

# Evidence for Asymptotic Safety

---

Ricci Flow in Quantum Gravity  
RWTH Aachen University

Yannick Kluth, September 9th, 2026



UNIVERSITY OF  
TORONTO



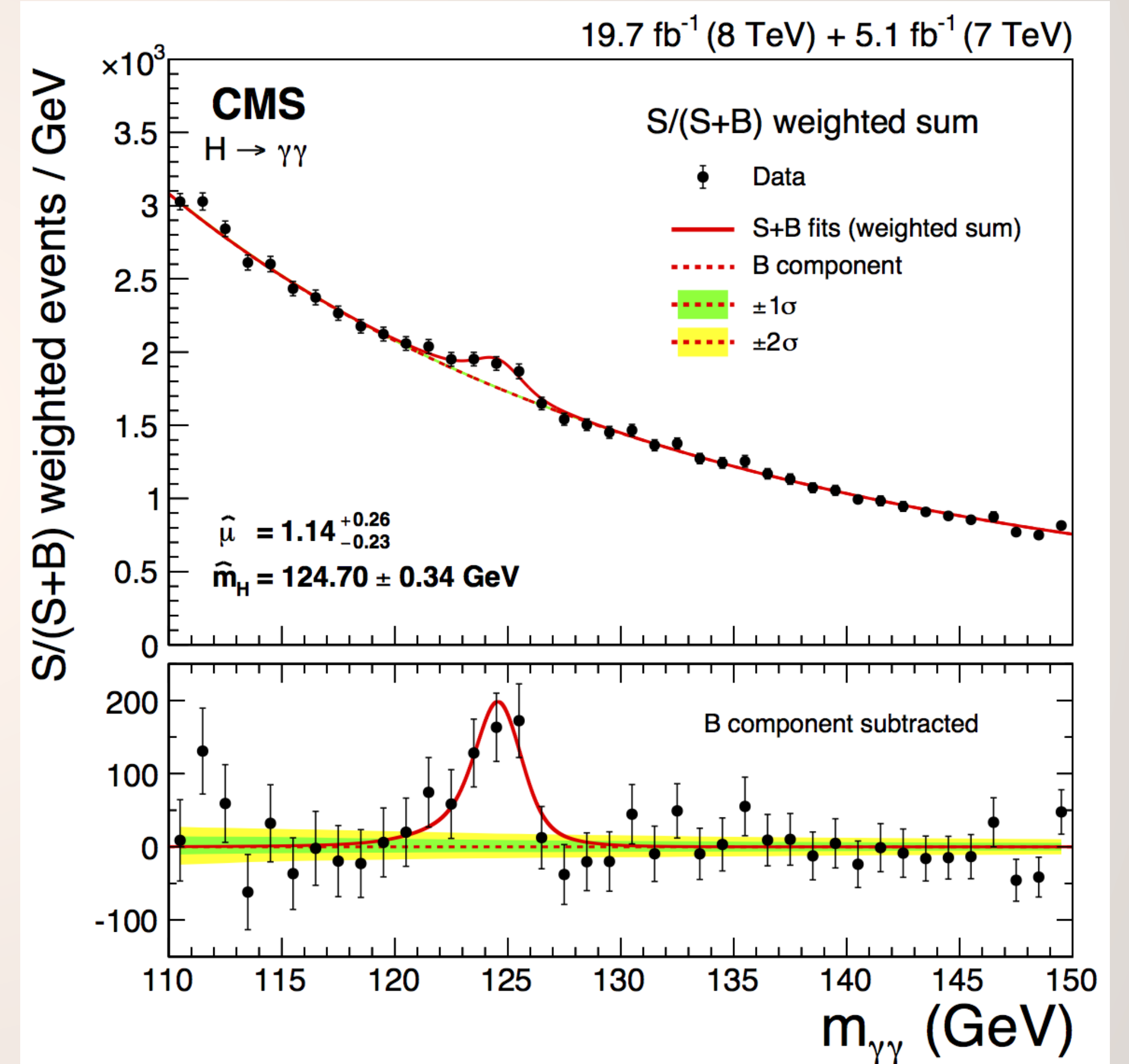
# QFT and the Standard Model

---

- QFT is incredibly successful

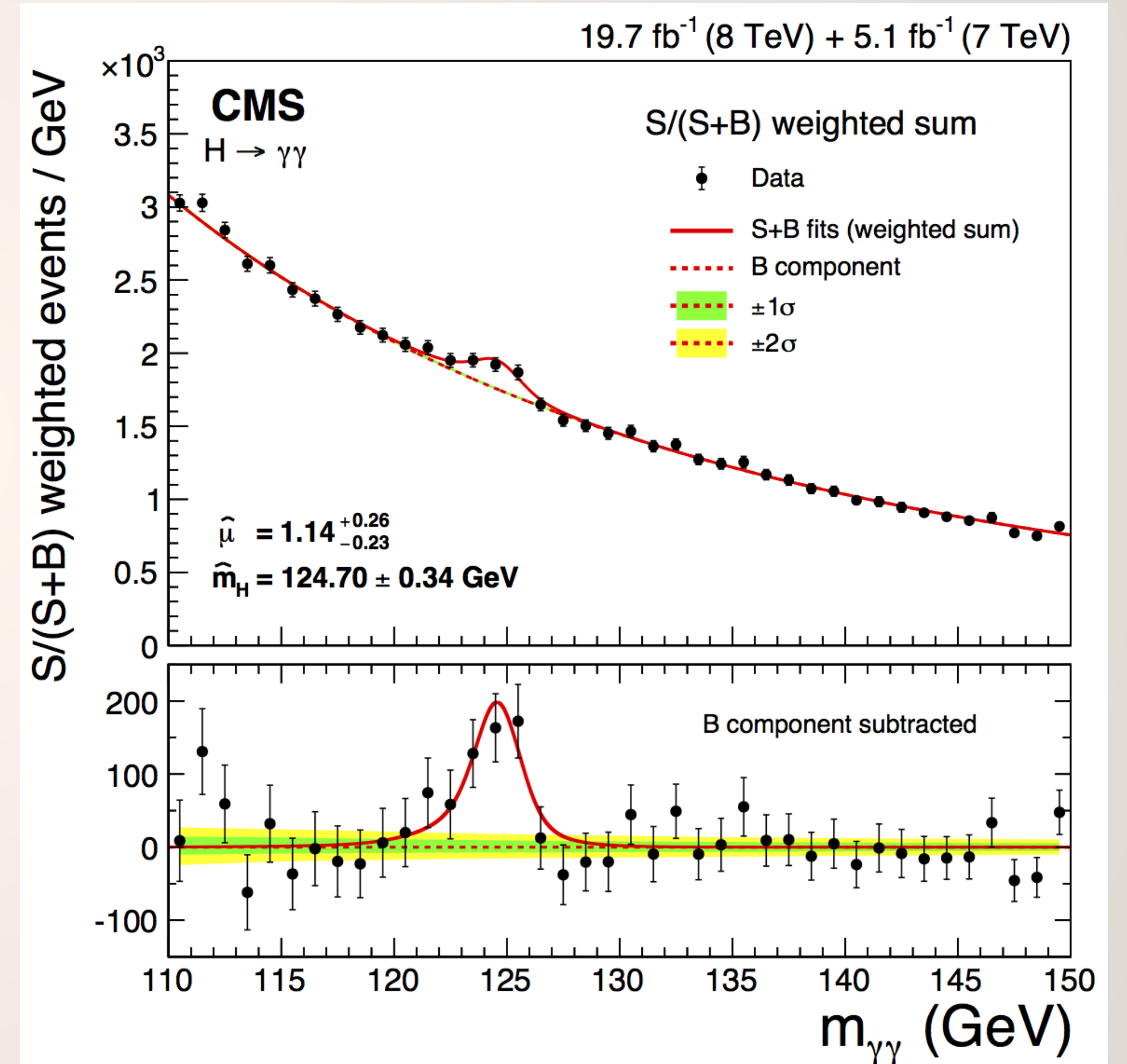
# QFT and the Standard Model

- QFT is incredibly successful
  - Standard Model of particle physics!



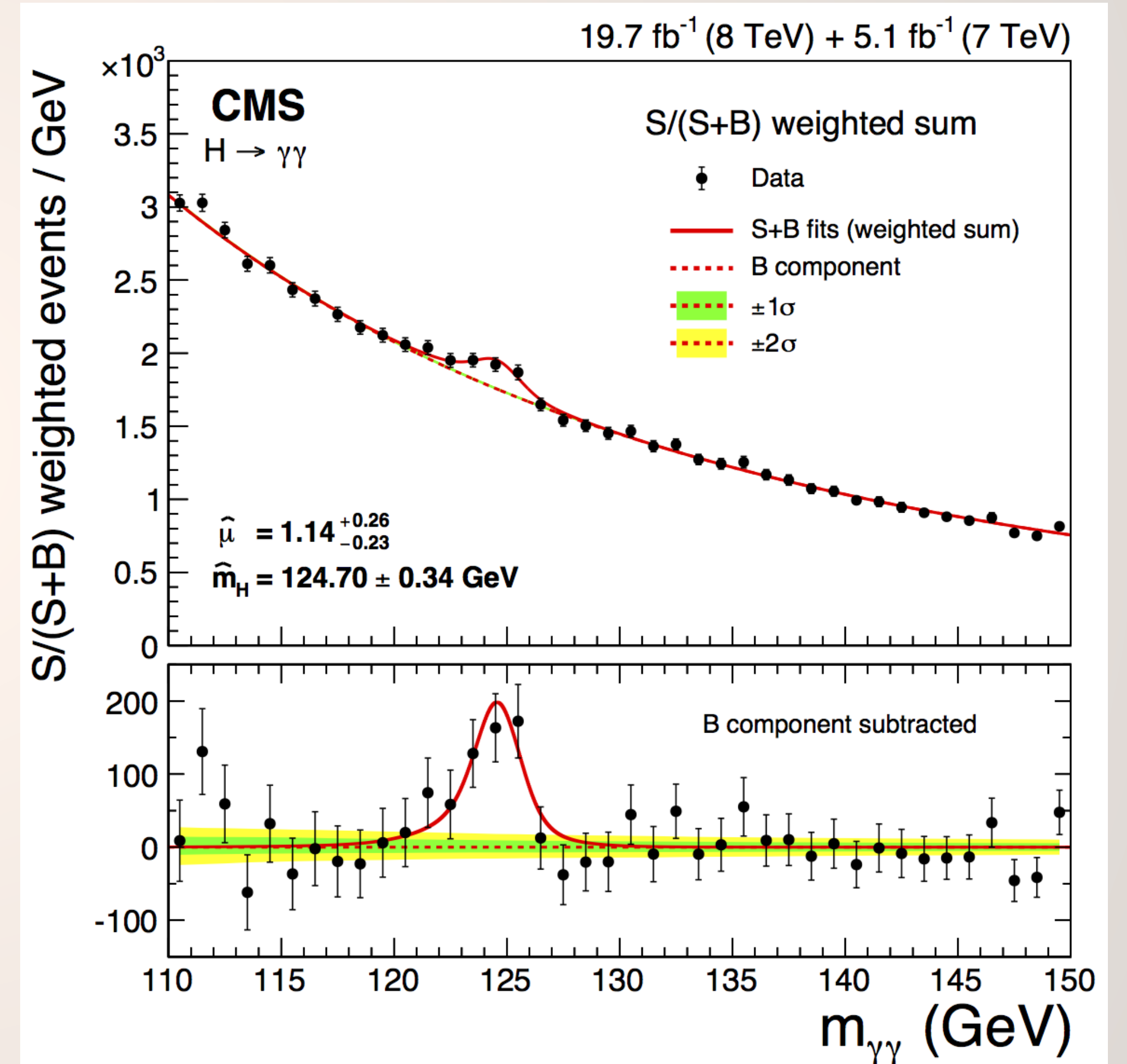
# QFT and the Standard Model

- QFT is incredibly successful
  - Standard Model of particle physics!
  - Effective Field Theories



