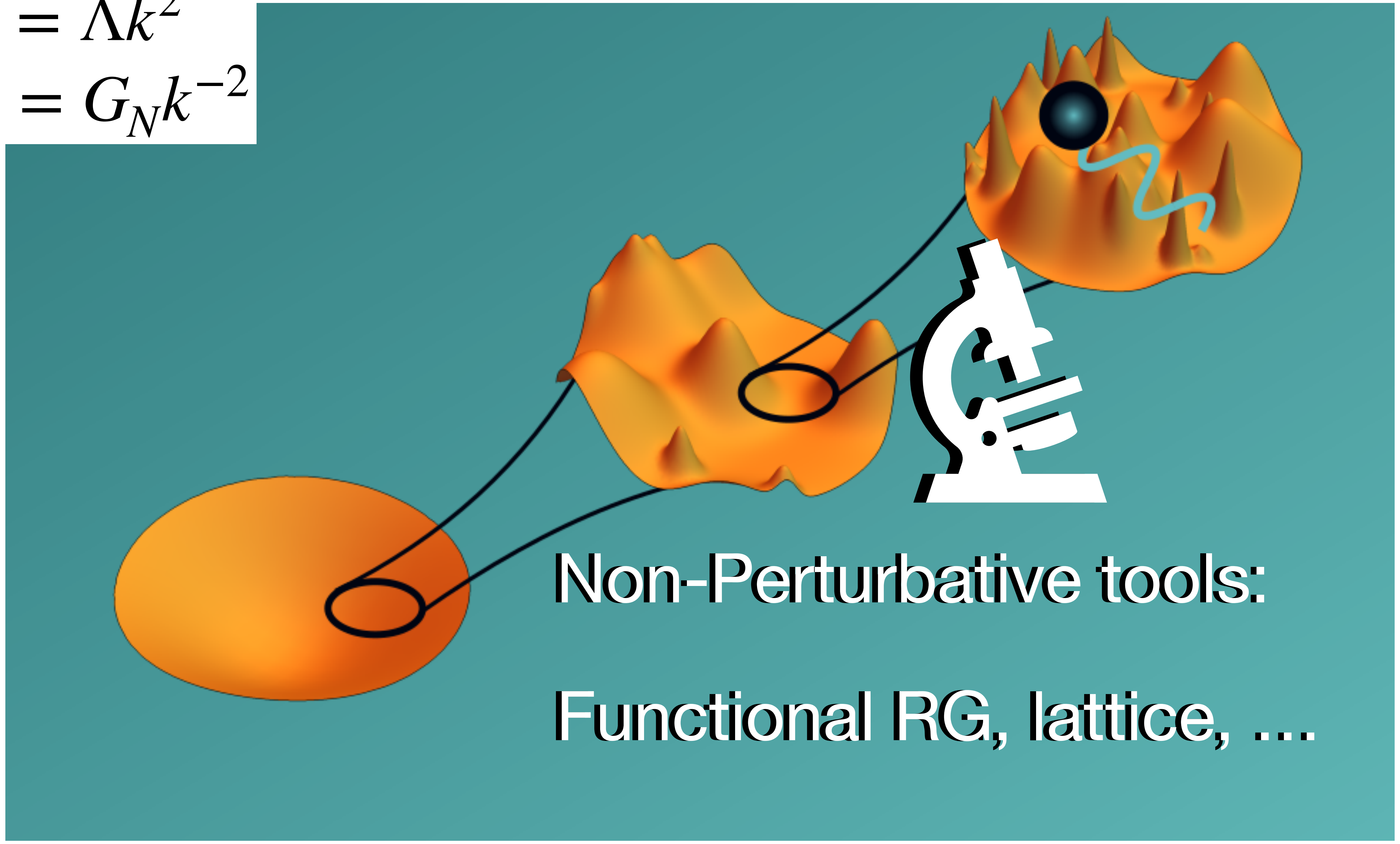


Image from Asymptotic Safety Seminars

Asymptotic Safety ~~Conjecture~~ Evidence

- Extended operators: $R^2, Ric^2, C^2, f(R), \ldots, R^{67}, \ldots$

[Dietz, Morris '13] [Ohta, Percacci, Vacca '15] [Falls, Ohta '16] [Demmel, Saueressig, Zanusso '15] [Gies, Knorr, Lippoldt, Saueressig '16] [Falls, Litim, Schröder '18] [Kluth, Litim '20] [Knorr '21] [Morris, Stulga '22] [Baldazzi, Falls, Kluth, Knorr '23]

- Coupling to Matter: Minimal Matter, Standard Model, BSM …

[Donà, Eichhorn, Percacci '13]] [Meibohm, Pawlowski, Reichert '15] [Eichhorn, Held '17] [Eichhorn, Schiffer '19] [Eichhorn, Schiffer '22] [Pastor-Gutiérrez, Pawlowski, Reichert '23]

- Lorentzian Studies

[Bonanno, Denz, Pawlowski, Reichert '22] [D'Angelo, Reijzner '23] [Fehre, Litim, Pawlowski, Reichert '23] [D'Angelo '23] [Wessely, Pawlowski, Reichert '25] [D'Angelo, Ferrero, Fröb '25] [Saueressig, Wang '25] [Kher, King, Litim, Reichert '25]

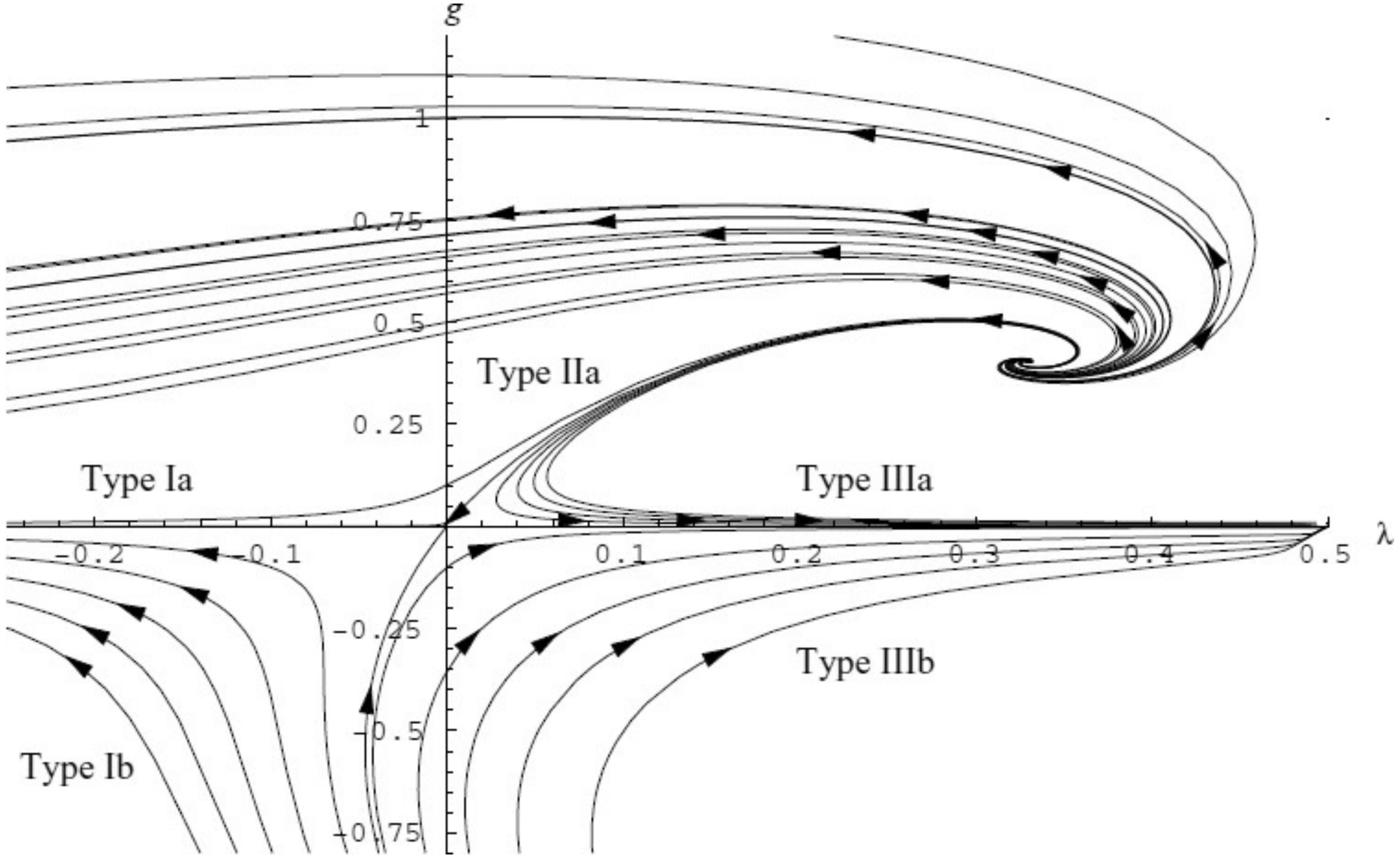
& much more…

Recent review: Eichhorn '26

Asymptotic Safety ~~Conjecture~~ Evidence

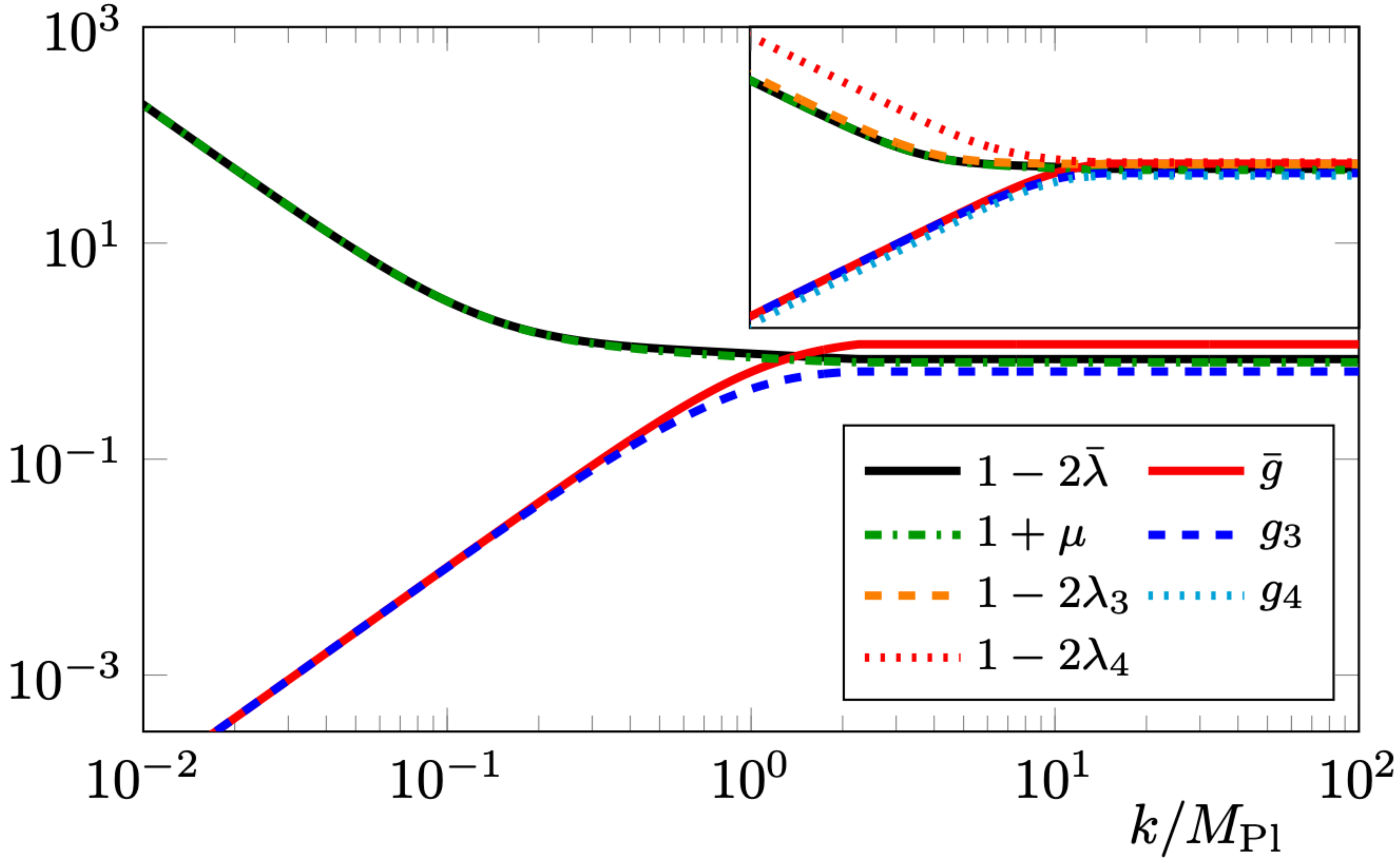
Phase diagram

[Reuter, Saueressig '01] [Niedermaier, Reuter '06]



Phase Diagram

[Denz, Pawlowski, Reichert '16]

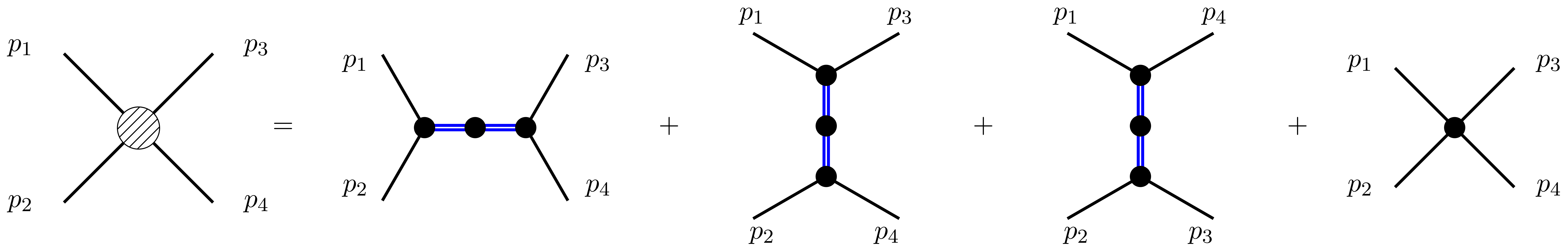


Running Couplings

Scattering in QG

$\phi\phi \to \phi\phi$

2-to-2-scalars: tree level diagrams $\Gamma_k = S_{EH} + S_{gf} + S_{gh} + \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\phi\partial^\mu\phi$



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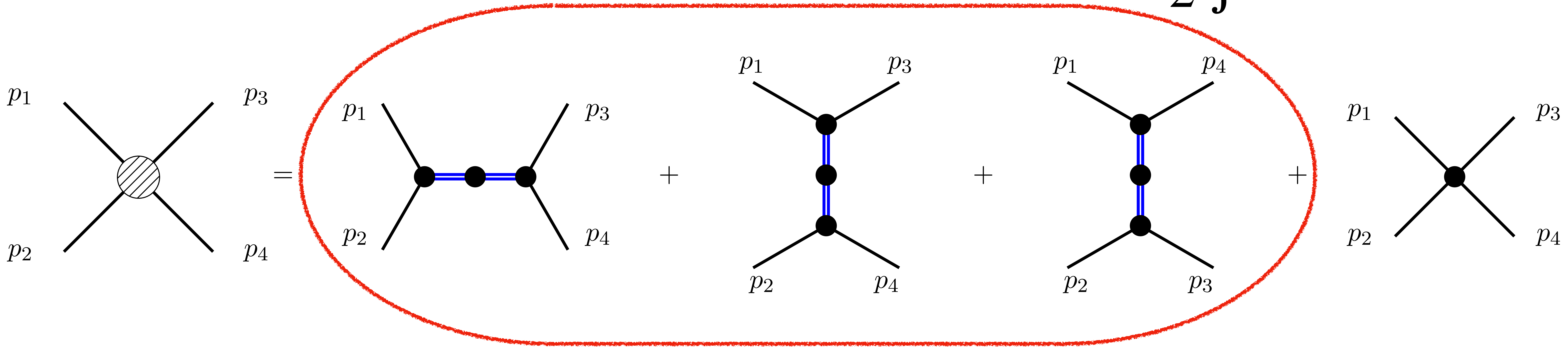

Gravity mediated

Scattering in QG

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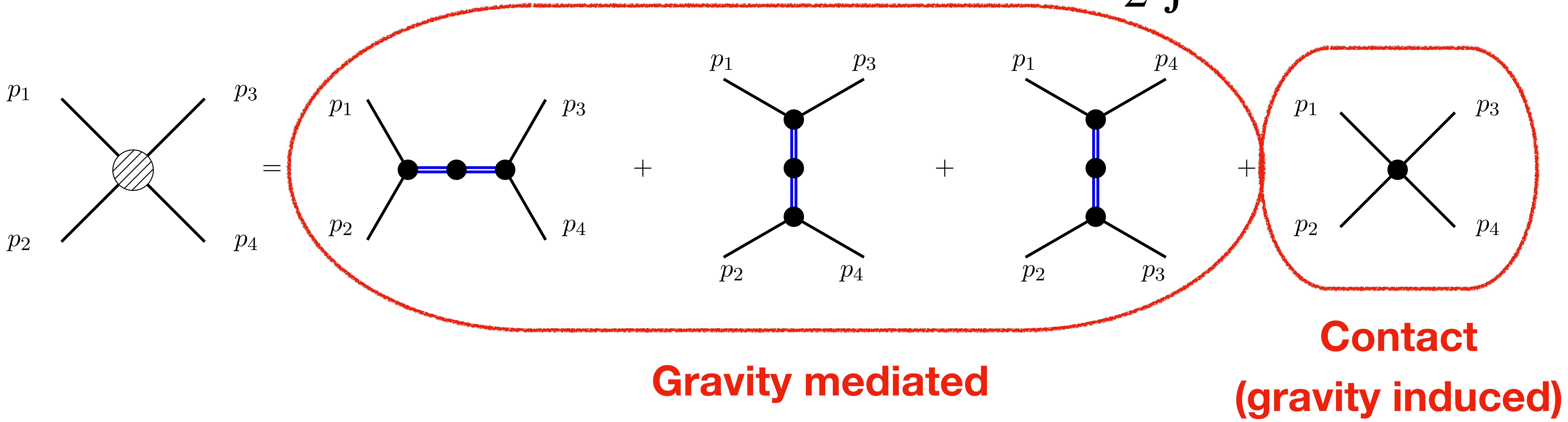
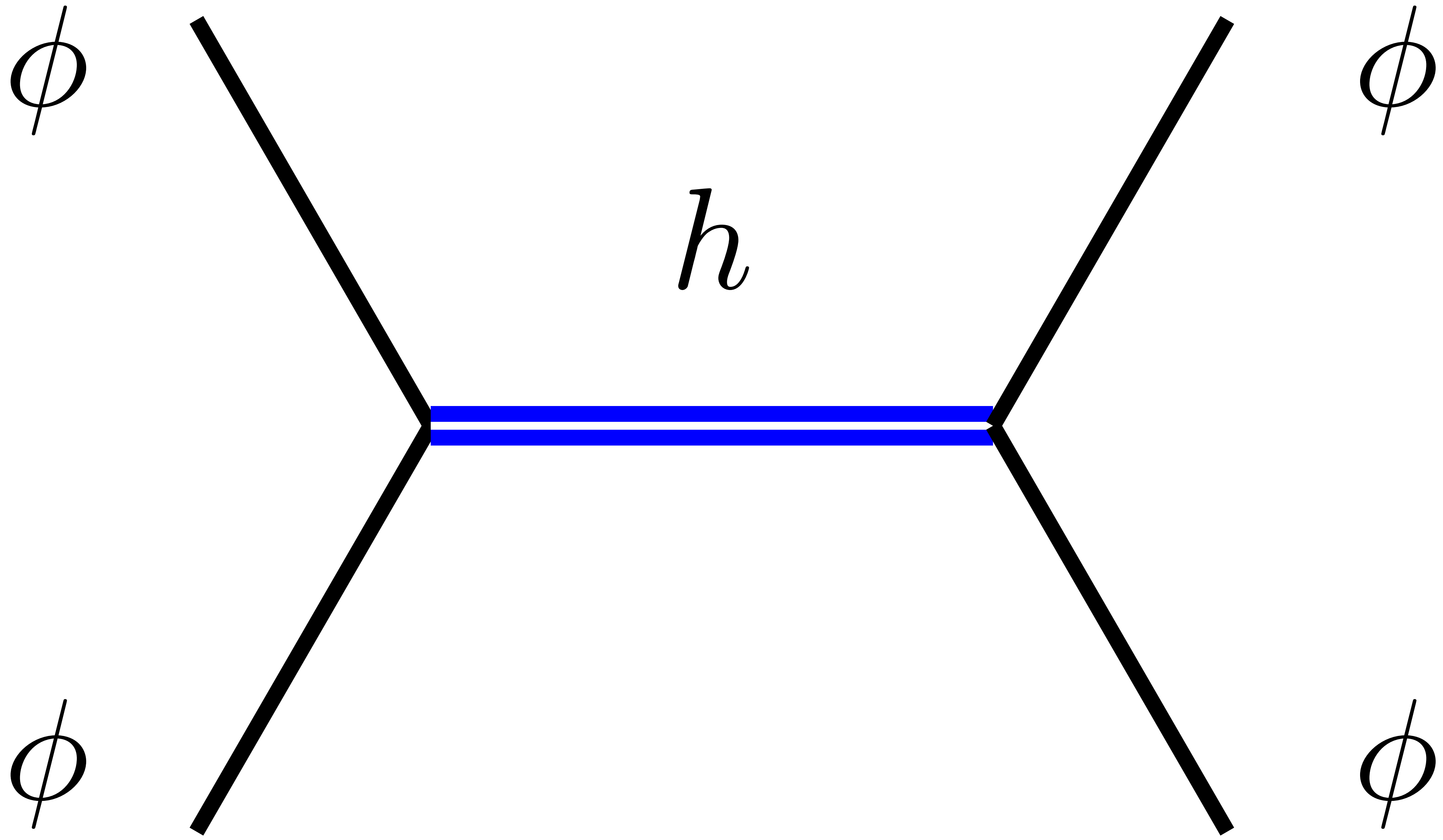


Scattering in QG

The cartoon

Graviton mediated process



Classical $\mathscr{A} \sim$

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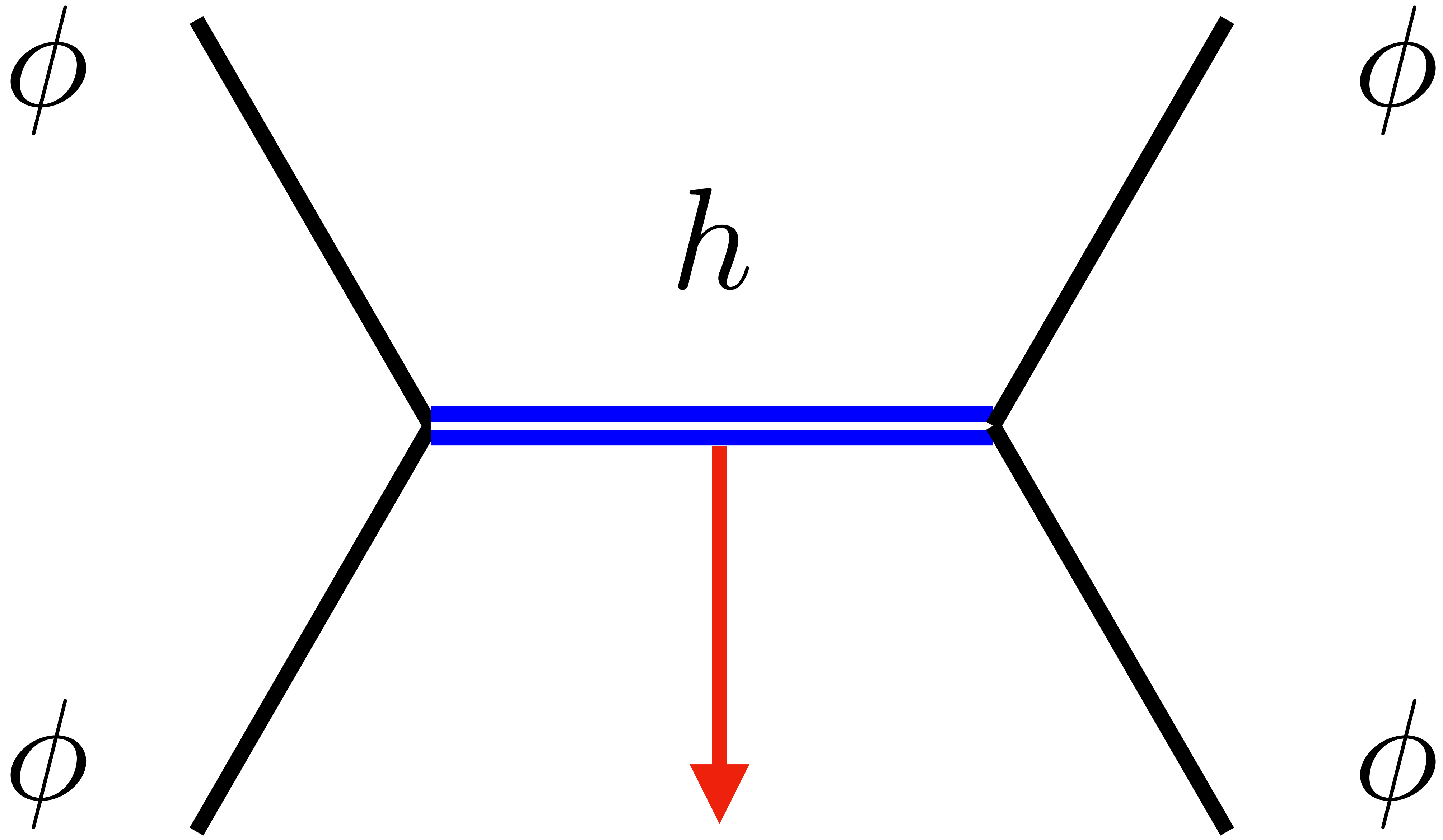

