Line of Fixed Points and U(1) Chiral Symmetry Breaking in a 3D NJL-type Model

Freddie King

Based on work with **Charlie Cresswell-Hogg** and **Daniel F. Litim**:
F. King, C. Cresswell-Hogg and D. F. Litim, *Line of Fixed Points and U(1) Chiral Symmetry Breaking in a 3D NJL-type Model* (in preparation).

Four-fermion models

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$$\mathcal{L}_{GN} = \bar{\psi}_i \left(i\partial\!\!\!/ - m \right) \psi_i + \frac{g}{N} \left(\bar{\psi}_i \psi_i \right)^2$$

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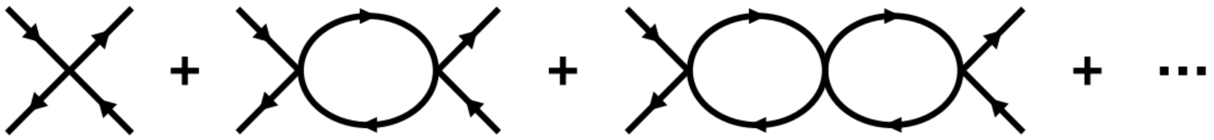

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Regulator, R_k

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→ non-perturbatively renormalizable in d=3,4

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→ Interacting UV fixed point

→ finite # undetermined parameters!

(→finite dimensional UV critical surface)

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  → Line of UV fixed points:

  Line of Fixed Points in Gross-Neveu Theories : 2207.10115
  Critical Fermions with Spontaneously Broken Scale Symmetry : 2212.06815
  Fermion Mass Generation without Symmetry Breaking: 2406.00100

  → marginal $(\bar{\psi}\psi)^3$ coupling

  → endpoint with spontaneous scale symmetry breaking

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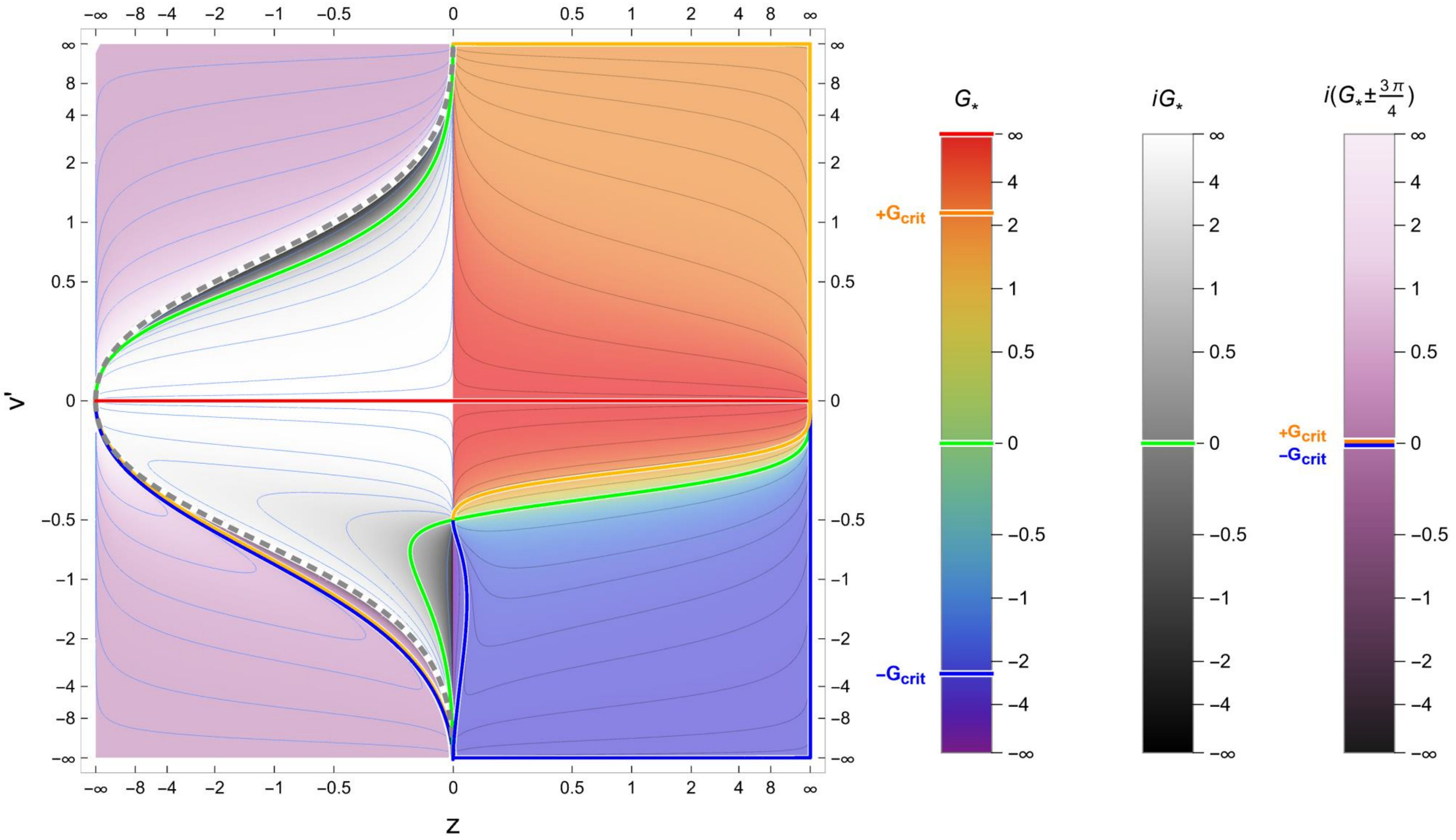

