Scattering Black Holes and Neutron Stars in Worldline Quantum Field Theory

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Gravitational Waves from Perturbative QFT

- Gravitational waves are a key prediction of GR.
- Worldline QFT used to compute gauge invariant classical scattering observables.
- Perturbation theory valid when $\lambda_c \ll r_{int} \ll |b|$, expansion in G.
- Finite size effects incorporated via worldline EFT.

Goldberger, Rothstein ’06 Mogull, Plefka, Steinhoff ‘20

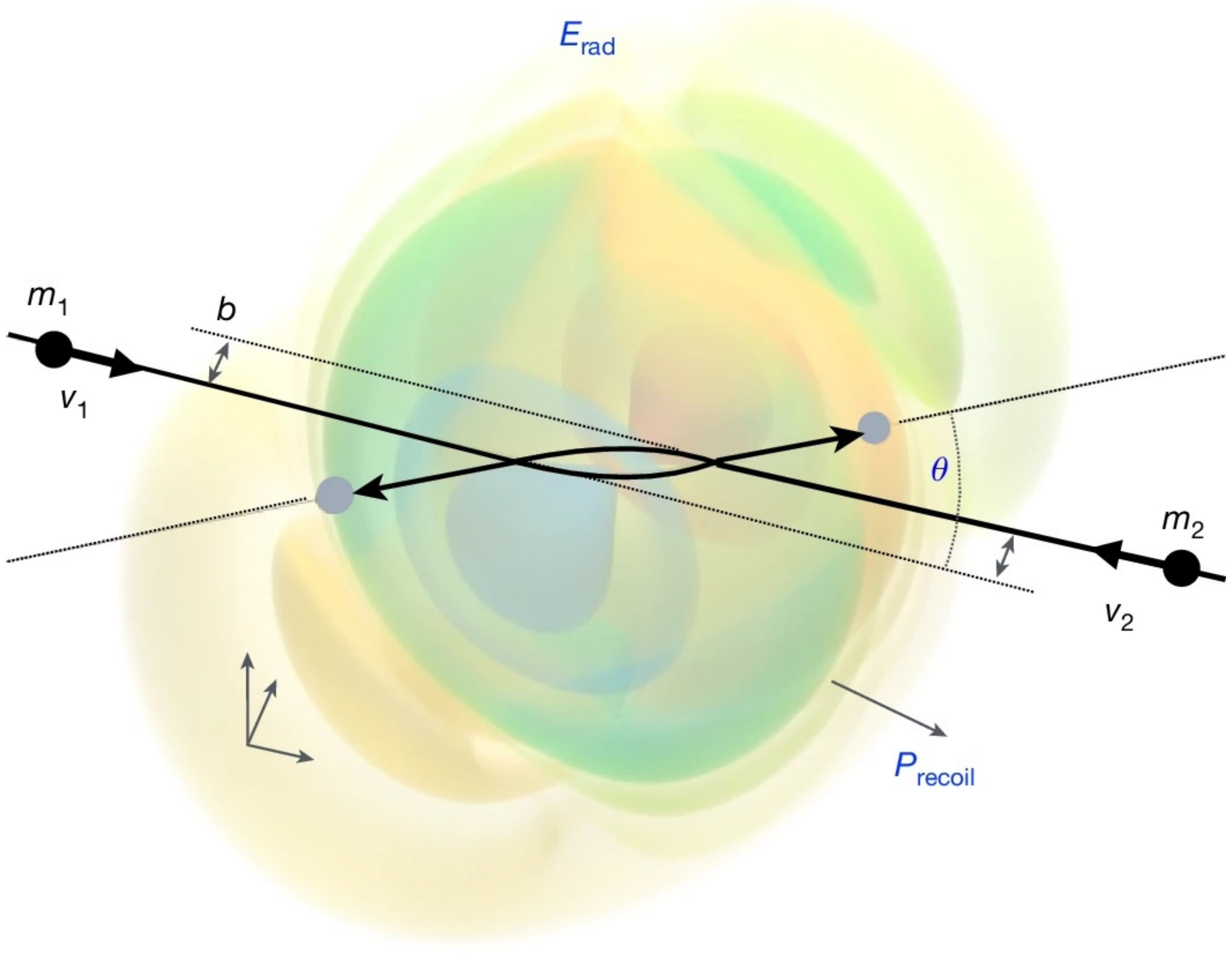


Image credit: Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch ’25

Worldline QFT in a Nutshell

$$Z = \int DhDx_iDQe^{\frac{i}{\hbar}(S_{EH}+S_{NS}+S_{BH}+S_{g.f}+S_{TSC})}$$

- Start with action and define the hybrid path integral.
- Tidally induced d.o.f encoded in $Q^{\mu\nu}$.
- Quantise fluctuations around the asymptotic background.