Classical $\mathscr{A} \sim \quad \times \quad \frac{1}{s} \quad \times$

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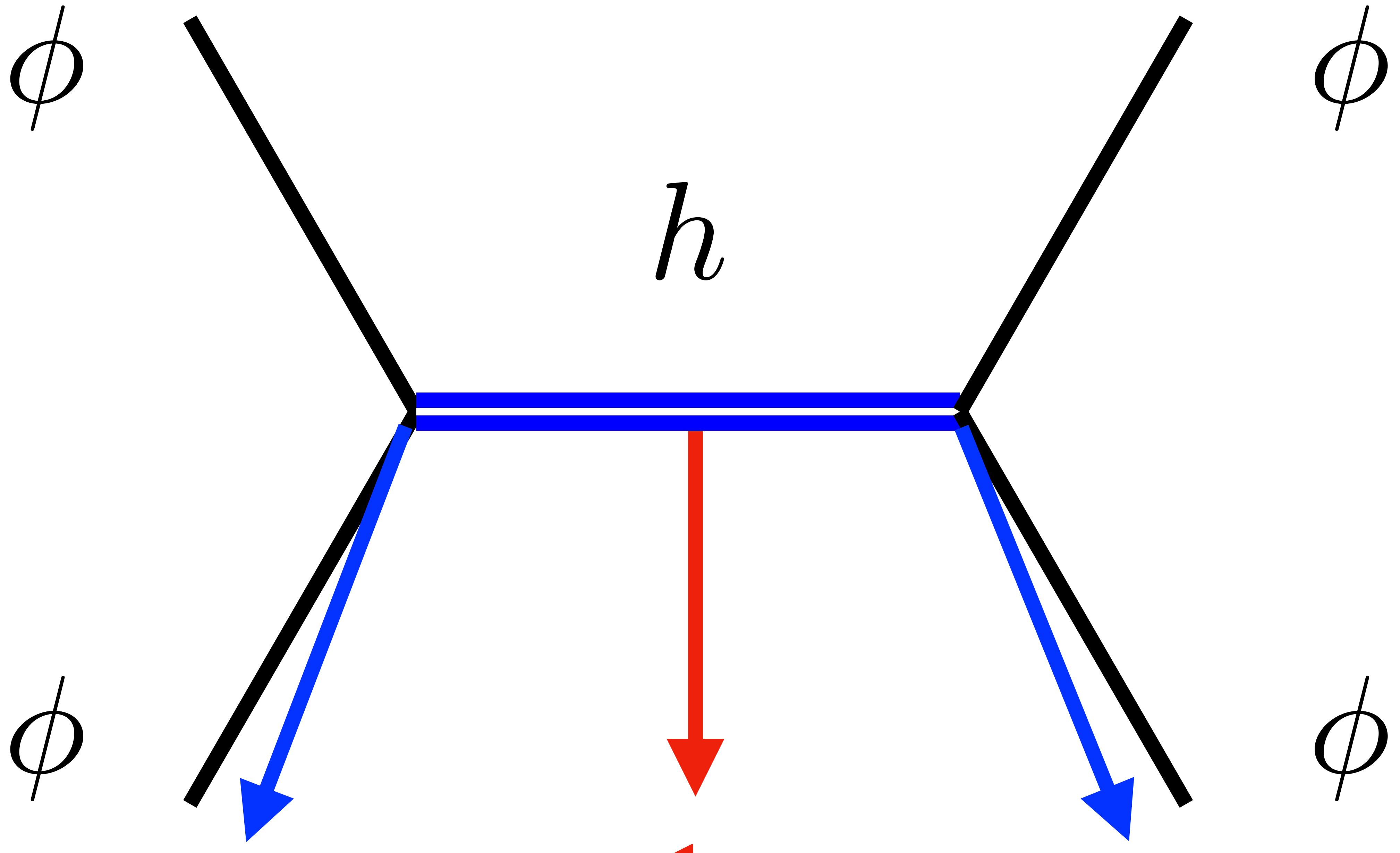

Classical $\qquad \mathscr{A} \sim \sqrt{G_N}\, s \times \frac{1}{s} \times \sqrt{G_N}\, s$

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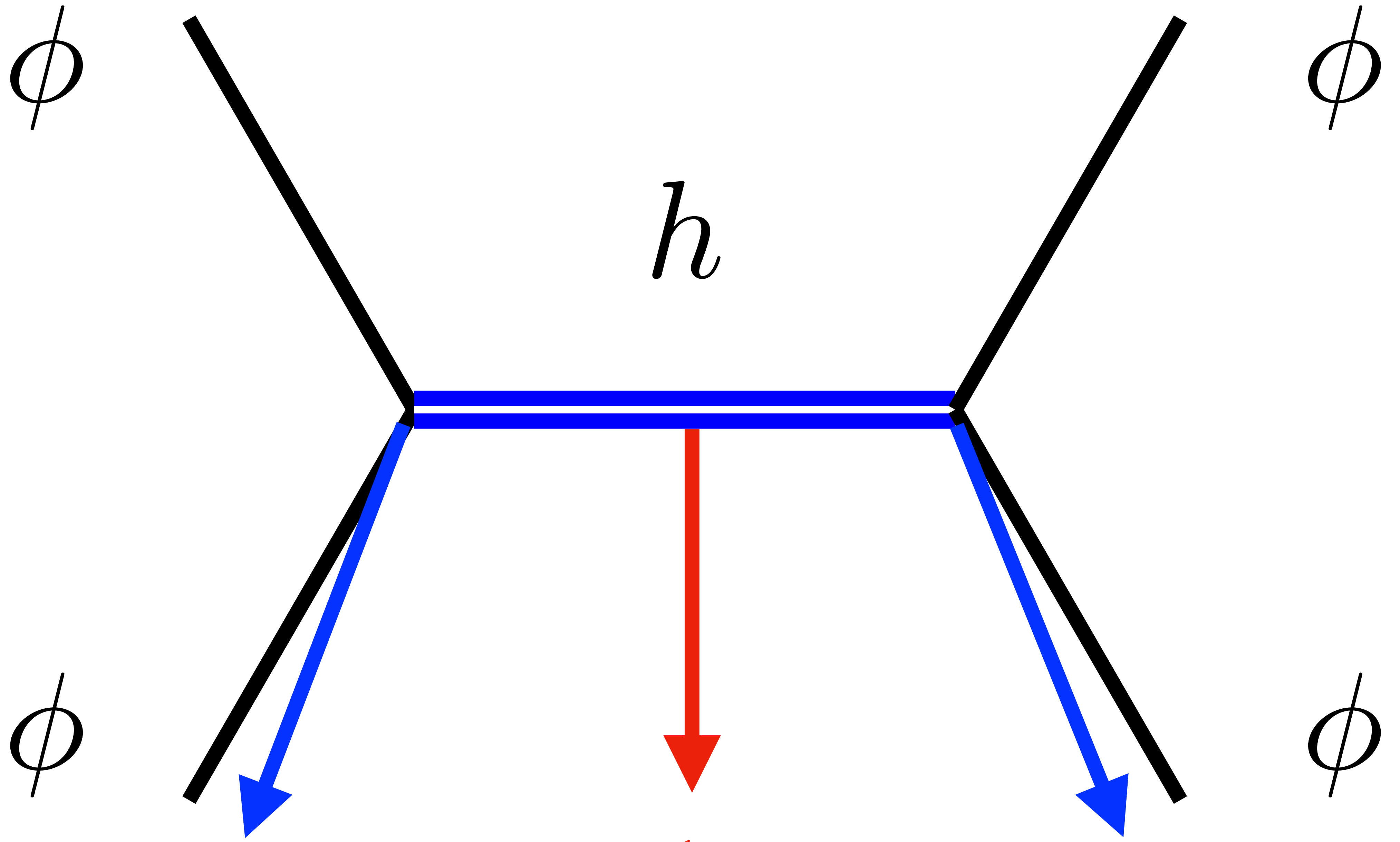

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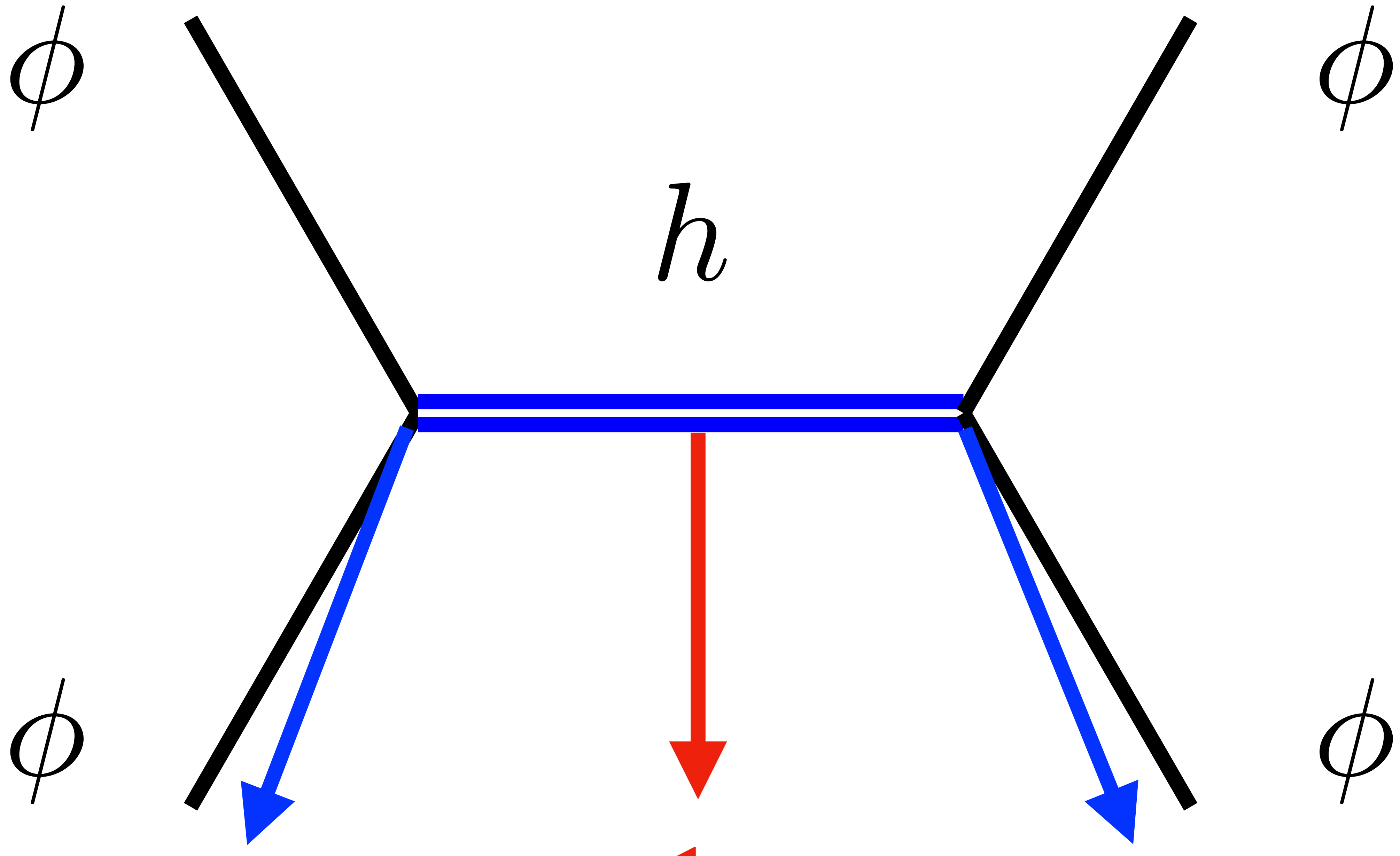

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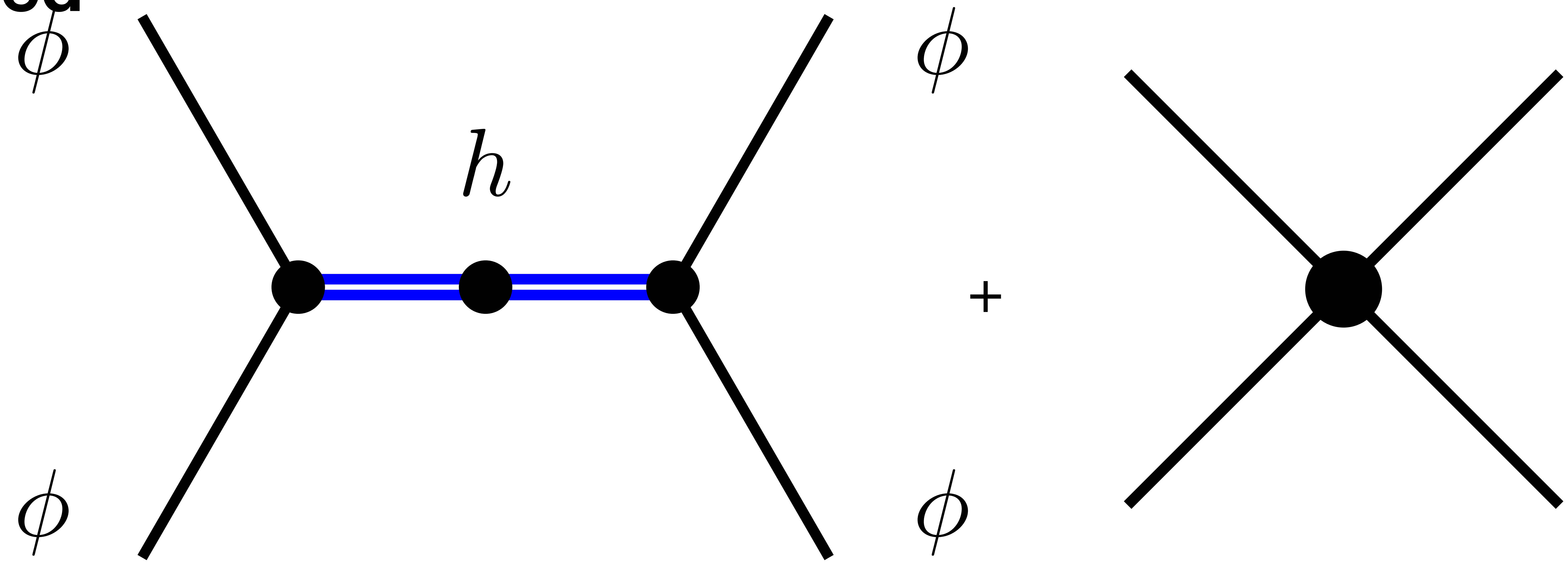
"The cure" $G_N = G_N(k) \sim \frac{1}{k^2}$ for $k > M_{pl}$ ➡ $\mathscr{A}$ **has a chance to be finite!**

Scattering in QG

The cartoon upgraded

Graviton mediated process

$$\phi\phi \to \phi\phi$$



1PI version $\mathscr{A} \sim$

Scattering in QG

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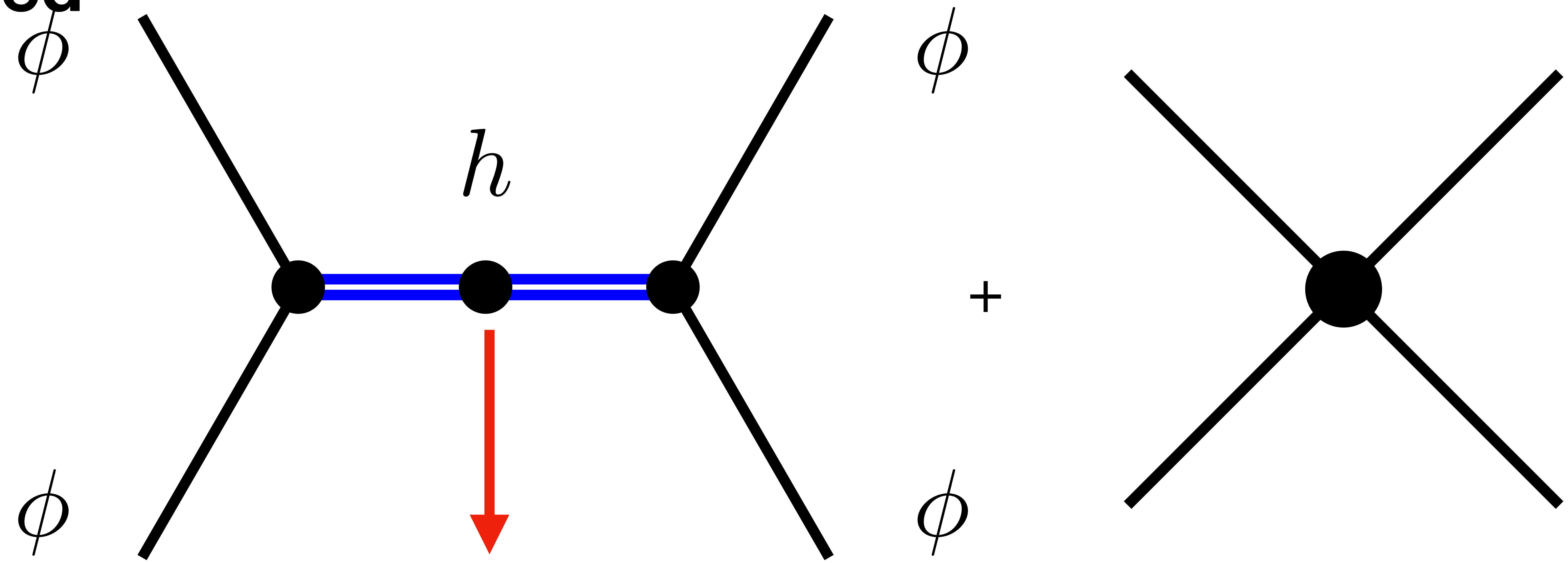

1PI version $\mathscr{A} \sim \quad \times \; \bar{\mathcal{G}}_{hh} \; \times$

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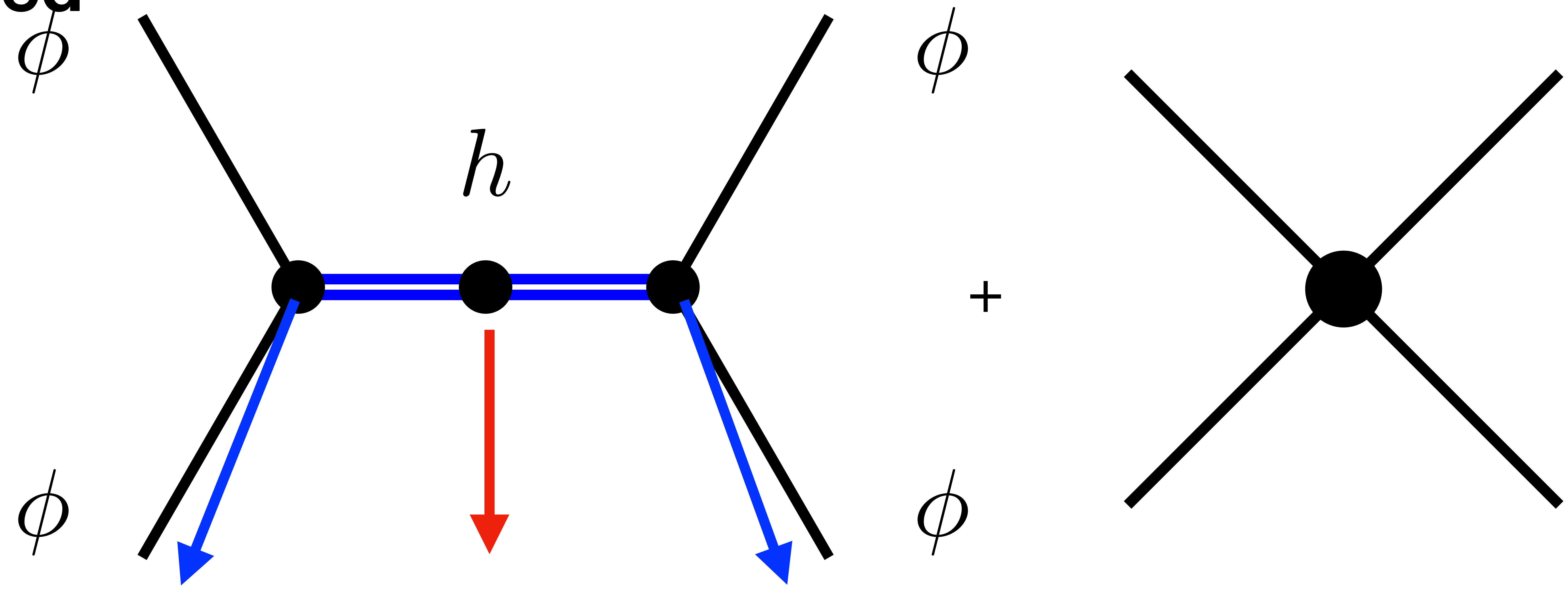

1PI version $\mathscr{A} \sim \quad \bar{\Gamma}^{\phi\phi h} \times \bar{\mathscr{G}}_{hh} \times \bar{\Gamma}^{\phi\phi h}$

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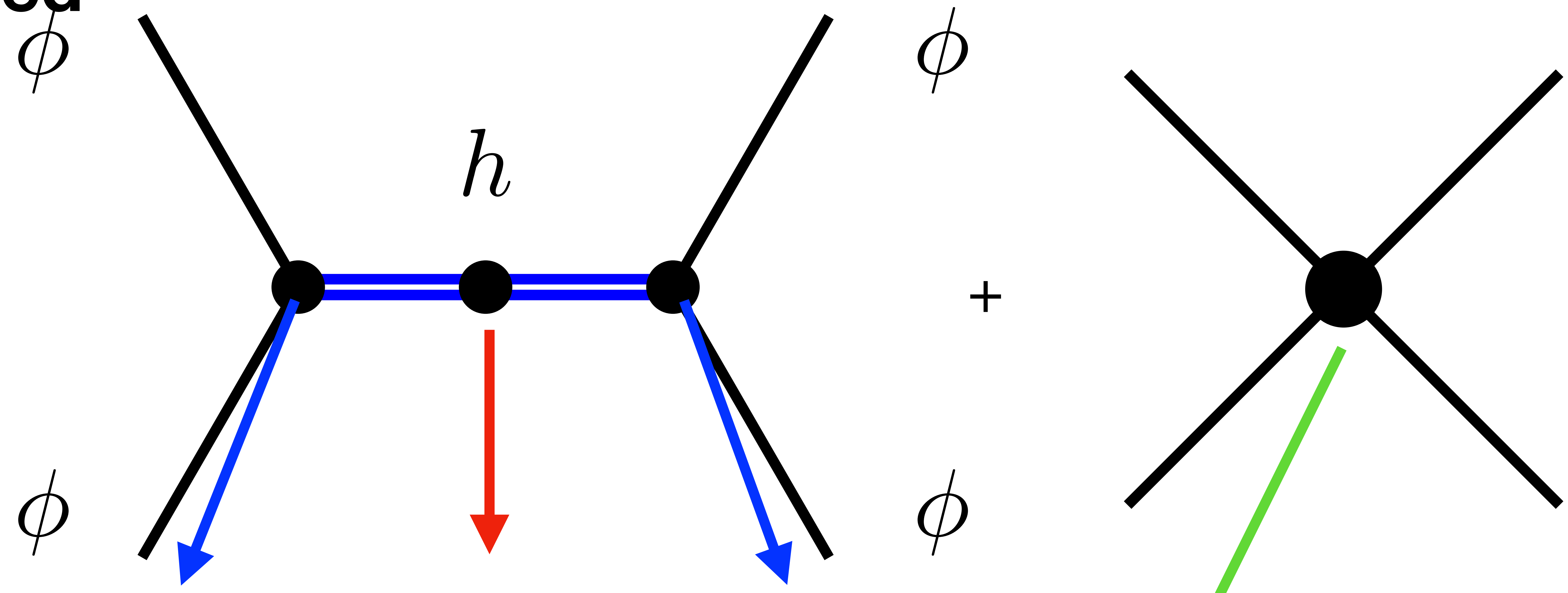
$$\phi\phi \to \phi\phi$$



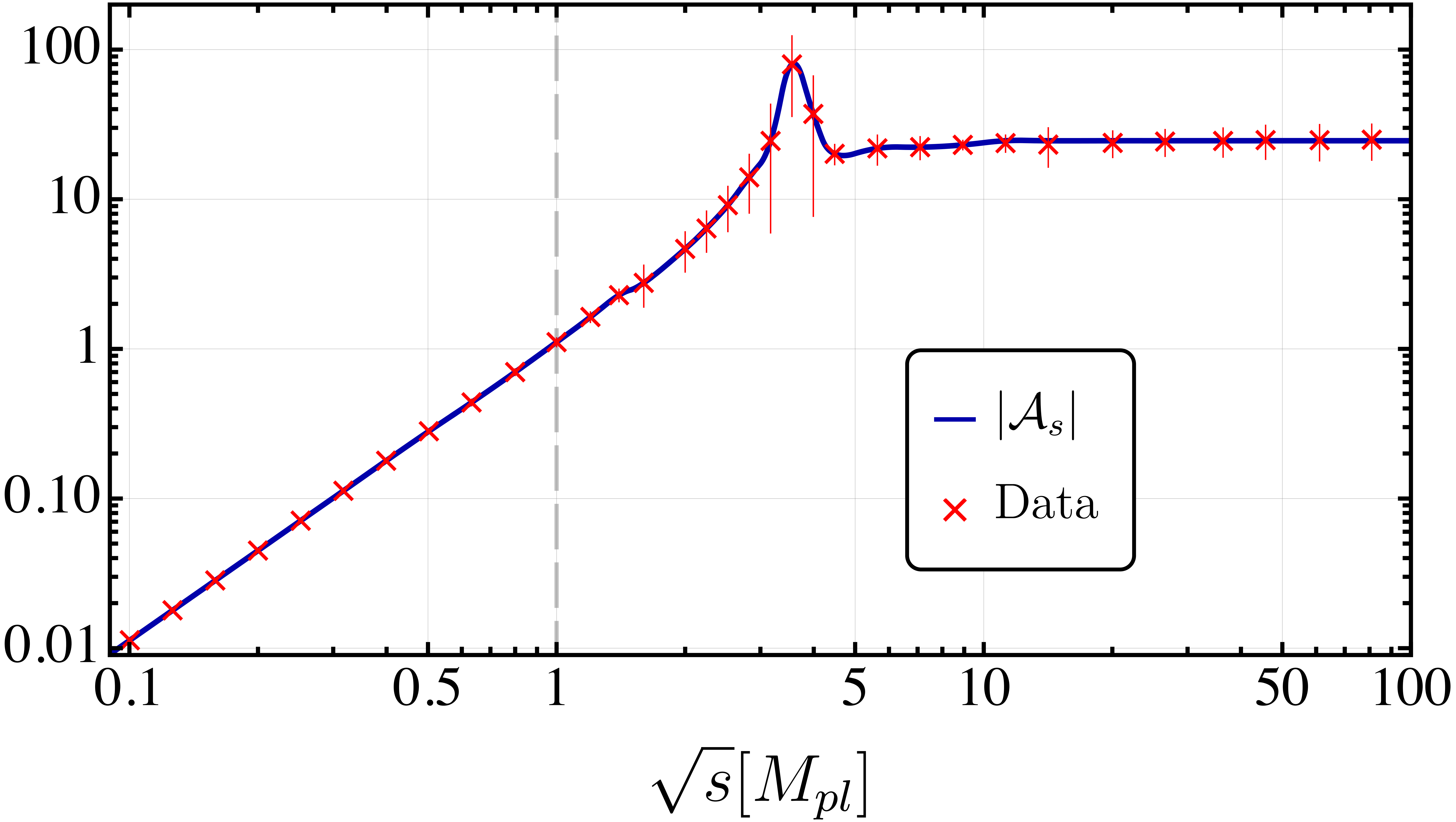
1PI version $\mathscr{A} \sim \quad \bar{\Gamma}^{\phi\phi h} \times \bar{\mathcal{G}}_{hh} \times \bar{\Gamma}^{\phi\phi h}$

