Towards a Post-Quantum Theory of Gravity

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Motivation

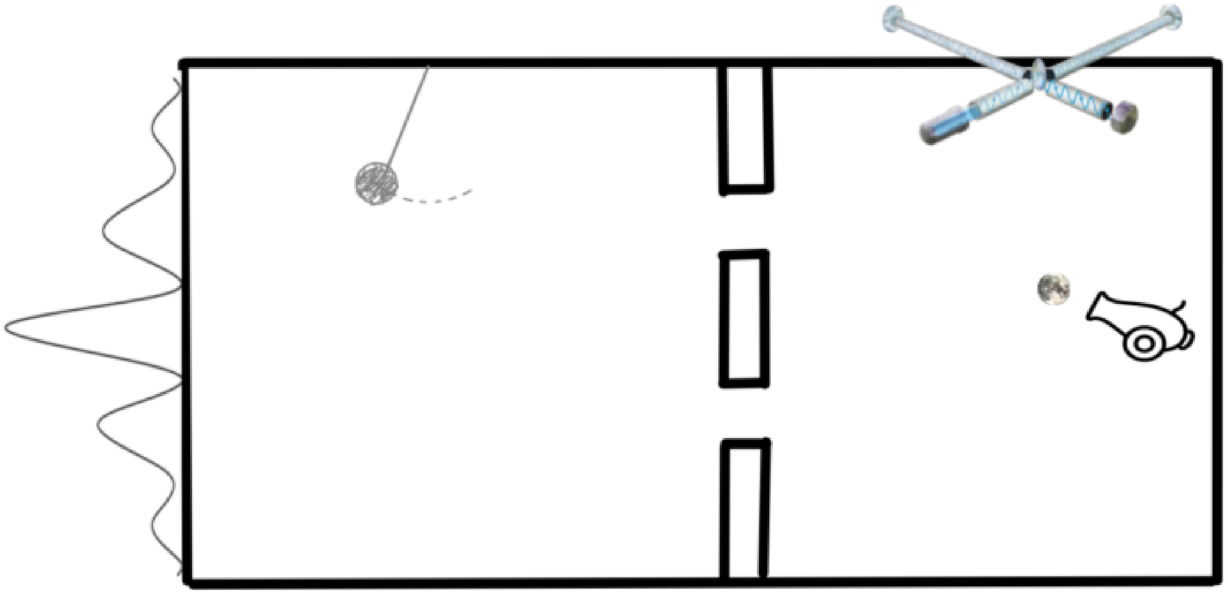
- Einstein's equation:

$$G_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

- But the left-hand side is a c-number field and the right-hand side is an operator

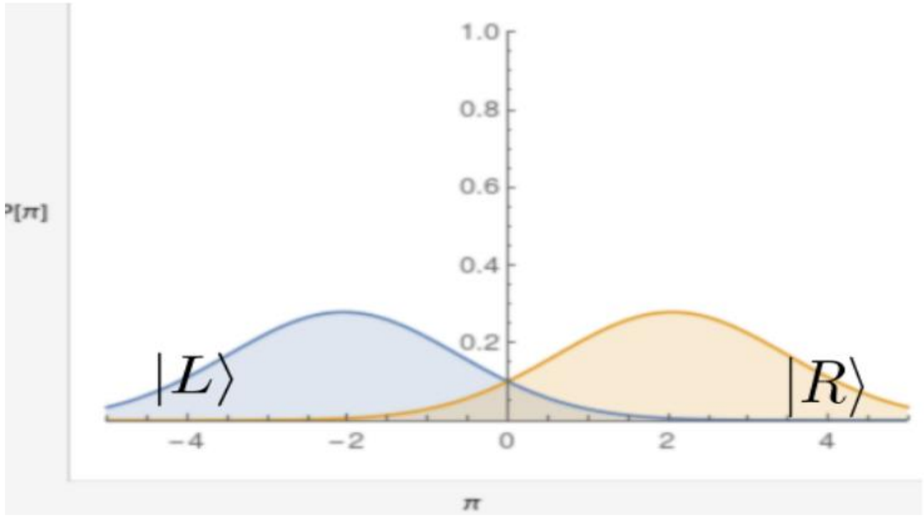
$$G_{\mu\nu}[g] \ \ (c\text{−number}) = \frac{8\pi G_N}{c^4} \hat{T}_{\mu\nu} \ \ (\text{operator})$$