$$S_{NS} = \int d\sigma_1 \left[-\frac{1}{2e_1}\dot{x}_{1\mu}\dot{x}_1^{\mu} - \frac{1}{2}e_1 m_1^2 + e_1\left(\frac{1}{4\lambda_E\omega_f^2 e_1^2}\frac{DQ^{\mu\nu}}{d\tau}\frac{DQ_{\mu\nu}}{d\tau} - \frac{1}{4\lambda_E}Q_{\mu\nu}Q^{\mu\nu} - \frac{1}{2e_1^2}C_{\mu\rho\nu\sigma}\dot{x}_1^{\rho}\dot{x}_1^{\sigma_1}Q^{\mu\nu} \right) \right]$$

* $S_{g.f}$ is a bulk gauge fixing term. S_{TSC} ensures observables at cubic order in $Q^{\mu\nu}$ satisfy physical constraints on the d.o.f.

Steinhoff, Hinderer, Buonanno, Taracchini '16   Flanagan, Hinderer '07

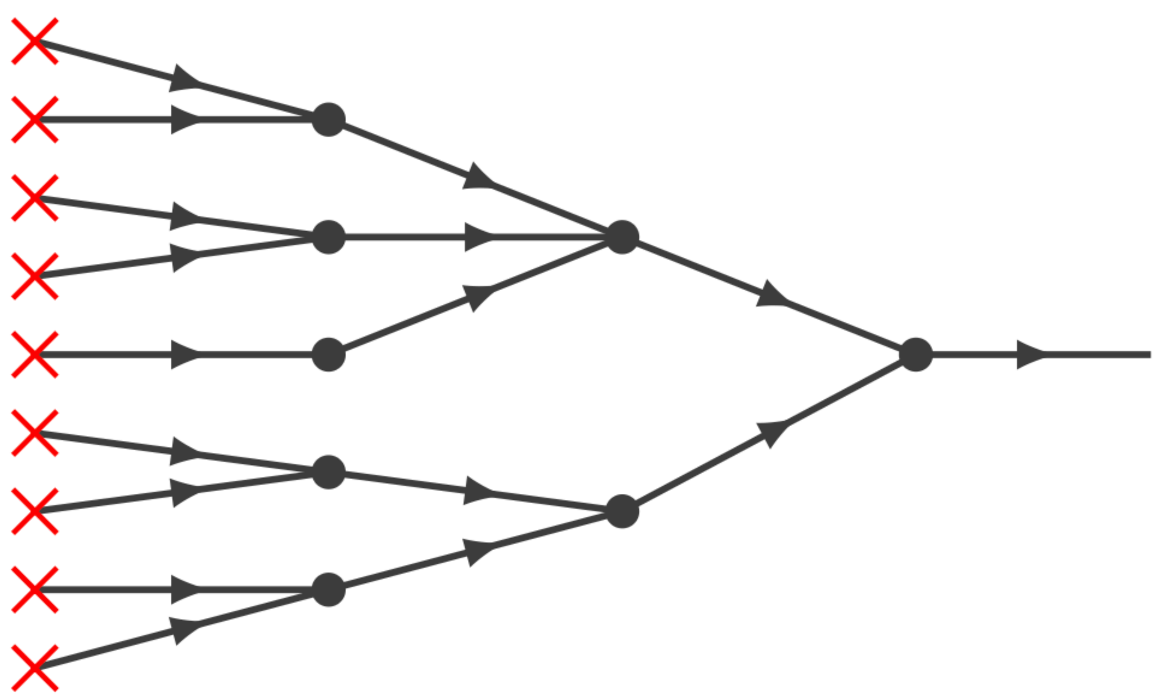
$$x_i^{\mu}(\tau_i) = b_i^{\mu} + v_i\tau_i + z_i^{\mu}(\tau_i)$$

$$S_{EH} = -\frac{1}{16\pi G_d}\int d^dx\sqrt{-g}R \qquad S_{BH} = -\frac{1}{2}\int d\sigma_2\left(\frac{1}{e_2}\dot{x}_{2\mu}\dot{x}_2^{\mu} + m_2^2 e_2\right)$$

$$Q^{\mu\nu}(\tau_2) = Q_0^{\mu\mu}e^{i\omega_f\tau_2} + h.c. + \zeta^{\mu\nu}(\tau_2)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_d}h_{\mu\nu}$$