$$= G_N(s)\frac{1}{s}\left(t^2 - 4tu + u^2 - s^2\right) + \Gamma^{(4)}$$

RESULTS

RESULT

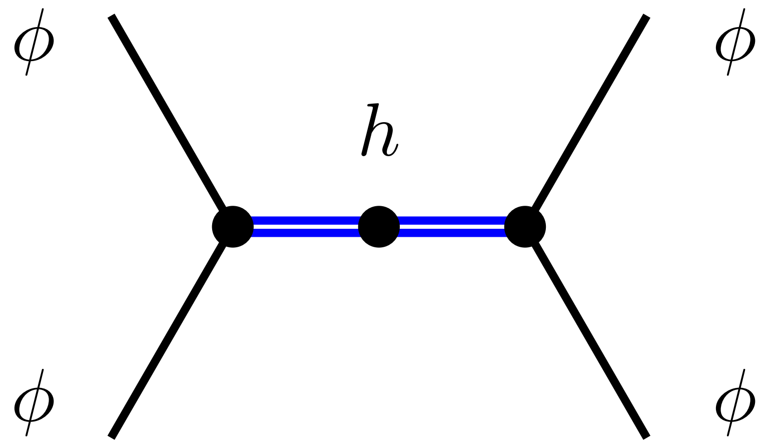
Meidated Amplitude

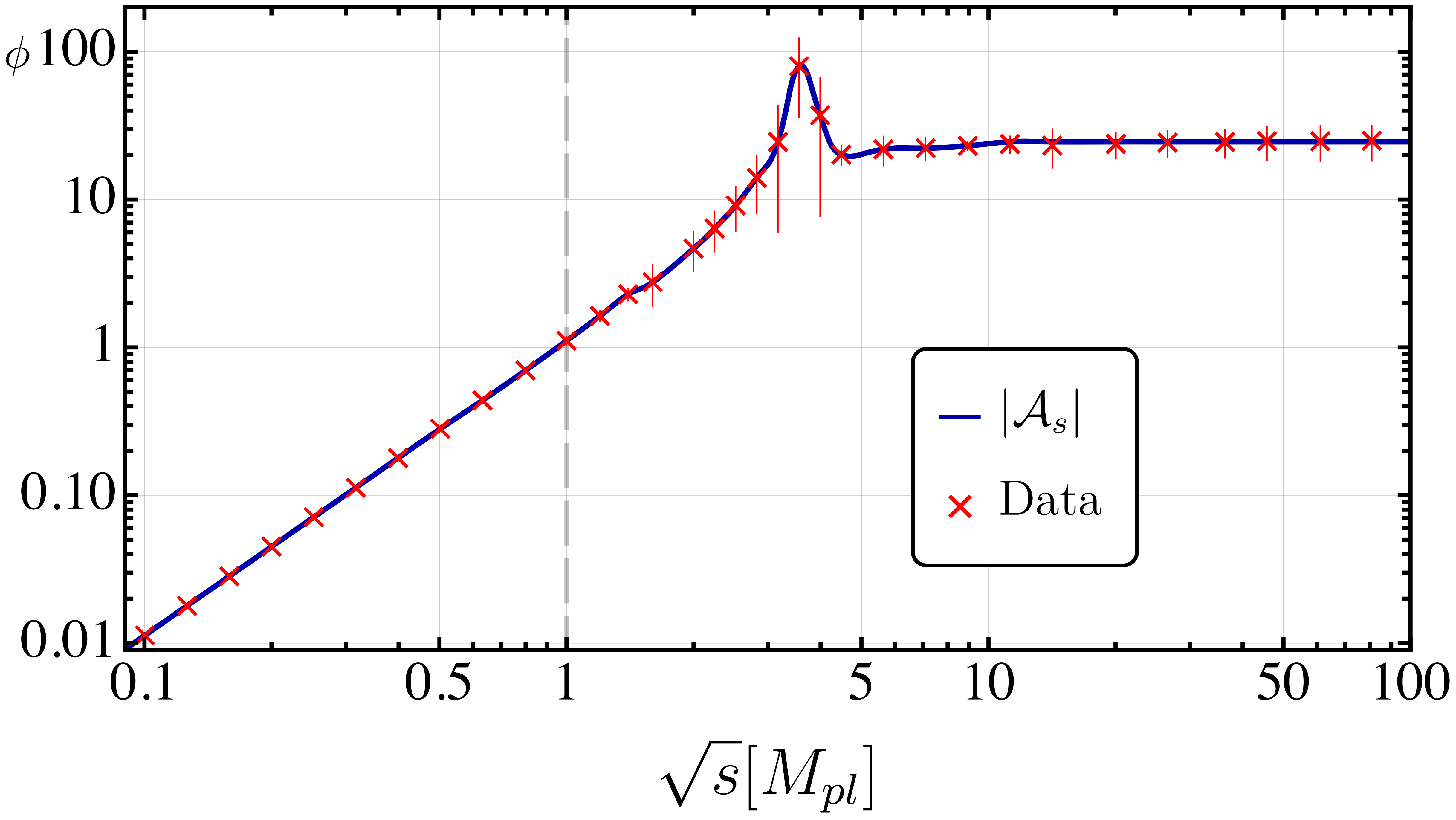


RESULT

Meidated Amplitude

- $s-$channel

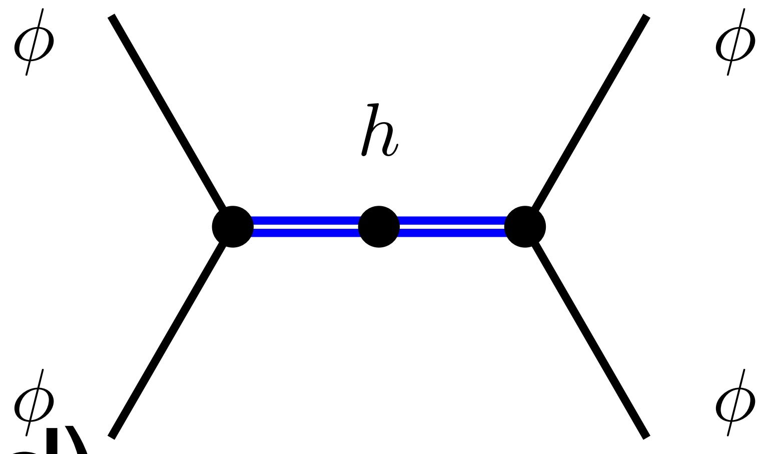


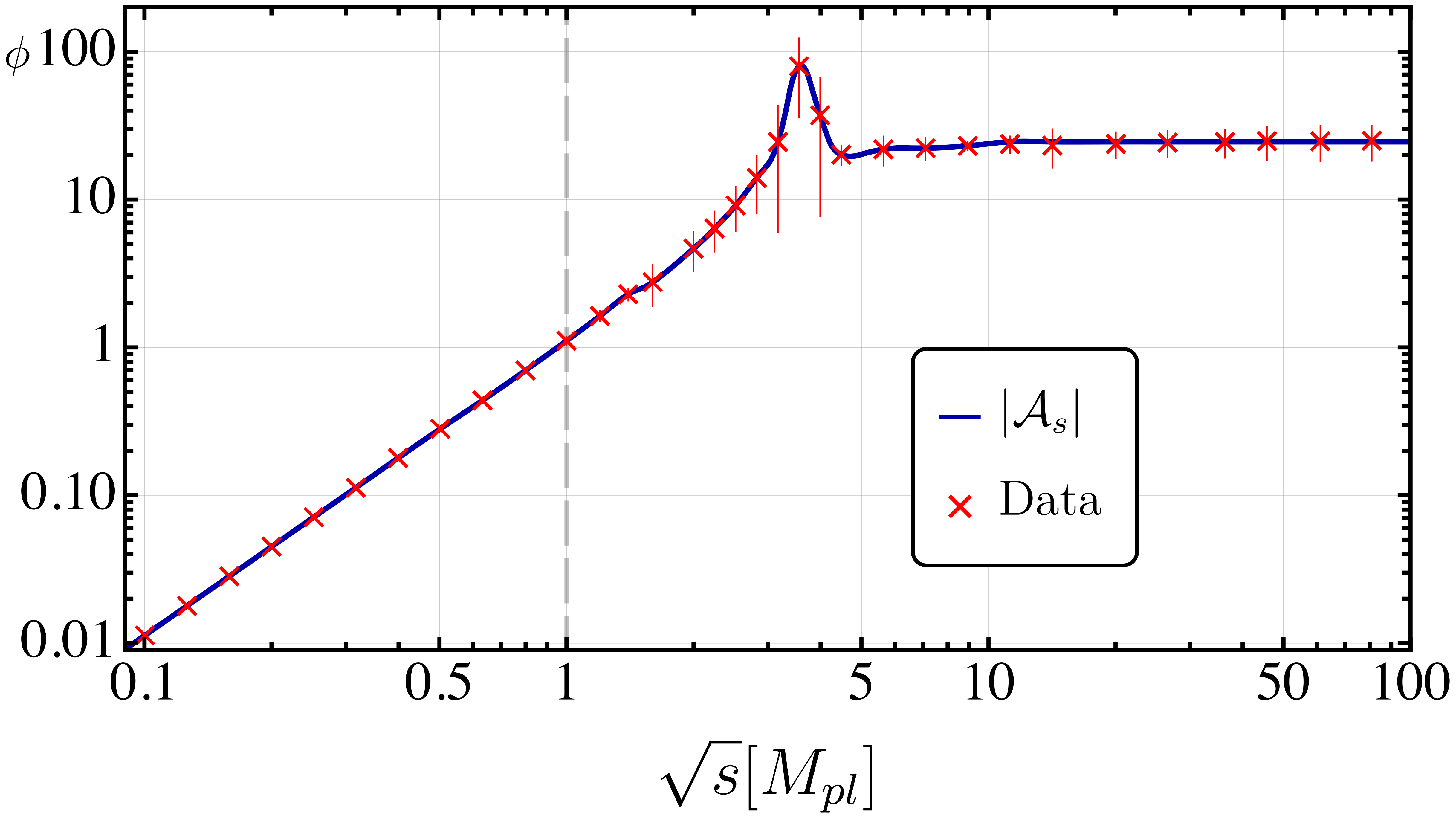


RESULT

Meidated Amplitude

- $s-$channel
- Lorentzian (reconstructed)





RESULT

Meidated Amplitude

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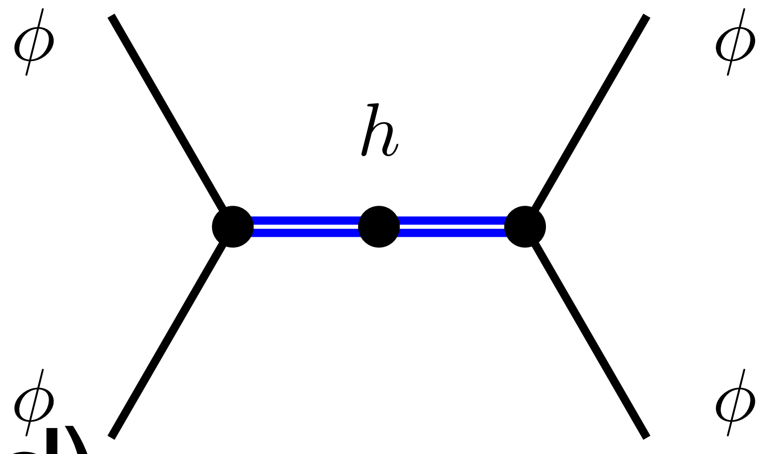
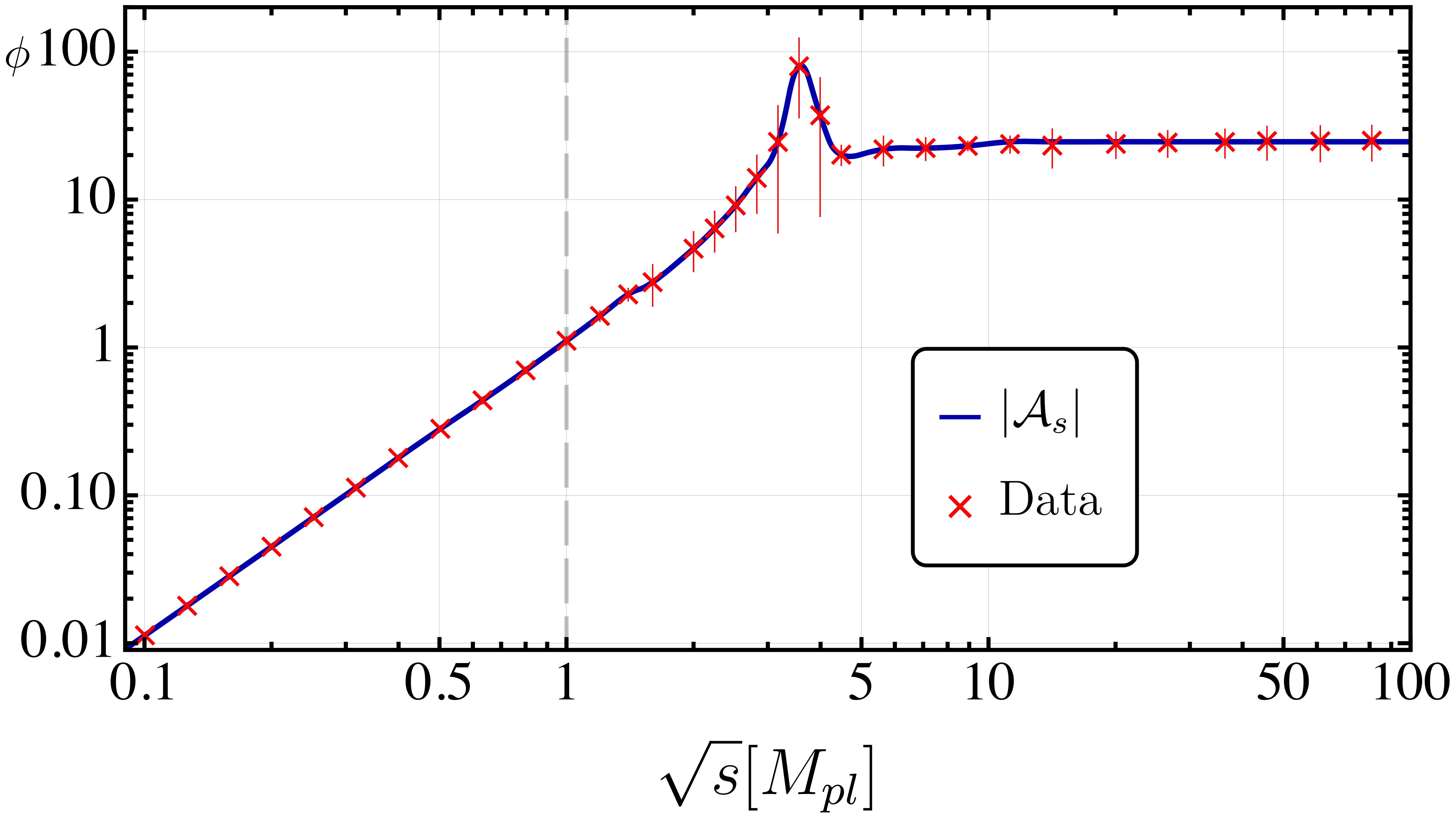
- Lorentzian (reconstructed)
- On-Shell $m_\phi^2 = 0$





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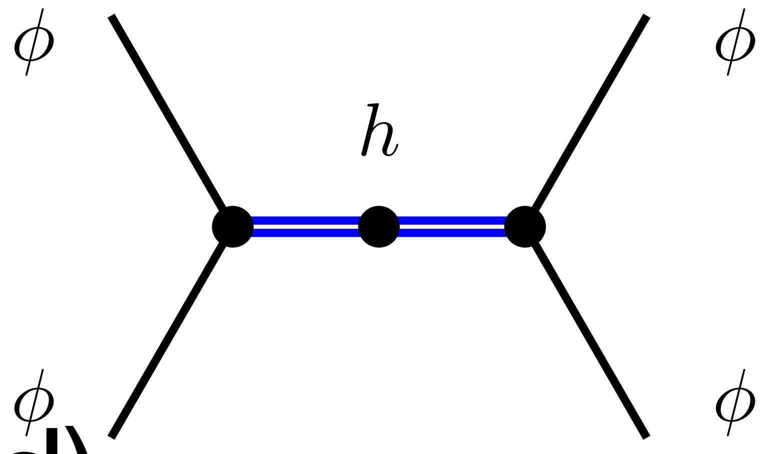
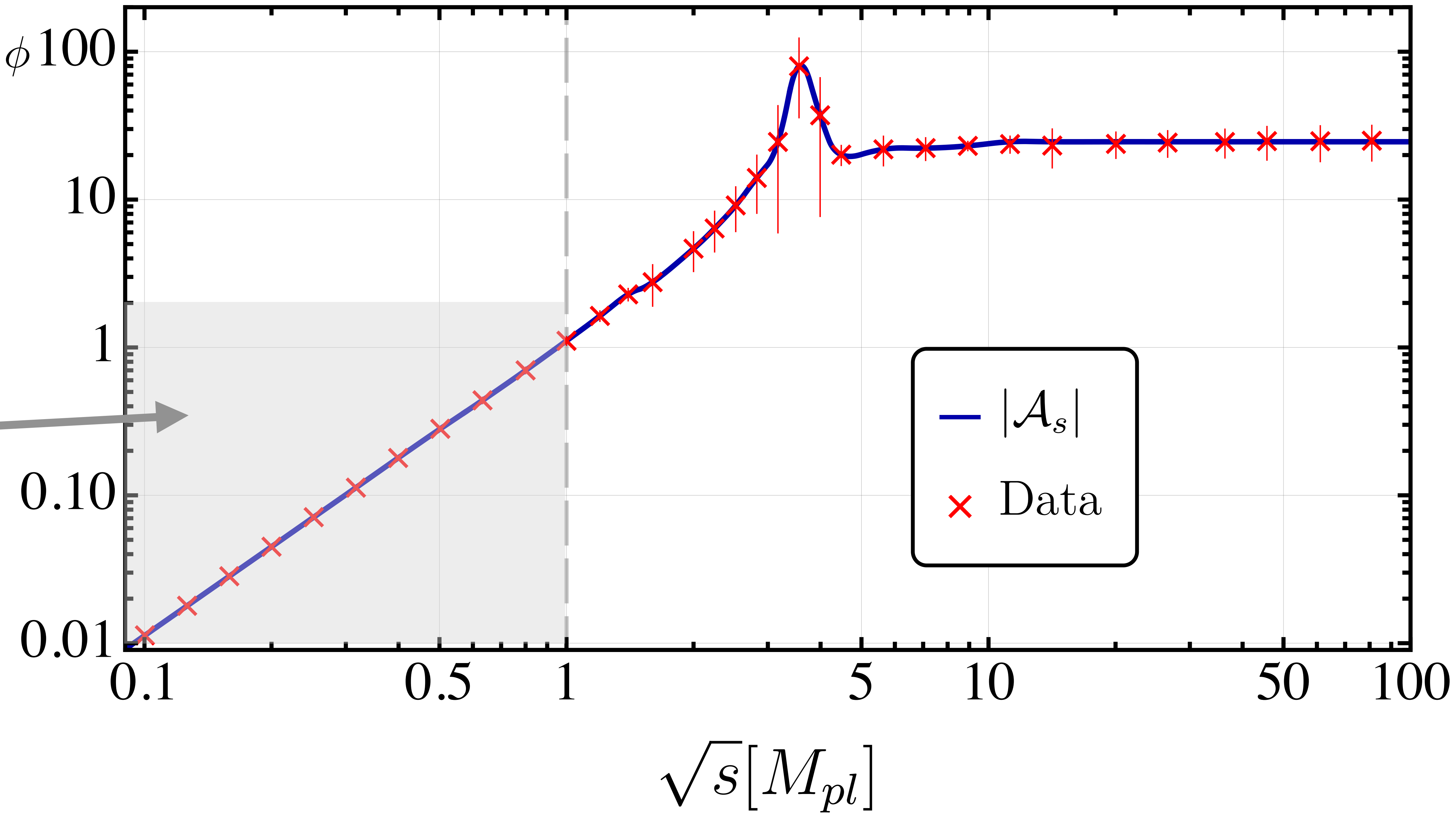
- $s-$channel
- Lorentzian (reconstructed)
- On-Shell $m_\phi^2 = 0$
- GR recovered





RESULT

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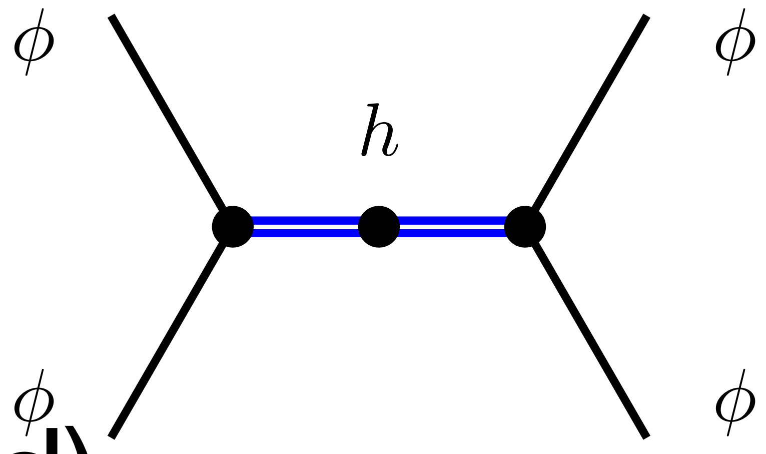
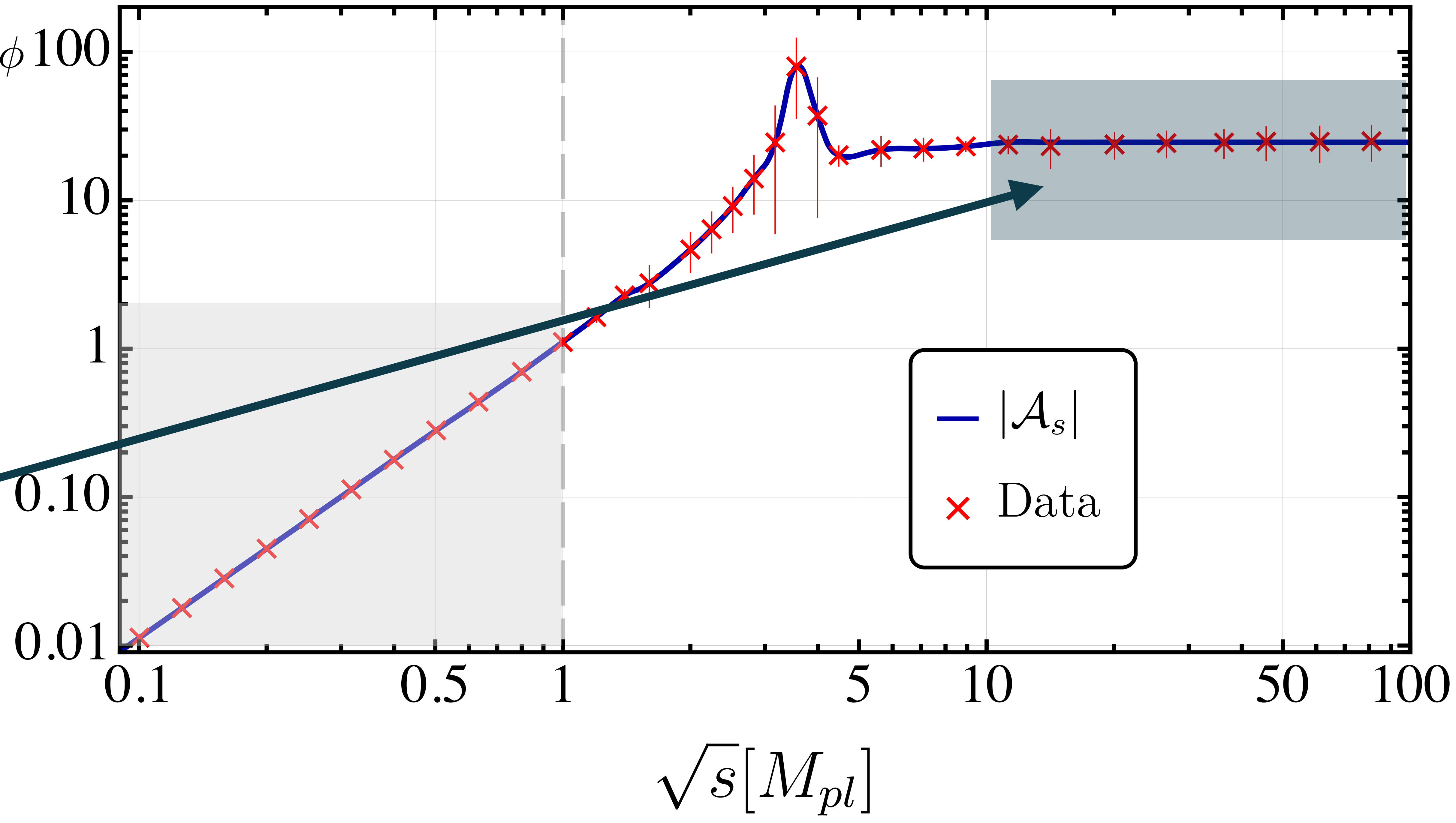
- Fixed point $\mathscr{A}$ **FINITE**





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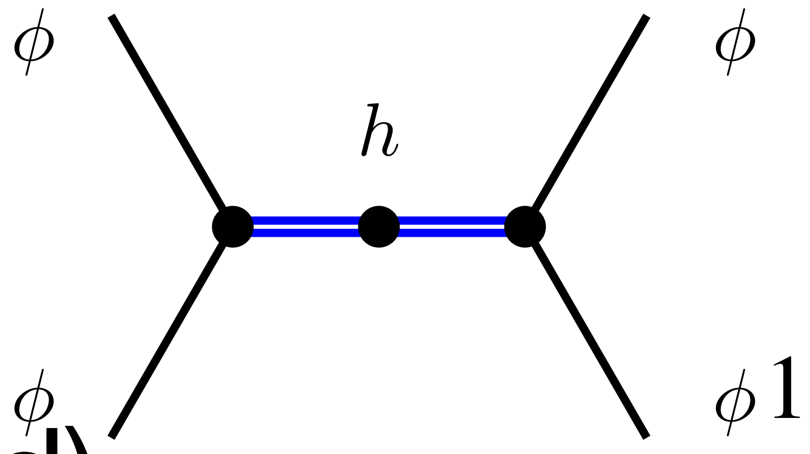
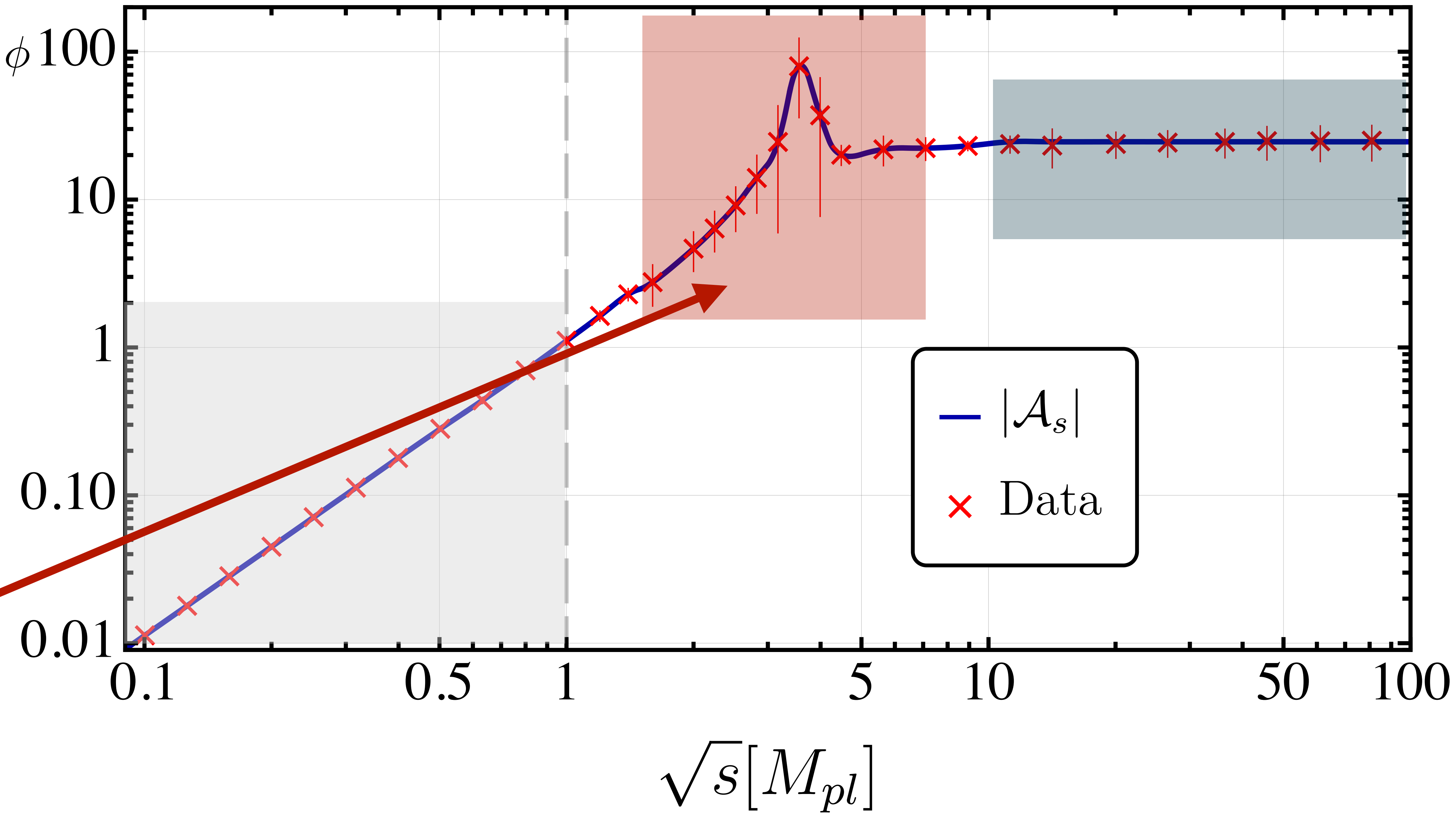
- Graviball resonance?