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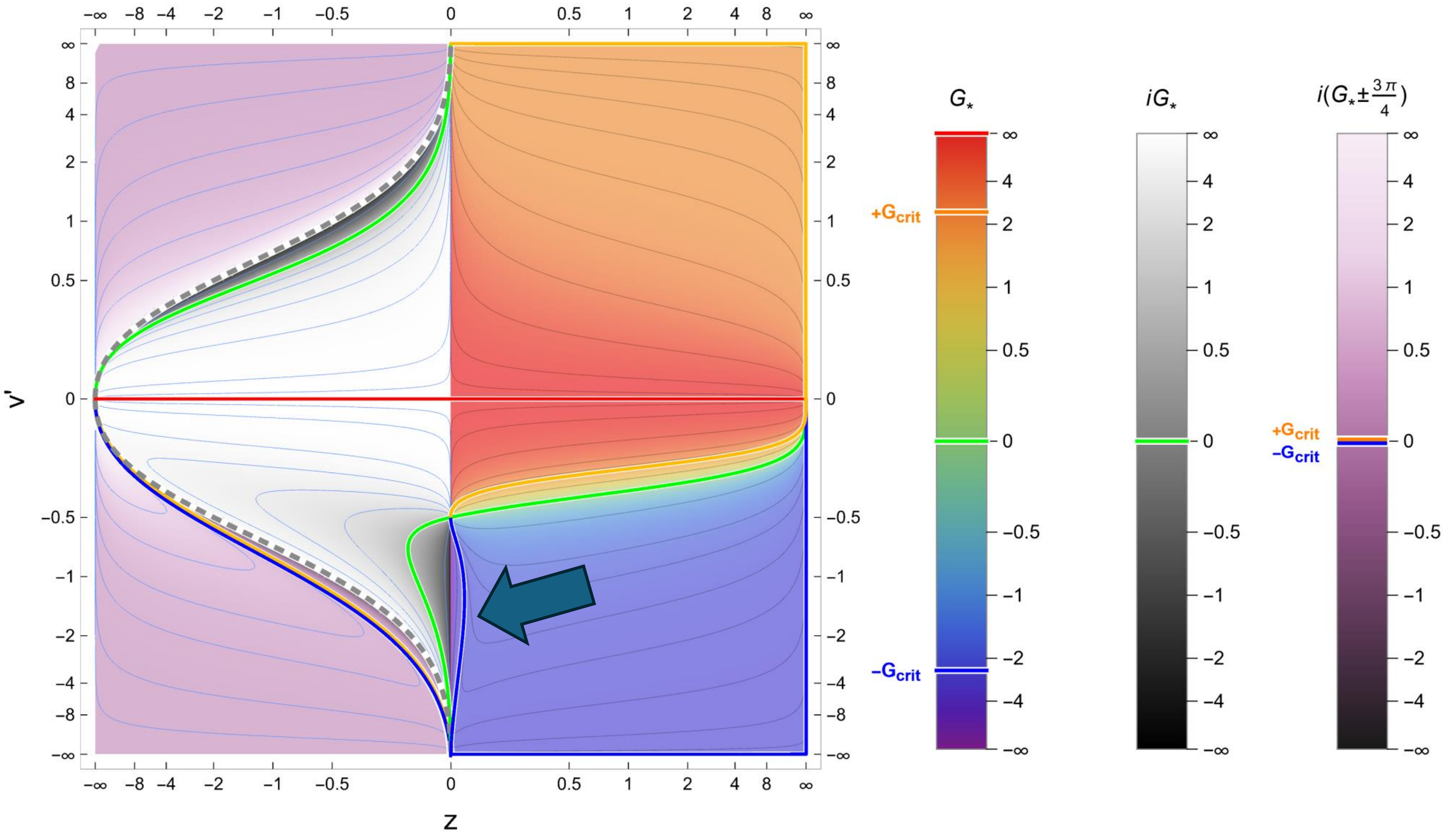