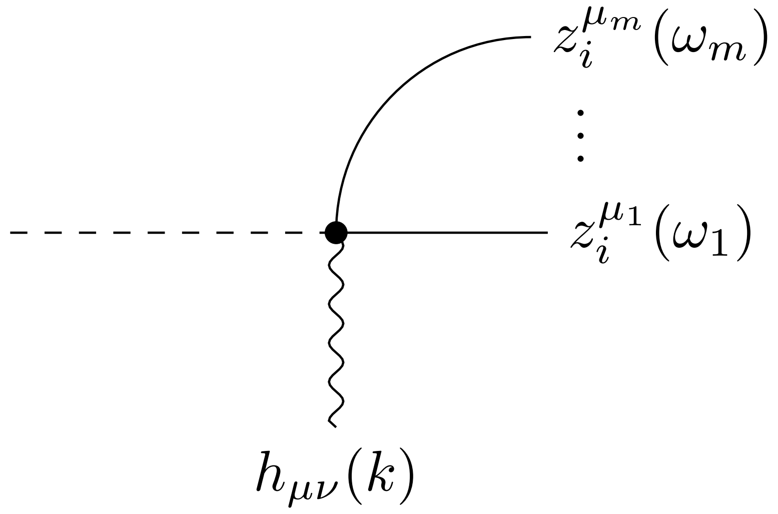
- Swap Feynman propagators for causal propagators oriented towards the outgoing leg.
- Tree level 1-pt functions solve the classical equations of motion.

