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# A Kilonova Toy Model

## How to cook a simple Kilonova

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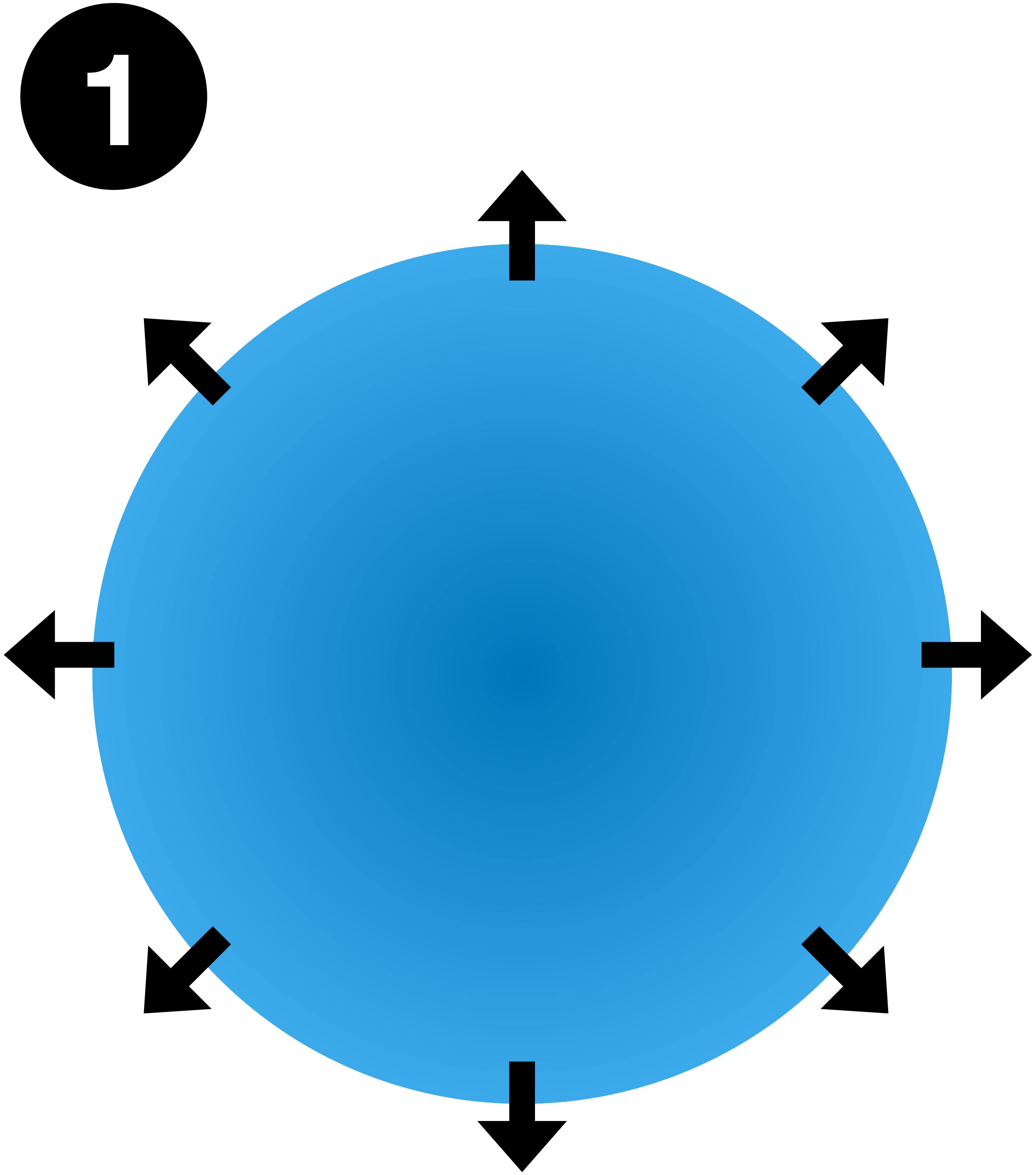
Model based on *Metzger 2020, LRR, 23, 1, Section 4.*