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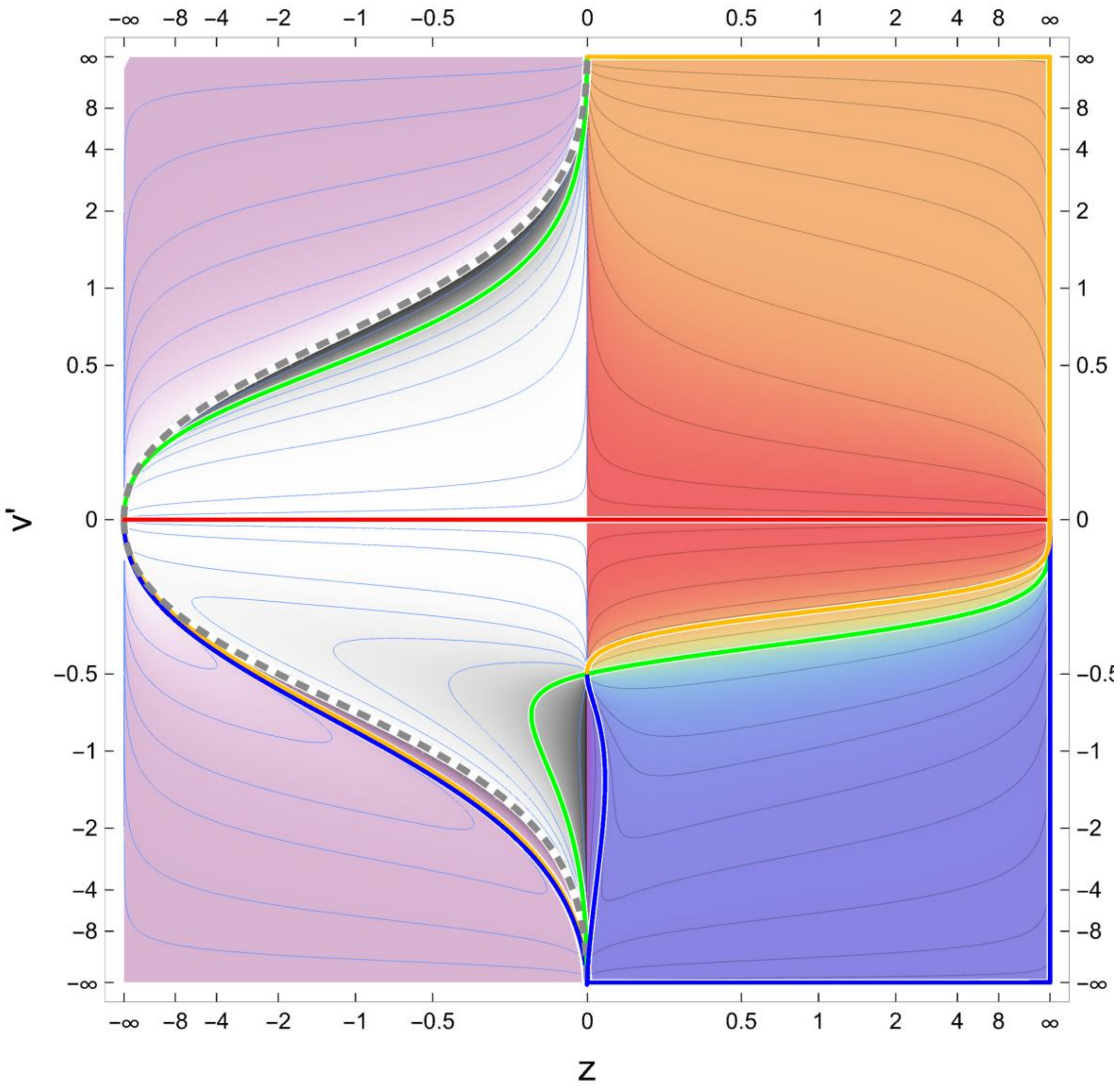