RESULT

Full Amplitude

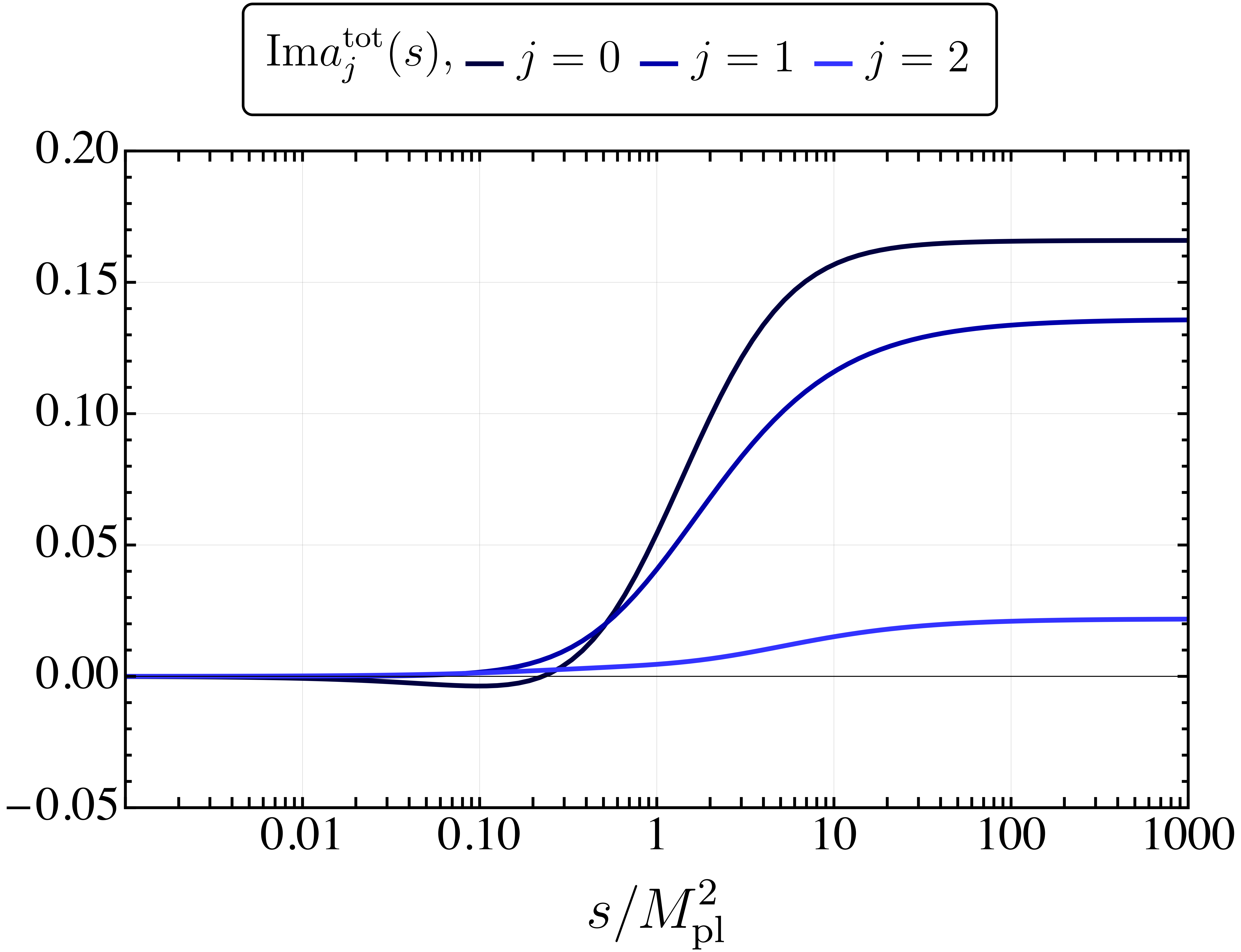
- Partial Waves:

$$\mathscr{A}(s,\theta) = 32\pi \sum_j a_j(s) P_j(\cos\theta)$$

- Optical theorem:

$$0 \leq \text{Im } a_j(s) \leq 1$$

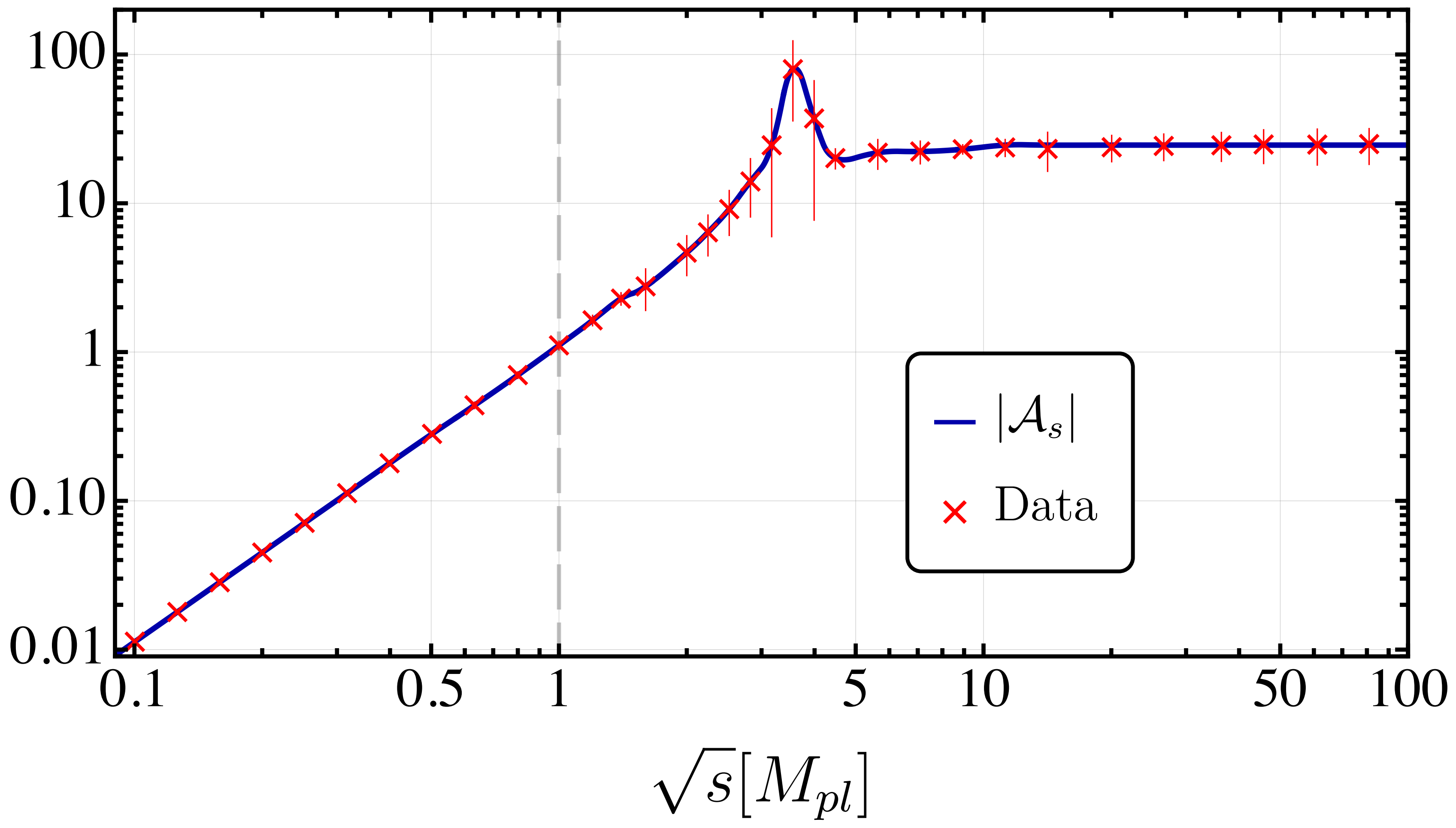
- $j > 2, a_j \equiv 0$

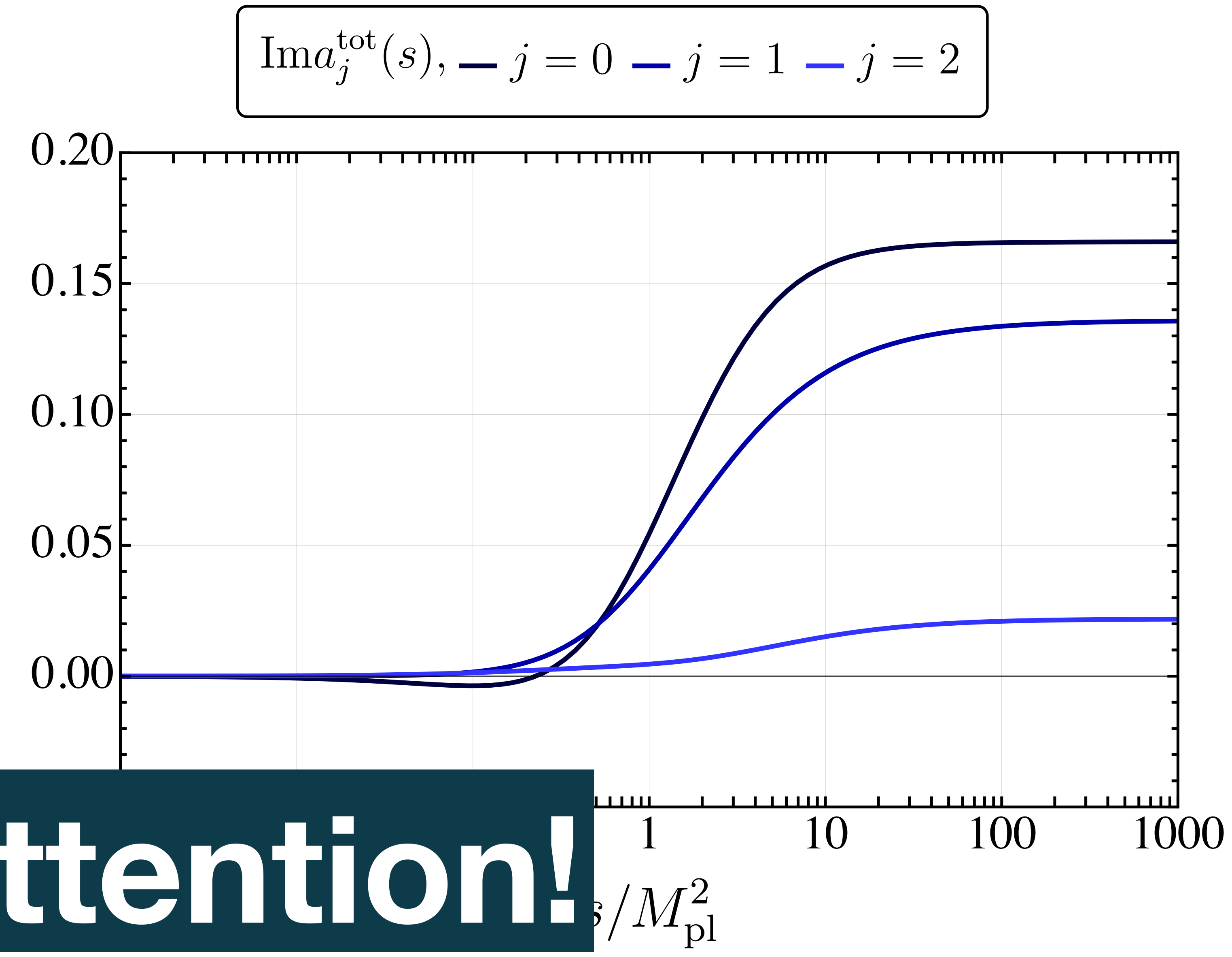


Conclusions & Outlook

- $\mathscr{A}_s$ **FINITE**
- $\mathscr{A}_4(s,t)$ **FINITE**
- **Graviton bound state?**

$\left\{\right.$ Indications for Unitarity





- $t-$**channel: for** $s \to \infty$, $\mathscr{A}_t \propto V_L(t)\, s^2$
- **Eikonal**

Thanks for your attention!

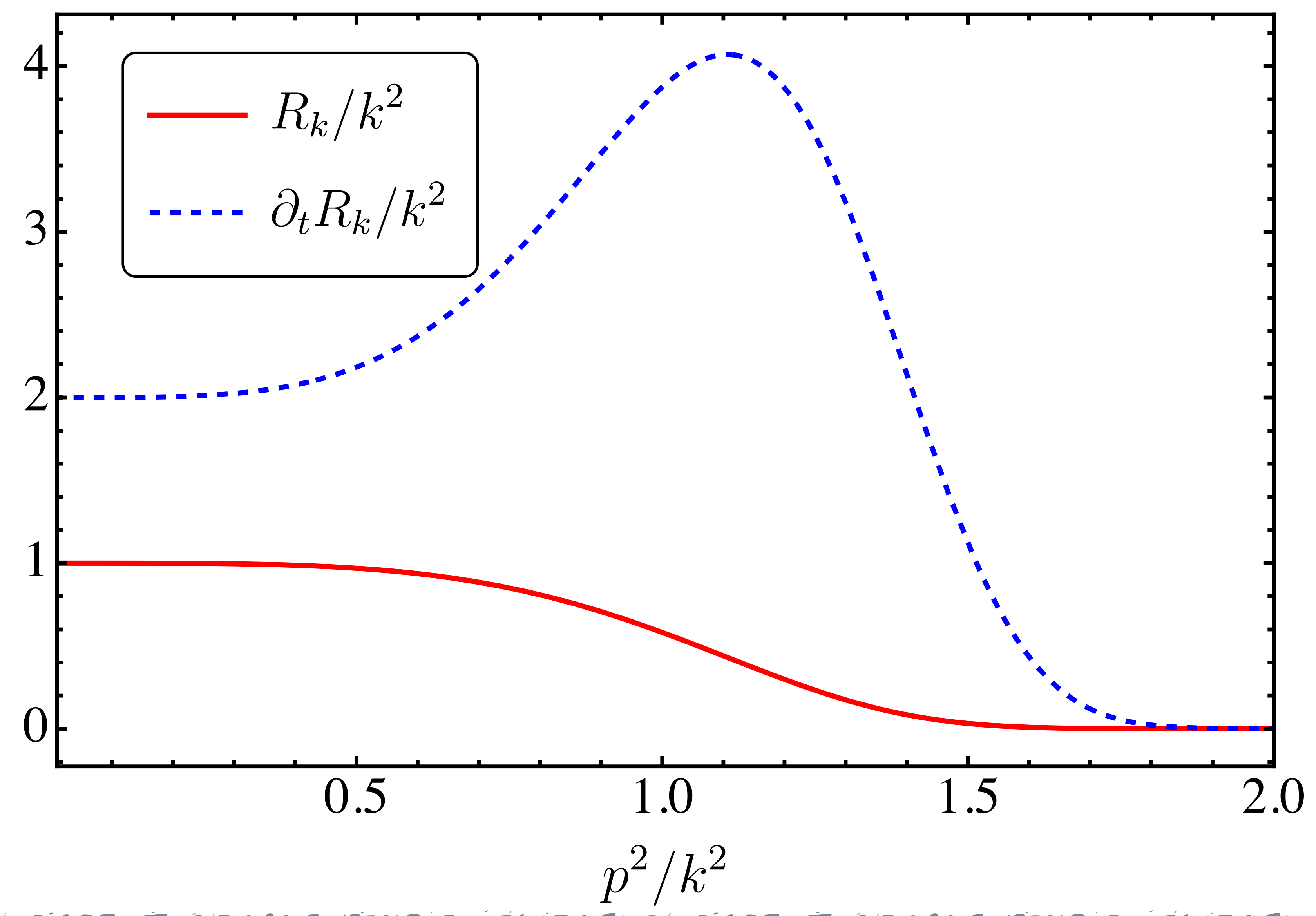
(More) DETAILS 2603.10168 or a.portas-chiesa@sussex.ac.uk

BACKUP

Asymptotically Safe QG in a nutshell

Functional RG

- Suppress IR modes $Z_k[J] = \int \mathcal{D}\phi_{p^2 \geq k^2}\ e^{-S[\phi] \int_x J \cdot \phi}$
- Regulated measure $\int \mathcal{D}\phi_{p^2 \geq k^2} = \int \mathcal{D}\phi\ e^{-\frac{1}{2}\int_p \phi R_k \phi}$



Wetterich Equation:

- IR finite
- UV finite

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}\right]$$

- Turns path integral in PDE
- Γ_k scale dependent effective action

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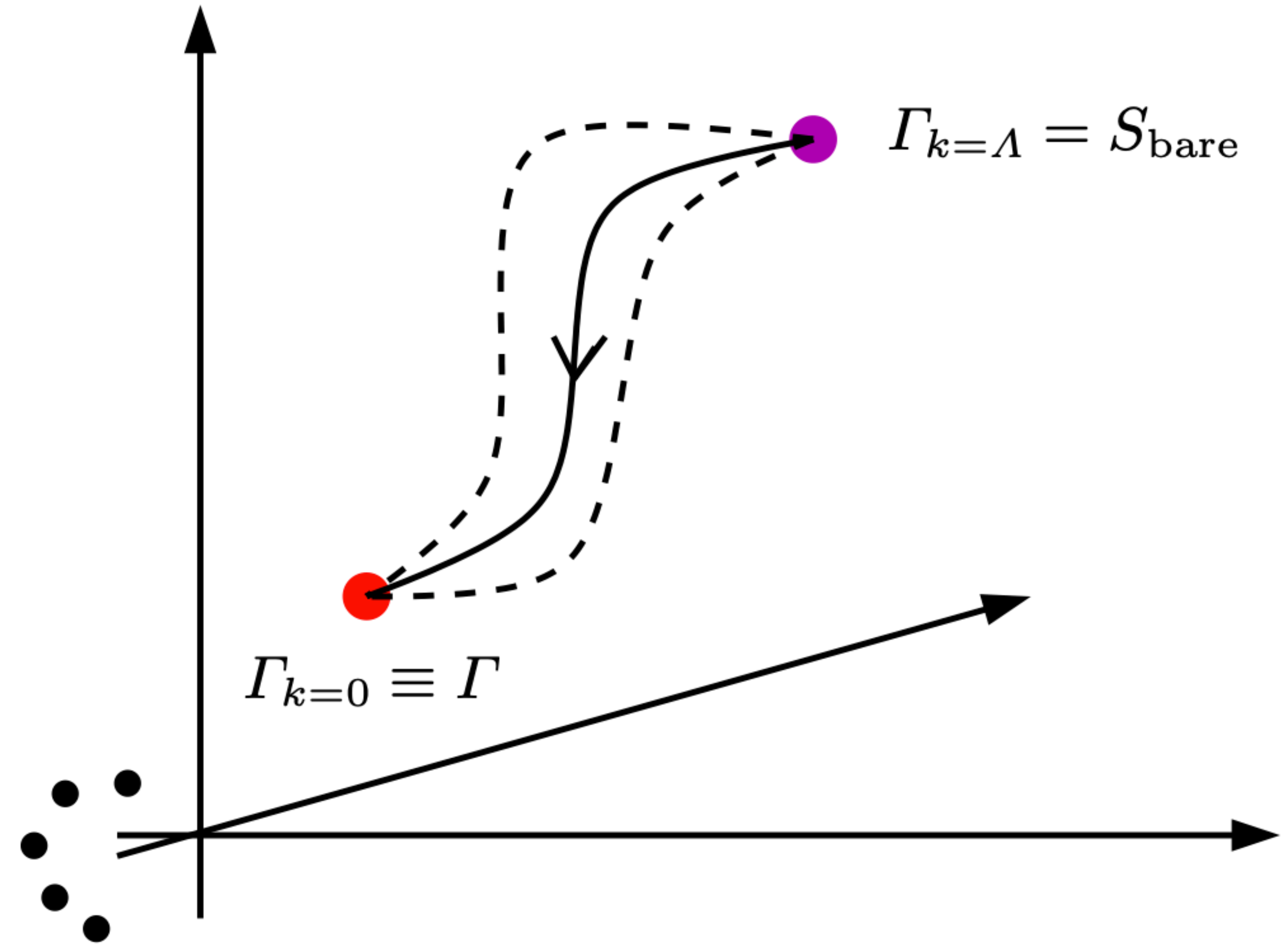
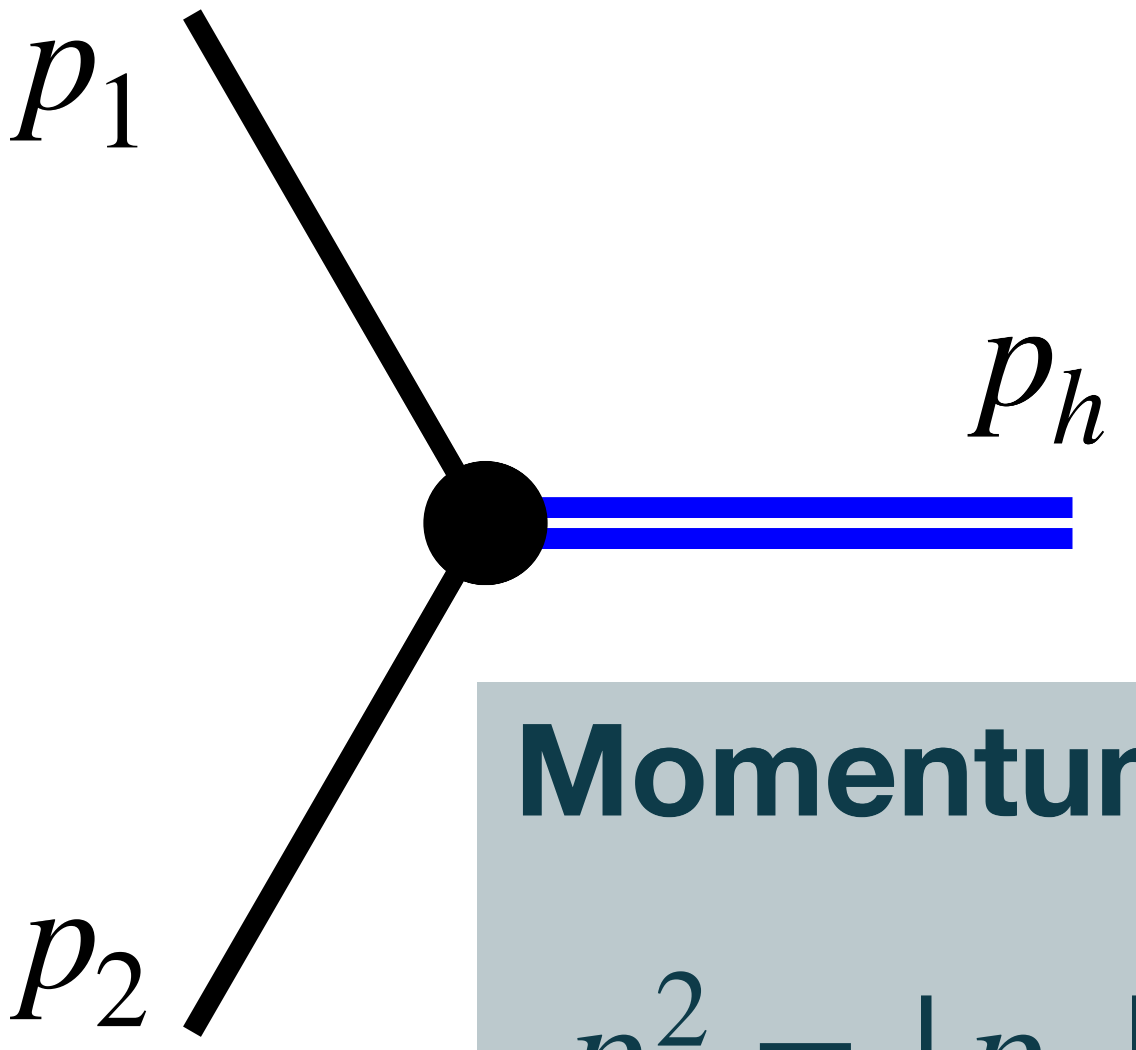
Γ_k interpolates between

- $k \to \infty$ bare action
- $k \to 0$ Quantum effective action Γ



Scalar-Graviton vertex in AS

How to



$\sim \Gamma_k^{\phi\phi h}(p_1, p_2, p_h)$

Momentum variables

$$p^2 = |p_1|^2 = |p_2|^2$$

$$p_1 \cdot p_2 = p^2 z$$

$$|p_h|^2 = 2p^2(1+z)$$

Tensor structure: min. coupled scalars

$$\Gamma_k = S_{EH} + S_{gf} + S_{gh} + \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\phi\partial^\mu\phi$$

Wetterich Equation

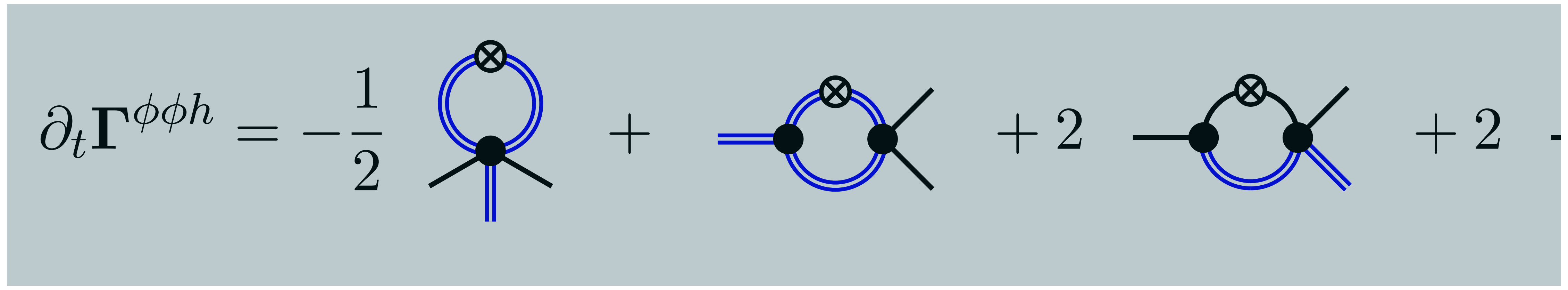
$$\partial_t\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}\right] \Rightarrow \partial_t\Gamma_k^{\phi\phi h}(p_1, p_2, p_h)$$