## Ingredients

We start with a given mass $M$ of material with electron fraction $Y_e$ expanding with velocities in the interval $[v_0, v_{max}]$

The matter is in homologous expansion, namely: $v \propto R$

The mass is distributed as a power-law of velocity, such that the mass with a velocity higher than $v$ is:

$$M_v = M\left(\frac{v}{v_0}\right)^{-\delta}$$

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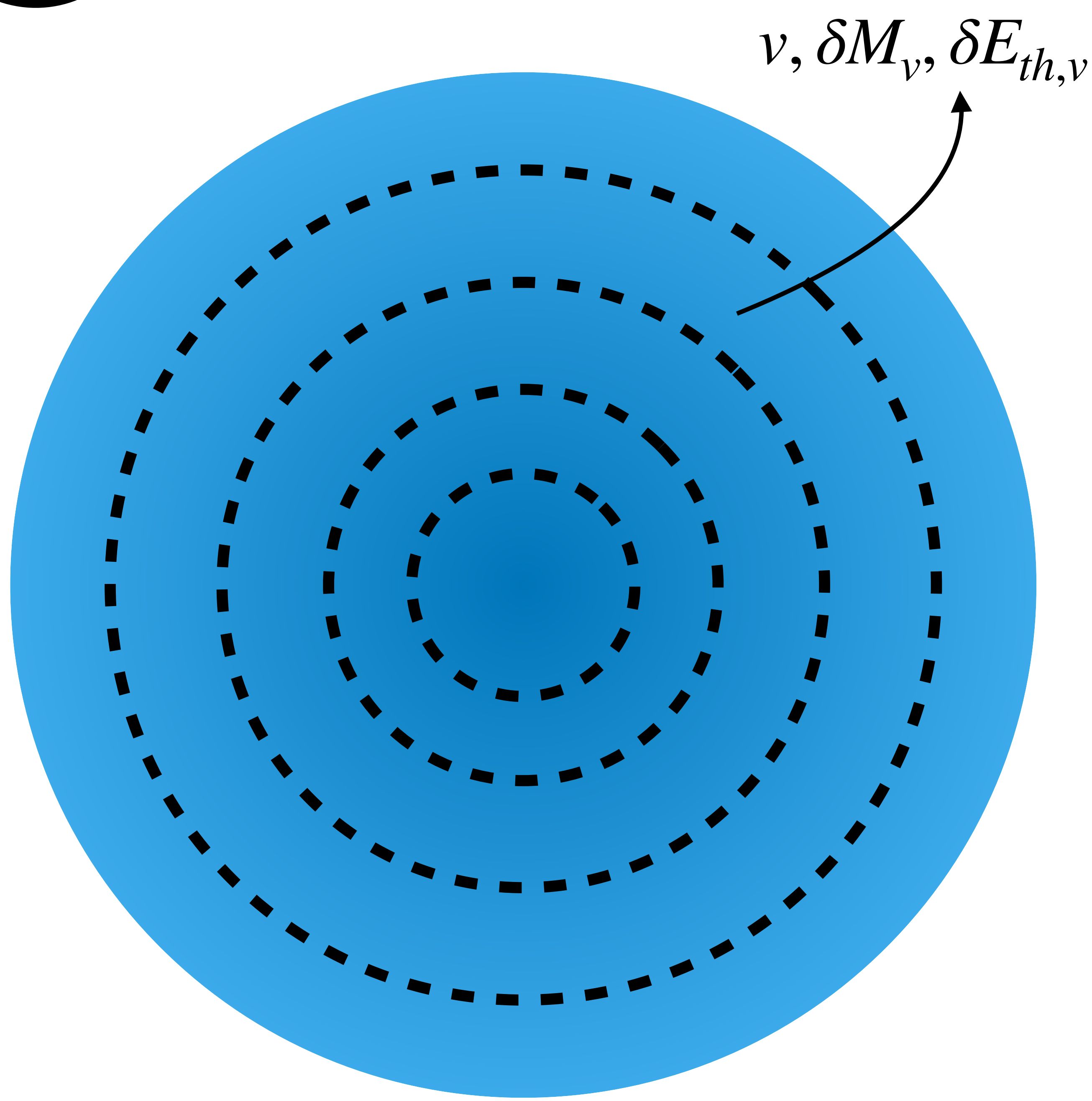


The thermal energy of each shell evolves according to the energy conservation:

$$\frac{d}{dt}(\delta E_{th,v}) = -\frac{\delta E_{th,v}}{R_v}\frac{dR_v}{dt} - L_v + Q_v$$

Adiabatic loss ($pdV$)

Radiative loss (what we observe!)