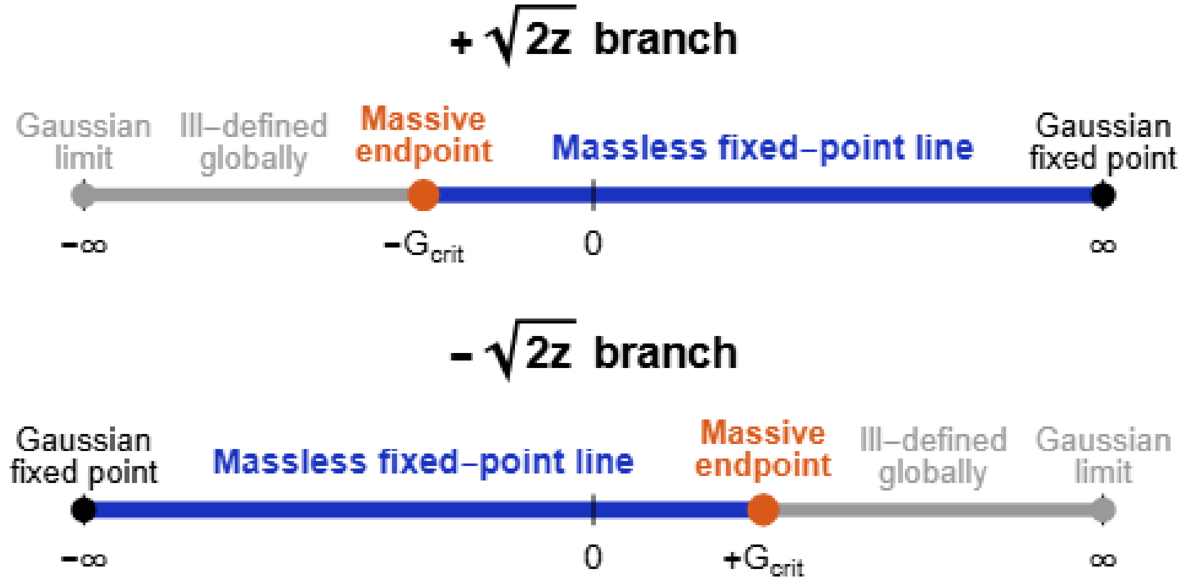

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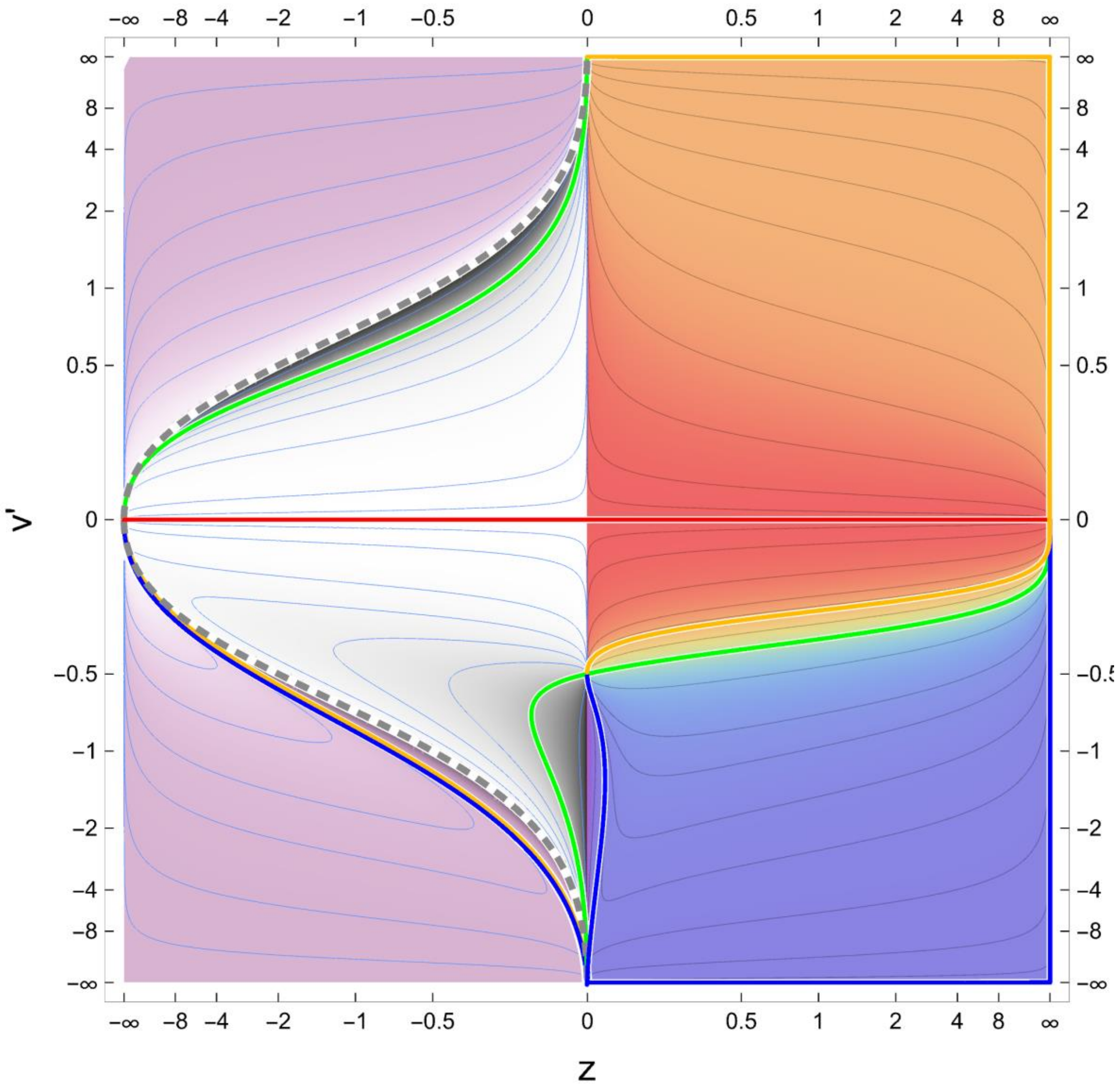