# QFT and the Standard Model

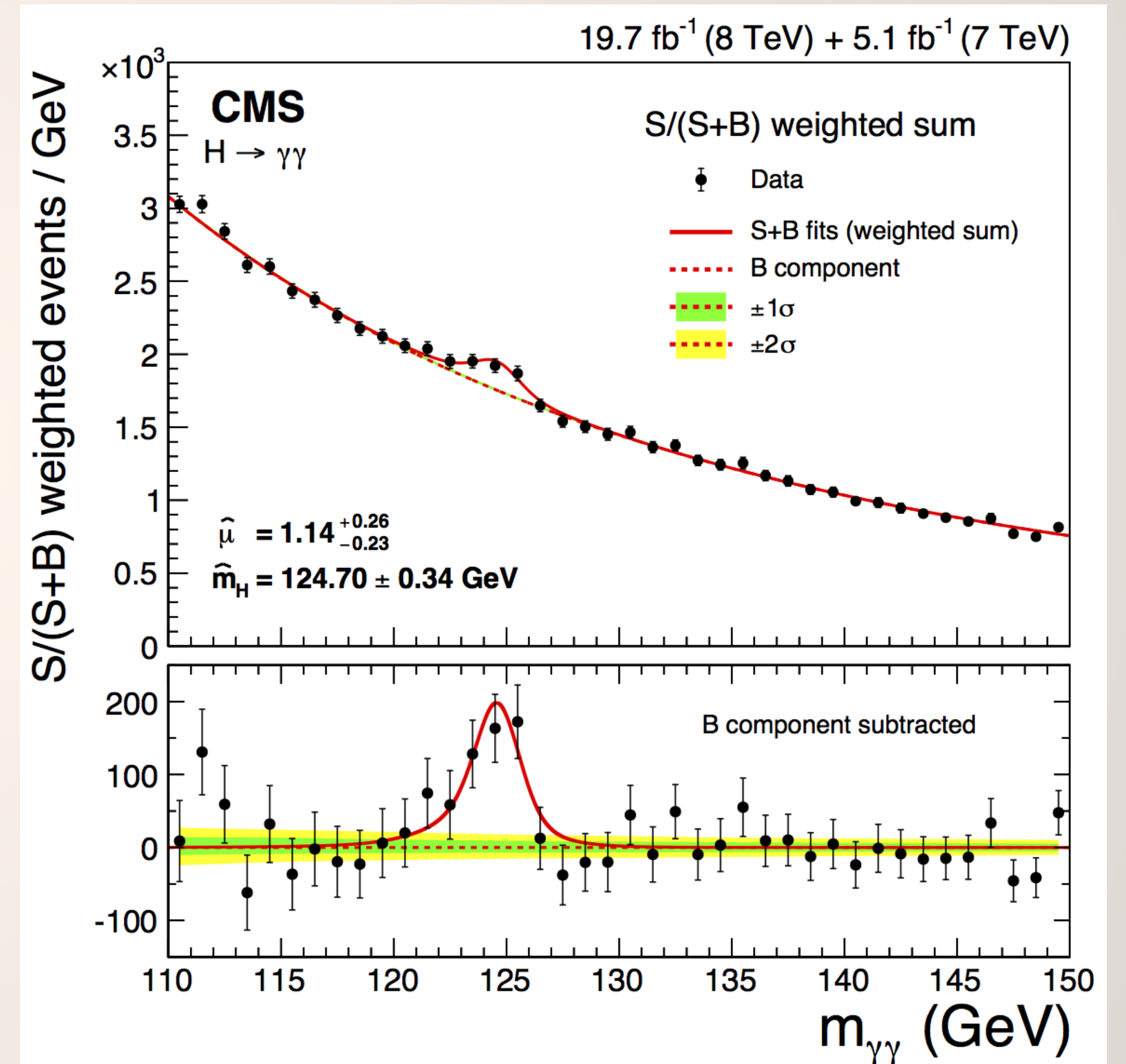
- QFT is incredibly successful
  - Standard Model of particle physics!
  - Effective Field Theories
  - Condensed Matter Physics



# QFT and the Standard Model

- QFT is incredibly successful
  - Standard Model of particle physics!
  - Effective Field Theories
  - Condensed Matter Physics

Is nature fundamentally a QFT?



# Non-Renormalizability of Quantum Gravity

---

- Classical Gravity described by Einstein-Hilbert Lagrangian:

$$S_{EH} = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} (R - 2\Lambda)$$

# Non-Renormalizability of Quantum Gravity

---

- Classical Gravity described by Einstein-Hilbert Lagrangian:

$$S_{EH} = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} (R - 2\Lambda)$$

- UV divergencies induce counterterms in a quantum field theory of gravity:

$$S = \int d^d x \sqrt{-g} \left[ \frac{1}{16\pi G_N} R - \frac{\Lambda}{8\pi G_N} \right. \quad \text{Tree-Level}$$
$$+ \lambda_{1,1} R^2 + \lambda_{1,2} R_{\mu\nu} R^{\mu\nu} + \sigma_E E \quad \text{One-Loop}$$
$$\left. + \lambda_{3,1} R^3 + \dots + \lambda_{C^3} C^{\rho\sigma}_{\mu\nu} C^{\mu\nu}_{\alpha\beta} C^{\alpha\beta}_{\rho\sigma} + \dots \right] \quad \text{Two-Loop}$$

# Non-Renormalizability of Quantum Gravity

---

- Classical Gravity described by Einstein-Hilbert Lagrangian:

$$S_{EH} = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} (R - 2\Lambda)$$

- UV divergencies induce counterterms in a quantum field theory of gravity:

$$S = \int d^d x \sqrt{-g} \left[ \frac{1}{16\pi G_N} R - \frac{\Lambda}{8\pi G_N} \right. \quad \text{Tree-Level}$$

$$\left. + \lambda_{1,1} R^2 + \lambda_{1,2} R_{\mu\nu} R^{\mu\nu} + \sigma_E E \quad \text{One-Loop} \right.$$

$$\left. + \lambda_{3,1} R^3 + \dots + \lambda_{C^3} C^{\rho\sigma}_{\mu\nu} C^{\mu\nu}_{\alpha\beta} C^{\alpha\beta}_{\rho\sigma} + \dots \right] \quad \text{Two-Loop}$$

[Goroff and Sagnotti, 1985]  
[van de Ven, 1992]

First on-shell divergence beyond EH

# Non-Renormalizability of Quantum Gravity

- Classical Gravity described by Einstein-Hilbert Lagrangian:

$$S_{EH} = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} (R - 2\Lambda)$$

- UV divergencies induce counterterms in a quantum field theory of gravity:

$$S = \int d^d x \sqrt{-g} \left[ \frac{1}{16\pi G_N} R - \frac{\Lambda}{8\pi G_N} \right.$$

Tree-Level

**Do we have to give up QFT?**

$$+ \lambda_{1,1} R^2 + \lambda_{1,2} R_{\mu\nu} R^{\mu\nu} + \sigma_E E$$

One-Loop

$$+ \lambda_{3,1} R^3 + \dots + \lambda_{C^3} C^{\rho\sigma}_{\mu\nu} C^{\mu\nu}_{\alpha\beta} C^{\alpha\beta}_{\rho\sigma} + \dots \Big]$$

Two-Loop

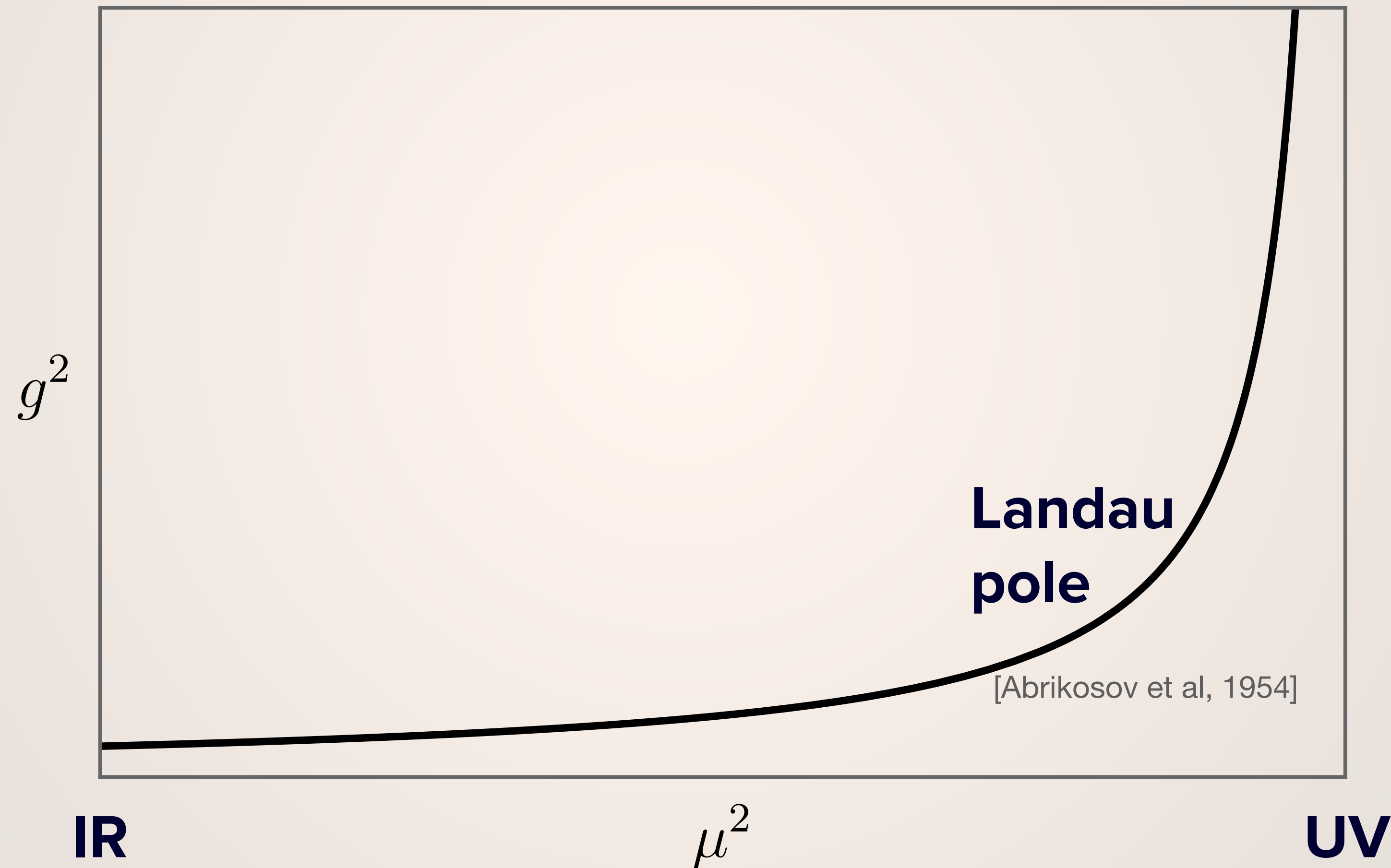
[Goroff and Sagnotti, 1985]