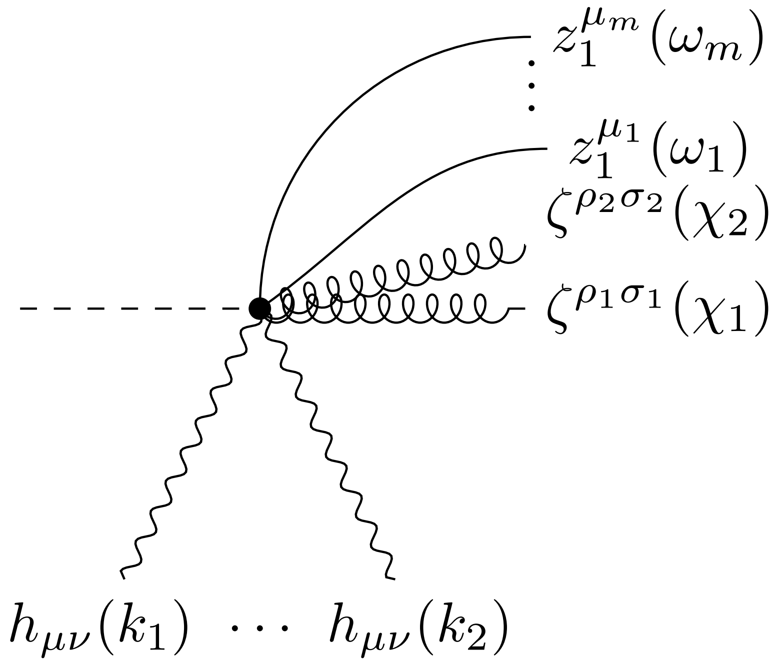


Jakobsen, Mogull, Plefka, Sauer '22

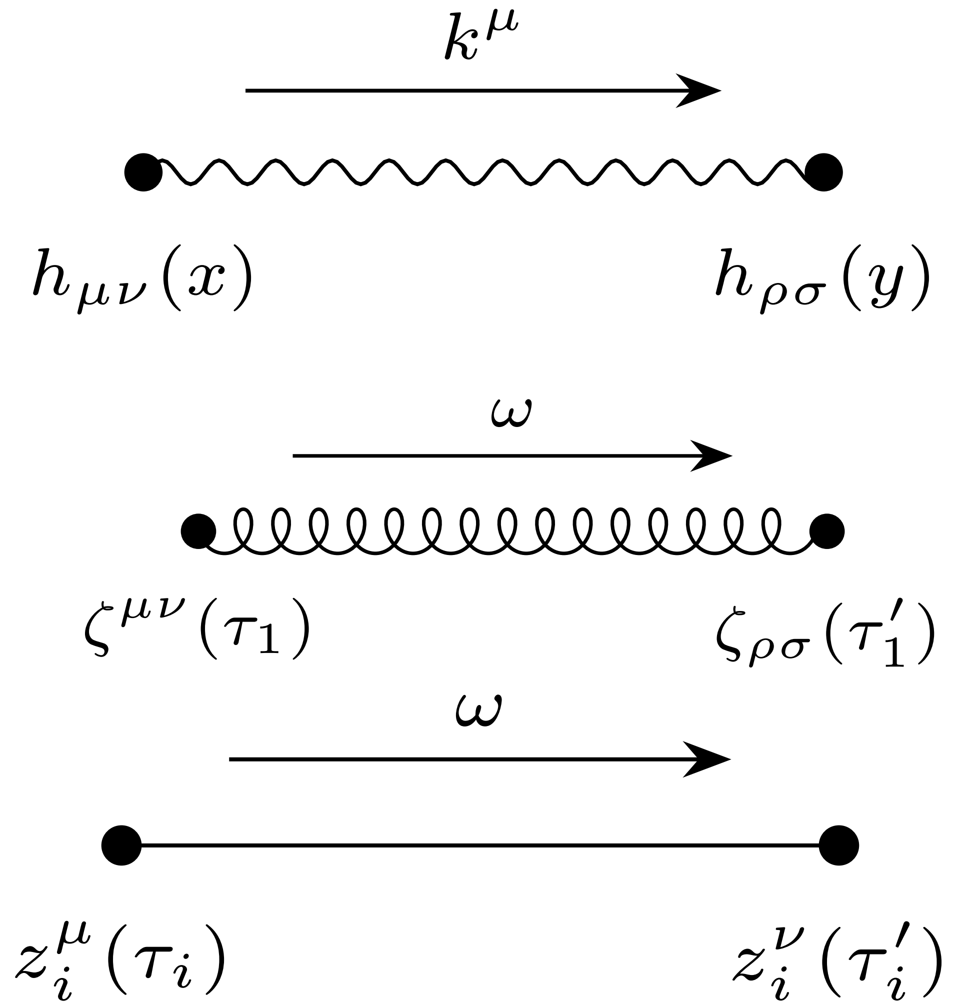
Classical Observables

- Some of the Feynman rules to compute observables:





Mogull, Plefka, Steinhoff '20



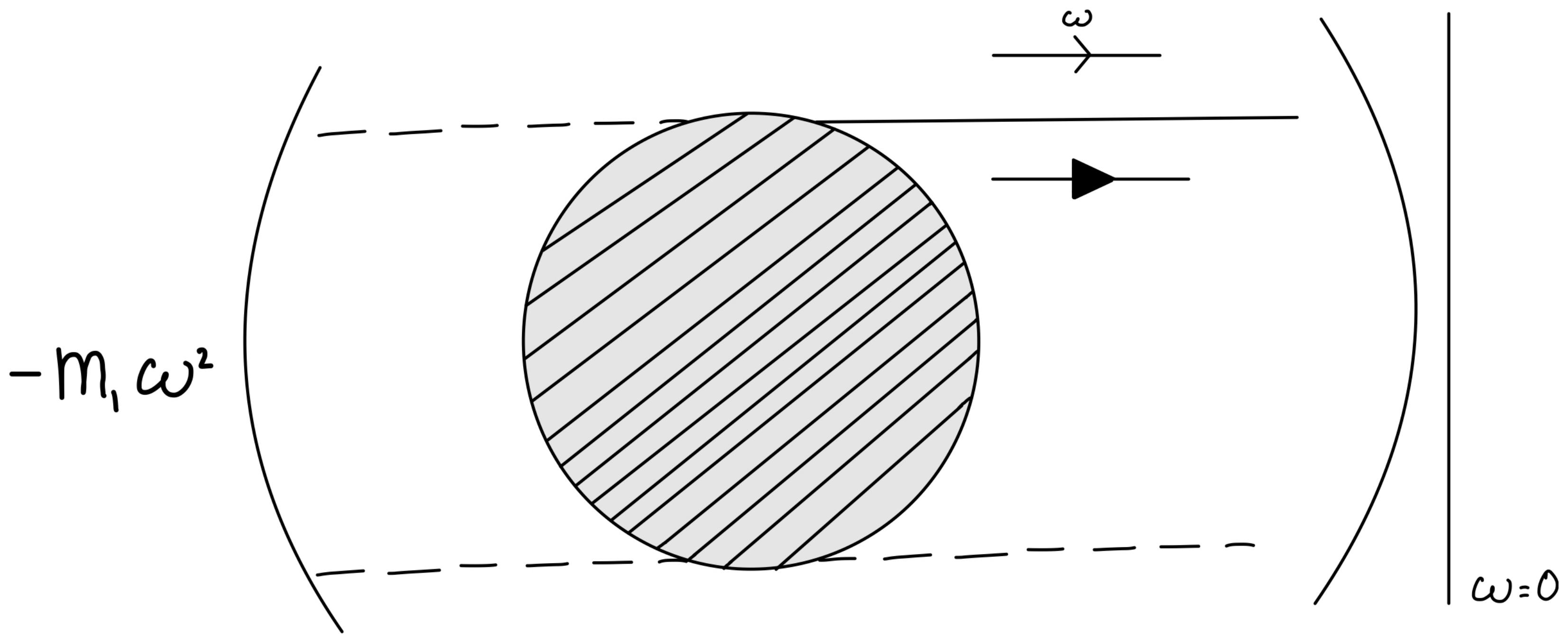
$$= i\int_k \frac{P_{\mu\nu;\rho\sigma}}{k^2 + ik^0 0^+} e^{-ik\cdot(x-y)}$$

$$= i\frac{2\lambda_E\omega_f^2}{m_1}\int_\omega \frac{\delta_\rho^{(\mu}\delta_\sigma^{\nu)}}{(\omega + i0^+)^2 - \omega_f^2} e^{-i\omega(\tau_1-\tau_1')}$$

$$= -\frac{i}{m_i}\int_\omega \frac{\eta^{\mu\nu}}{(\omega + i0^+)^2} e^{-i\omega(\tau_i-\tau_i')}$$

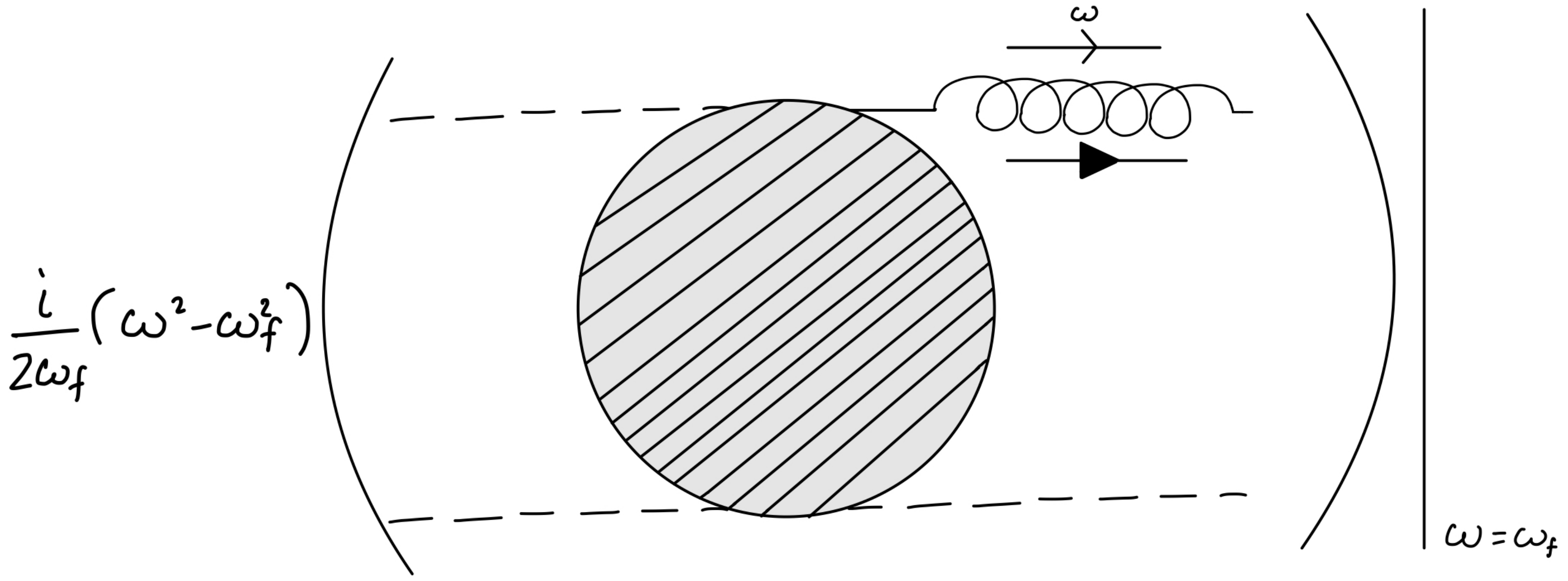
- The impulse $p_i^\mu \to p_i^\mu + \Delta p_i^\mu$