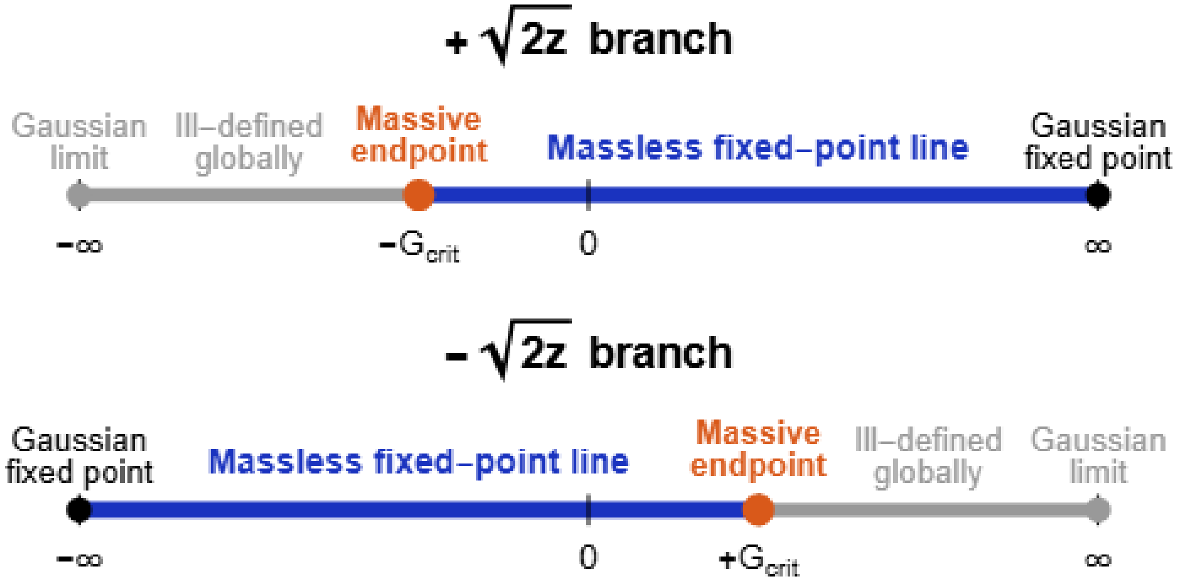

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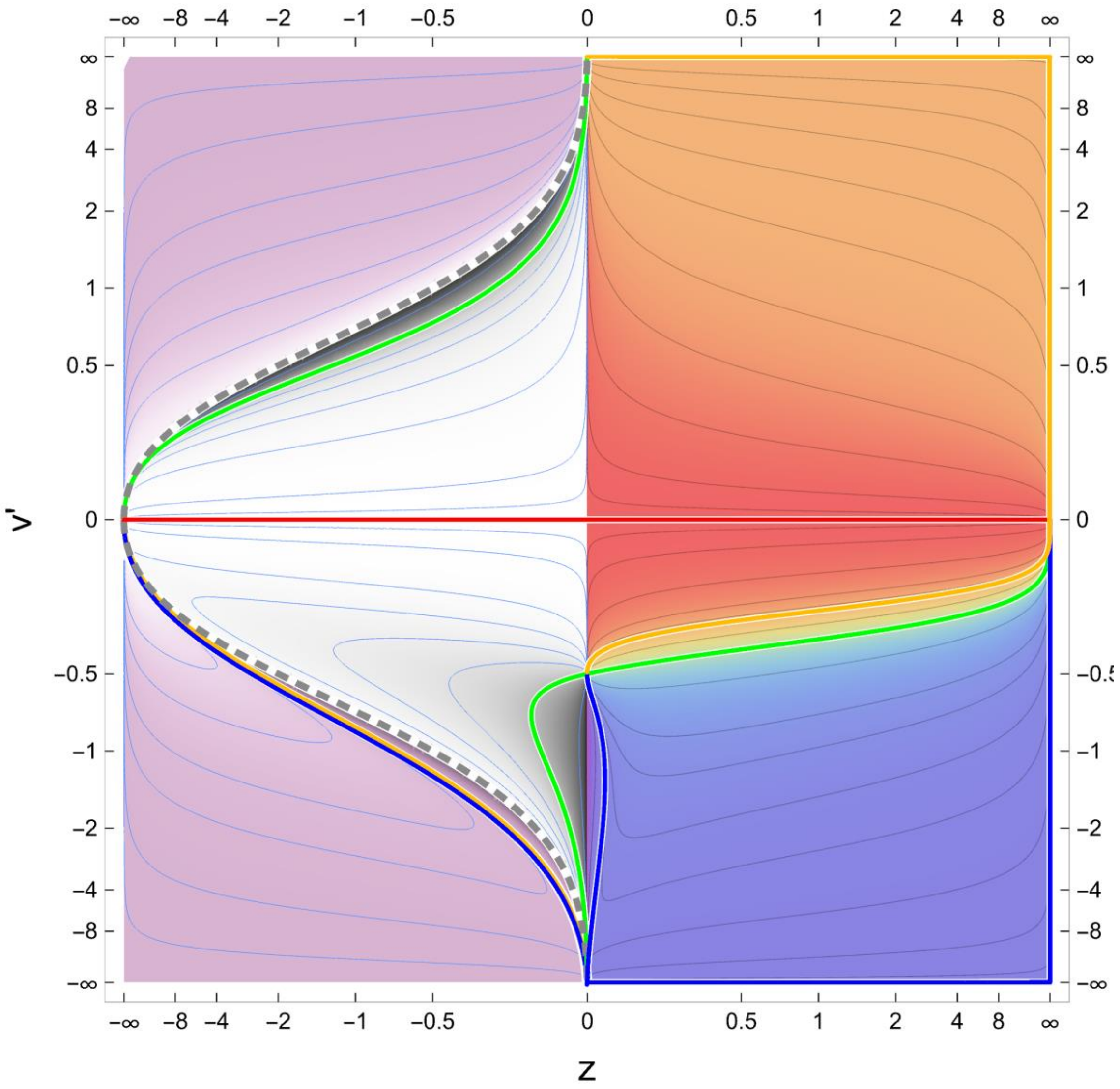

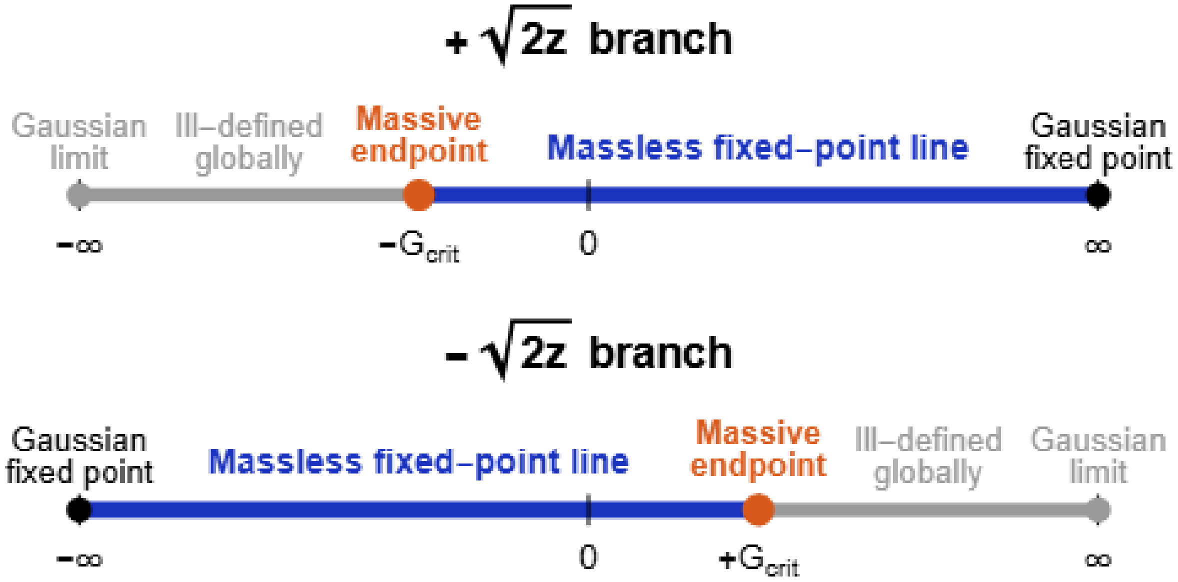

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→ Endpoint at $\pm G_{crit}$ has massive fermions + scale SSB