[van de Ven, 1992]

First on-shell divergence beyond EH

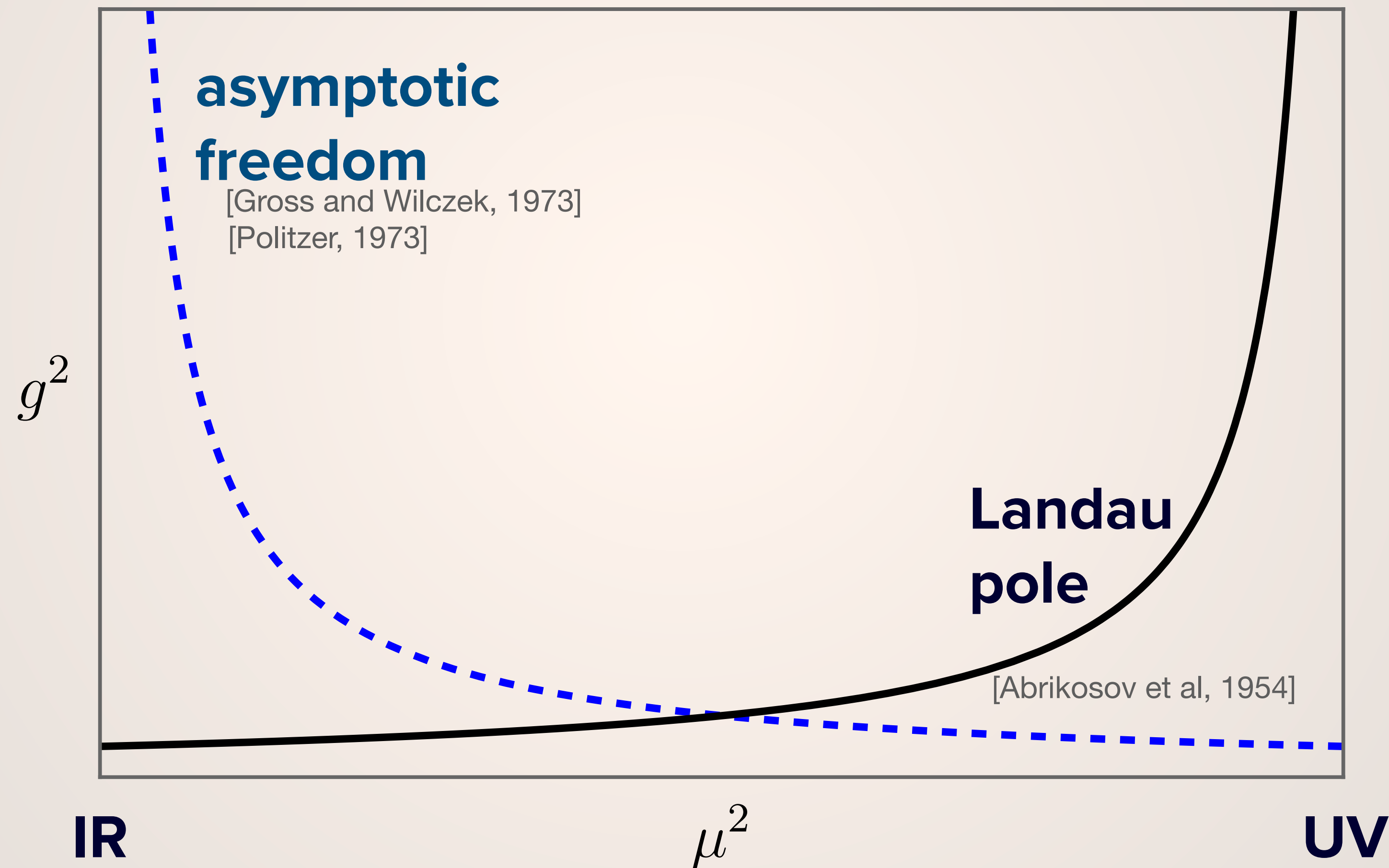
# Are there UV complete QFTs?