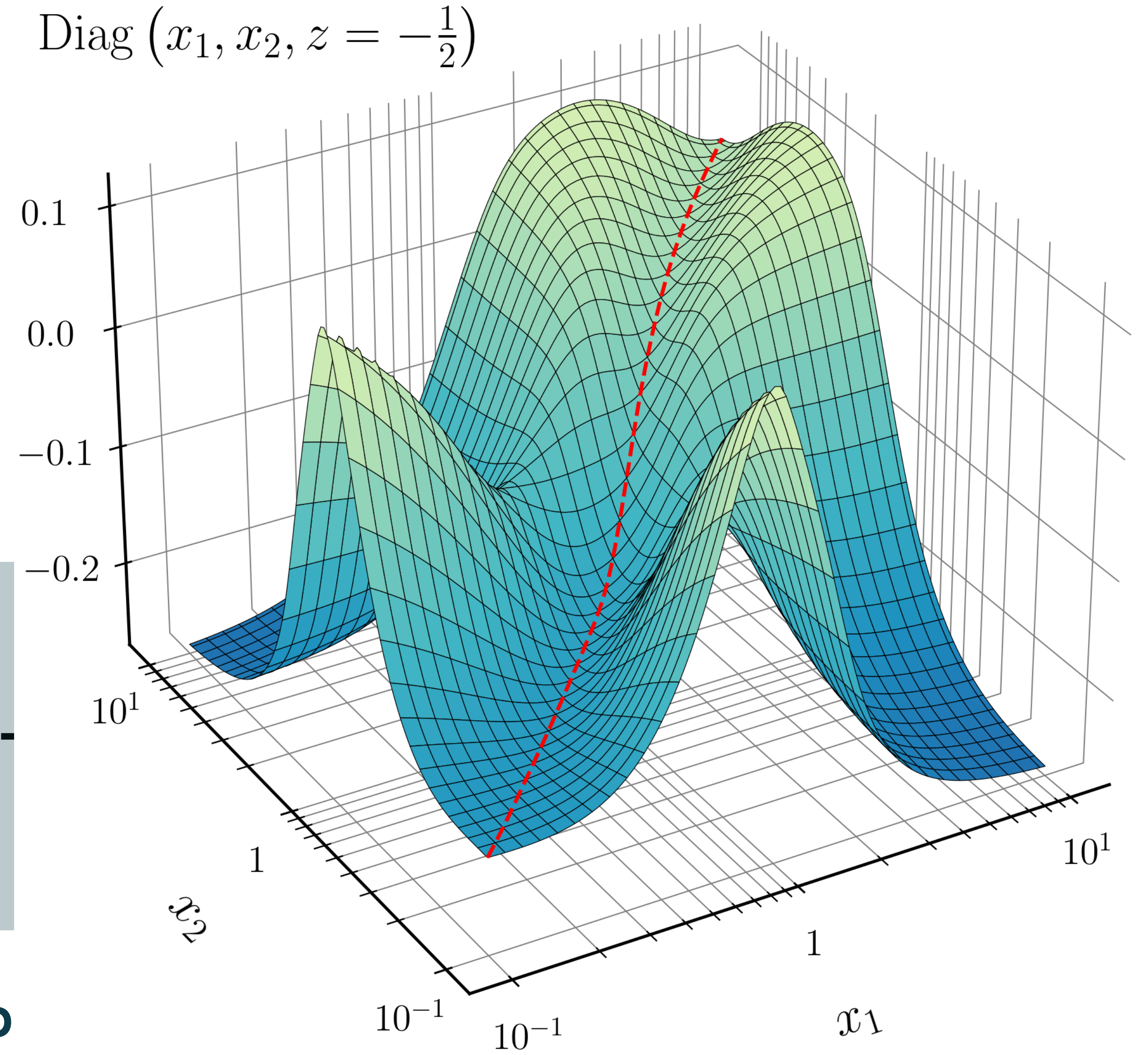
⚠ **Euclidean**

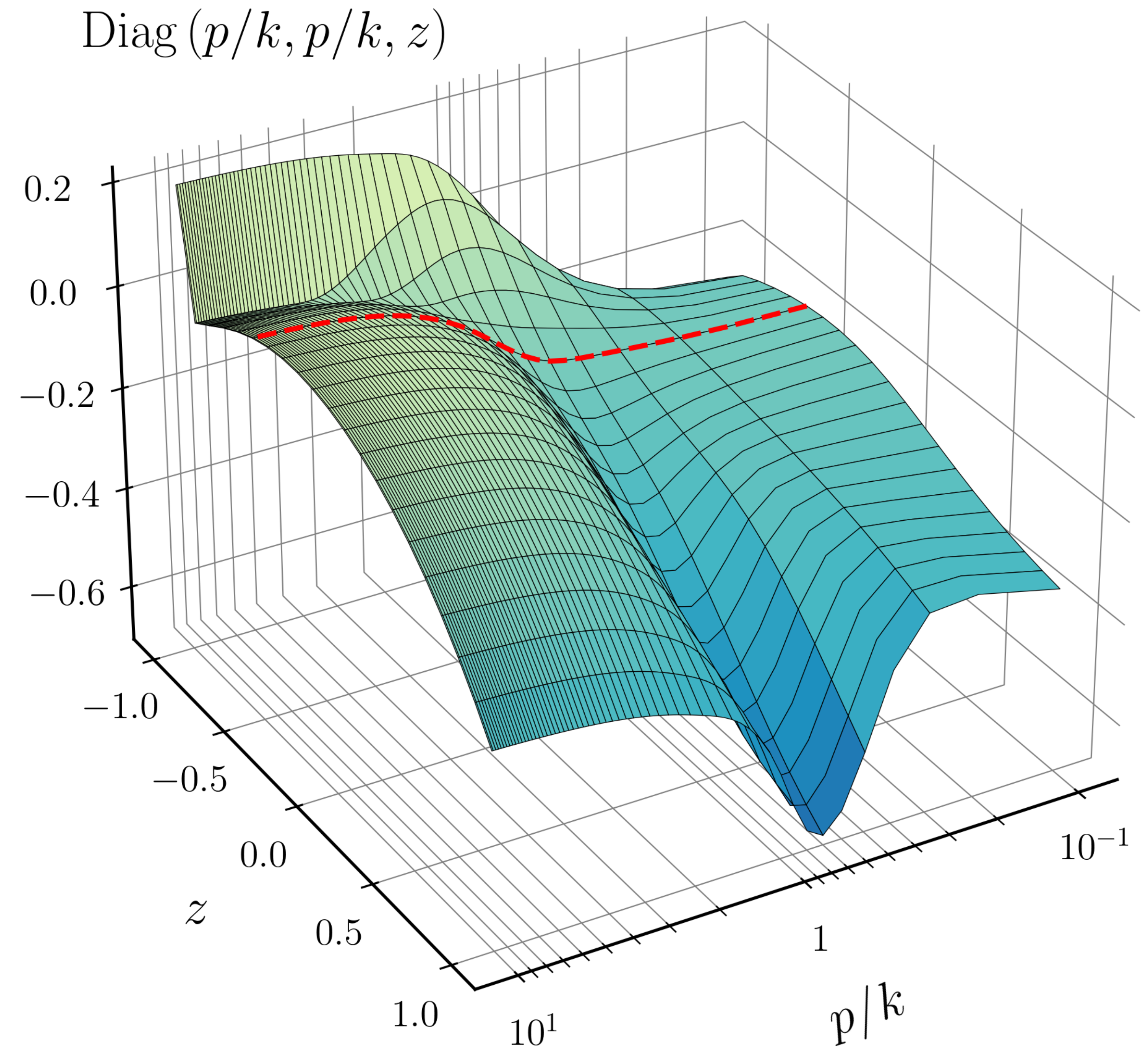
Scalar-Graviton vertex in AS

Flow Equation

Diagrams generated in $\partial_t \Gamma_k^{\phi\phi h}$







Flow Equation

See Also [Eichhorn, Labus, P

$$\partial_t g_{\phi\phi h,k}(p,z) = (2+\eta_{h,k}) g_{\phi\phi h,k}(p,z) + g^2_{\phi\phi h,k}(p,z)\, \text{Diag}_k(p,z)$$

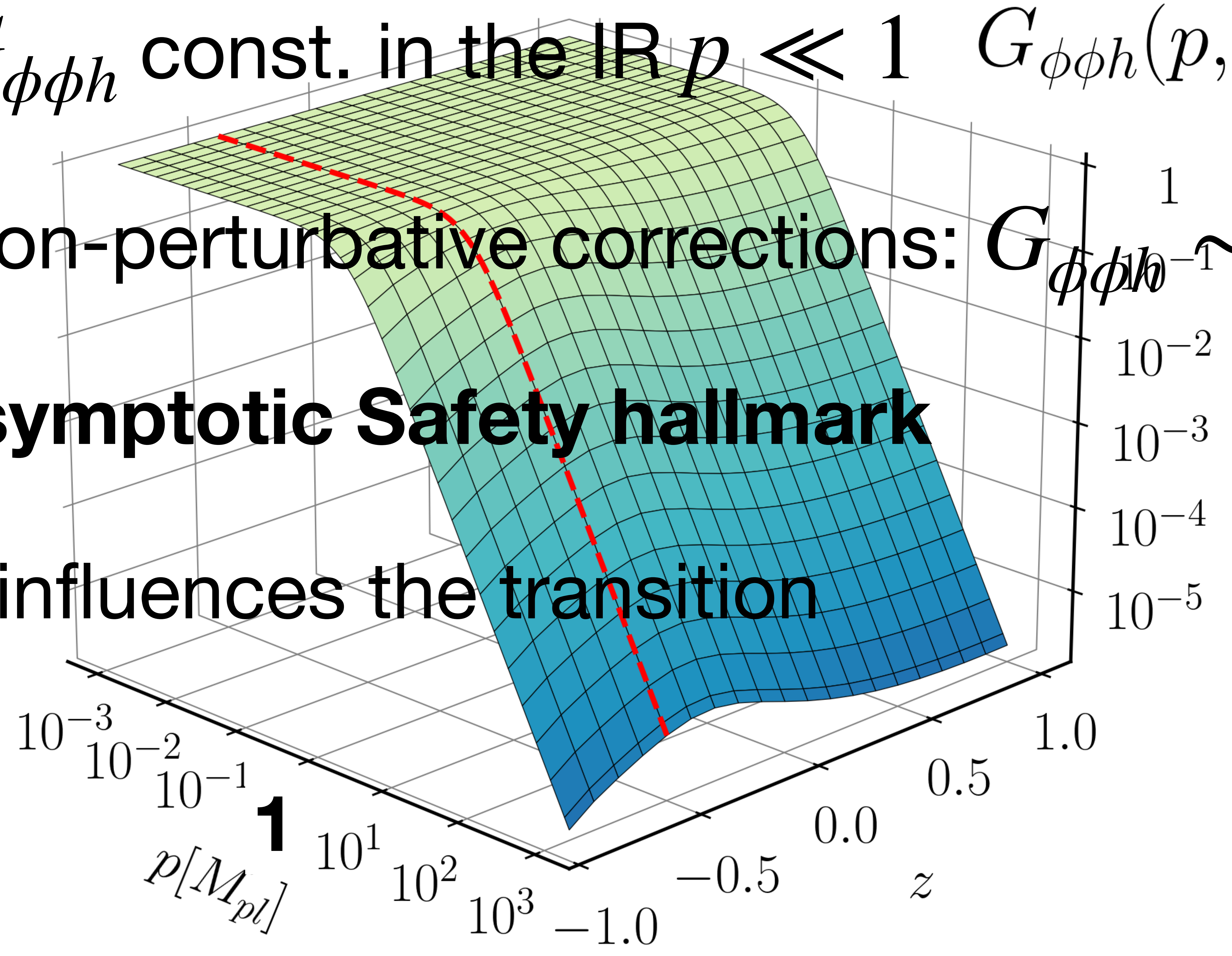
$(p/k, z)$ **dependence** $\to k = 0$ **(physics)**

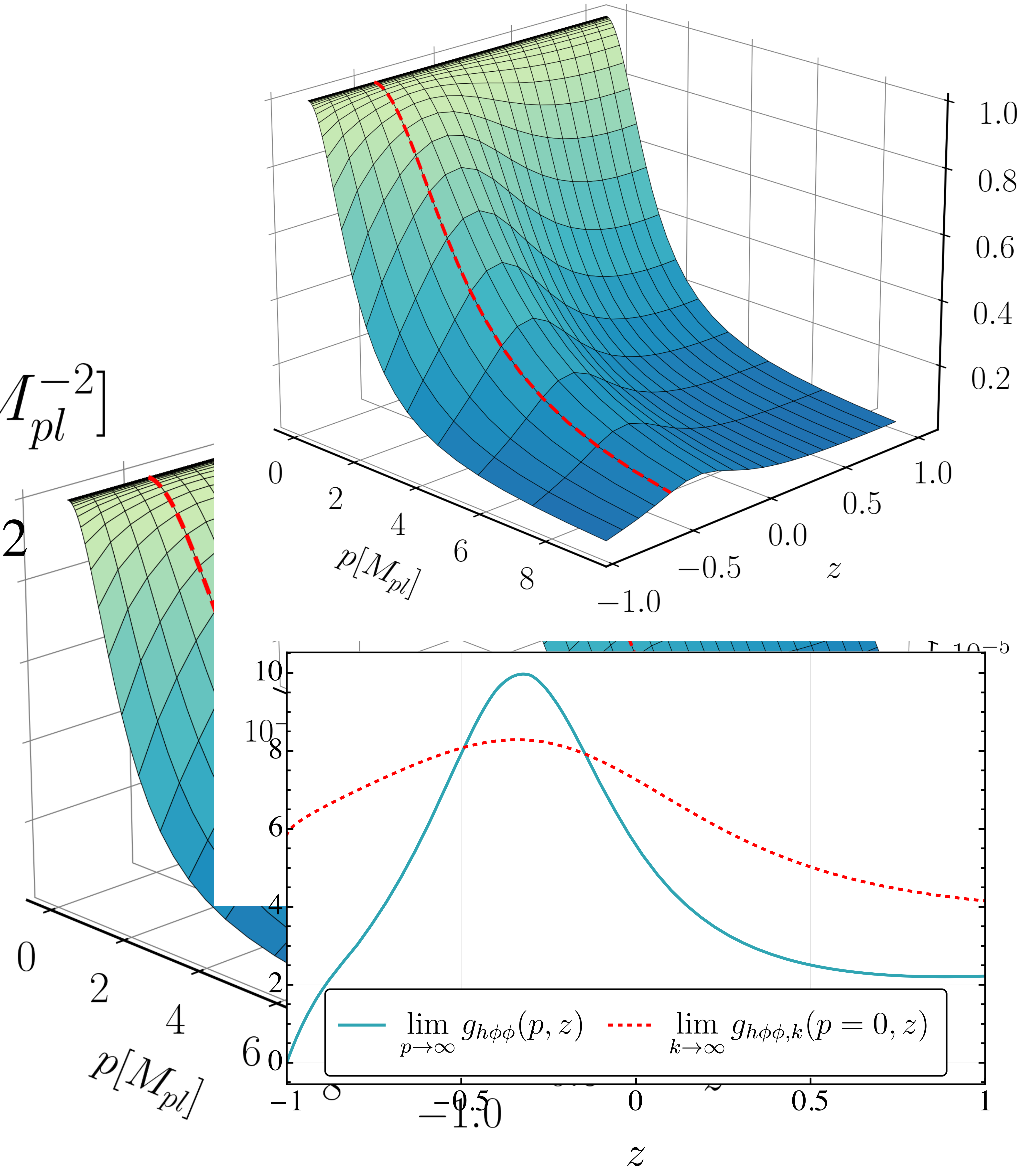
RESULTS
Newton Coupling

- $G_{\phi\phi h}$ const. in the IR $p \ll 1$
- Non-perturbative corrections: $G_{\phi\phi h} \sim p^{-2}$

Asymptotic Safety hallmark

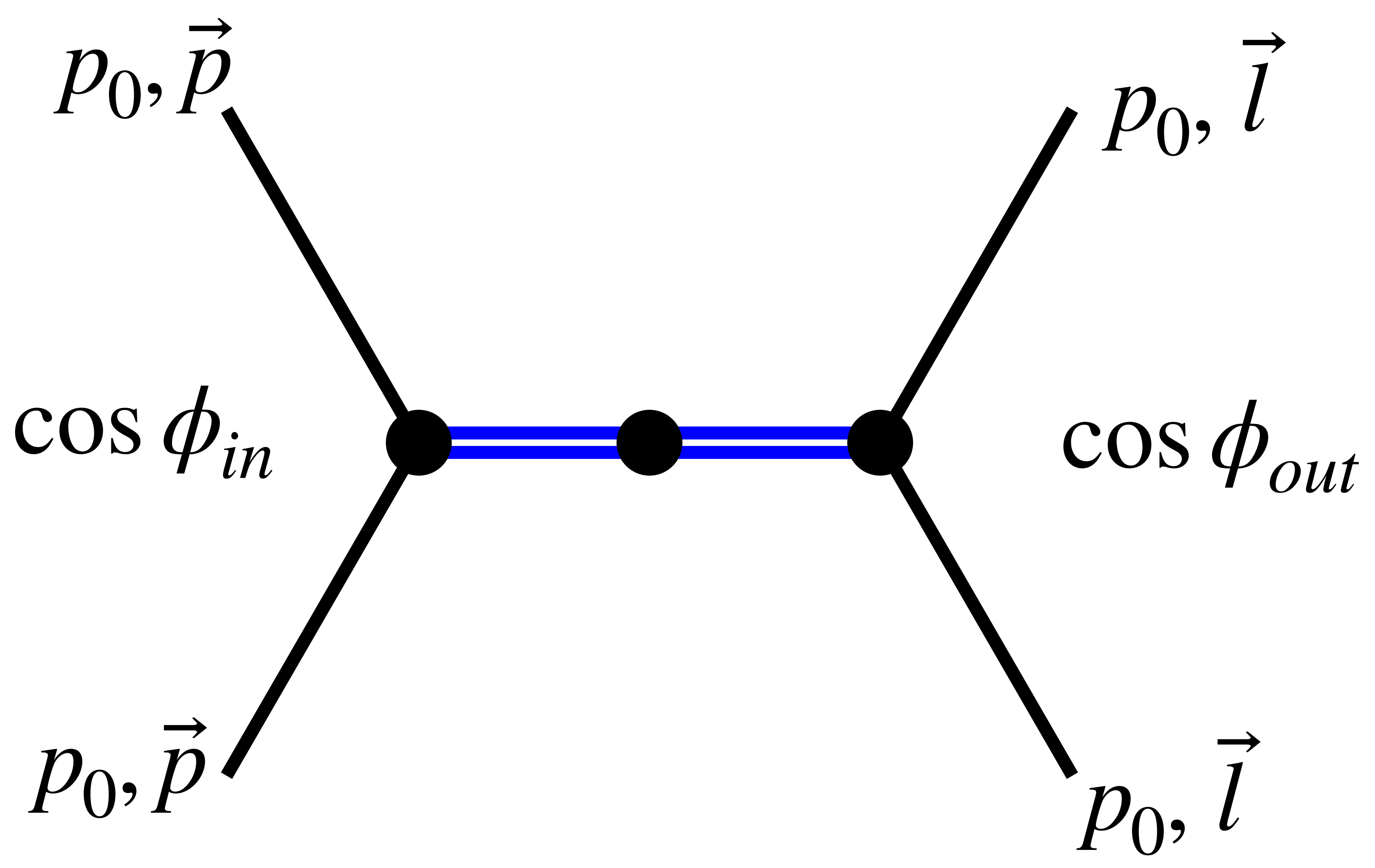
- z influences the transition





Interlude

Momentum Variables



- (p, z) **Euclidean** $\Rightarrow$ **Wick rotation?**
- Relate to $p_0, \vec{p}, \vec{l}$ + angles ϕ_{in}, ϕ_{out} in **CoM frame**

$$\mathscr{A} = \sqrt{G_{\phi\phi h}(p_0, ||\vec{p}||, \phi_{in})}\sqrt{G_{\phi\phi h}(p_0, ||\vec{l}||, \phi_{out})} \times T.C. = V_E(p_0) \times T.C.$$

- Reconstruct **only** p_0 **Multivariable reconstruction**
- On-shell: $p_0^2 = ||\vec{p}||^2$

RESULTS

Amplitude & Cross Section

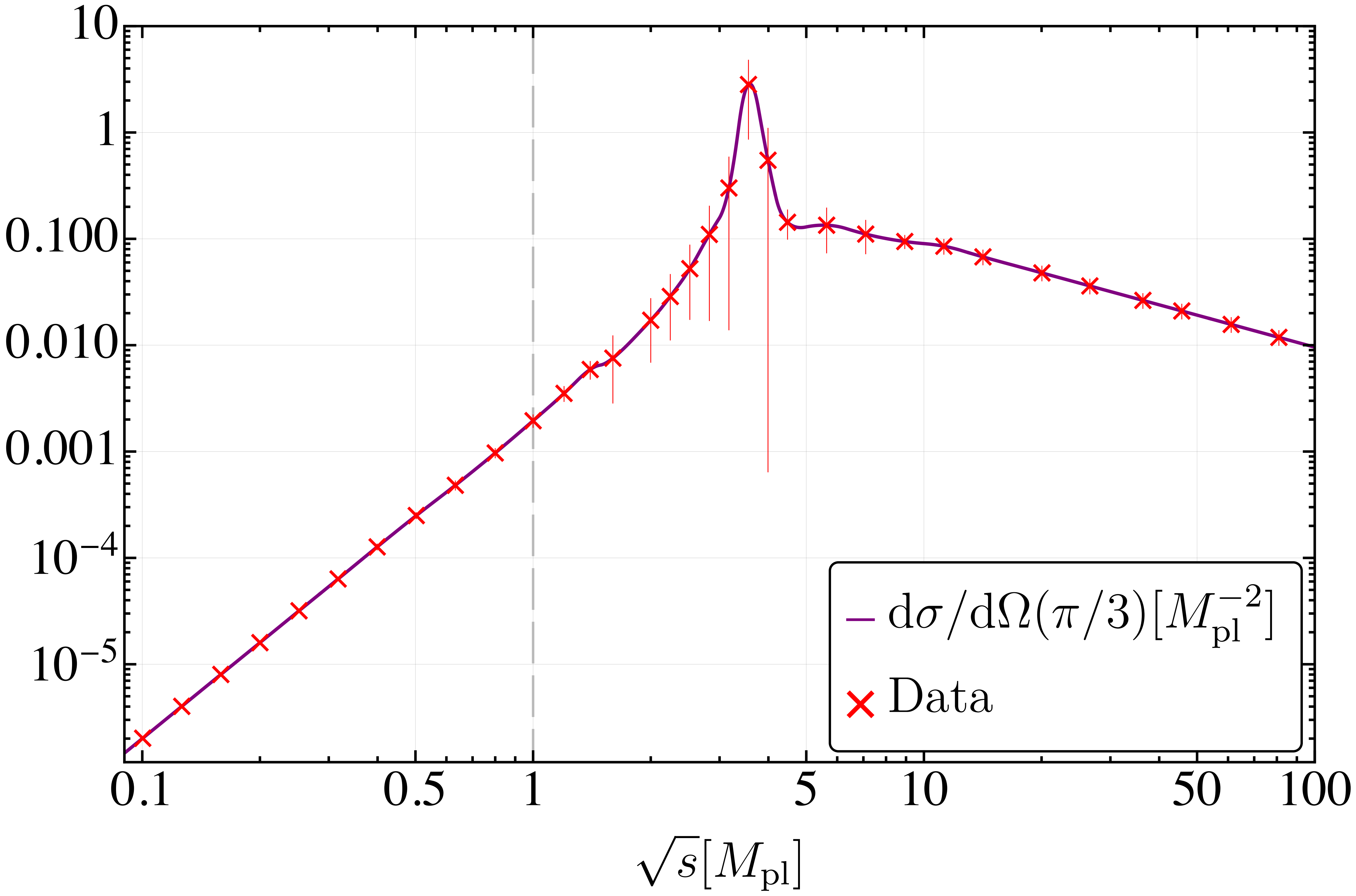
Numerical Nightmare

- Reconstruct and $p_0 \to -ip_0$
- On-shell $\to$ s, t, u dependence

$$\mathcal{A}_s = V_L(s)\frac{1}{s}\left(t^2 - 4tu + u^2 - s^2\right)$$

- Cross Section

$$\frac{d\sigma_s}{d\Omega} = \frac{1}{64\pi^2 s}\,|\mathcal{A}_s|^2$$



RESULTS

$$\mathcal{A}_s = V_L(s)\frac{1}{s}(t^2 - 4tu + u^2 - s^2)$$

RGi comparison

See also [Pastor-Gutiérrez, Pawlowski, Reichert, Ruisi '24]

- Standard RGi $p = 0, k = \sqrt{s}$

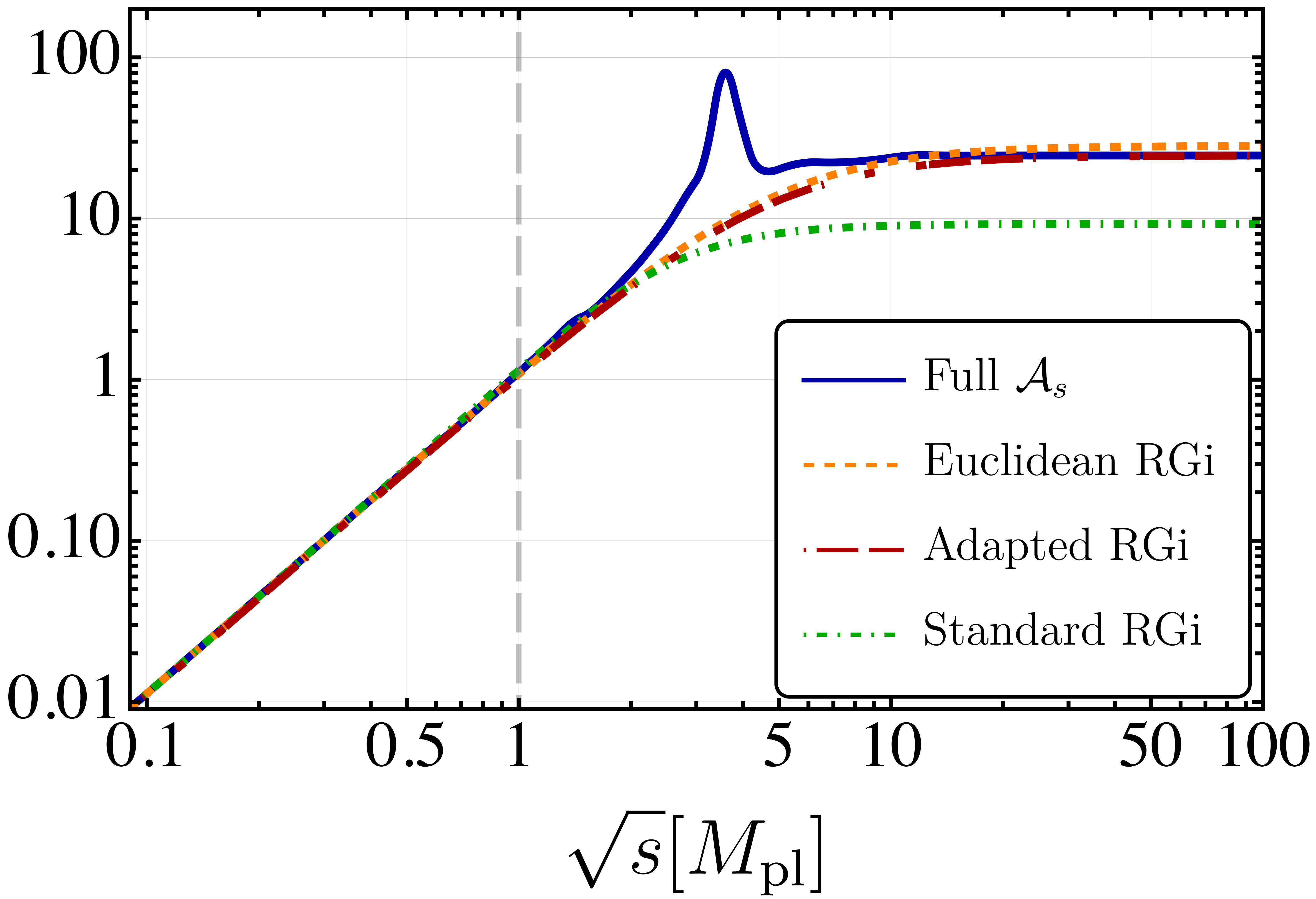
$$V_L(s) \longrightarrow \boxed{G(\sqrt{s},0)}$$

- Euclidean RGi $k \to 0, p^2 = s$

$$V_L(s) \longrightarrow \boxed{G(0,\sqrt{s})}$$

- Adapted RGi: use fix point of V_L

$$V_{\mathrm{RG}}(s) = \boxed{\frac{V^*}{s + V^* \mathrm{M}_{\mathrm{pl}}^2}}$$



Qualitative agreement

Miss peak structure!