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And last: bosonisation

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→ Exchange fermionic potential $V(Z)$ for a bosonic potential $U(P)$ and Yukawa interactions

$$P := \phi^\dagger \phi$$

Preserves $U(1)_A$

$$\bar{\psi}\left(\phi_1 + i\phi_2\gamma^5\right)\psi$$

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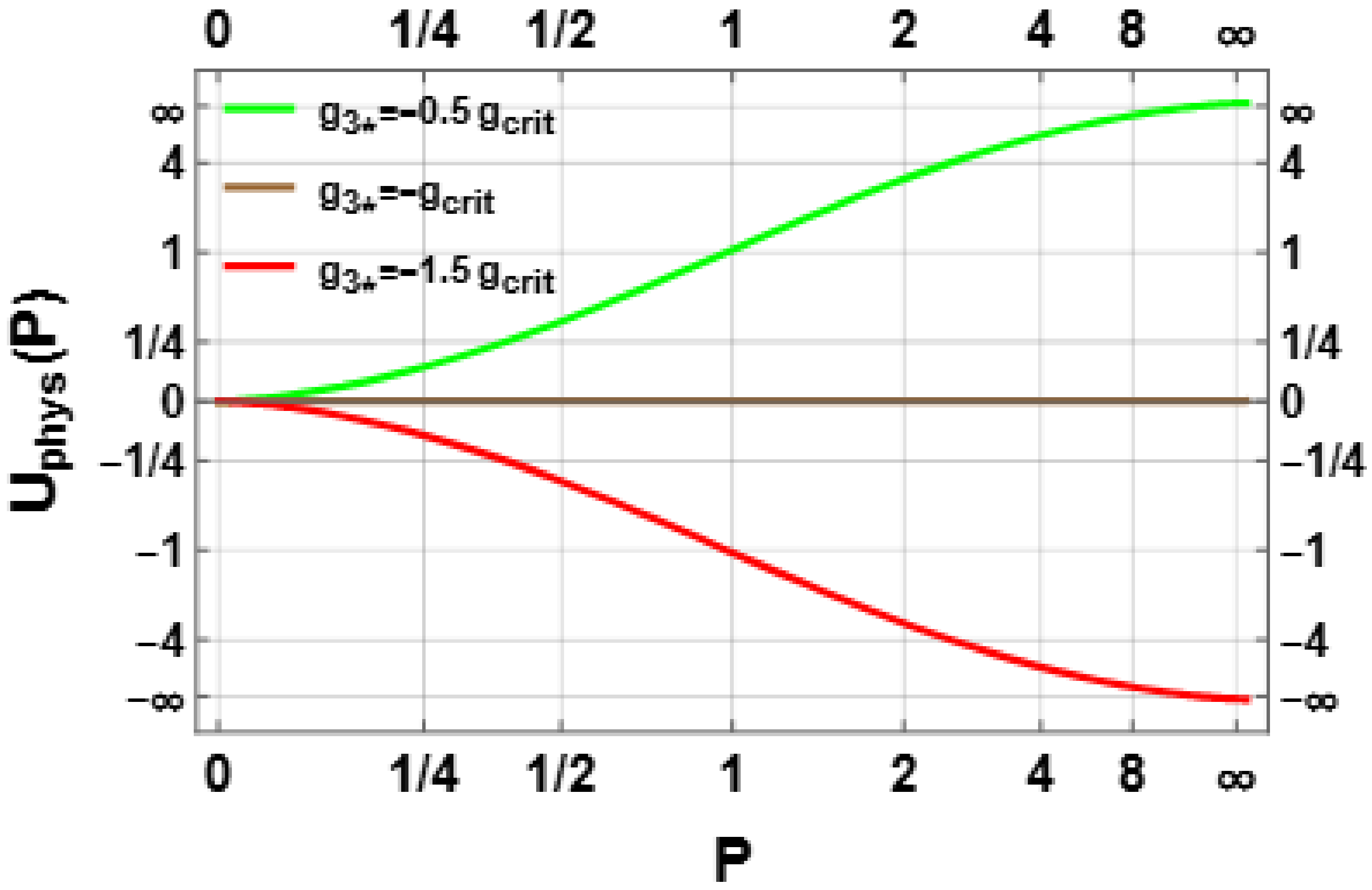