---

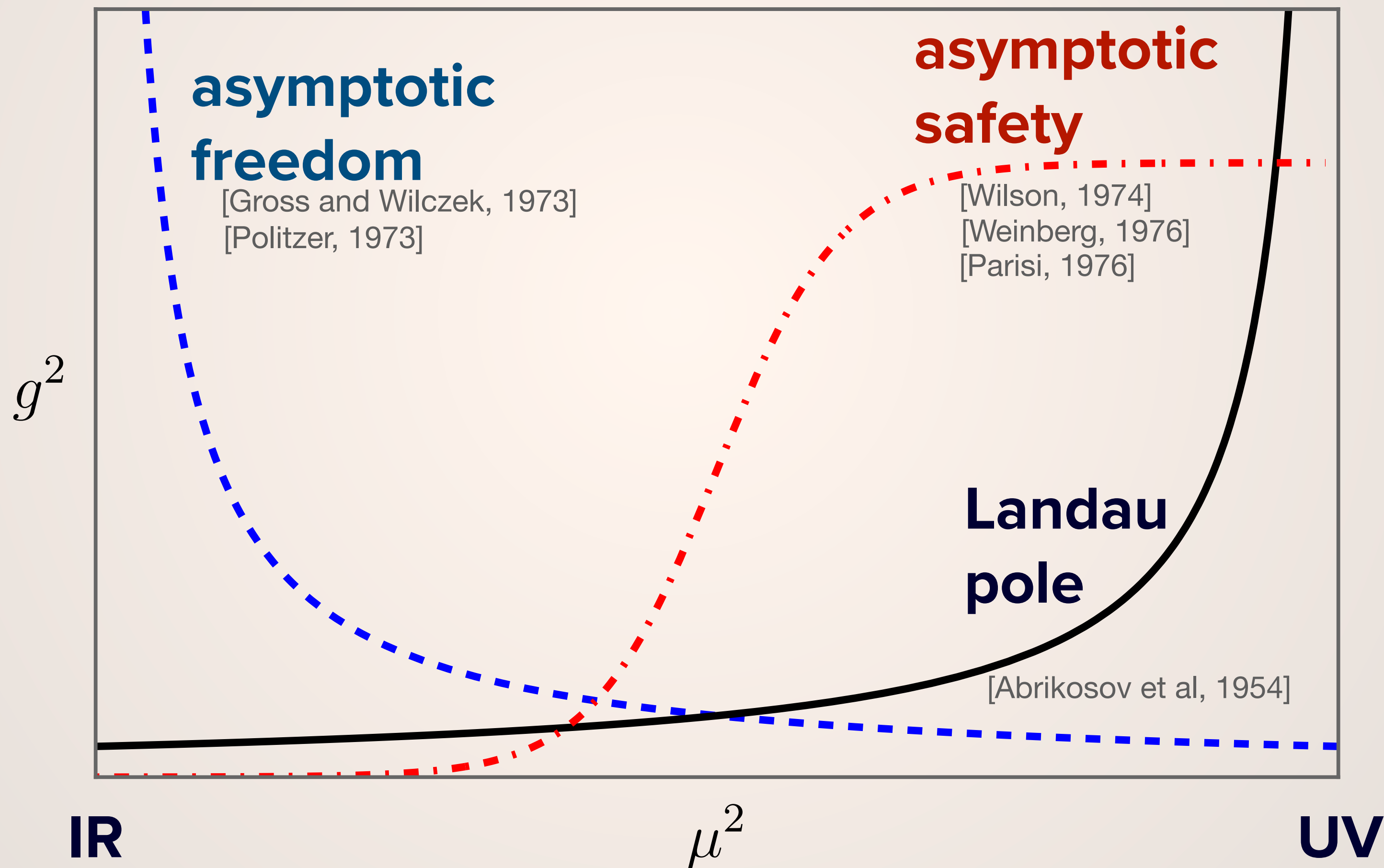


# Are there UV complete QFTs?

---

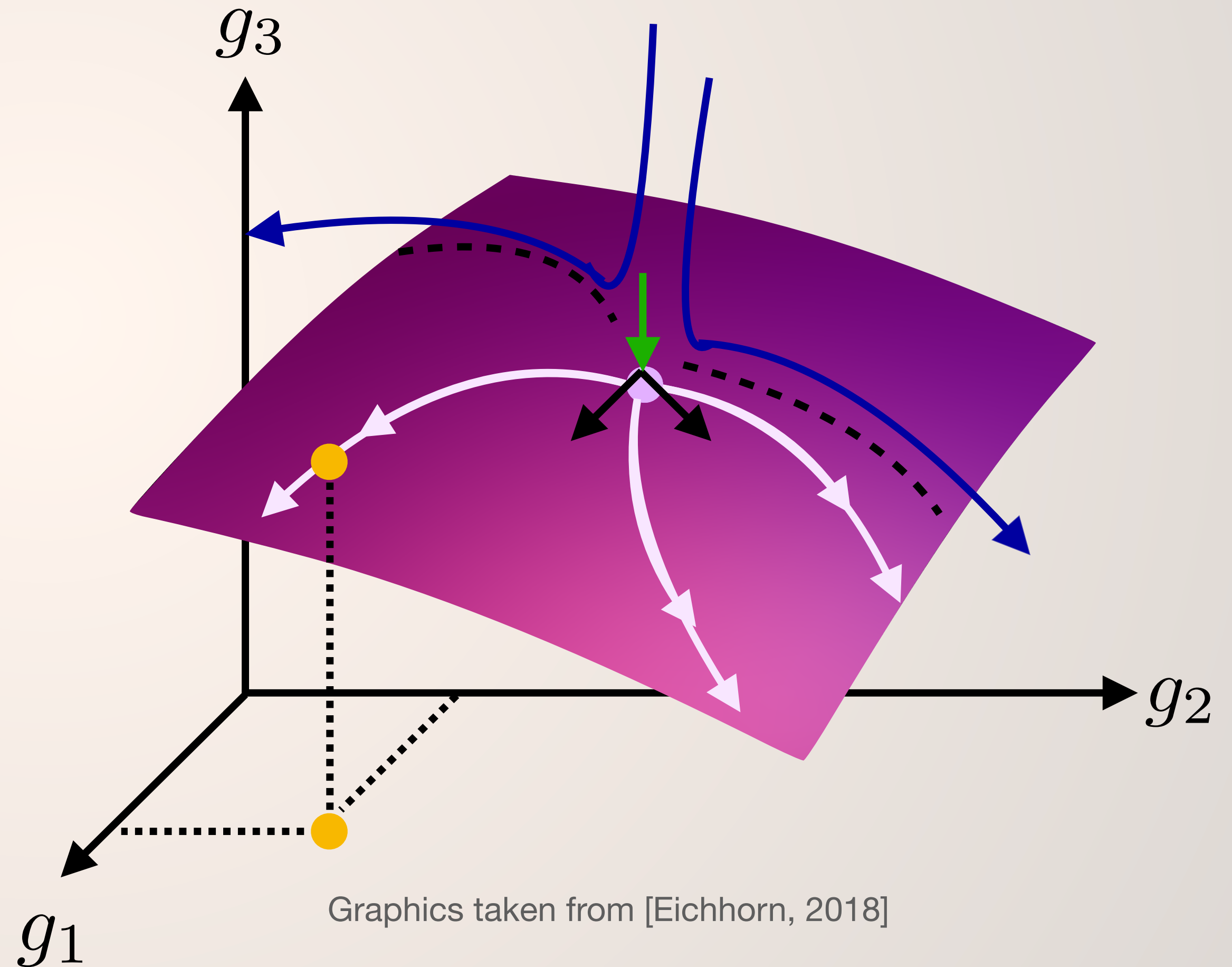


# Are there UV complete QFTs?



# Predictive Power of Asymptotic Safety

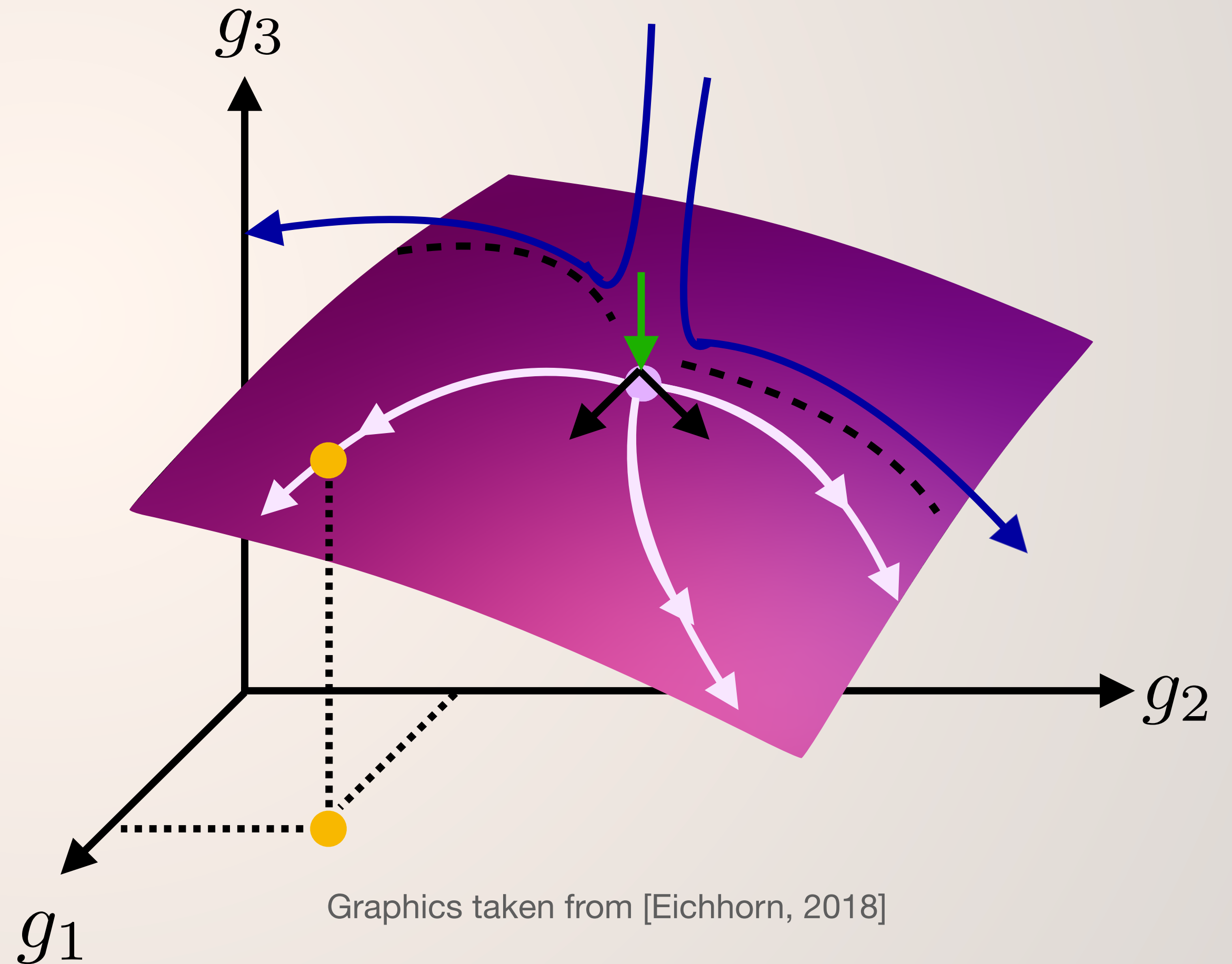
- Couplings must flow into UV fixed point
- Not all trajectories reach UV fixed point
- **Only trajectories on UV critical surface (purple) are asymptotically safe**



# Predictive Power of Asymptotic Safety

- Count parameters by linearizing flow around fixed point

$$\beta_i = \left. \frac{\partial \beta_i}{\partial \lambda_j} \right|_* (\lambda_j - \lambda_j^*) + \dots$$

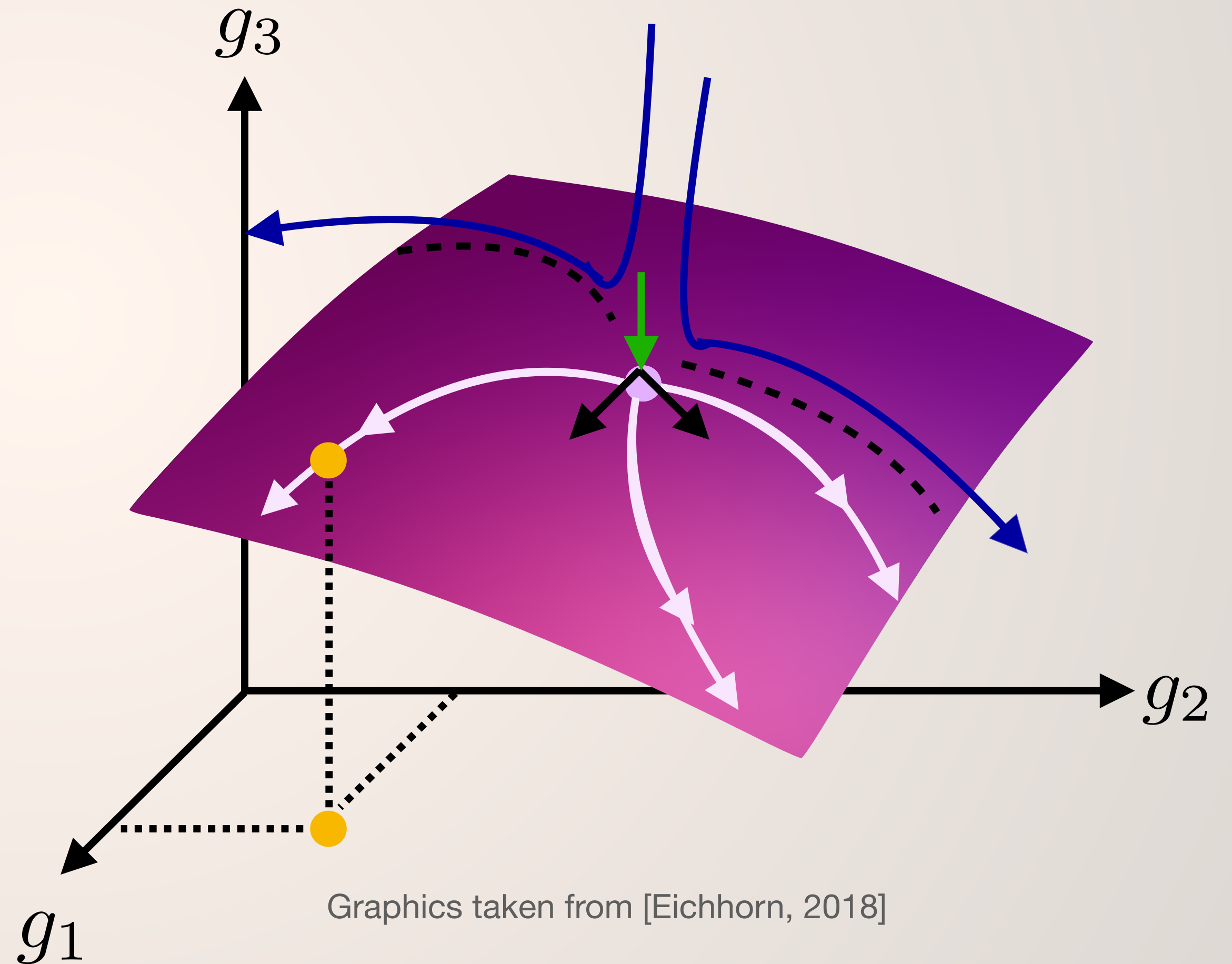


# Predictive Power of Asymptotic Safety

- Count parameters by linearizing flow around fixed point

$$\left. \frac{\partial \beta_i}{\partial \lambda_j} \right|_* v_j^{(\ell)} = \theta_\ell v_i^{(\ell)}$$

- $\theta_\ell > 0$ : irrelevant parameter
- $\theta_\ell < 0$ : **relevant parameter**