Heating

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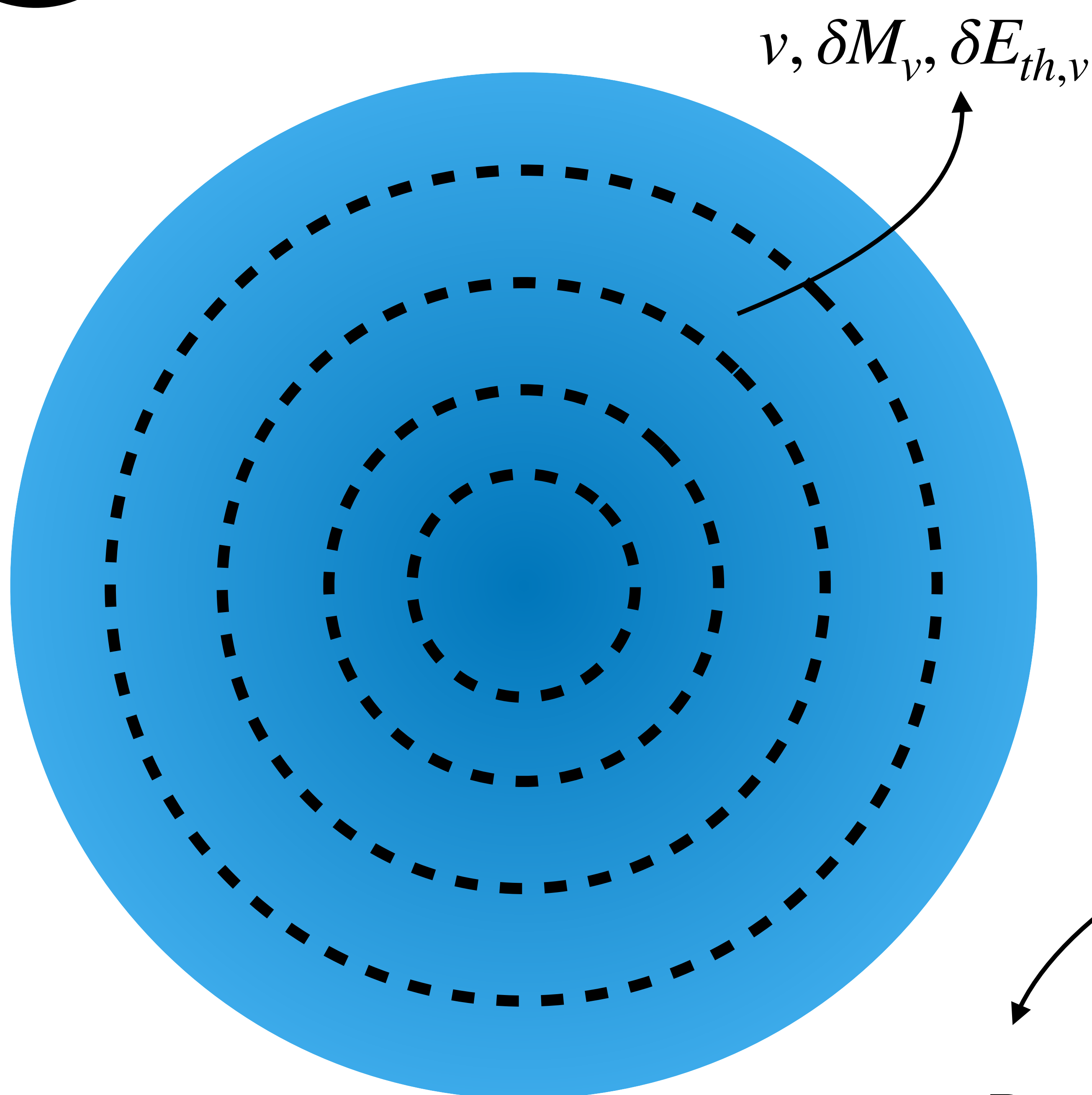