The split-slit gedanken experiment



- A massive particle through the left slit sources a different field than through the right.
- If that field is classical it can be measured to arbitrary accuracy
- Measuring it gives which path information: no interference pattern
- Feynman at Chapel Hill (1957), and Aharonov: gravity must be quantised.

The hidden assumption



$$P[\pi\,|\,L] \ \text{vs.}\ \ P[\pi\,|\,R]: \qquad \text{overlap} > 0 \iff \text{no perfect which-path readout}$$

- The classical field need not be sharp. It can be stochastic.
- Then |L⟩ and |R⟩ give overlapping distributions for the conjugate momentum π of the pendulum.
- Overlapping distributions carry no perfect which-path record, so the interference pattern survives.
- Classical gravity can only be stochastic

Semi-classical gravity is not good enough

- Source the metric via the expectation value.

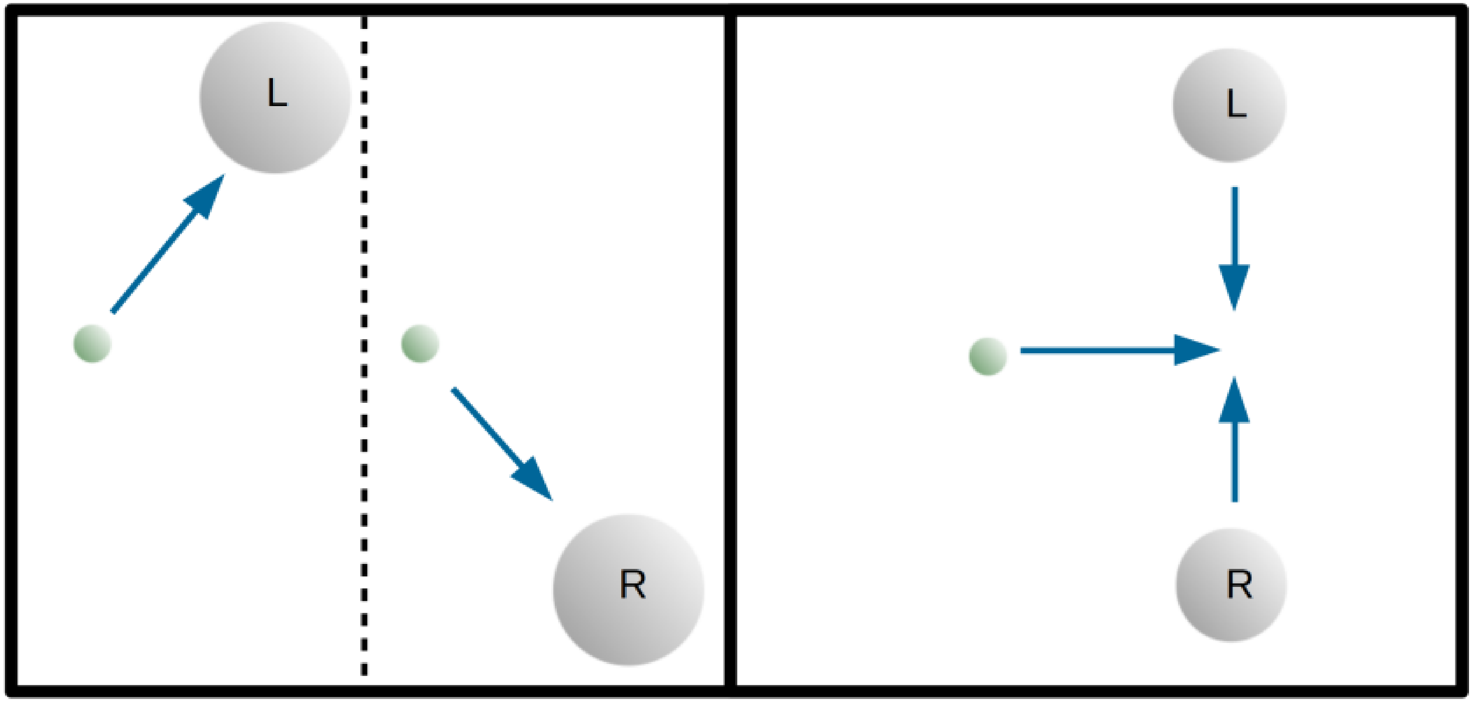
$$G_{\mu\nu}[g] \;=\; \frac{8\pi G_N}{c^4} \langle \psi | \hat{T}_{\mu\nu} | \psi \rangle$$