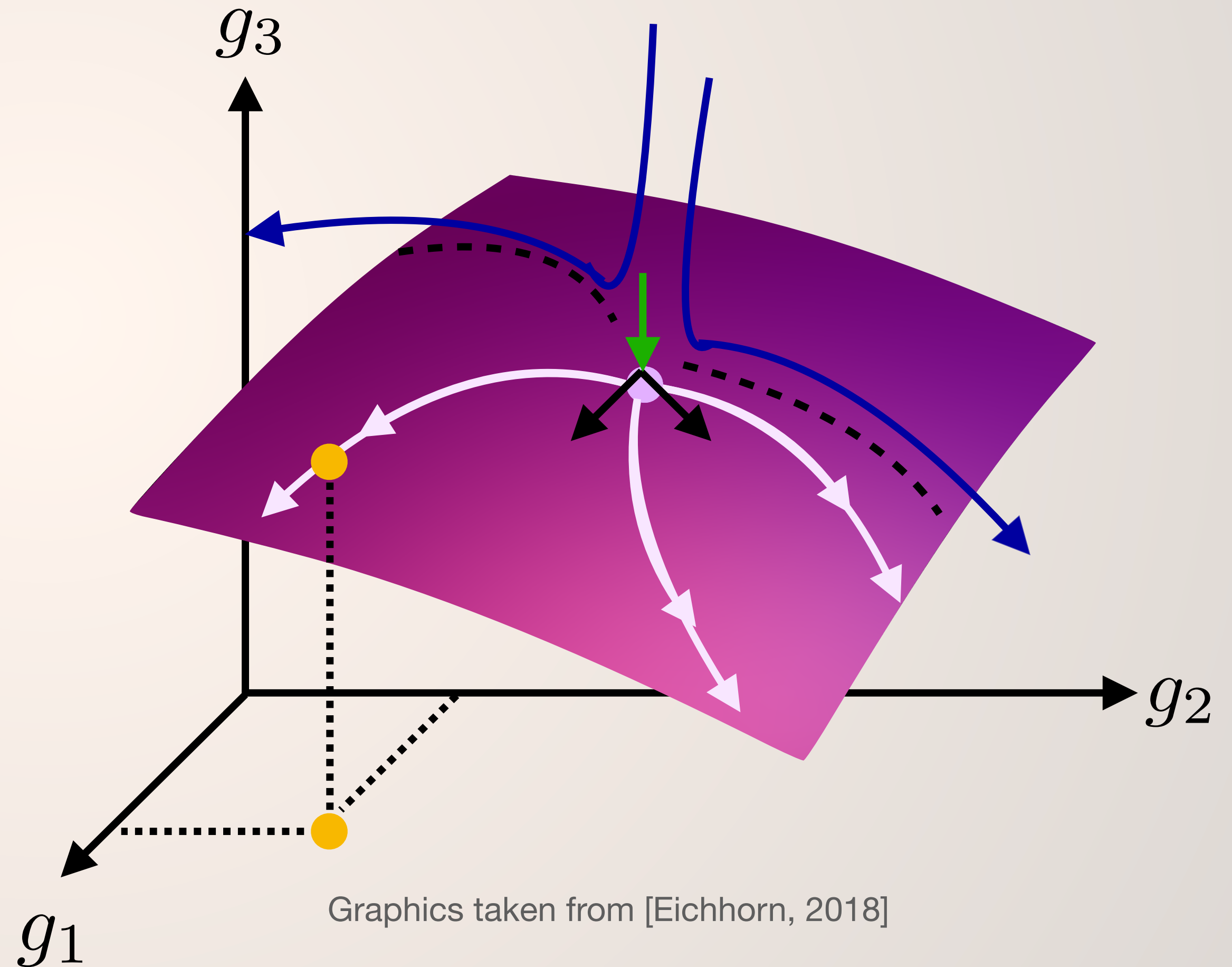
# Predictive Power of Asymptotic Safety

- Count parameters by linearizing flow around fixed point

$$\left. \frac{\partial \beta_i}{\partial \lambda_j} \right|_* v_j^{(\ell)} = \theta_\ell v_i^{(\ell)}$$

- $\theta_\ell > 0$ : irrelevant parameter
- $\theta_\ell < 0$ : relevant parameter

**Does QG have a UV fixed point with finitely many relevant directions?**



# Are there any asymptotically safe theories?

---

# Are there any asymptotically safe theories?

---

- **Gauge-Yukawa Theories in 4d** [Litim and Sannino, 2014]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + [\bar{\psi}i\not{D}\psi] - y [\bar{\psi}\phi\psi] + [\partial^\mu\phi^\dagger\partial_\mu\phi] - m^2 [\phi^\dagger\phi] - u [\phi^\dagger\phi\phi^\dagger\phi] - v [\phi^\dagger\phi] [\phi^\dagger\phi]$$

- Conventionally Renormalizable

- Rigorous UV fixed point under perturbative control  $\alpha_g \propto \epsilon$  with  $\epsilon = \frac{n_F}{N_c} - \frac{11}{2}$

# Are there any asymptotically safe theories?

---

- **Gauge-Yukawa Theories in 4d** [Litim and Sannino, 2014]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + [\bar{\psi}i\not{D}\psi] - y [\bar{\psi}\phi\psi] + [\partial^\mu\phi^\dagger\partial_\mu\phi] - m^2 [\phi^\dagger\phi] - u [\phi^\dagger\phi\phi^\dagger\phi] - v [\phi^\dagger\phi] [\phi^\dagger\phi]$$

- Conventionally Renormalizable

- Rigorous UV fixed point under perturbative control  $\alpha_g \propto \epsilon$  with  $\epsilon = \frac{n_F}{N_c} - \frac{11}{2}$

- **Gross-Neveu model in 3d** [Gross and Neveu, 1974]

$$\mathcal{L}_{\text{GN}} = \bar{\psi}_0 (i\not{\partial} - M_0) \psi_0 + \frac{G_0}{2}(\bar{\psi}_0\psi_0)^2$$

# Are there any asymptotically safe theories?

- **Gauge-Yukawa Theories in 4d** [Litim and Sannino, 2014]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + [\bar{\psi}i\not{D}\psi] - y [\bar{\psi}\phi\psi] + [\partial^\mu\phi^\dagger\partial_\mu\phi] - m^2 [\phi^\dagger\phi] - u [\phi^\dagger\phi\phi^\dagger\phi] - v [\phi^\dagger\phi] [\phi^\dagger\phi]$$

- Conventionally Renormalizable

- Rigorous UV fixed point under perturbative control  $\alpha_g \propto \epsilon$  with  $\epsilon = \frac{n_F}{N_c} - \frac{11}{2}$

- **Gross-Neveu model in 3d** [Gross and Neveu, 1974]

$$\mathcal{L}_{\text{GN}} = \bar{\psi}_0 (i\not{\partial} - M_0) \psi_0 + \frac{G_0}{2}(\bar{\psi}_0\psi_0)^2$$

- Rigorous UV fixed point despite being conventionally non-renormalizable
  - Large N, conformal bootstrap, FRG, ... [Rosenstein and Warr and Park, 1989]  
[de Calan et al, 1991]

# Are there any asymptotically safe theories?

- **Gauge-Yukawa Theories in 4d** [Litim and Sannino, 2014]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + [\bar{\psi}i\not{D}\psi] - y [\bar{\psi}\phi\psi] + [\partial^\mu\phi^\dagger\partial_\mu\phi] - m^2 [\phi^\dagger\phi] - u [\phi^\dagger\phi\phi^\dagger\phi] - v [\phi^\dagger\phi] [\phi^\dagger\phi]$$

- Conventionally Renormalizable

- Rigorous UV fixed point under perturbative control  $\alpha_g \propto \epsilon$  with  $\epsilon = \frac{n_F}{N_c} - \frac{11}{2}$

- **Gross-Neveu model in 3d** [Gross and Neveu, 1974]

$$\mathcal{L}_{\text{GN}} = \bar{\psi}_0 (i\not{\partial} - M_0) \psi_0 + \frac{G_0}{2}(\bar{\psi}_0\psi_0)^2$$

- Rigorous UV fixed point despite being conventionally non-renormalizable
  - Large N, conformal bootstrap, FRG, ... [Rosenstein and Warr and Park, 1989]
  - **Blueprint for Quantum Gravity** [de Calan et al, 1991]

# Quantum Gravity in $d = 2 + \varepsilon$

---

- First indications for asymptotic safety from perturbation theory in  $d = 2 + \varepsilon$

[Kawai and Ninomiya, 1990]

[Jack and Jones, 1991]

[Kawai and Kitazawa and Ninomiya, 1993]

$$\mu \frac{dg}{d\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}(g^3)$$

·  
·  
·

# Quantum Gravity in $d = 2 + \varepsilon$

---

- First indications for asymptotic safety from perturbation theory in  $d = 2 + \varepsilon$

[Kawai and Ninomiya, 1990]

[Jack and Jones, 1991]

[Kawai and Kitazawa and Ninomiya, 1993]

$$\mu \frac{dg}{d\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}(g^3)$$

·  
·  
·

- Extrapolate to  $\varepsilon \rightarrow 2$ 
  - UV fixed point!

# Quantum Gravity in $d = 2 + \varepsilon$

---

- First indications for asymptotic safety from perturbation theory in  $d = 2 + \varepsilon$

[Kawai and Ninomiya, 1990]

[Jack and Jones, 1991]

[Kawai and Kitazawa and Ninomiya, 1993]

$$\mu \frac{dg}{d\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}(g^3)$$

·  
·  
·

- Extrapolate to  $\varepsilon \rightarrow 2$  **Are we allowed to do this?**
  - UV fixed point!

# Running Couplings in Quantum Gravity

---

- $\beta$ -functions in **4d** one-loop quantum gravity in **dimensional regularization with MS**

# Running Couplings in Quantum Gravity

---

- $\beta$ -functions in **4d** one-loop quantum gravity in **dimensional regularization with MS**
  - No power-law divergences

$$\mu \frac{d\lambda}{d\mu} = -\lambda (2 + A_3 g \lambda) + \mathcal{O}(g^2)$$

$$\mu \frac{dg}{d\mu} = g (2 - B_2 g \lambda) + \mathcal{O}(g^3)$$

# Running Couplings in Quantum Gravity

---

- $\beta$ -functions in **4d** one-loop quantum gravity in **dimensional regularization with MS**
  - No power-law divergences

$$\mu \frac{d\lambda}{d\mu} = -\lambda (2 + A_3 g \lambda) + \mathcal{O}(g^2)$$

$$\mu \frac{dg}{d\mu} = g (2 - B_2 g \lambda) + \mathcal{O}(g^3)$$

- $A_3 \neq -B_2$
- **No one-loop fixed point**

# **Asymptotic Safety from the FRG**

---

# Functional Renormalization Group

---

- Introduce scale dependent Legendre effective action  $\Gamma_k$  (effective average action)
- RG-scale dependence introduced by IR regulator in path integral

$$\mathcal{J}[J, \bar{g}] = \int \mathcal{D}h_{\mu\nu} e^{-\int d^d x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k^{\mu\nu\rho\sigma}[\bar{g}] h_{\rho\sigma} - \mathcal{S}[\bar{g}+h] + \int d^d x \sqrt{\bar{g}} J^{\mu\nu} h_{\mu\nu}}$$

# Functional Renormalization Group

---

- Introduce scale dependent Legendre effective action  $\Gamma_k$  (effective average action)
- RG-scale dependence introduced by IR regulator in path integral

$$\mathcal{J}[J, \bar{g}] = \int \mathcal{D}h_{\mu\nu} e^{-\int d^d x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k^{\mu\nu\rho\sigma}[\bar{g}] h_{\rho\sigma} - \mathcal{S}[\bar{g}+h] + \int d^d x \sqrt{\bar{g}} J^{\mu\nu} h_{\mu\nu}}$$

- Functional Renormalization Group

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left\{ \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \right\} \quad [\text{Wetterich, 1993}]$$

# Functional Renormalization Group

---

- Introduce scale dependent Legendre effective action  $\Gamma_k$  (effective average action)
- RG-scale dependence introduced by IR regulator in path integral

$$\mathcal{J}[J, \bar{g}] = \int \mathcal{D}h_{\mu\nu} e^{-\int d^d x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k^{\mu\nu\rho\sigma}[\bar{g}] h_{\rho\sigma} - \mathcal{S}[\bar{g}+h] + \int d^d x \sqrt{\bar{g}} J^{\mu\nu} h_{\mu\nu}}$$

- Functional Renormalization Group

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left\{ \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \right\} \quad [\text{Wetterich, 1993}]$$

- Reproduces Perturbation Theory
- Successful in non-perturbative territory (QCD, Wilson-Fisher, ...)