The luminosity in the shell is the thermal energy over the characteristic time (diffusion time/light crossing time)

$$L_v = \frac{\delta E_{th,v}}{t_{diff,v} + t_{lc}}$$

$$t_{diff,v} = \tau_v \frac{R_v}{c} = \frac{3M_v k_v}{4\pi\beta R_v c}$$

$$t_{lc} = \frac{R_v}{c}$$

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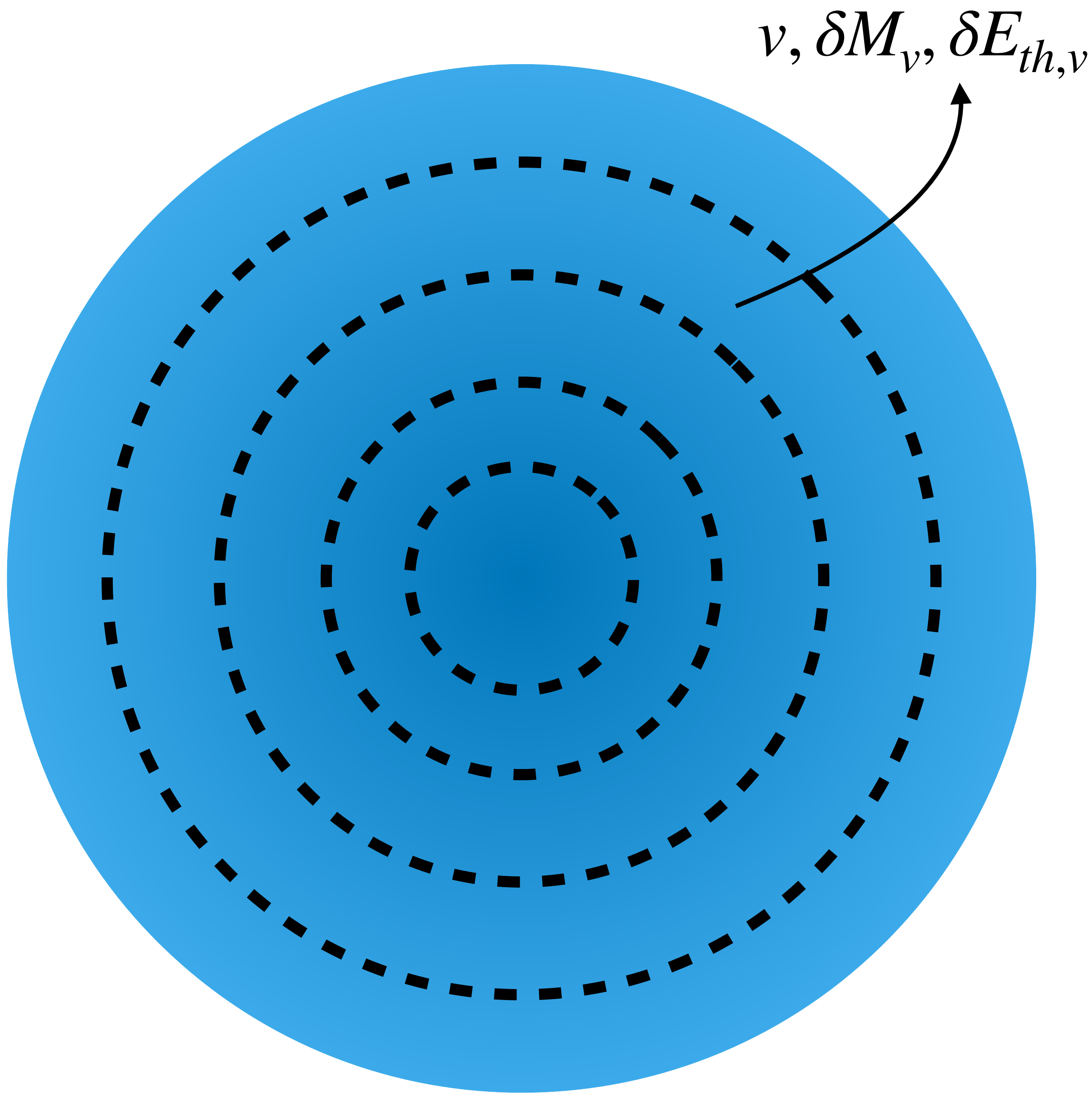
We need to solve the equations:




$$\begin{cases} \dfrac{dR_v}{dt} = v \\[2ex] \dfrac{d}{dt}(\delta E_{th,v}) = -\dfrac{\delta E_{th,v}}{R_v}\dfrac{dR_v}{dt} - L_v + Q_v \end{cases}$$

For $\delta E_{th,v}$ to find $L_v$

$$L_v = \frac{\delta E_{th,v}}{t_{diff,v} + t_{lc}}$$

And sum over the shell to find the total luminosity

$$L_{tot} = \sum L_v$$

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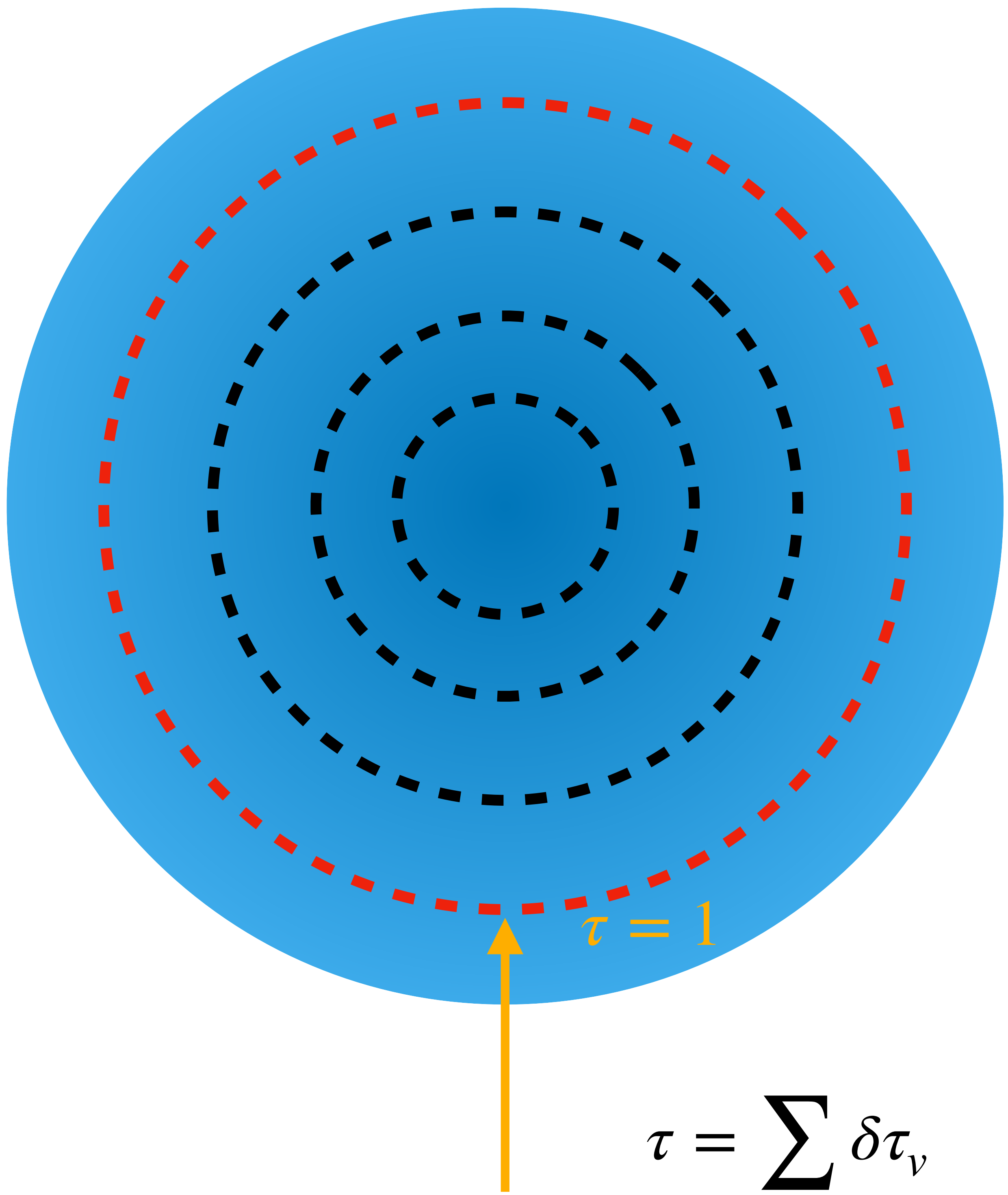
Once we have the total luminosity we can find the temperature (Stefan-Boltzmann law)