- In a superposition the field then sits at the mean. The mass is here and there; the field is at neither.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle) \quad \Longrightarrow \quad \langle \hat{T}_{\mu\nu} \rangle = \frac{1}{2}\left(T^{L}_{\mu\nu} + T^{R}_{\mu\nu}\right)$$

- Non-linear in the state, super-luminal signalling

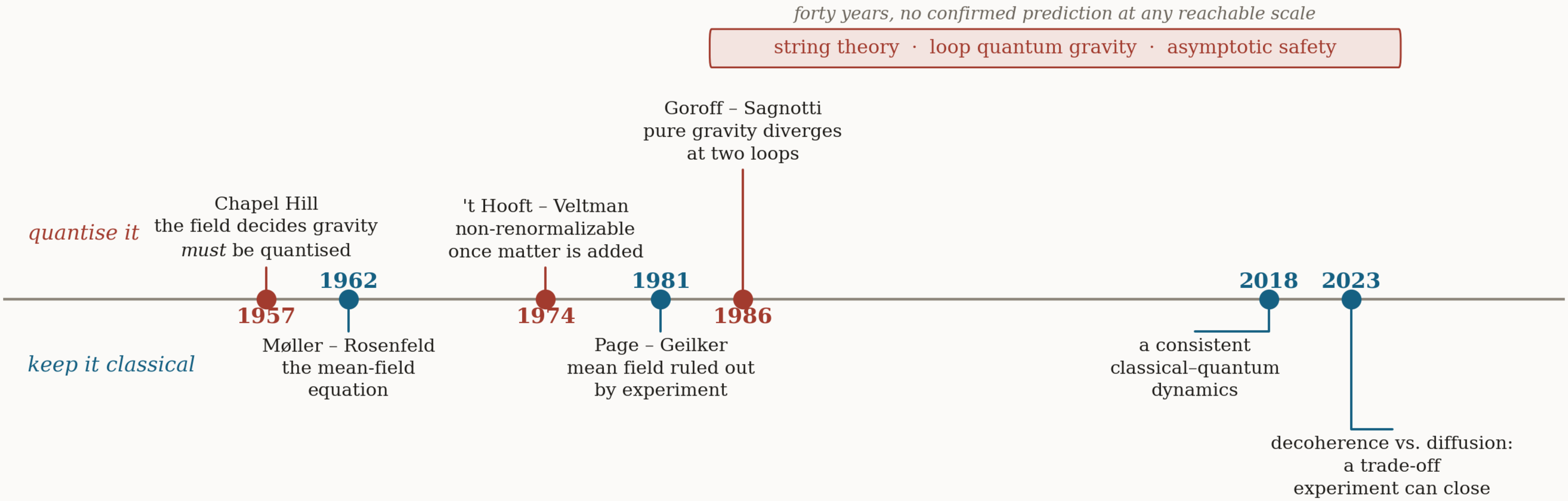
Here, not there



This or that

Not this!