# Non-Perturbative Approximations

---

- FRG is non-perturbative in couplings
  - Various approximation schemes can be applied that are not based on expansions in coupling

# Non-Perturbative Approximations

---

- FRG is non-perturbative in couplings
  - Various approximation schemes can be applied that are not based on expansions in coupling
- Approximate by truncating form of effective action:

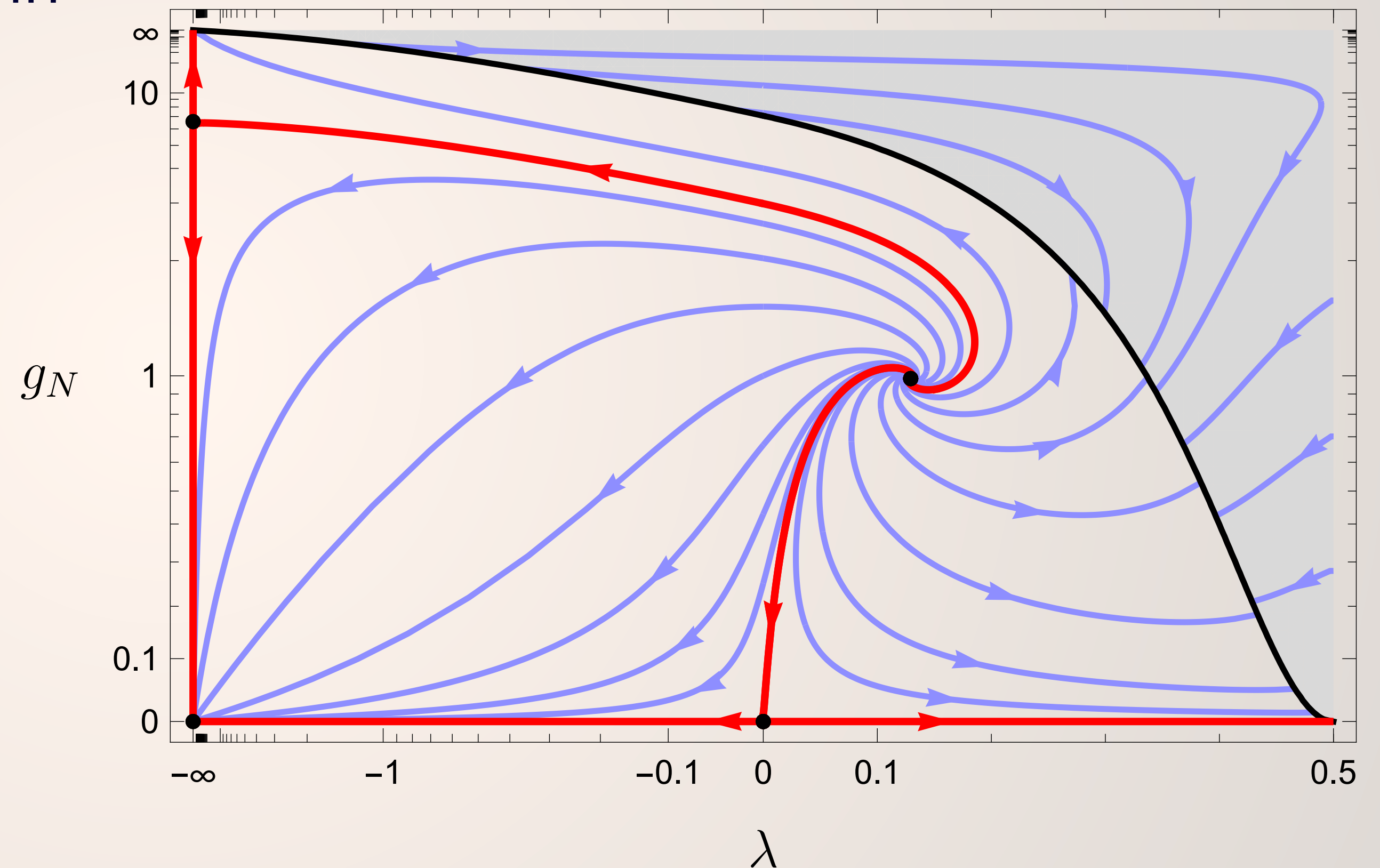
$$\Gamma_k = \int d^d x \sqrt{g} \left[ \frac{\Lambda}{8\pi G_N} - \frac{R}{16\pi G_N} + \lambda_{R^2} R^2 + \lambda_{R^{\mu\nu} R_{\mu\nu}} R^{\mu\nu} R_{\mu\nu} + \lambda_E E \right. \\ \left. + \lambda_{R^3} R^3 + \lambda_{C^3} C^3 + \dots \right. \\ \left. + \lambda_{R^4} R^4 + \dots \right]$$

# Einstein-Hilbert Gravity

- Fixed Point can already be seen in Einstein-Hilbert [Reuter, 1996]

$$\lambda^* = 0.11867 \quad g_N^* = 0.94305$$

$$\theta = -2.4113 \pm 1.9471I$$



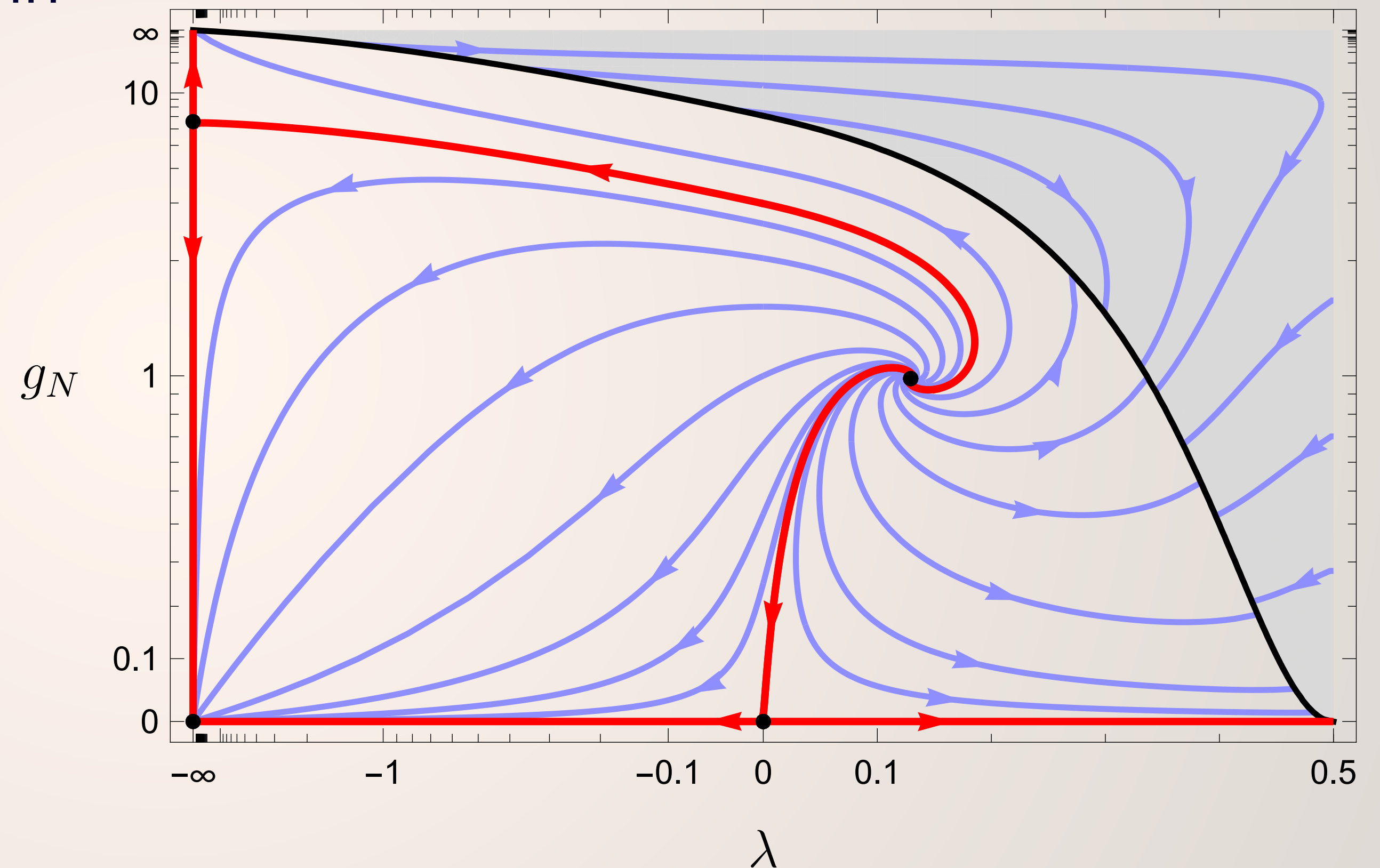
# Einstein-Hilbert Gravity

- Fixed Point can already be seen in Einstein-Hilbert [Reuter, 1996]

$$\lambda^* = 0.11867 \quad g_N^* = 0.94305$$

$$\theta = -2.4113 \pm 1.9471i$$

- Is this a predictive theory of quantum gravity?