Approximations

Massage the Flow eq.

- $\eta_\phi \equiv 0$ at LO in $\alpha = 0, \beta = 1$ gauge
- Neglect matter self-interactions
- Couplings $g_{n,m}$ loop independent
- Identify all $g_{n,m} \equiv g_{h\phi\phi}$
- Neglect $\mu = \lambda_i = 0$
- $\eta_h(q) \equiv 0$

$$\partial_t g_{k,h\phi\phi}(p_1,p_2) = \left(2 + \sum_j \eta_{k,j}(p_j^2)\right) g_{k,h\phi\phi}(p_1,p_2) + $$
$$+2g^{1/2}_{k,h\phi\phi}(p_1,p_2)\,\frac{\partial_t \Gamma^{h\phi\phi}_{\mu\nu}\mathcal{T}^{h\phi\phi,\mu\nu}}{\mathcal{N}k^{-1}},$$

$$\partial_t g_{h\phi\phi,k}(p,z) = (2 + \eta_{h,k})g_{h\phi\phi,k}(p,z) + g^2_{h\phi\phi,k}(p,z)\,\mathrm{Diag}_k(p,z)$$

Momentum Variables

Full glory

$$z = \frac{p_1 \cdot p_2}{|p_1||p_2|}$$

$$p_{\text{in}}^2 = p_0^2 + |\vec{p}\,|^2 \qquad z_{\text{in}} = 1 + \frac{|\vec{p}\,|^2}{|\vec{p}\,|^2 + p_0^2}\Big(\cos\left(\phi_{\text{in}}\right) - 1\Big)$$

$$p_{\text{out}}^2 = l_0^2 + |\,\vec{l}\,|^2 \qquad z_{\text{out}} = 1 + \frac{|\,\vec{l}\,|^2}{|\,\vec{l}\,|^2 + l_0^2}\Big(\cos\left(\phi_{\text{out}}\right) - 1\Big)$$

$$l_0^2 = \frac{1}{2}\Big(2p_0^2 + |\vec{p}\,|^2\left(1 + \cos\phi_{\text{in}}\right) - |\,\vec{l}\,|^2(1 + \cos\phi_{\text{out}})\Big)$$

$$\vec{p} \cdot \vec{l} = |\vec{p}\,||\,\vec{l}\,|\cos\theta_{\text{sc}}$$

$$s = 2p_{\text{in}}^2(1 + z_{\text{in}}) = 2p_{\text{out}}^2(1 + z_{\text{out}})$$

$$t = p_{\text{in}}^2 + p_{\text{out}}^2 + 2(p_0 l_0 + |\vec{p}\,||\,\vec{l}\,|\cos\theta_{\text{sc}})$$

$$u = 2p_{\text{in}}^2 + 2p_{\text{out}}^2 - s - t$$

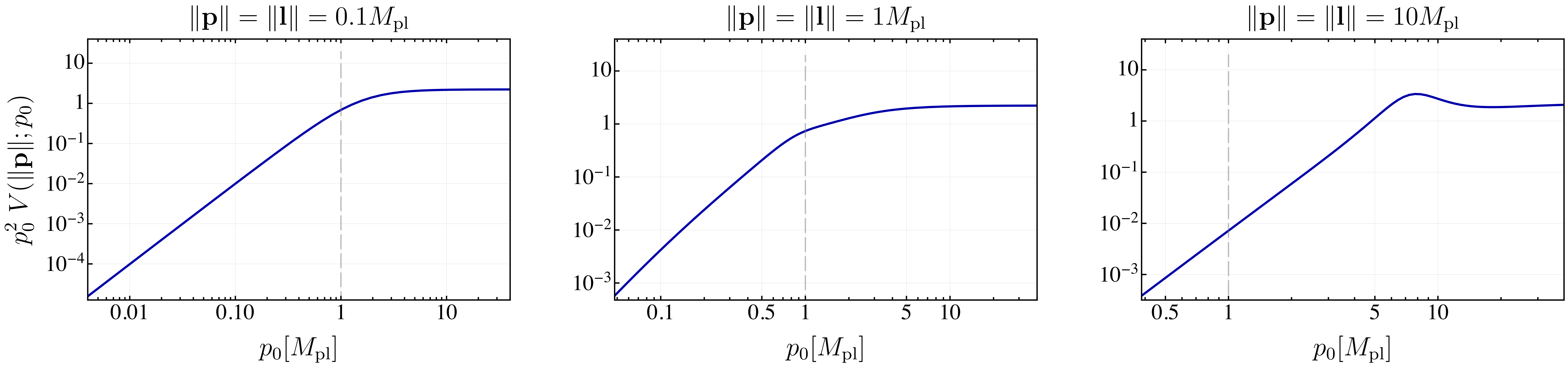
RESULTS

Euclidean Vertex

Quantum corrections, **Physical AS**

$$\mathscr{A} = V_E(p_0) \times T.C.$$

$$= \frac{1}{s}(t^2 - 4tu + u^2 - s^2)$$



Reconstruction

Technical details - Pipeline

$$C_{\mathrm{N}}(x) = \cfrac{y_1}{1 + \cfrac{a_1(x - x_1)}{1 + \cfrac{a_2(x - x_2)}{\vdots \; \ldots a_{\mathrm{N}-1}(x - x_{\mathrm{N}-1})}}}$$

- Assume a continued fraction for $V_E(|\vec{p}|; p_0)$
- Get the coefficient → **Hard Part**
- Derive the spectral function $\rho(|\vec{p}|; \lambda) = 2\, \Im V(|\vec{p}|; -\mathrm{i}(\lambda + \mathrm{i}0^+))$
- Compute $V_{rec}(|\vec{p}|; p_0) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(|\vec{p}|; \lambda)}{\lambda^2 + p_0^2}$.
- Compute $V_L(|\vec{p}|) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(|\vec{p}|; \lambda)}{\lambda^2 - |\vec{p}|^2 + i0^+}$ **On-shell Lorentzian Vertex**

Reconstruction

Technical details - Algorithm

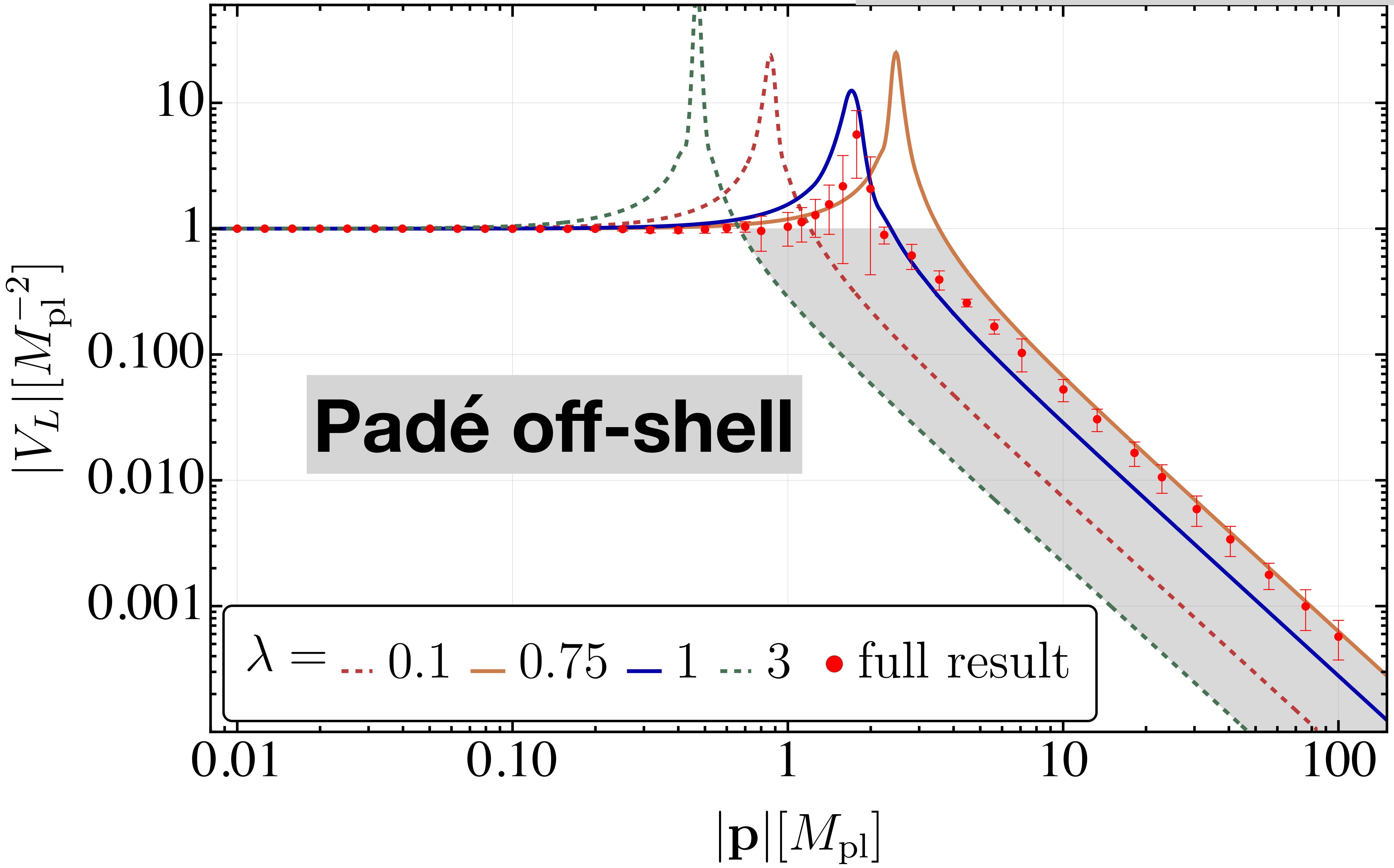
1. Select $N < M$ points from data & sample $(p_0^{(i)}, V(|\vec{p}|; p_0^{(i)}))$
2. Compute $C_N(p_0)$, $\rho_M(\lambda)$ and $V_{rec}(p_0)$
3. Compute $\chi^2 = \frac{1}{M}\sum_{i=1}^{M}\left(V\left(|\vec{p}|; p_0^{(i)}\right) - V_{rec,N}\left(|\vec{p}|; p_0^{(i)}\right)\right)^2$
4. Reject if $\chi^2 > \chi_T^2$ and select other N random points.
5. Accept otherwise and compute $V_L(|\vec{p}|)$
6. Repeat 1-5 for each $|\vec{p}|$
7. Repeat 1-6 $L = 80$ times ⇒ average 6 error bars!

Comparison

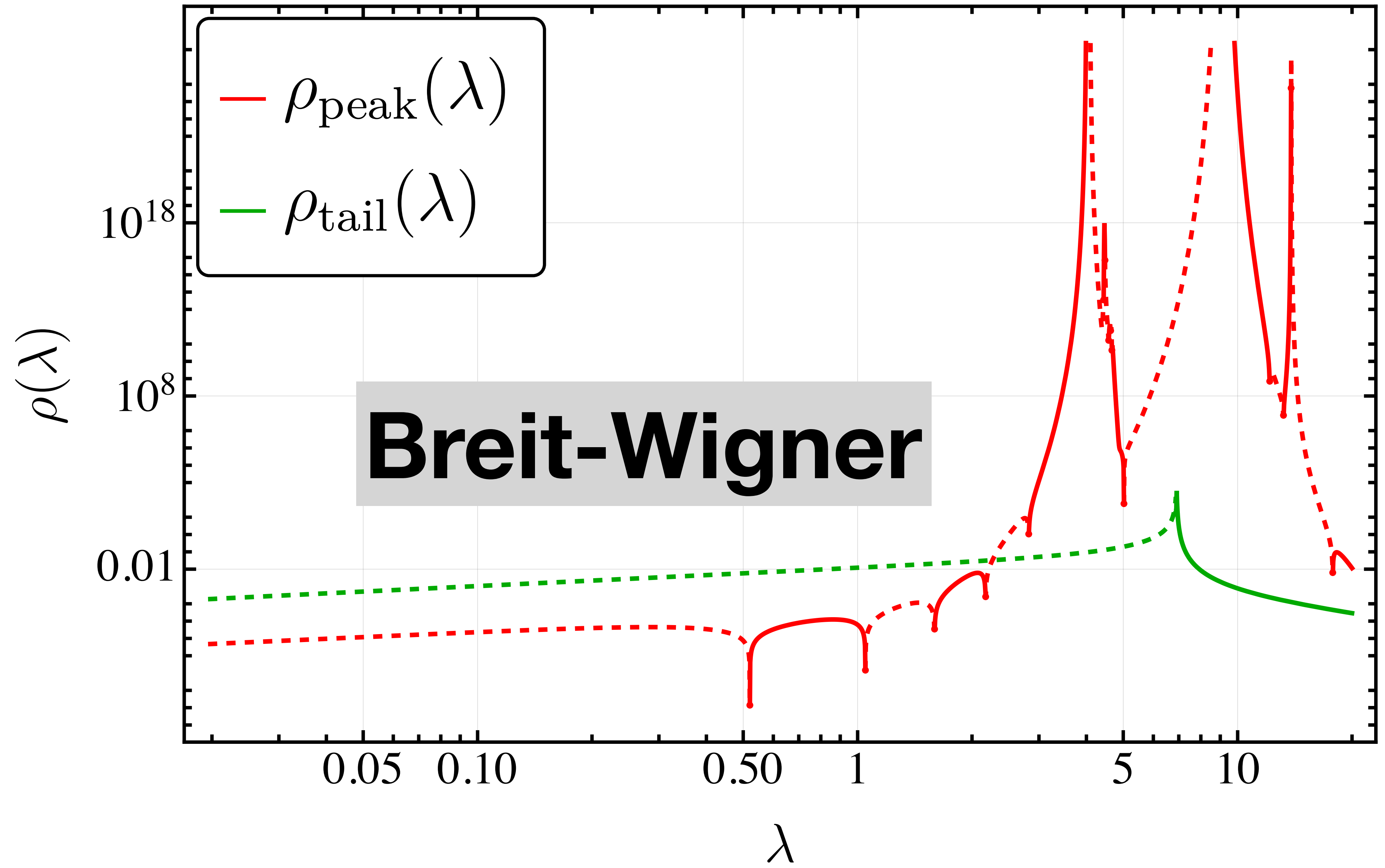
Other reconstructions

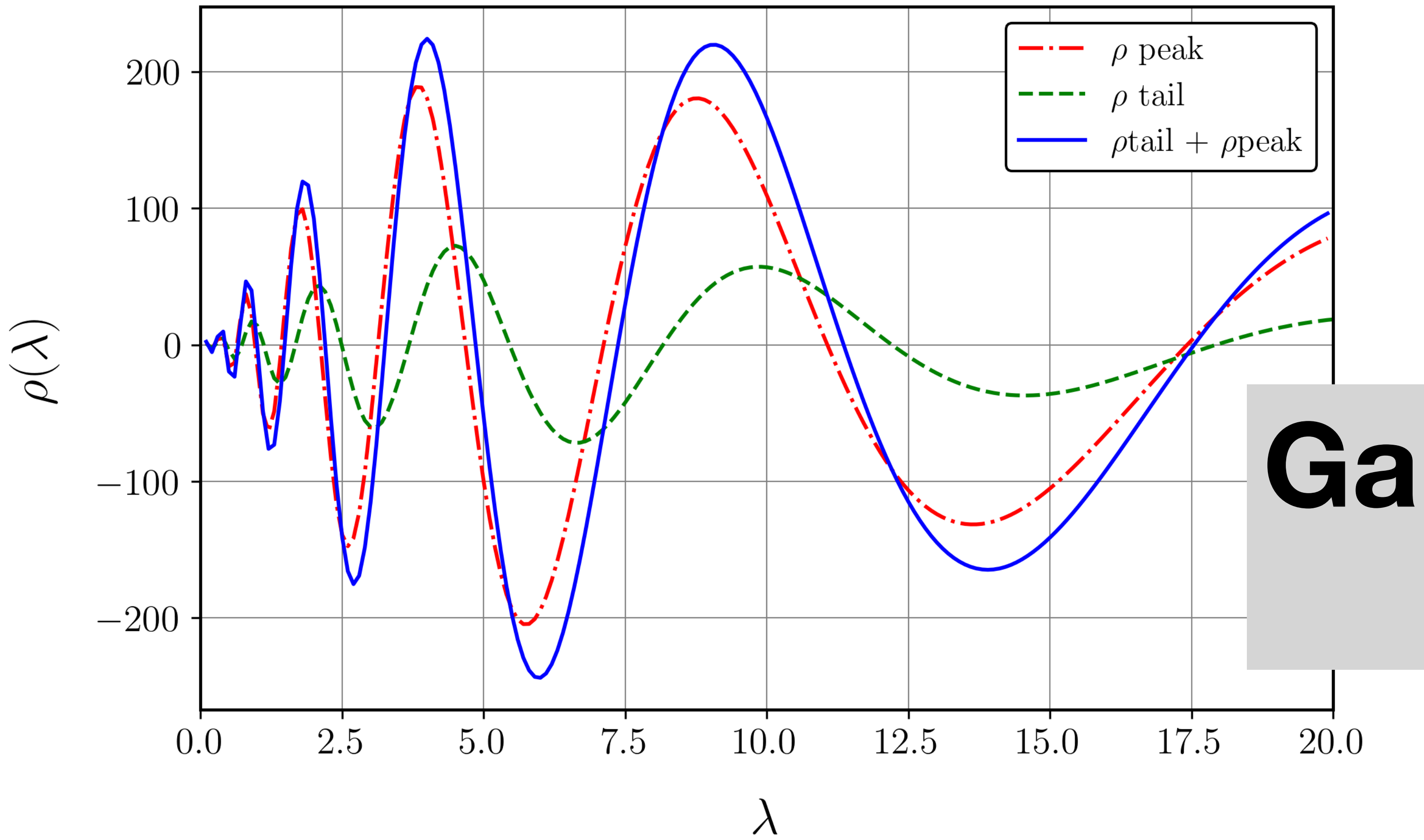
$$p_E^2 = (1+\lambda^2)\,|\vec{p}\,|^2,\;\; z = \frac{-1+\lambda}{1+\lambda}$$

$$V_{L,\lambda}(p_0) = \frac{1+\sum_{n=1}^{N} a_n p_0^n}{1+\sum_{n=1}^{N} b_{n+2} p_0^{n+2}}$$



$$V_{BW}(p_0) = \sum_{i=1}^{3} N_i \prod_{k=1}^{5} \left(\frac{1}{(p_0+\Gamma_{i,k})^2 + M_{i,k}^2} \right)^{\delta_{i,k}}$$





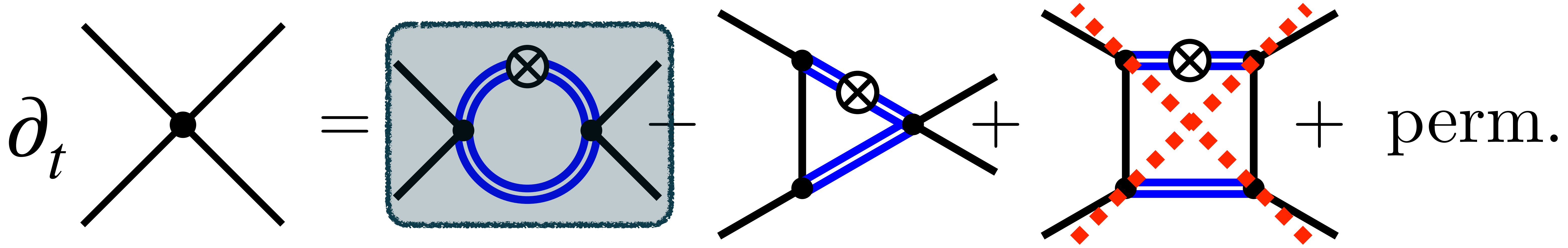
Gaussian Process Regression

Lorentzian $\Gamma^{4\phi}$

[APC, Reichert, in prep.]

“Through the looking glass“

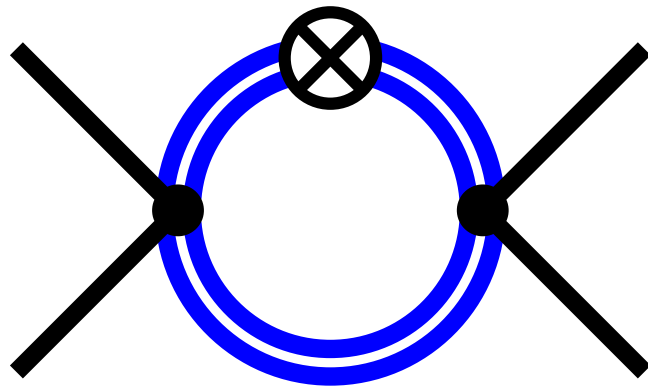
- Lorentzian fRG [Braun et al. ’22] [Fehre, Litim, Pawlowski, Reichert. ’21] [Kher, King, Litim, Reichert. ’25] [Pawlowski, Reichert, Wessely ‘25] [Assant, Litim, Reichert, in prep.]
- On-shell, Lorentzian flow $\partial_t \Gamma_k^{4\phi}(s,t)$
- $\Gamma_k \supset \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\phi\partial^\mu\phi$



Lorentzian $\Gamma^{4\phi}$

"Through the looking glass"

[APC, Reichert, in prep.]

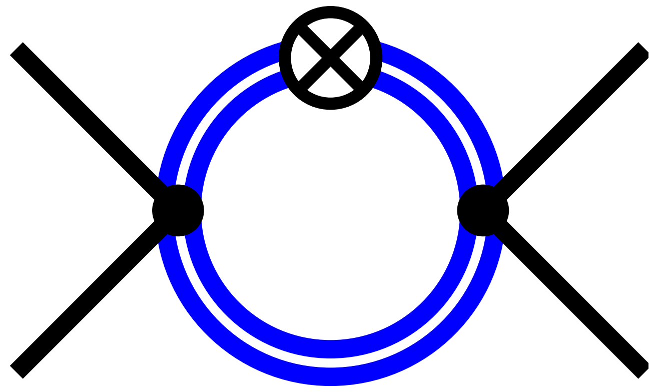


$$= \int_q \Gamma_k^{\phi\phi hh}(p_1, p_2, q)\Gamma_k^{\phi\phi hh}(p_3, p_4, q) f(q, p_1, p_2, p_3, p_4) \times \mathcal{G}_k(q + p_1 + p_2)(\mathcal{G}_k \partial_t R_k \mathcal{G}_k)(q)$$

IR

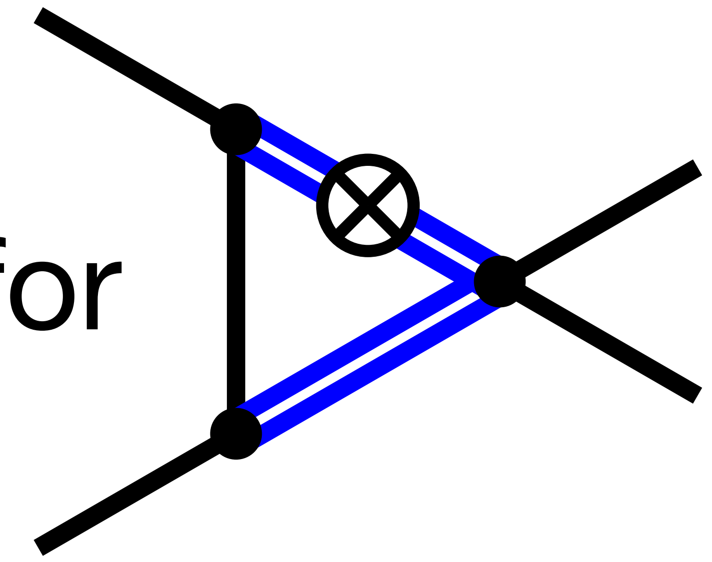
$$G^{\phi\phi hh}(0) = G_N$$

Main approximation: $\Gamma_k^{\phi\phi hh}(\mathbf{p}, q) \approx G_k^{\phi\phi hh}(\mathbf{p},0)\, T^{\mu\nu\rho\sigma}(\mathbf{p}, q)$



$\approx G_k^{\phi\phi hh}(s, t)^2\, \mathrm{Diag}_k(s, t)$

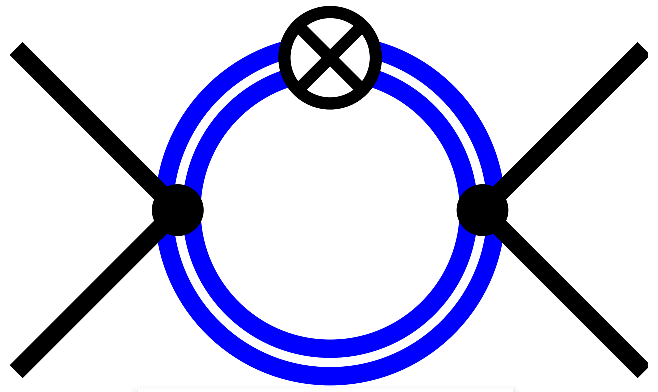
Mutatis Mutandis for



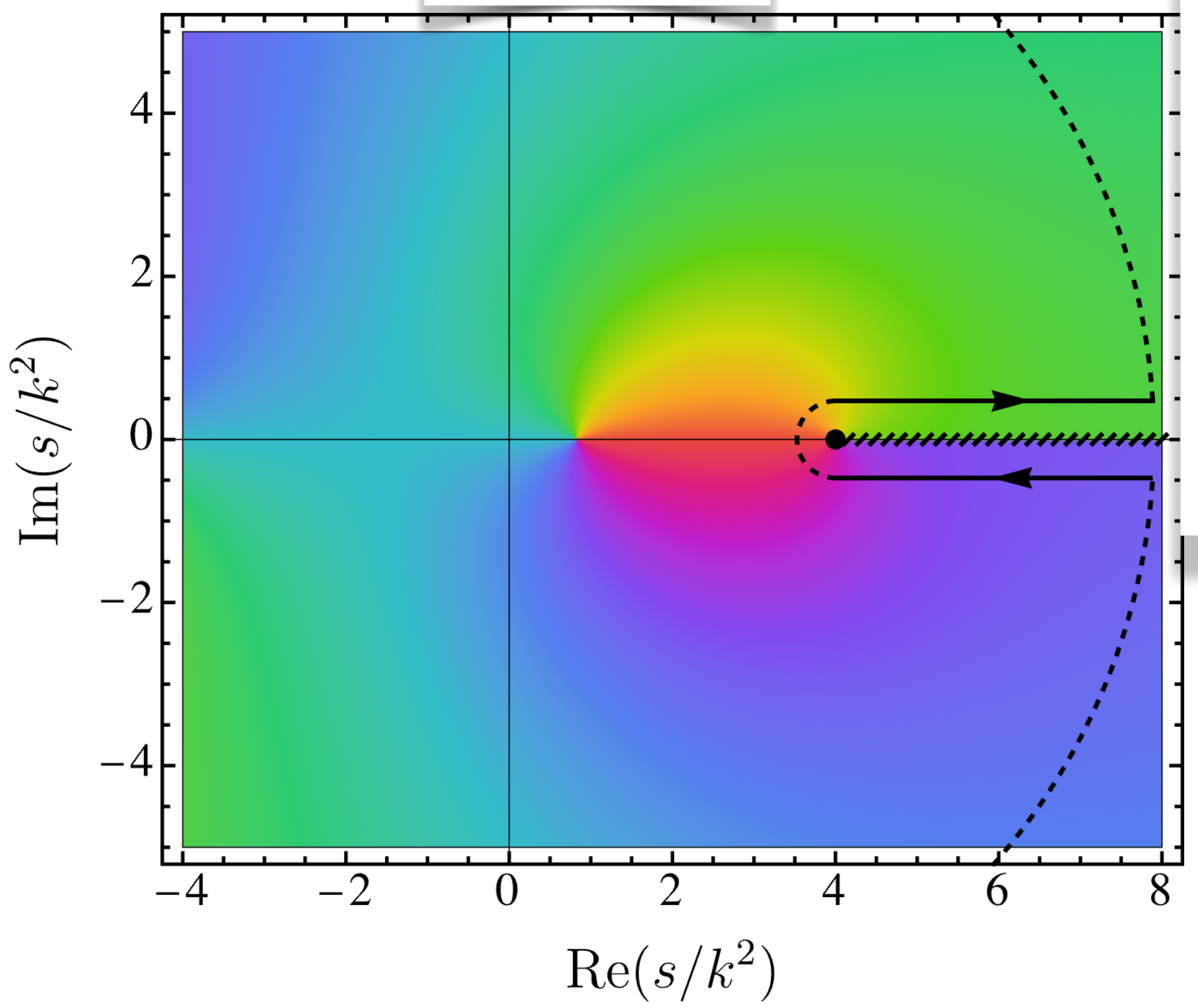
Lorentzian $\Gamma^{4\phi}$

“Through the looking glass“

[APC, Reichert, in prep.]