A classical field may be in the L configuration or in the R configuration, with probabilities.
What it cannot be is the average of the two.



70 years since Chapel Hill

- 1957, UNC Chapel Hill: the field committed to quantising gravity on the strength of a gedanken experiment.
- Perturbative quantisation: EH is non-renormalizable, QG is non-unitary etc.
- No direct evidence of the quantum nature of gravity

Consistent classical–quantum dynamics

- Evading the no-go theorem: the coupling can be neither unitary nor deterministic: drift sourced by average fields must cause quantum fields to decohere
- Consistency requires the map on the hybrid state ϱ(z,t) to be completely positive and trace preserving.

$$\frac{\partial \varrho(z,t)}{\partial t} = \int dz' \left[W^{\mu\nu}(z|z')\, L_\mu\, \varrho(z')\, L_\nu^\dagger - \frac{1}{2}\left\{ W^{\mu\nu}(z'|z)\, L_\nu^\dagger L_\mu,\ \varrho(z) \right\}_+ \right] - i\left[\hat{H}(z), \varrho\right]$$

- Expanding the transition rates in moments of the classical jump gives three coefficients:

$$\frac{\partial \varrho}{\partial t} = -i[\hat{H}, \varrho] - \partial_i\left(D_1^{i\mu} L_\mu \varrho\right) + \frac{1}{2}\partial_i \partial_j\left(D_2^{ij} \varrho\right) + D_0^{\mu\nu}\left(L_\mu \varrho L_\nu^\dagger - \frac{1}{2}\{L_\nu^\dagger L_\mu, \varrho\}\right)$$

- D_1: back-reaction — the classical field drifts under the quantum source.
- D_2: diffusion — the classical field is noisy.
- D_0: decoherence — the quantum state loses coherence in the source.

The classical–quantum state

- A density matrix over the quantum fields, indexed by the classical configuration:

$$\hat{\varrho}(z,\dot{z},t) := \int d\phi^+ d\phi^- \; \varrho(\phi^+,\phi^-,z,\dot{z},t)\; |\phi^+\rangle\langle\phi^-|$$

with components

$$\varrho(\phi^+,\phi^-,z,\dot{z},t) := \langle\phi^+|\hat{\varrho}(z,\dot{z},t)|\phi^-\rangle$$

- ϕ^+ and ϕ^- are the ket and bra fields, and may themselves depend on z, ż and t. The components evolve by a path integral:

$$\varrho(\phi^+,\phi^-,z_f,\dot{z}_f,t_f) = \frac{1}{\mathcal{N}}\int D\phi^+ D\phi^- Dz\; e^{-\mathcal{I}_{CQ}}\; \varrho(\phi^+,\phi^-,z_i,\dot{z}_i,t_i)$$

- with 1/N a normalisation constant, and the action splitting into three pieces:

$$\mathcal{I}_{CQ} = -i\,\mathcal{I}_Q(\phi^+,\phi^-) + \mathcal{I}_{FV} + \mathcal{I}_{Diff}$$

The Classical Quantum Action

- The imaginary part, $I_Q(\phi^+,\phi^-)$, — the unitary dynamics of the bra and ket fields.
- I_{FV} — the Feynman–Vernon, or Schwinger–Keldysh, terms; these decohere. The first two pieces alone are the standard open-quantum-systems formalism.
- I_{Diff} — the diffusive part, which carries the quantum back-reaction on the classical field. It is what is new.