# Curvature Invariants in QG

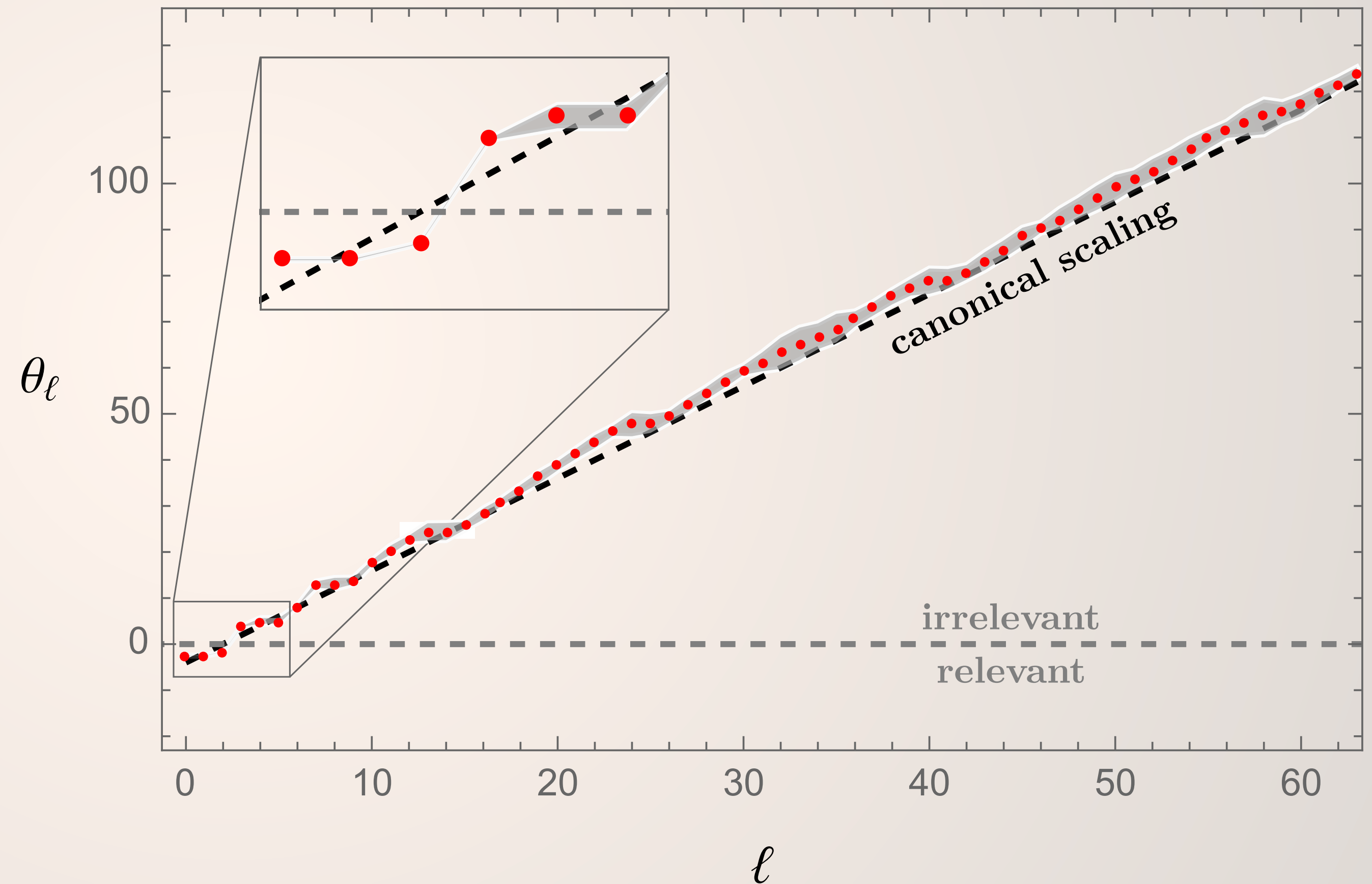
|       | Curvature Invariants                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| dim-0 | $\mathbb{1}$                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| dim-2 | $R$                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| dim-4 | $R^2, R_{\mu\nu}R^{\mu\nu}, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, \square R$                                                                                                                                                                                                                                                                                                                                                                       |
| dim-6 | $R^3, RR_{\mu\nu}R^{\mu\nu}, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, R_{\mu\nu}R^{\nu\rho}R^\mu_\rho, R_{\mu\rho}R_{\nu\sigma}R^{\mu\nu\rho\sigma}, R^\mu_\nu R_{\nu\alpha\beta\gamma}R^{\mu\alpha\beta\gamma},$<br>$R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu}, R_{\mu\nu\rho\sigma}R^\mu_\alpha{}^\rho_\beta R^{\nu\alpha\sigma\beta}, \nabla_\mu R \nabla^\mu R, \nabla_\rho R_{\mu\nu} \nabla^\rho R^{\mu\nu}$ |
|       | $\vdots$                                                                                                                                                                                                                                                                                                                                                                                                                                               |

# Predictivity of AS

- Fixed point persists in models with larger number of curvatures, e.g.  $f(R)$  gravity

$$f(R) = \sum_{\ell=0}^{\infty} \lambda_{\ell} R^{\ell}$$

- 3 relevant directions
- Eigenvalues follow canonical scaling**



# Predictivity of AS

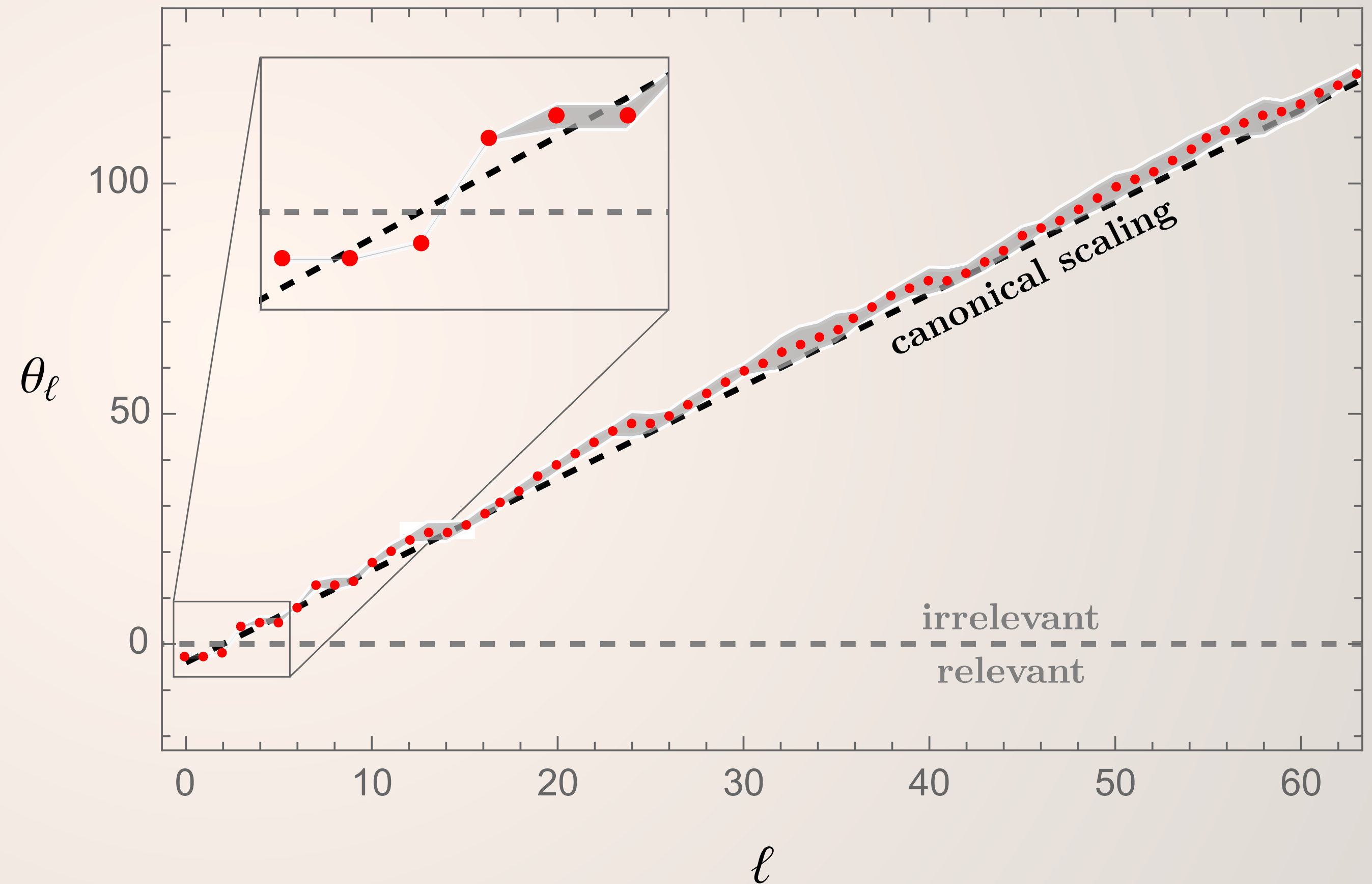
- Fixed point persists in models with larger number of curvatures, e.g.  $f(R)$  gravity

$$f(R) = \sum_{\ell=0}^{\infty} \lambda_{\ell} R^{\ell}$$

- 3 relevant directions

- Eigenvalues follow canonical scaling**

**small quantum corrections?**



# AS in cubic Gravity

---

- Using field redefinitions, a complete derivative expansion to sixth order was performed
  - Action only needs to include on-shell terms

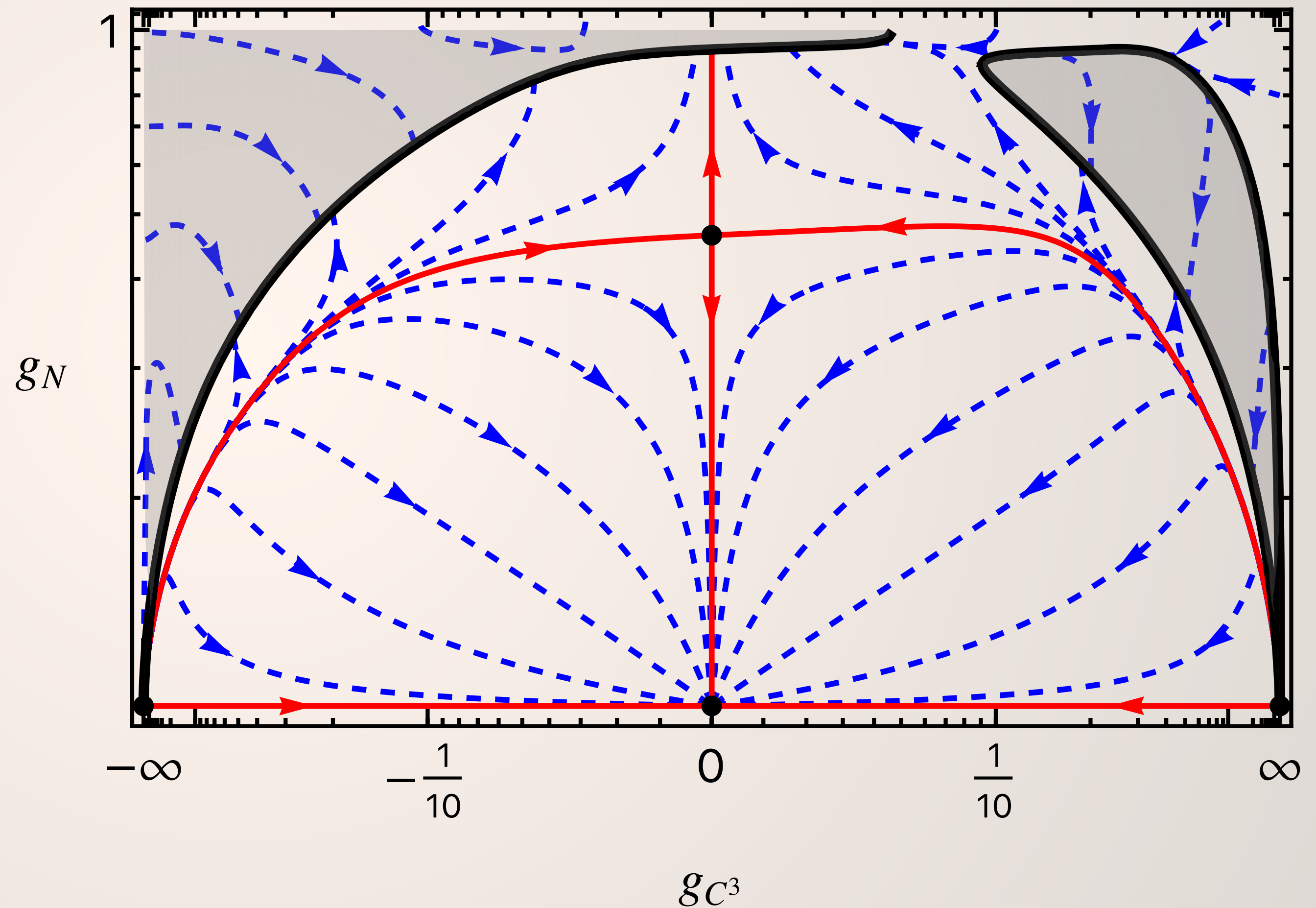
$$\Gamma_k = \int d^4x \sqrt{g} \left[ \frac{\rho}{8\pi} - \frac{1}{16\pi G_N} R + \sigma_E E + G_{C^3} C^{\rho\sigma}_{\mu\nu} C^{\mu\nu}_{\alpha\beta} C^{\alpha\beta}_{\rho\sigma} \right]$$

- Off-shell terms absorbed by field redefinitions [Ben Ali-Zinati and Baldazzi and Falls, 2021]

$$\langle \partial_t g_{\mu\nu} \rangle = \gamma_g g_{\mu\nu} + \gamma_R R g_{\mu\nu} + \gamma_S S_{\mu\nu} + \gamma_{R^2} R^2 g_{\mu\nu} + \dots$$

# AS in cubic Gravity

- Unique non-trivial fixed point  
 $g_N^* = 0.364$   
 $g_{C^3}^* = 4.490 \cdot 10^{-7}$
- Only one fixed point!