$t = 0$

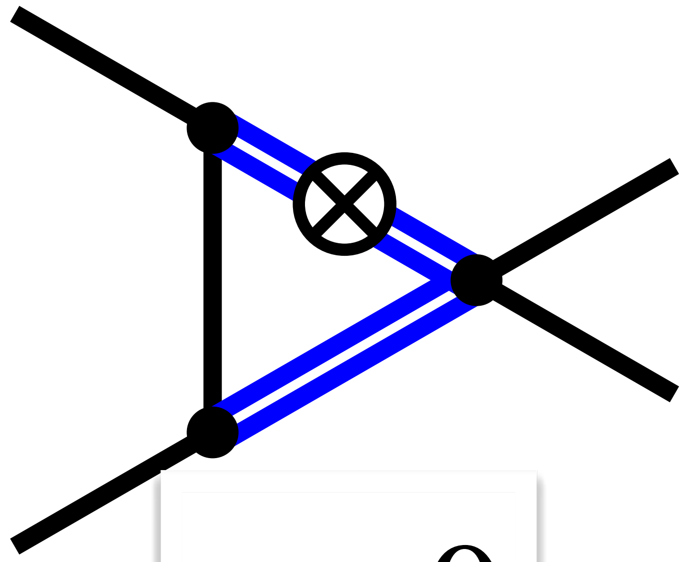


“Flowing cut” $\partial_t \, \mathrm{Im}\left(\Gamma^{4\phi}\right) =$

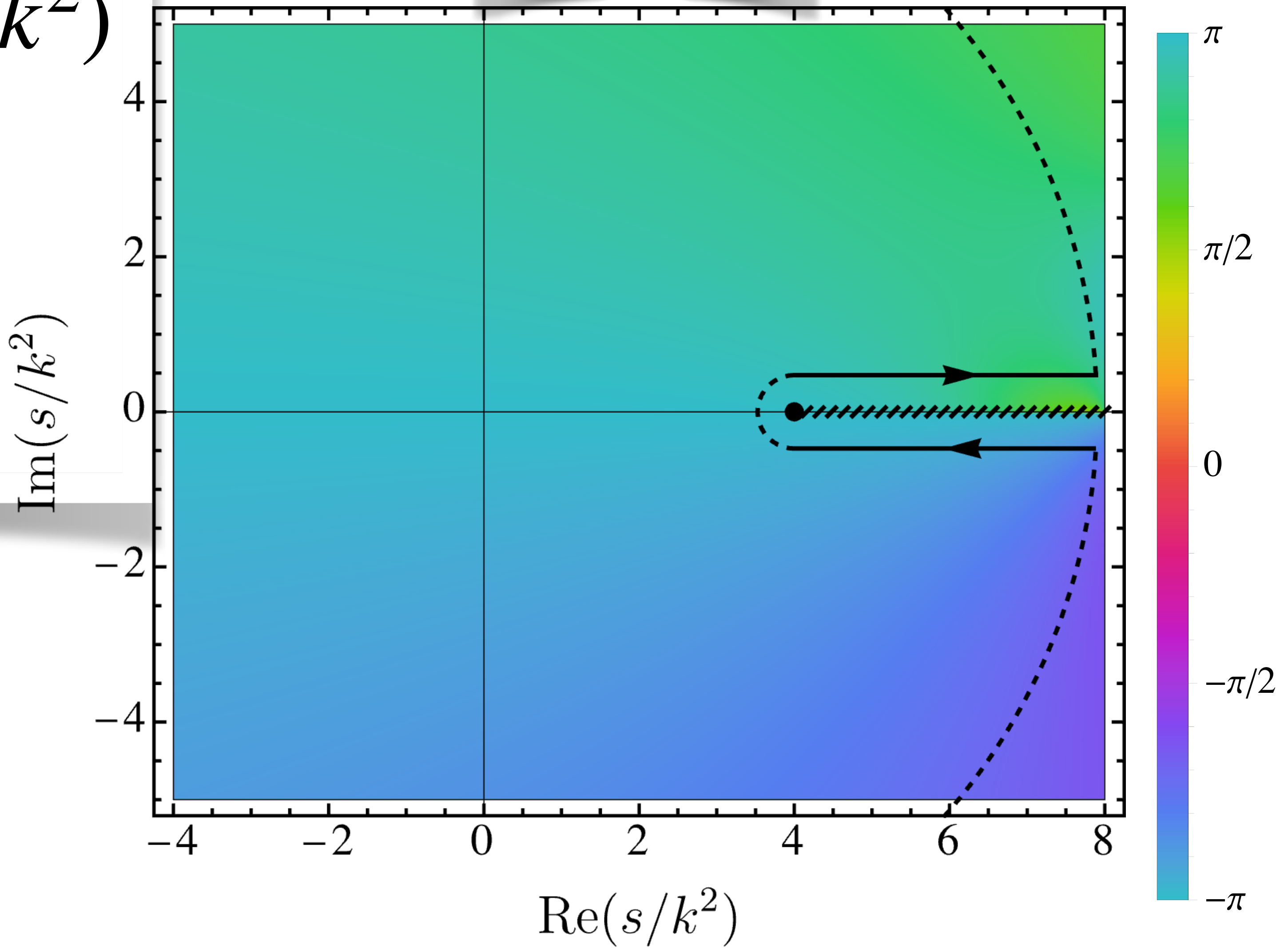
$$\left(G_k^{\phi\phi hh}\right)^2 \frac{32\pi\left(-4k^6 + 4k^4 s + k^2(s+2t)^2\right)}{\sqrt{s\left(s-4k^2\right)}}\Theta(s-4k^2)$$

$+$Triangle(s)

$$s + t + u = 4k^2$$



$t = 0$

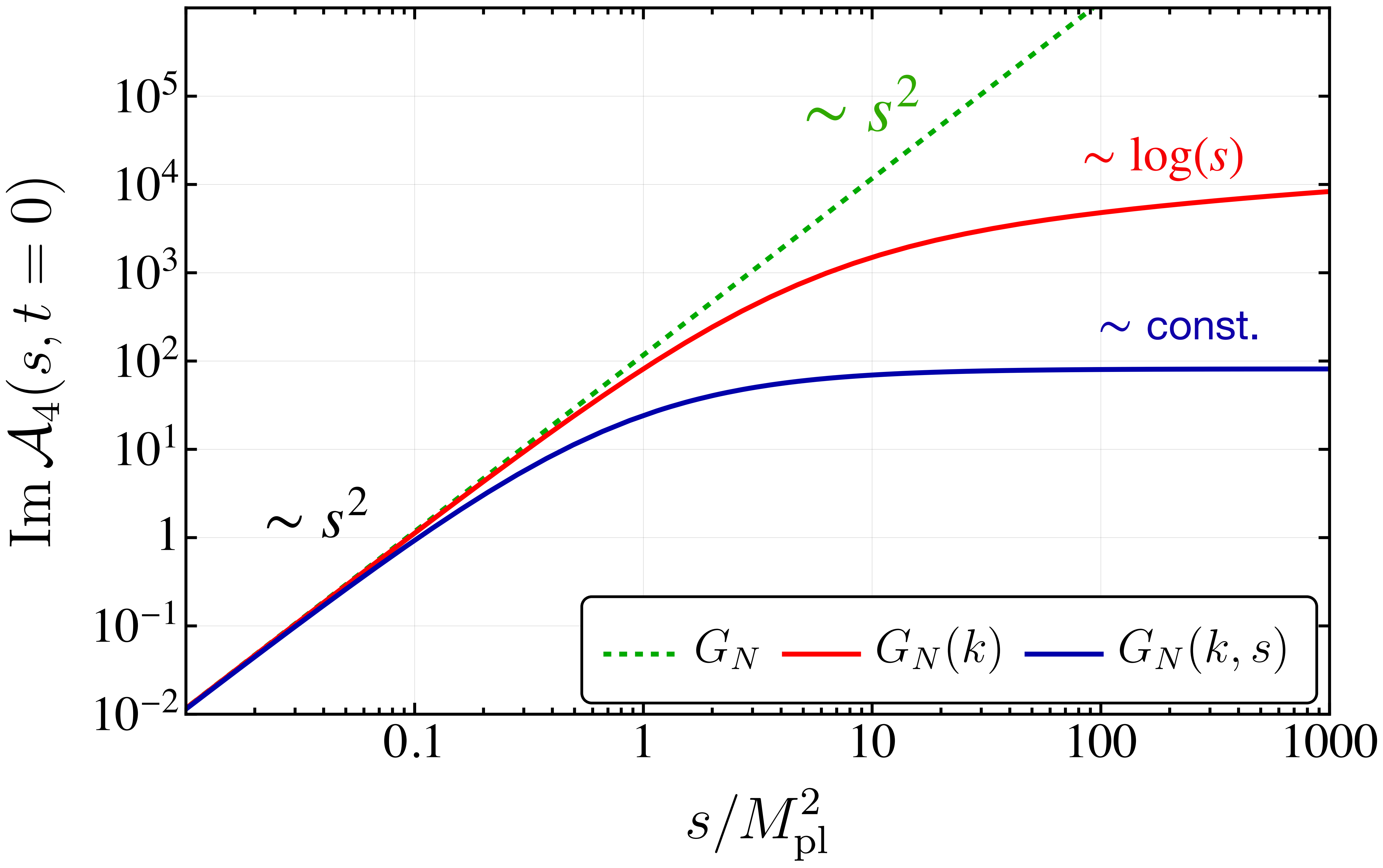


Lorentzian $\Gamma^{4\phi}$

"Through the looking glass"

[APC, Reichert, in prep.]

$t \to 0$ Amplitude



Structure:

$$\partial_t \,\mathrm{Im}\,\Gamma^{4\phi}(s,t) = G_N(\mathbf{x})^2 \;\mathrm{Im}\;\mathrm{Diag}_k(s,t)$$

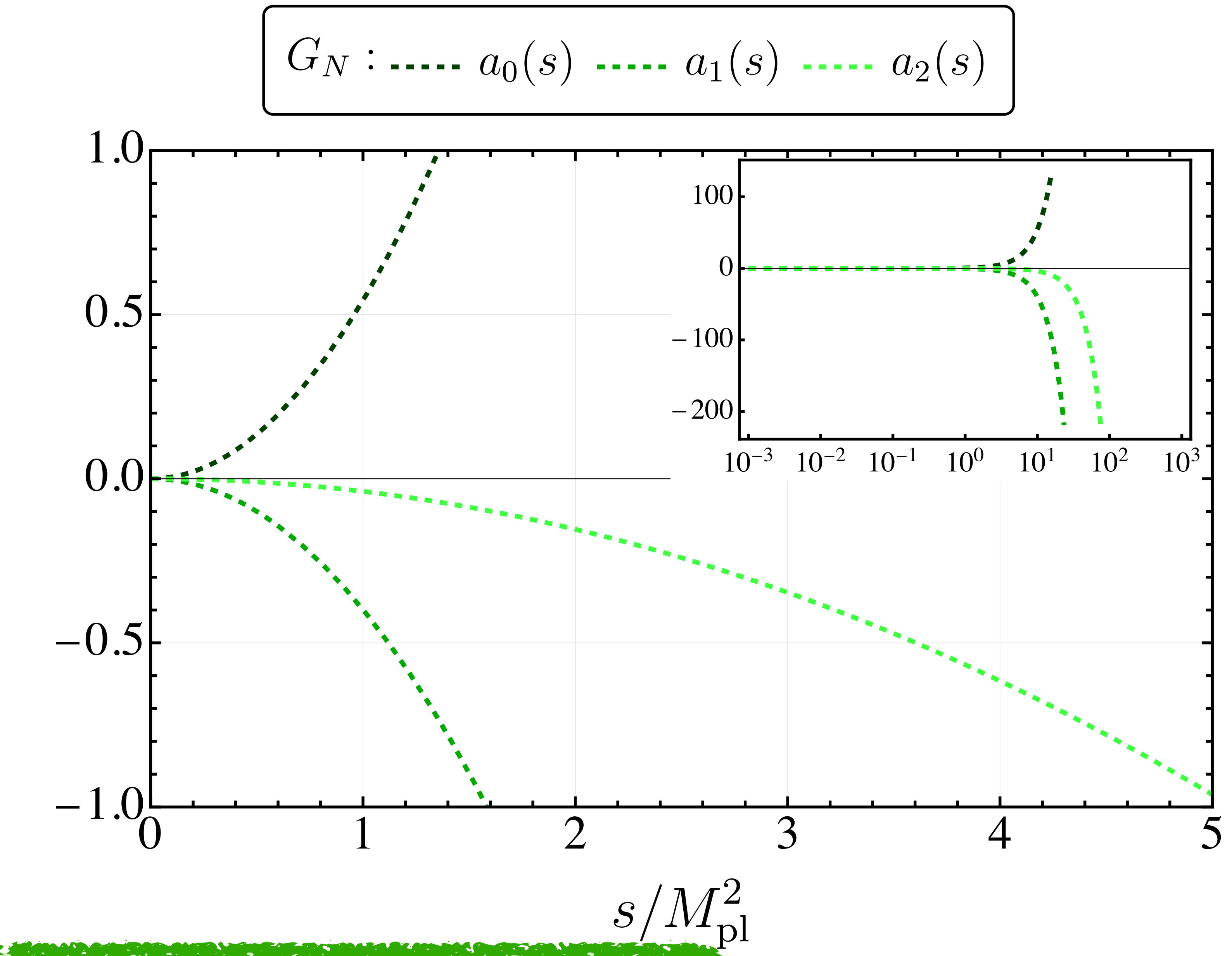
Approximations:

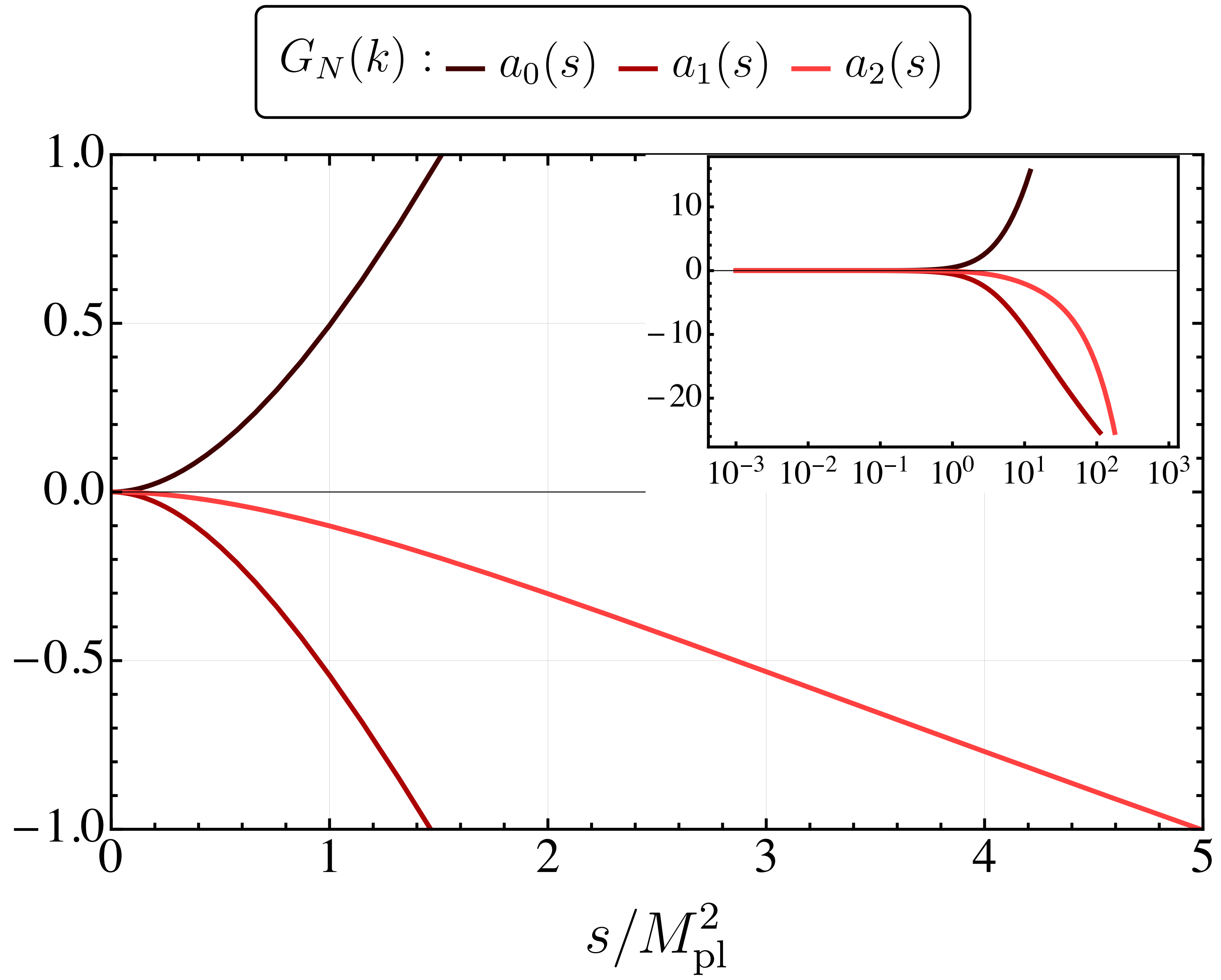
1. $G_N(\mathbf{x}) \equiv G_N$
2. $G_N(\mathbf{x}) = G_N(k) = \dfrac{g^*}{k^2 + g^* M_{\rm pl}^2}$
3. $G_N(\mathbf{x}) = G_N(k,s) = \dfrac{g^*}{k^2 + s + g^* M_{\rm pl}^2}$

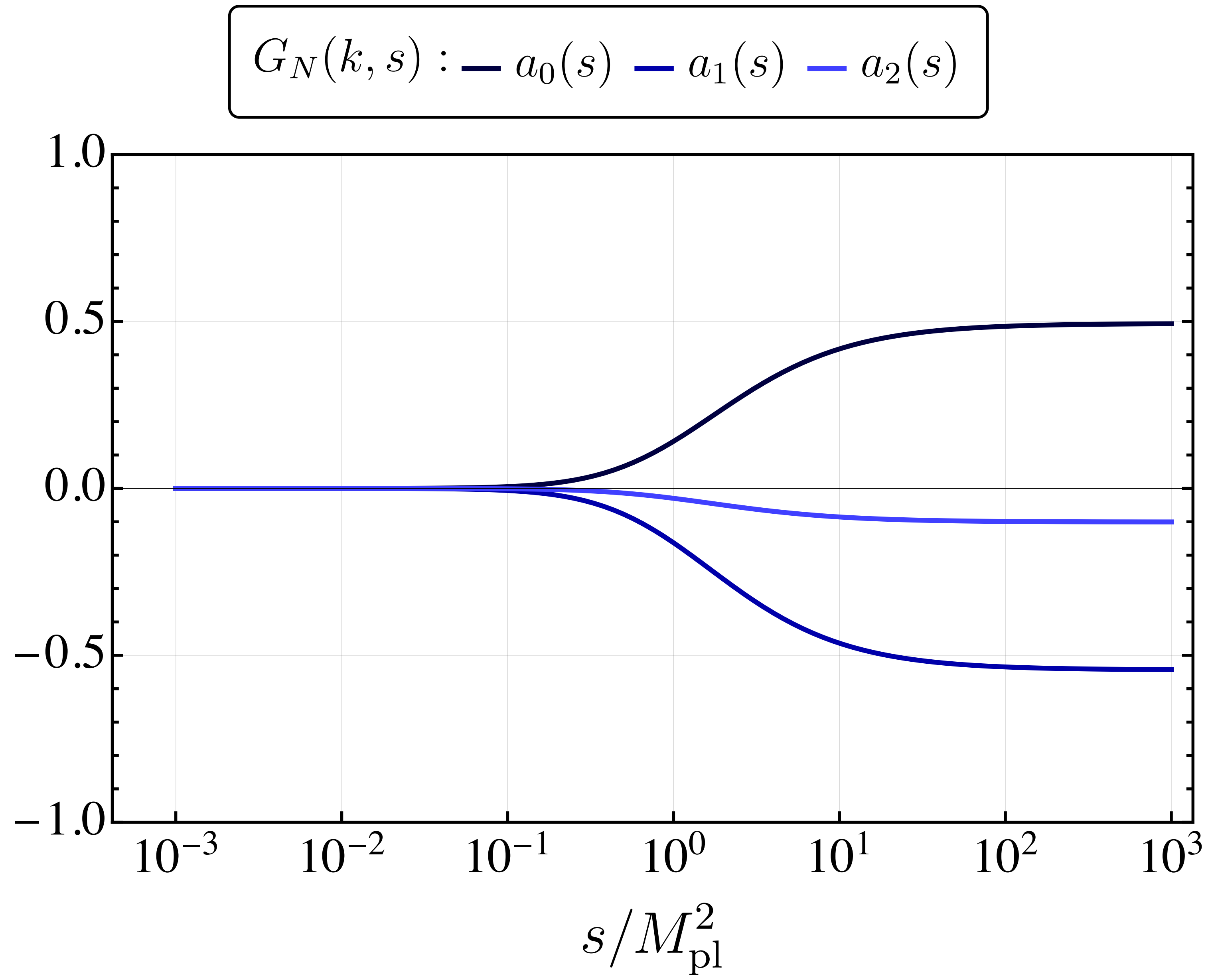
Educated by Euclidean results

Lorentzian $\Gamma^{4\phi}$

[APC, Reichert, in prep.]