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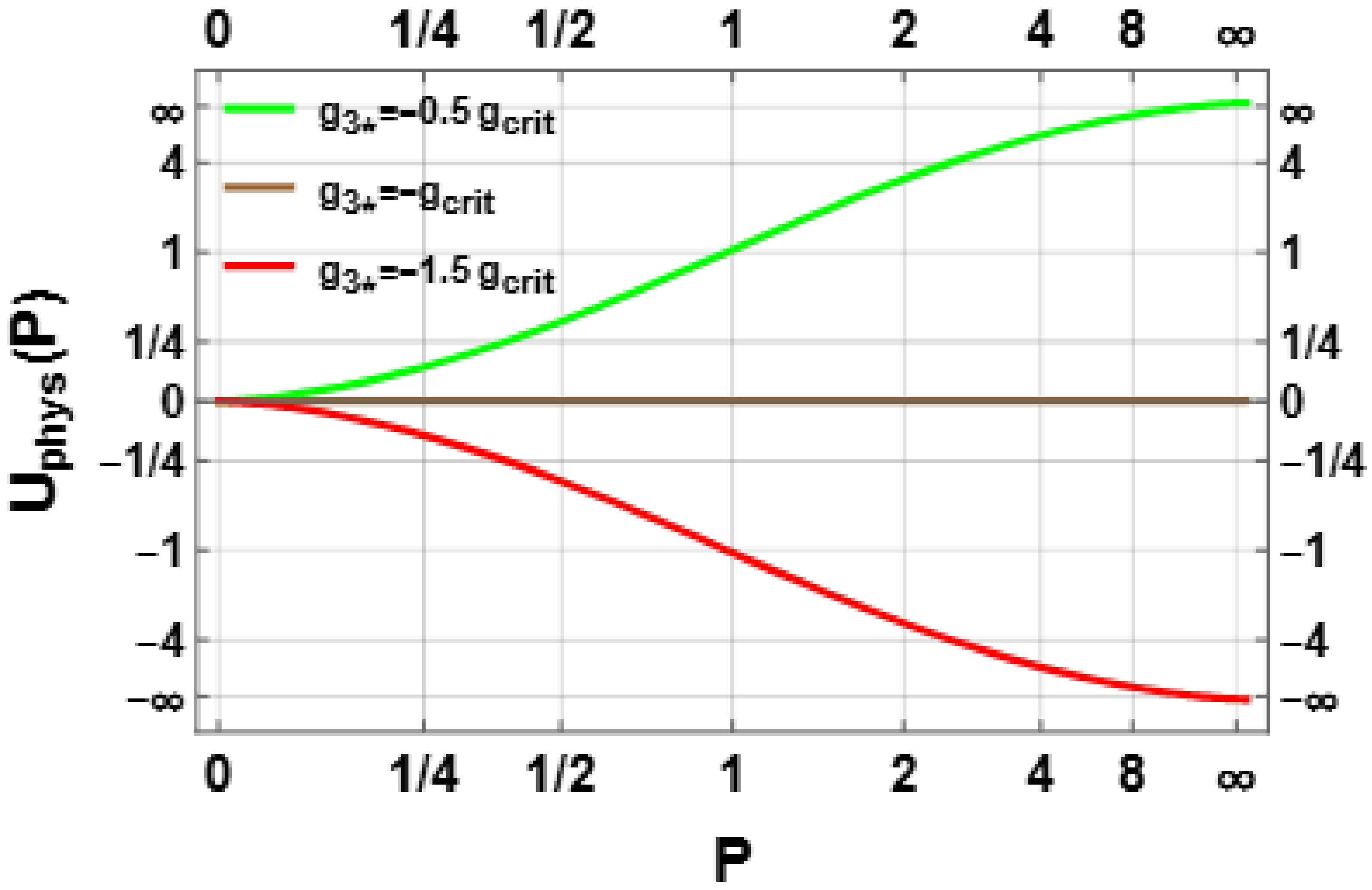

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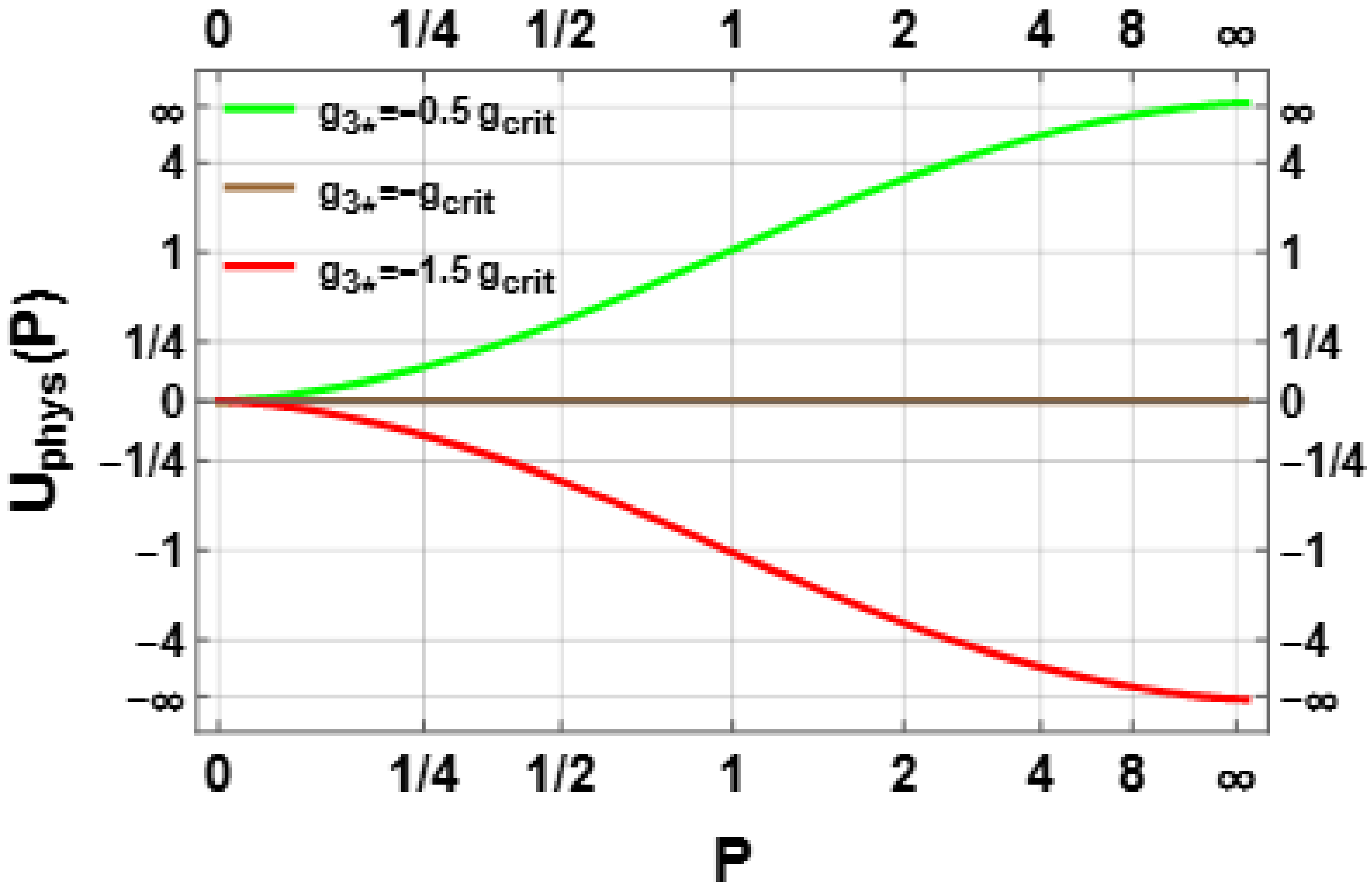