- A simple example, with ϕ and z a field and a classical variable (for fields, dt → d^4x):

$$\mathcal{I}_{CQ} = \int_{t_i}^{t_f} dt \quad - i\left((\dot{\phi}^+)^2 - (\dot{\phi}^-)^2\right) - \frac{D_0}{8}\left((\phi^+)^2 - (\phi^-)^2\right)^2$$
$$-\frac{1}{2D_2}\left(\ddot{z} - \frac{D_1}{2}\left((\phi^+)^2 + (\phi^-)^2\right)\right)^2$$

- The imaginary part is I_Q; the D_0 term is Feynman–Vernon, decohering the field at a strength set by D_0.
- The last term is a classical–quantum Onsager–Machlup functional: an equation of motion, squared. Small D_2 suppresses departures from it — stochastic kicks to the acceleration $\ddot{z}$.
- If D_2 is not positive (not PSD, for a matrix of equations) the action is unbounded and wild paths dominate. D_1 sets the strength of the back-reaction.

The gravitational action

$$\mathcal{I}_{CQ}[g_{\mu\nu}, T^{\mu\nu +}, T^{\mu\nu -}] = \int d^4x \Big[-i\sqrt{-g}\left(\mathcal{L}_Q[T^{\mu\nu +}] - \mathcal{L}_Q[T^{\mu\nu -}]\right) - \frac{\sqrt{-g}}{8}(T^{\mu\nu +} - T^{\mu\nu -}) D_{0,\,\mu\nu\rho\sigma}(T^{\rho\sigma +} - T^{\rho\sigma -})$$
$$- \frac{\sqrt{-g}}{2}\left(G^{\mu\nu} + \Lambda g^{\mu\nu} - 8\pi G_N \frac{T^{\mu\nu +} + T^{\mu\nu -}}{2}\right) D^{-1}_{2,\,\mu\nu\rho\sigma}\left(G^{\rho\sigma} + \Lambda g^{\rho\sigma} - 8\pi G_N \frac{T^{\rho\sigma +} + T^{\rho\sigma -}}{2}\right)\Big]$$

governing the evolution of the state through

$$\varrho(T^+, T^-, g, t_f) = \frac{1}{\mathcal{N}} \int D\varphi^+ D\varphi^- D'g \; e^{-\mathcal{I}_{CQ}} \, \varrho(T^+, T^-, g(z_i, \dot{z}_i, t_i))$$

- $T^{\pm}$ is a function of $\phi^{\pm}$; G is the Einstein tensor of the metric g, and the gravitational path integral runs over continuous configurations, up to diffeomorphisms.
- The Feynman–Vernon term decoheres the density matrix, damping its off-diagonal elements exponentially.
- The last term is the Onsager–Machlup functional for Einstein's equations, with the classical $G^{\mu\nu}$ sourced by the average of the bra and ket fields, $\bar{T}^{\mu\nu} = ½(T^{\mu\nu +} + T^{\mu\nu -})$.
- The classical field diffuses around its equations of motion, the covariance of the stochastic force being the diffusion matrix D_2.

Drift, noise and Girsanov's theorem

- The classical field drifts under the quantum source and diffuses:

$$dz^i = D_1^i[\hat{T}]dt \;+\; \sqrt{D_2}\; dW^i, \qquad \langle dW^i dW^j\rangle = \delta^{ij}dt$$

- Girsanov: for $D_2 > 0$ the measures with and without the drift are equivalent — mutually absolutely continuous, not orthogonal:

$$\frac{dP_{D_1}}{dP_0}\Big|_{[0,\,T]} = \exp\left[\frac{1}{D_2}\int_0^T D_1\, dz - \frac{1}{2D_2}\int_0^T D_1^2\, dt\right] \;>\; 0 \quad \text{a.s.}$$

- No record of finite length separates them with certainty. The drift can only be estimated, and the variance of that estimate falls off only as 1/T:

- Over any finite interval the branch driving the field is therefore undetermined, and the source that acts is the average of the two:

$$\bar{T}^{\mu\nu}[\phi^+,\phi^-] = \tfrac{1}{2}\left(T^{\mu\nu}[\phi^+] + T^{\mu\nu}[\phi^-]\right)$$

- Only as $T \to \infty$ do the measures become mutually singular.