$$T_{eff} = \left(\frac{L_{tot}}{4\pi\sigma_{SB}R_{phot}^2}\right)^{1/4}$$

But we need the **radius of the photosphere**!

We find it by integrating the optical depth from outside to inside, until it is = 1

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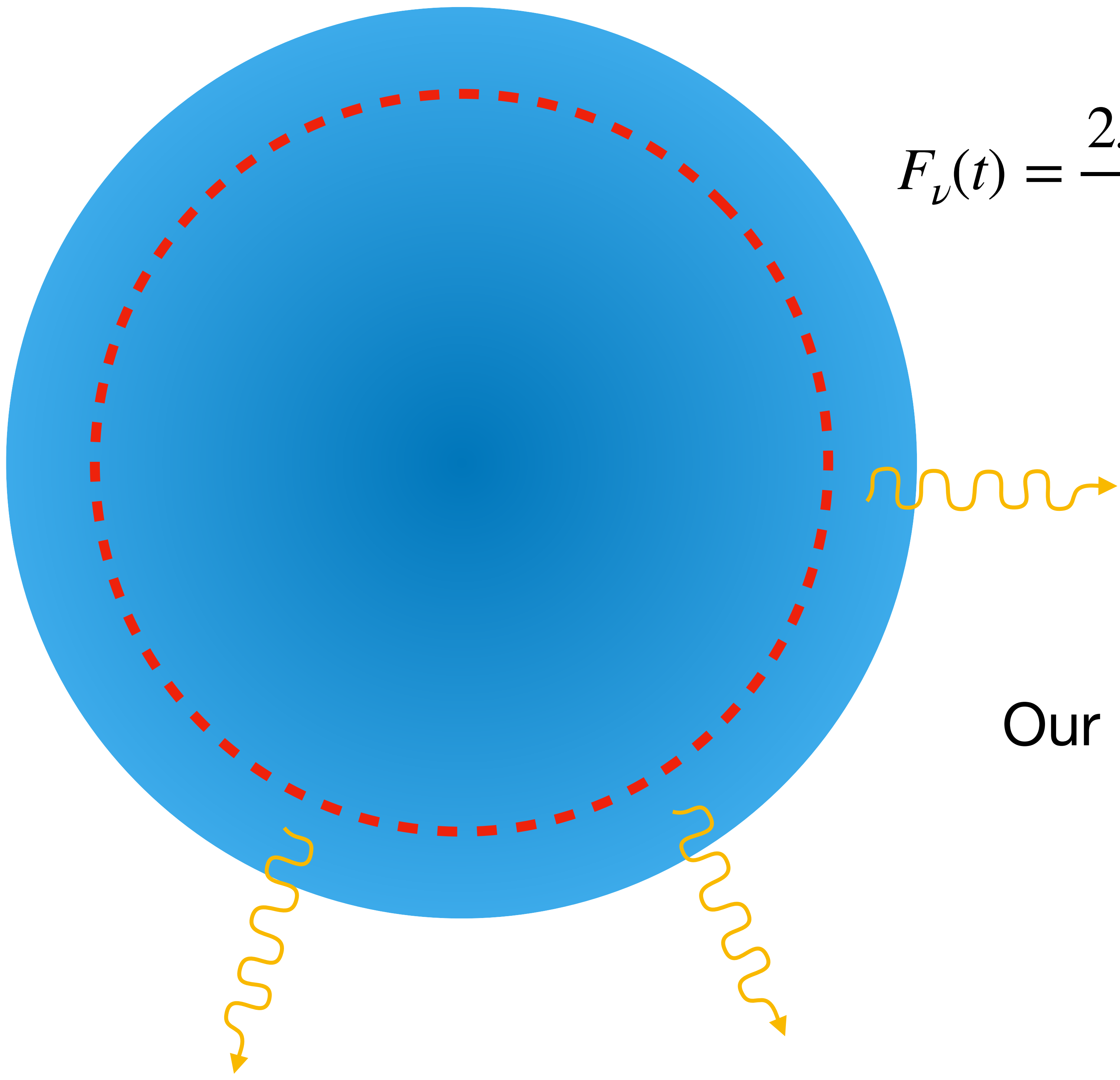
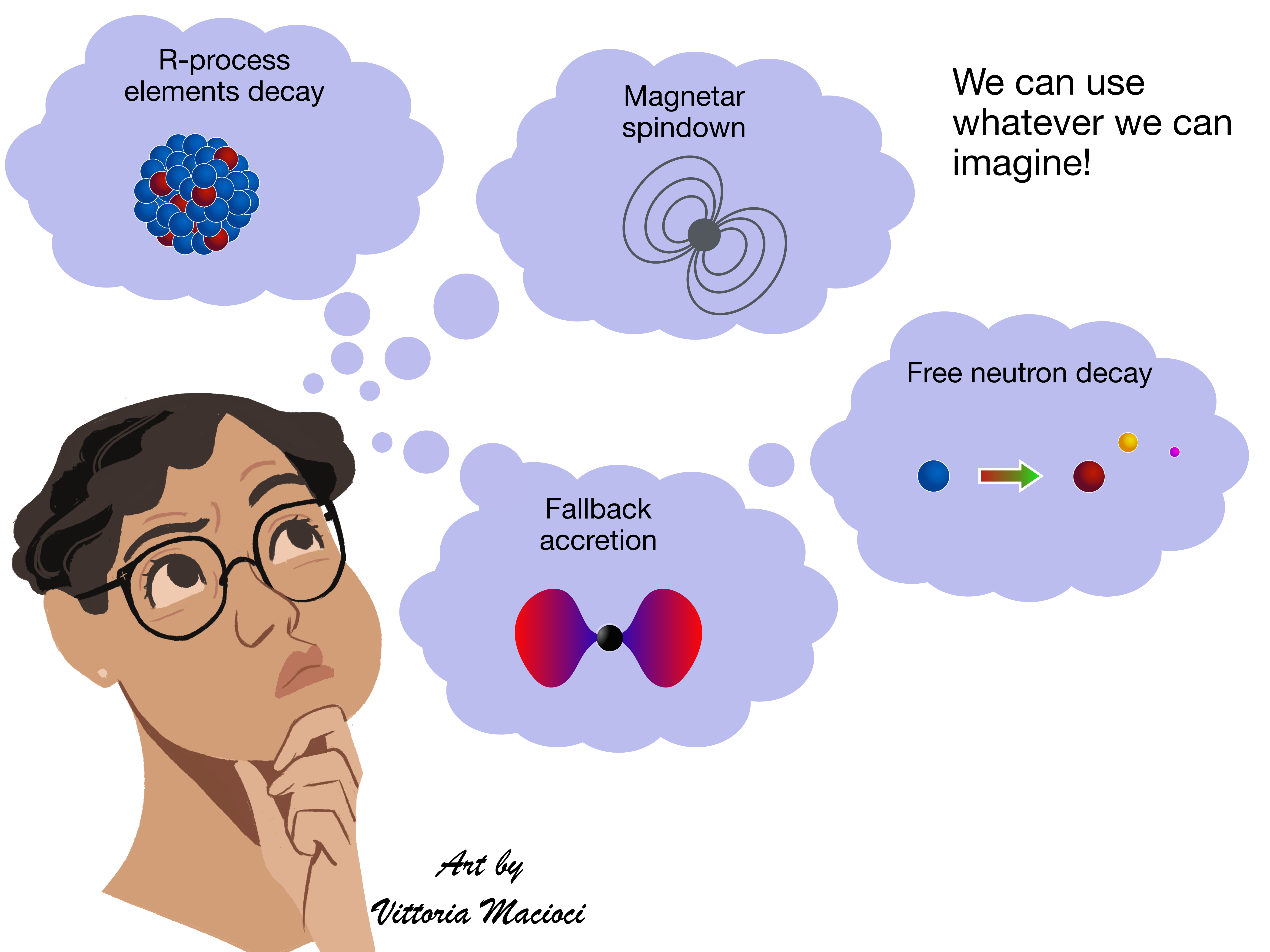
Now, given a distance, we have a flux for a given frequency!