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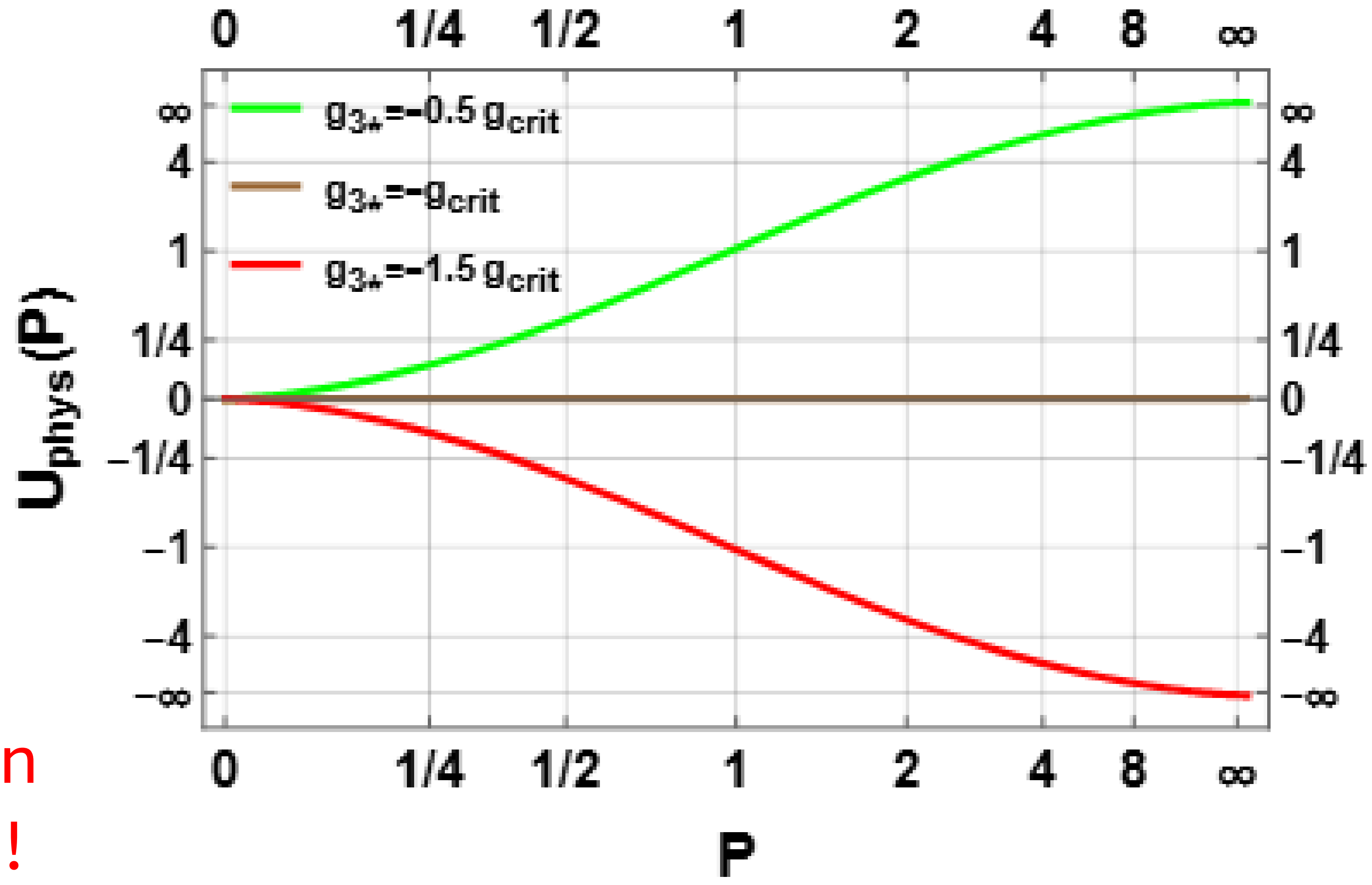
$\Longrightarrow$ (toy) dilaton + pion goldstone modes!



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