Complete positivity and the decoherence–diffusion trade-off

- Complete positivity of the hybrid map amounts to three conditions:

$$D_0 \succcurlyeq 0, \qquad D_2 \succcurlyeq 0, \qquad 2D_0 \succcurlyeq D_1^\dagger D_2^{-1} D_1$$

- With back-reaction fixed by Newton's constant, $D_1 \propto 8\pi G_N$, the third condition is a trade-off:

$$D_0 D_2 \succcurlyeq \mathcal{O}((8\pi G_N)^2) \qquad \text{(decoherence} \times \text{diffusion is bounded below)}$$

- Decoherence and diffusion cannot both be small: an upper bound on one is a lower bound on the other.
- The classical equations of motion favour anti-correlation of the two branches; the decoherence term counteracts it.

- At saturation the entire positivity question collapses to positivity of the diffusion matrix:

$$D_0 = (8\pi G_N)^2 D_2^{-1} \quad \Longrightarrow \quad \text{complete positivity} \iff \text{supermetric} \succcurlyeq 0$$

Investigations: a renormalizable CQ action

- Local and Lorentz covariant, built from the linearised Einstein tensor and the averaged source:

$$\mathcal{L} = \tfrac{1}{2}\left[\alpha\, E_{\mu\nu}E^{\mu\nu} - \beta\,(E^{\mu}_{\mu})^2\right], \qquad E_{\mu\nu} = G^{(1)}_{\mu\nu} - 8\pi G_N \bar{T}_{\mu\nu}$$

- Four derivatives, so the propagator falls off like p^{-4} — power-counting renormalizable:

$$\mathcal{L} \sim \alpha\,(\partial^2 h)^2 \quad \Longrightarrow \quad D(p) \sim \tfrac{1}{\alpha\, p^4} \qquad (\text{power}-\text{counting renormalizable})$$

- Two couplings, α and β, fix the diffusion and the decoherence

Scalar channel: LISA Pathfinder

- The extra scalar mode diffuses, at a rate set by the two couplings:

$$D_{2,\,scalar} = \frac{1}{3}\left[\frac{1}{\alpha - 3\beta} + \frac{8}{\alpha}\right] \lesssim 10^{-56}$$

- The residual acceleration noise of LISA Pathfinder caps it, providing a floor for the couplings:

$$\alpha \gtrsim 3 \times 10^{56}, \qquad \alpha - 3\beta \gtrsim 3 \times 10^{55}$$

Tensor channel: the gravitational-wave background

- The transverse-traceless two-point function of the diffusing metric:

$$\langle h^{ij,\,TT} h^{kl,\,TT}\rangle(p) = \frac{1}{(2\pi)^4}\,\frac{\alpha^{-1} P^{ij,\,kl}}{(k^2-\omega^2)^2+4\omega^2\eta^2}$$

- Integrating it gives the stochastic background the diffusion must produce:

$$\frac{d\rho_{GW}}{d\ln\omega} \approx \frac{\alpha^{-1}c^2}{8\pi G_N (2\pi)^2}\left(\frac{\omega^3}{\eta} + \omega c\,\delta(0)\right)$$

$$\Omega_{GW}(25\,\mathrm{Hz}) \leq 10^{-9} \qquad \Longrightarrow \qquad D_{2,\,tensor} = \alpha^{-1} \lesssim 10^{-66}$$

An upper bound on diffusion is a lower bound on decoherence

- Write the trade-off in terms of the diffusion and the decoherence it forces:

$$\mathcal{D} \equiv D_{2,\,scalar}, \qquad \mathcal{C} \equiv \frac{2\alpha + (\alpha - 3\beta)}{6}$$