$$\Delta p_i^\mu = \lim_{\omega\to 0}\left(-m_i\omega^2\langle z_i(\omega)\rangle\right)$$

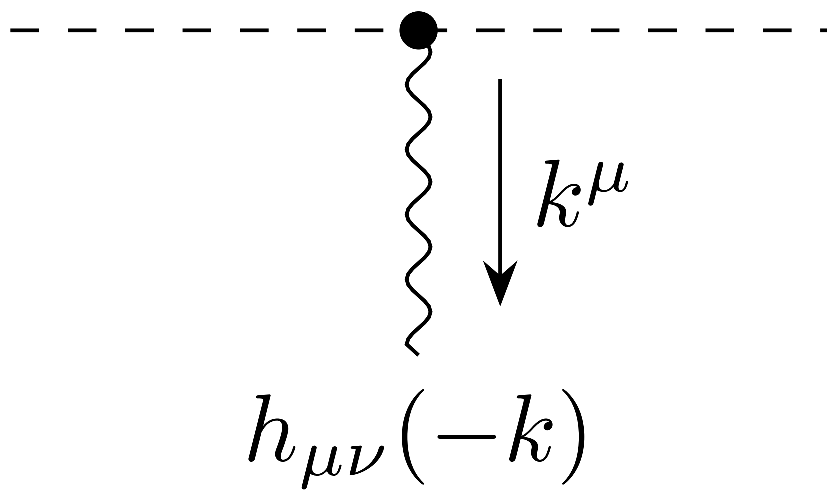


- The "wobble"-change in the amplitude $Q_0^{\mu\nu} \to Q_0^{\mu\nu} + \Delta\tilde{Q}^{\mu\nu}$ (and h.c.).

$$\Delta\tilde{Q}^{\mu\nu} = \lim_{\omega\to\omega_f}\frac{i}{2\omega_f}(\omega^2 - \omega_f^2)\langle\zeta^{\mu\nu}(-\omega)\rangle$$



Classical Loop Integrals



$$= -i\frac{m_1\sqrt{32\pi G}}{2\lambda_E\omega_f}e^{ik\cdot b_1}\bar{\delta}(k\cdot v_1)\left(\lambda_E\omega_f v_1^\mu v_1^\nu + 2(k_1\cdot Q_0^\dagger\cdot Q_0)^{(\mu}v_1^{\nu)}\right)$$