$$F_\nu(t) = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp\left[\frac{h\nu}{k_B T(t)}\right] - 1} \frac{R^2_{phot}(t)}{D^2}$$

Our Kilonova is ready!

# Wait… what about the heating?


R-process
elements decay
Magnetar
spindown
We can use
whatever we can
imagine!
Free neutron decay
Fallback
accretion
Art by
Vittoria Macioci

# Playing with the Toy Model

1) Do we really need heating? What about an initially hot bubble just expanding and radiating?

2) Playing with the opacity 1: make a blue and a red kilonova

3) Playing with the opacity 2: what about a radial distribution of electron fraction $Y_e$ ?

4) Add Barnes et al. 2016 thermalization efficiency

$$f_{th}(t) = 0.36 \left[ \exp(-at) + \frac{\ln(1+2t^d)}{2bt^d} \right]$$, (time in days; a, b, d already in the code)

5) Add a central engine

# Some possible central engine

"Millisecond magnetar" spindown

$$L_{sd} = 7 \times 10^{50} \mathrm{erg\,s^{-1}} \left(\frac{I}{1.3 \times 10^{45}\,\mathrm{gcm^2}}\right) \left(\frac{B}{10^{15}\,G}\right)^{-2} \left(\frac{P_0}{0.7\,\mathrm{ms}}\right)^{-4} \left(1 + \frac{t}{t_{sd}}\right)^{-2}$$

$$t_{sd} \simeq 150\,\mathrm{s} \left(\frac{I}{1.3 \times 10^{45}\,\mathrm{g\,cm^2}}\right) \left(\frac{B}{10^{15}\,G}\right)^{-2} \left(\frac{P_0}{0.7\,\mathrm{ms}}\right)^{2}$$

$$\dot{Q}_{sd} = \epsilon_{th} L_{sd}$$

Fallback accretion

$$\dot{Q}_{fb} = \epsilon_j \dot{M}_{fb} c^2 \simeq 2 \times 10^{51}\,\mathrm{erg\,s^{-1}} \left(\frac{\epsilon_j}{0.1}\right) \left(\frac{\dot{M}_{fb}(0.1\,s)}{10^{-3} M_\odot s^{-1}}\right) \left(\frac{t}{0.1\,s}\right)^{-5/3}$$

# Other decay channels

Nichel decay

$$\dot{\epsilon}_{Ni} = 3.9 \times 10^{10} e^{-t/t_{Ni}} + 6.78 \times 10^{9} \left[ e^{-t/t_{Co}} - e^{-t/t_{Ni}} \right]$$

$$t_{Ni} = 8.8\,\mathrm{days} \qquad t_{Co} = 113.6\,\mathrm{days}$$