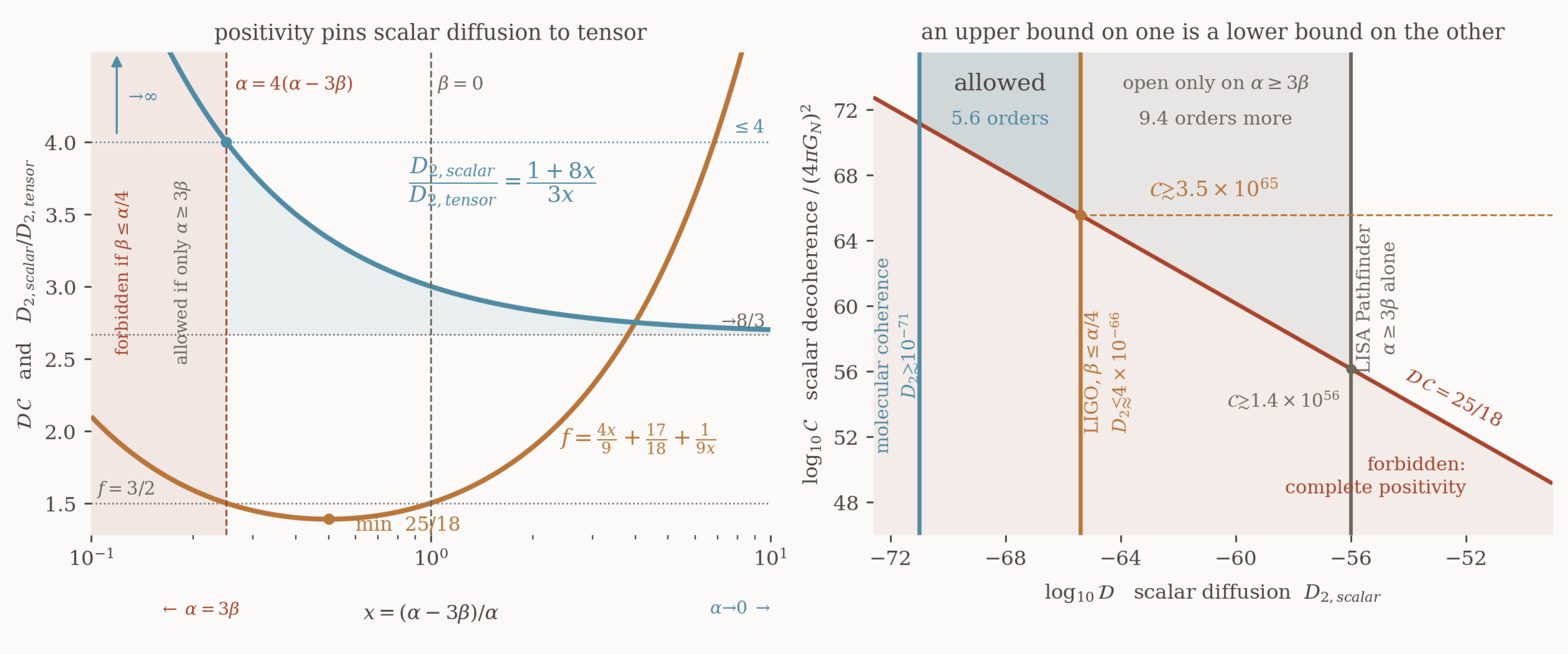
$$\mathcal{D}\,\mathcal{C} = f(x) = \frac{(1+8x)(2+x)}{18x} = \frac{4x}{9} + \frac{17}{18} + \frac{1}{9x} \;\geq\; \frac{25}{18}$$

- The product has a floor at 25/18, so the LIGO cap on the diffusion turns into a floor under the decoherence:

$$\mathcal{D} \lesssim 4 \times 10^{-66} \qquad \Longrightarrow \qquad \mathcal{C} \gtrsim \frac{25}{18} \times \frac{10^{66}}{4} \;\approx\; 3.5 \times 10^{65}$$

- Gravitationally induced decoherence predicts the lower bound for decoherence
- Decoherence based experiments set an upper bound

The trade-off



Left: positivity pins the ratio of scalar to tensor diffusion. Right: the excluded hyperbola, and the two windows that survive.

References

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