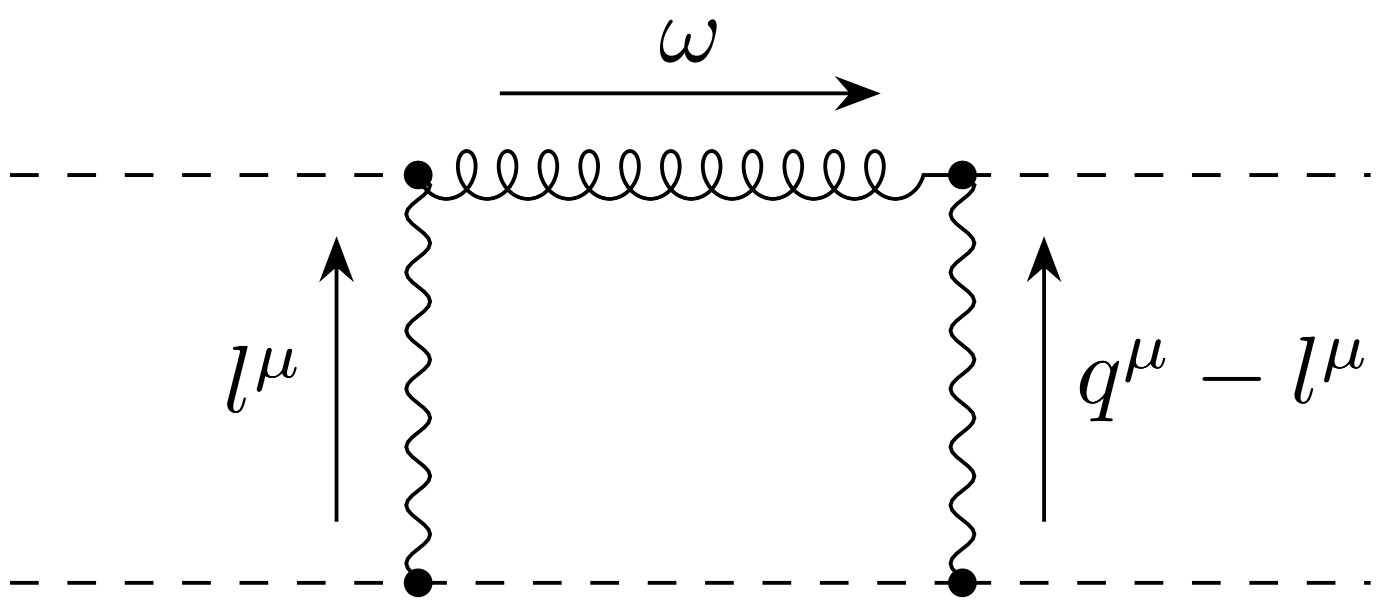
$$+\, i m_1\sqrt{32\pi G}e^{ik\cdot b_1}\bar{\delta}(\omega_f + k\cdot v_1)\,(\ldots) + (\omega_f \to -\omega_f, Q_0 \leftrightarrow Q_0^\dagger)$$

E.g. single graviton vertex. Worldline vertices conserve energy $\Longrightarrow$ loop integrals even at tree level.

- E.g. one integral family this diagram belongs to.

$$T_{n_1 n_2 n_3}(d, |q|, \gamma, \omega_f) = \int \frac{d^d l}{(2\pi)^d}\frac{2\pi\delta(l\cdot v_2)}{(l^2)^{n_1}((q-l)^2)^{n_2}((l\cdot v_1 + i0^+)^2 - \omega_f^2)^{n_3}}$$

$$= \frac{i^{2n_1+2n_2}}{(\gamma^2-1)^{2n_3}}\int\frac{d^{d-1}\mathbf{l}}{(2\pi)^{d-1}}\frac{1}{(\mathbf{l}^2)^{n_1}((\mathbf{q}+\mathbf{l})^2)^{n_2}((\mathbf{l}\cdot\mathbf{v}_1 + i0^+)^2 - (\gamma^2-1)^{-2}\omega_f^2)^{n_3}}$$



- Non-trivial integrals at leading-order. Loops begin appearing at next-to-leading order.

- Use of IBP identities, method of regions, canonical DEs to compute scalar loop integrals.

Conclusions

- WQFT efficiently computes classical observables.
- WEFT allows for implementation of finite size effects.
- Leading order (tree-level) observables computed. Finish next-to-leading (1-loop) order.
- Use the N-matrix, $S = e^{iN}$, to compute observables via Poisson brackets $\Delta O = e^{\{N,\cdot\}_{P.B}} O - O$.

Haddad, Jakobsen, Mogull, Plefka ‘25 Brandhuber, Brown, Pichini, Travaglini, Vives-Matasan ‘25 Damgaard, Plante, Vanhove ‘21

Title page image: Artwork by Sandbox Studio, Chicago with Corinne Mucha.“Nobel recognizes gravitational wave discovery”, Kathryn Jepsen, 2017, https://www.symmetrymagazine.org/article/nobel-recognizes-gravitational-wave-discovery