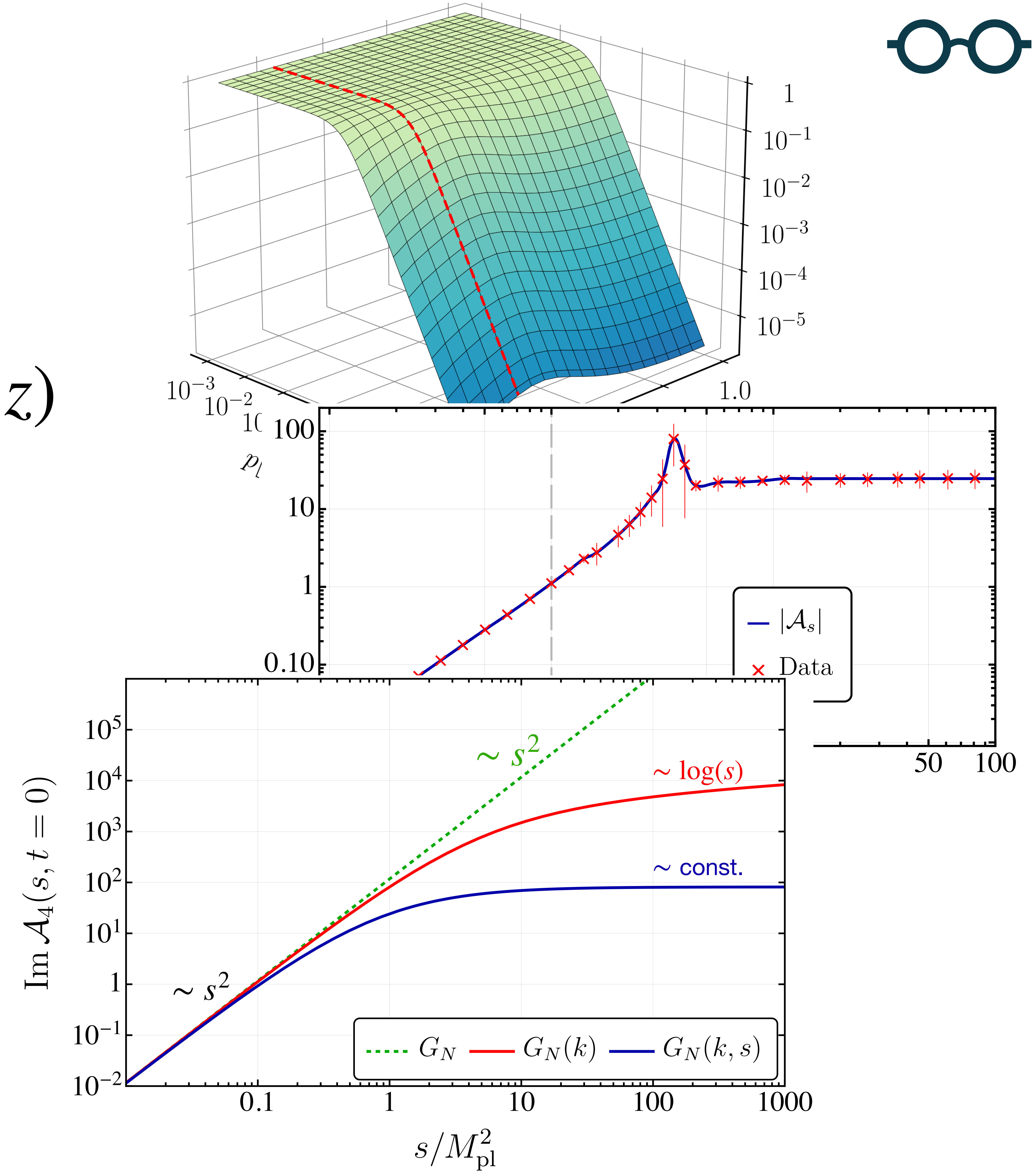
1. $G_N(\mathbf{x}) \equiv G_N$
2. $G_N(\mathbf{x}) = G_N(k) = \dfrac{g^*}{k^2 + g^* M_{\rm pl}^2}$
3. $G_N(\mathbf{x}) = G_N(k,s) = \dfrac{g^*}{k^2 + s + g^* M_{\rm pl}^2}$

Partial Waves: $t = -\dfrac{s}{2}(1-x)$ CoM frame

$$a_j(s) = \frac{1}{32\pi}\int_{-1}^{1} dx\, \mathscr{A}_4(s,x)$$

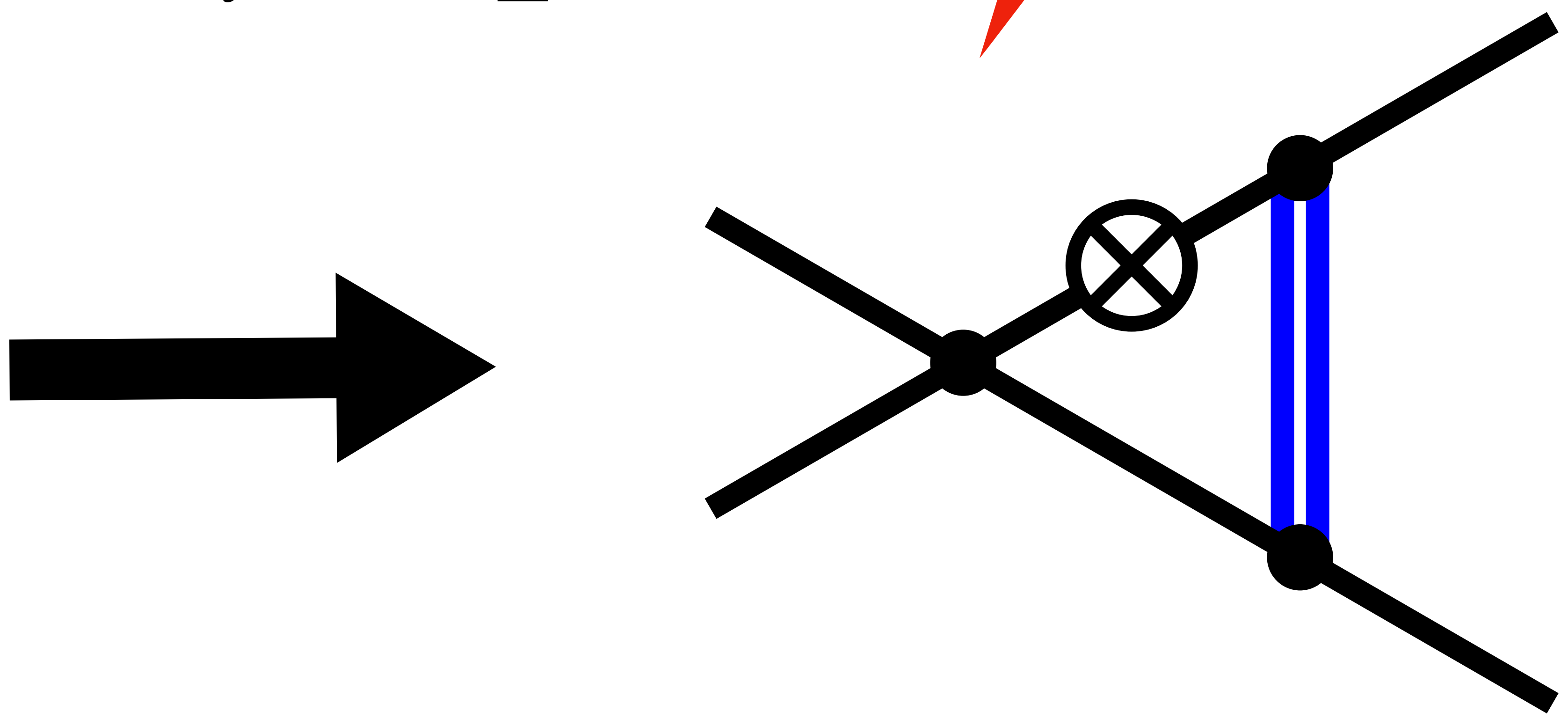
Conclusions

- **Full momentum dependence** $G_{\phi\phi h}(p, z)$
- $s-$**channel amplitude FINITE**
- **Full momentum dependence** $\Gamma^{4\phi}(s, t)$
- $\mathscr{A}_4(s, t)$ **FINITE**
- **Graviball?** [Pastor-Gutiérrez, Pawlowski, Reichert, Ruisi '24] [Maas, Plätzer, Pressler '25]



Outlook

- **Include all the channels**
  - $t-$channel: for $s \to \infty$, $\mathscr{A}_t \propto V_L(t)\, s^2$
- **Feedback** $\Gamma_k^{4\phi}(s,t)$

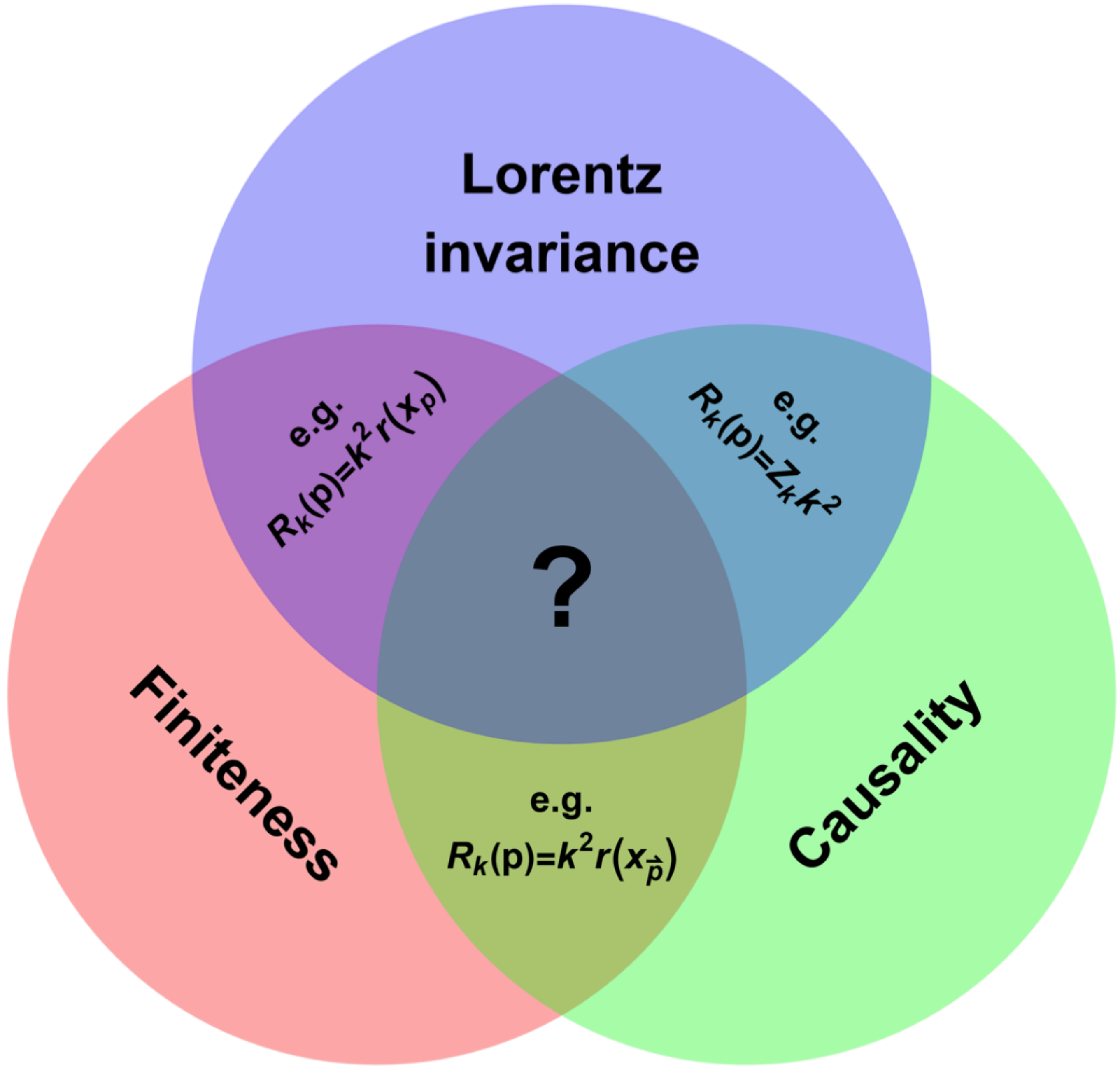


  - **Eikonal?**
- **Graviton bound state**

Spectral fRG

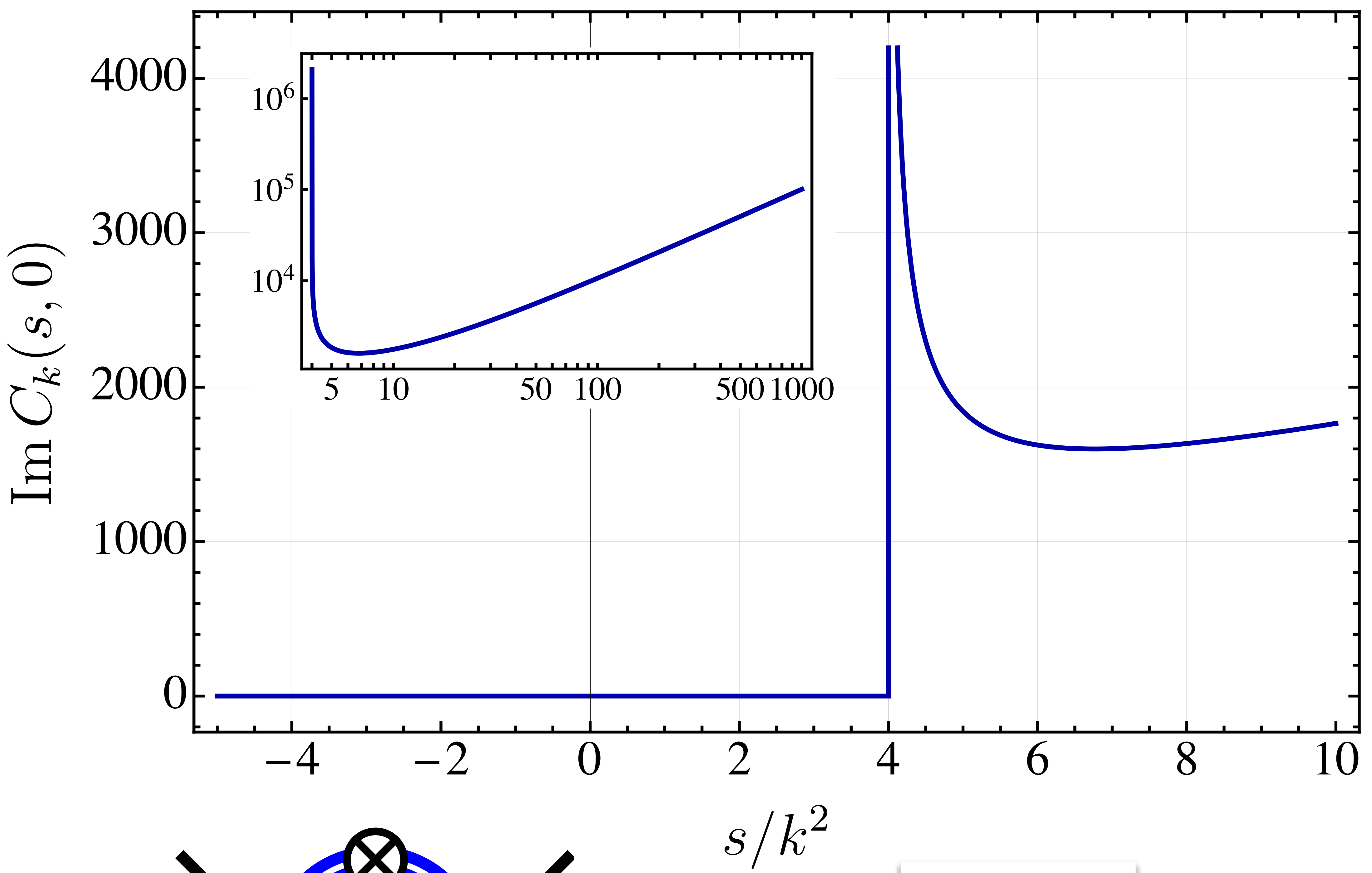
[Braun et al. ’22] [Fehre, Litim, Pawlowski, Reichert. ’21] [Kher, King, Litim, Reichert. ’25] [Pawlowski, Reichert, Wessely ‘25] [Assant, Litim, Reichert, in prep.]

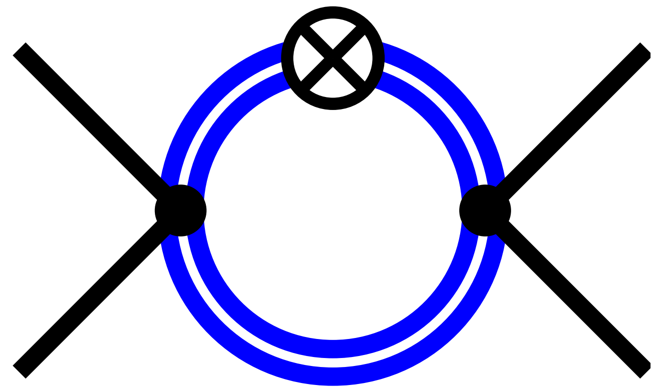
- Casual & Lorentz Invariant $R_k(p) = k^2$
- IR finite - UV needs “flowing counterterms”
- Wetterich Equation $\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \right] - \partial_t S_{ct,k}$
- On-shell scheme $\mu_h = 0, \eta_h = 0, \eta_s = 0$



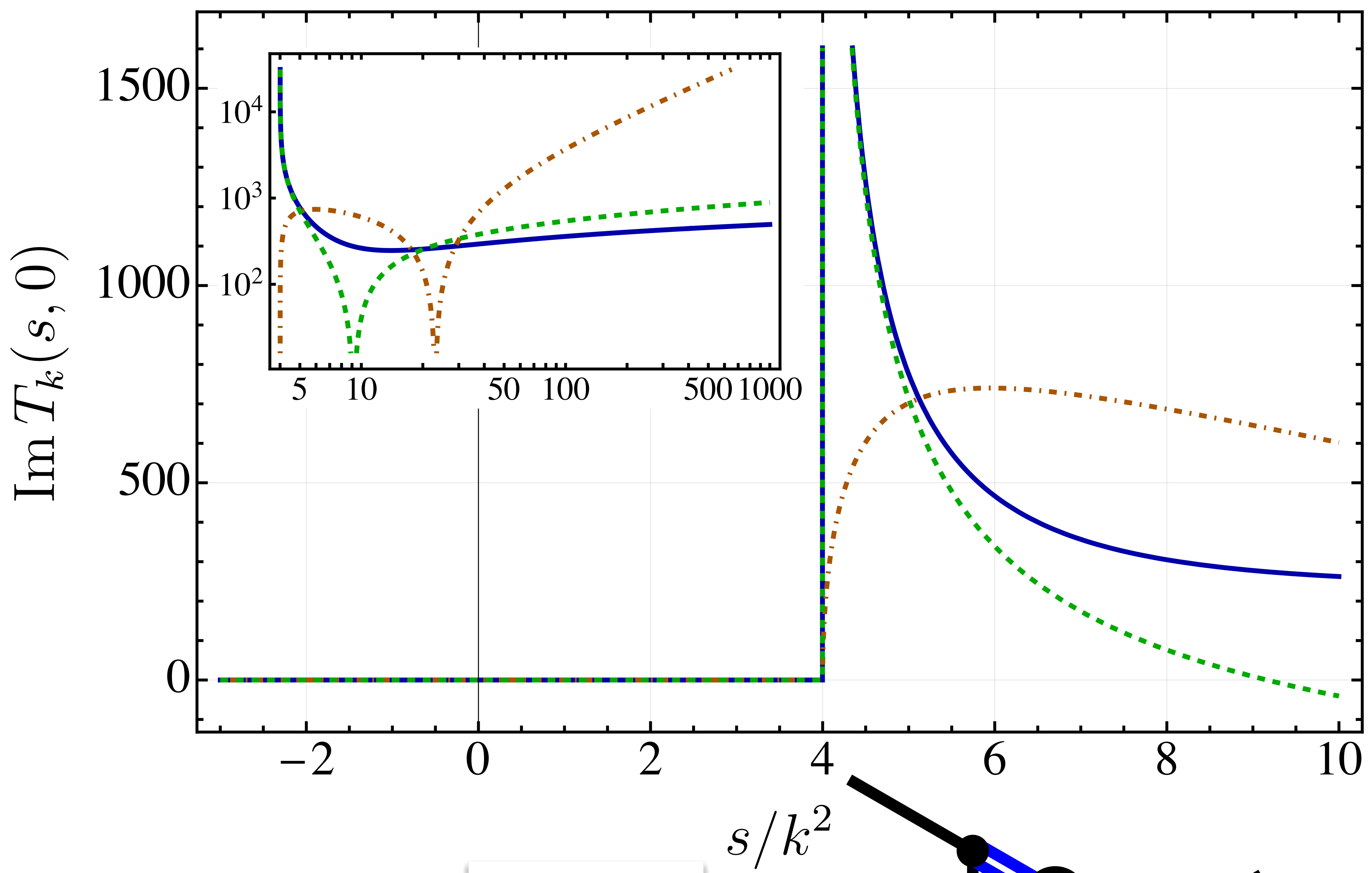
Flowing on the cut

Candy & Triangle

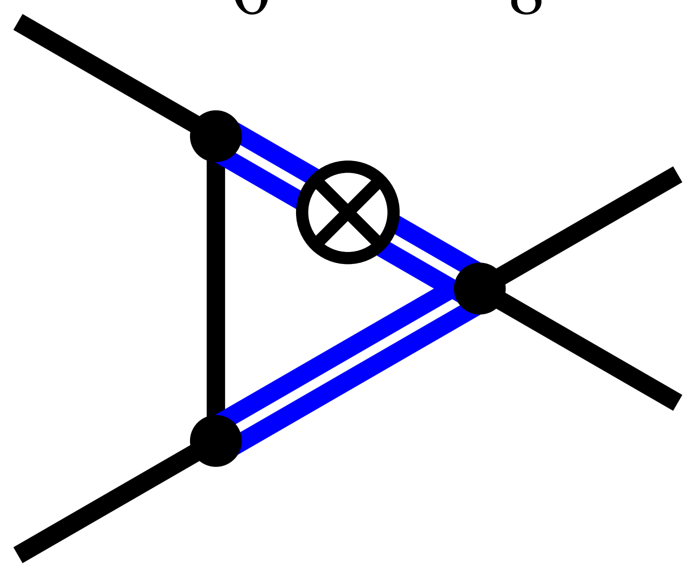




$t = 0$



$t = 0$



Box Diagram

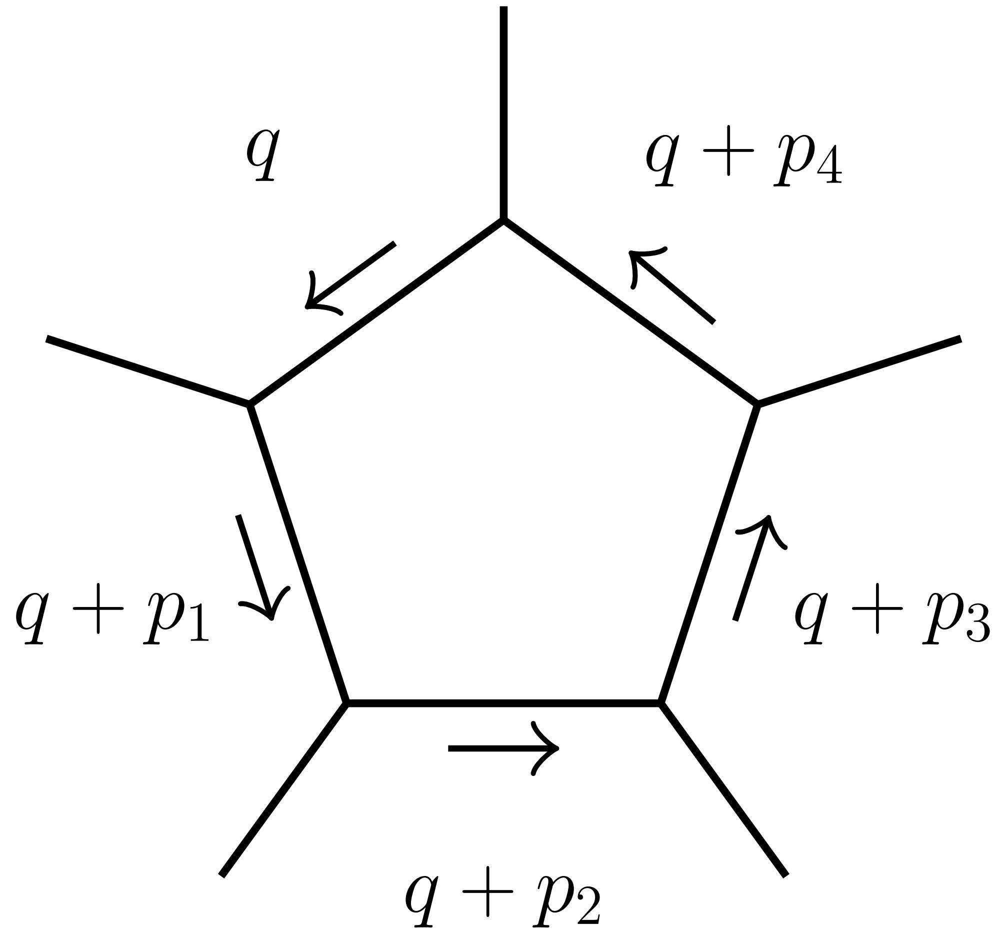
Basically a pentagon with

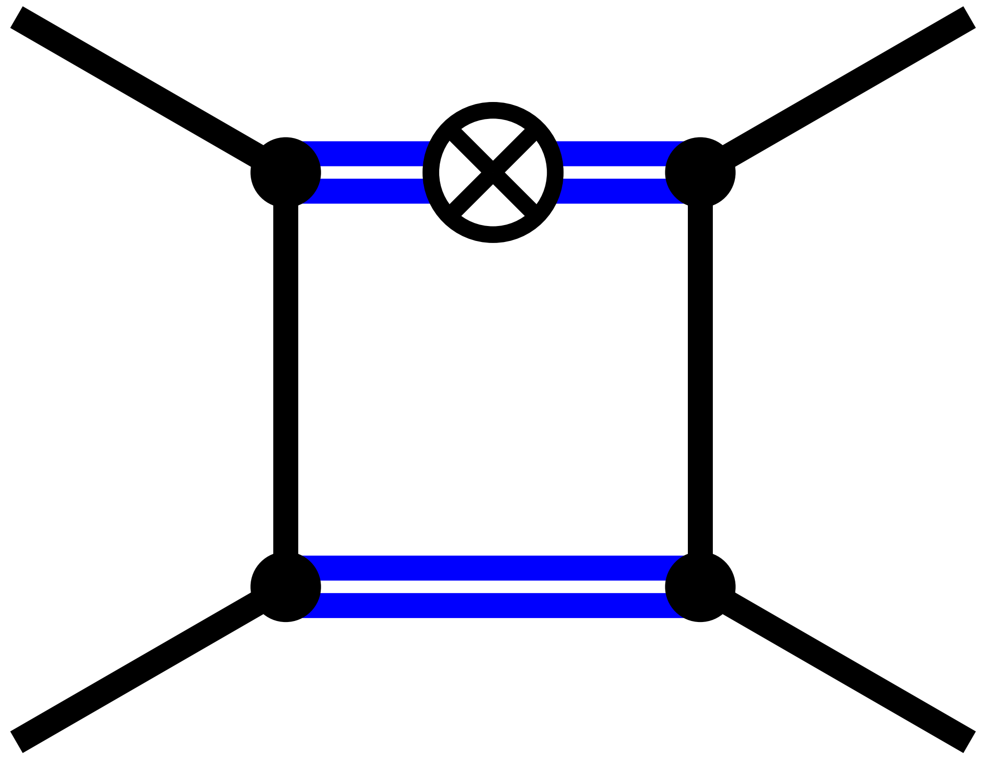
$p_4 \to 0$ (topology)

$$T^5 = \int_q \frac{1}{D_0 D_1 D_2 D_3 D_4}\,.$$

$$D_i = (q + p_i)^2 - m_i^2 + i\epsilon\,.$$

$$T^4(n) = \int_q \frac{1}{\prod_{i\neq n}^4 D_i}\,.$$





PaVe reduction $T^5 = \sum_{i=0}^{4} c_i(p_4)\, T^4(i)\,.$

Triangles on the cut

$$\mathrm{Im}T_k^{(1)}(s,0) = \frac{16\pi}{(\hat{s}-3)\sqrt{(\hat{s}-4)\hat{s}}}\Big[\hat{s}(3\hat{s}+16) + (\hat{s}-18)(\hat{s}-3)\log(\hat{s}-3) - 68\Big]\Theta(\hat{s}-4)\,,$$

$$\mathrm{Im}T_k^{(2)}(s,0) = \frac{16\pi}{(\hat{s}-3)\sqrt{(\hat{s}-4)\hat{s}}}\Big[-(\hat{s}-4)((\hat{s}-8)\hat{s}+8) + 4(\hat{s}-3)(\hat{s}+5)\log(\hat{s}-3)\Big]\Theta(\hat{s}-4)\,,$$

$$\mathrm{Im}T_k^{(3)}(s,0) = \frac{16\pi}{(\hat{s}-3)\sqrt{(\hat{s}-4)\hat{s}}}\Big[\hat{s}(3\hat{s}+16) + ((7-3\hat{s})\hat{s}+6)\log(\hat{s}-3) - 68\Big]\Theta(\hat{s}-4)$$

N.B: $\hat{s} = s/k$