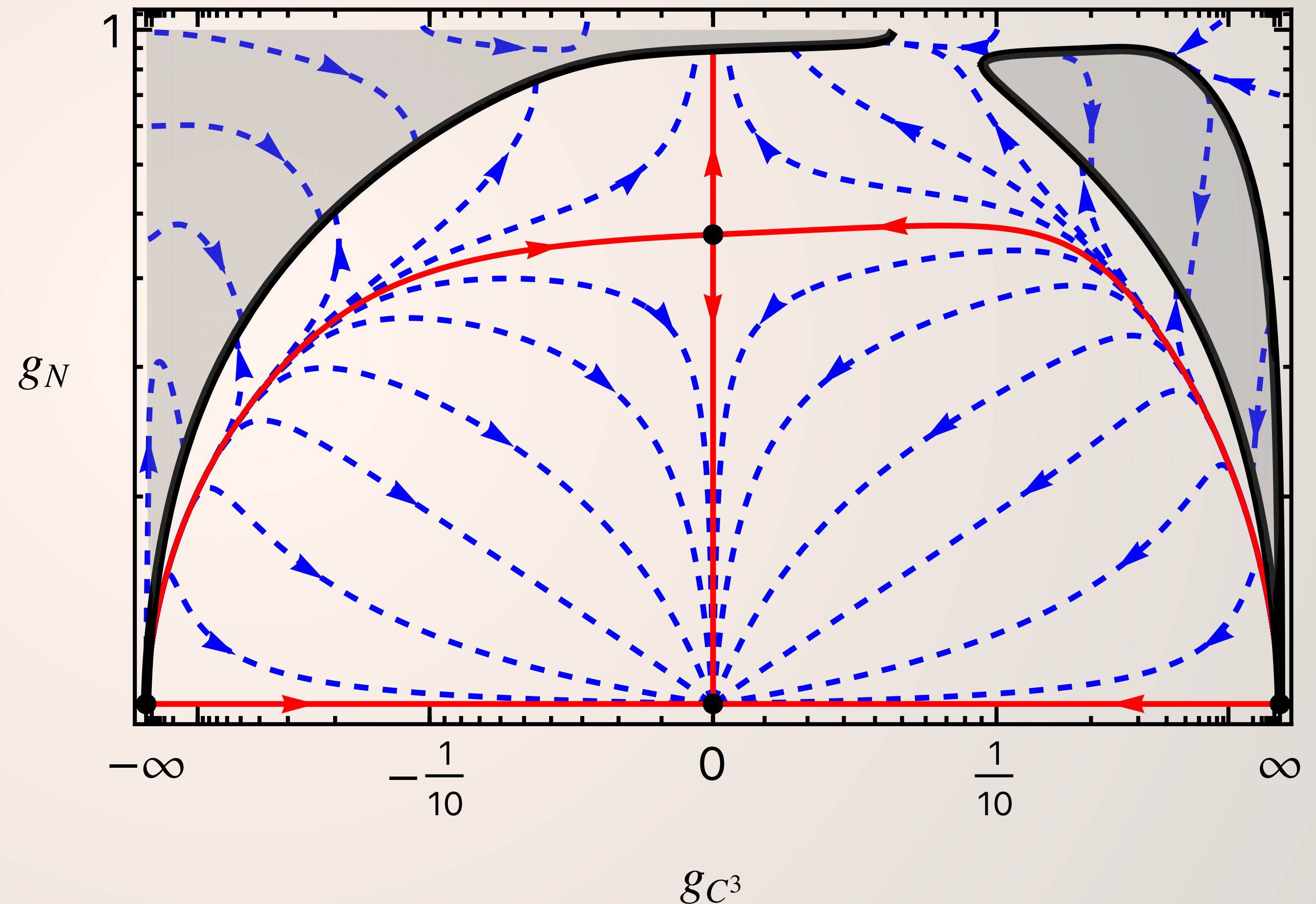
# AS in cubic Gravity

- Unique non-trivial fixed point

$$g_N^* = 0.364$$

$$g_{C^3}^* = 4.490 \cdot 10^{-7}$$

- Only one fixed point!



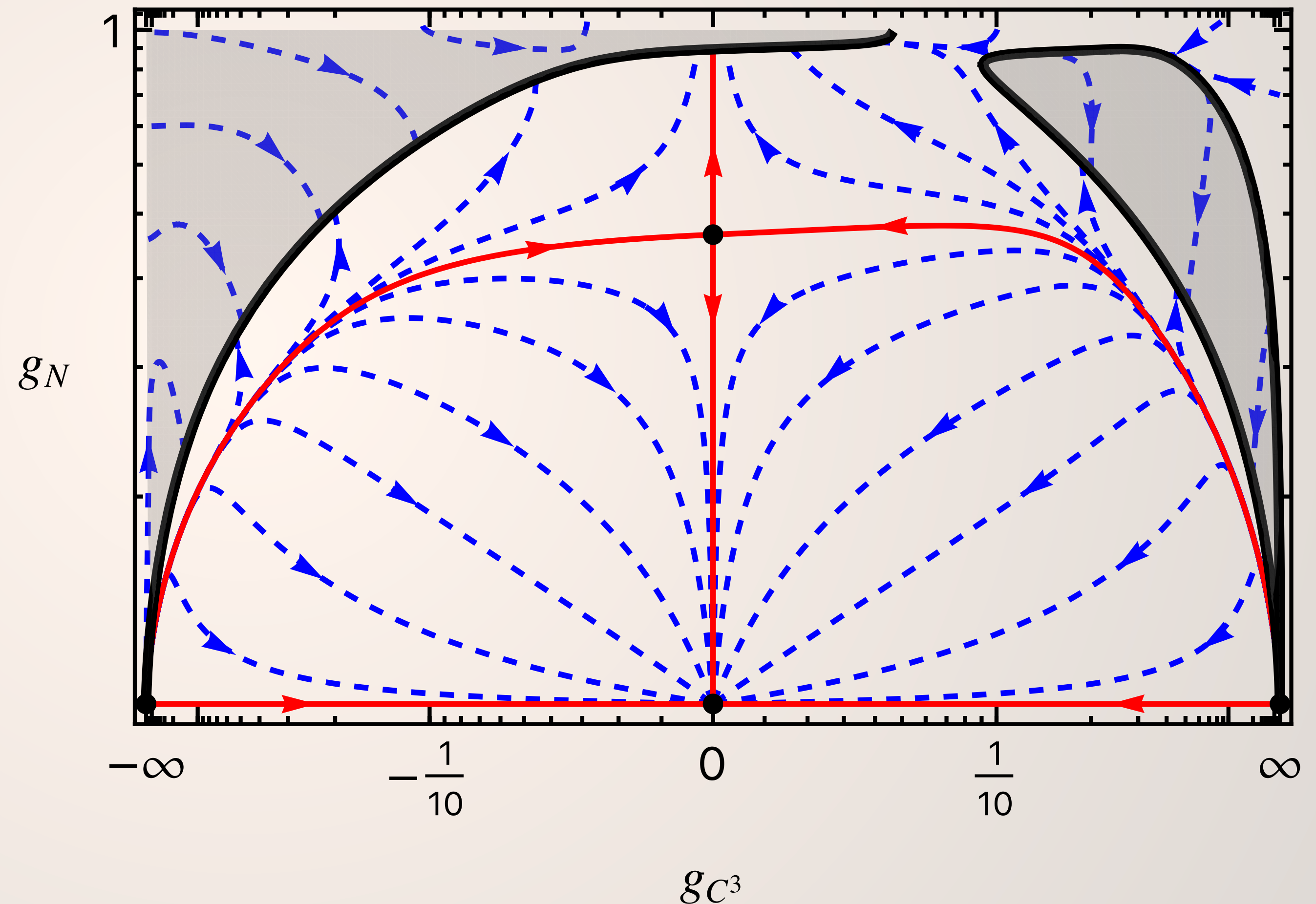
**Near-Perturbative?**

# AS in cubic Gravity

- Critical exponents are stable between different orders

| order                     | $\theta_1$ | $\theta_2$ |
|---------------------------|------------|------------|
| $\mathcal{O}(\partial^2)$ | 2.239      | —          |
| $\mathcal{O}(\partial^4)$ | 2.228      | —          |
| $\mathcal{O}(\partial^6)$ | 2.225      | -3.850     |

- Goroff-Sagnotti term irrelevant** [Gies et al, 2016]



# AS from the FRG

---

- Evidence for existence of fixed point in quantum gravity
  - Finite number of relevant directions
  - Operators with large mass dimensions are irrelevant
- Eigenvalues follow canonical scaling
- Perturbatively small fixed point for Goroff-Sagnotti coupling
- Challenges:
  - Diffeomorphism symmetry?
  - Lorentzian signature?
  - Unitarity?

# AS from the FRG

---

- Evidence for existence of fixed point in quantum gravity
  - Finite number of relevant directions
  - Operators with large mass dimensions are irrelevant
- **Eigenvalues follow canonical scaling**
- **Perturbatively small fixed point for Goroff-Sagnotti coupling**
- Challenges:
  - Diffeomorphism symmetry?
  - Lorentzian signature?
  - Unitarity?

# Perturbative Asymptotic Safety

---

# Running Couplings in Quantum Gravity

---

- $\beta$ -functions in 4d one-loop quantum gravity

The diagram illustrates the relationship between different types of divergences in the beta functions for the cosmological constant  $\lambda$  and the gravitational coupling  $g$ . Colored arrows point from the divergence labels to the corresponding terms in the equations.

$$\mu \frac{d\lambda}{d\mu} = -2\lambda - g \left( \overset{\text{quart div}}{A_1} + \overset{\text{quad div}}{A_2} \lambda + \overset{\text{log div}}{A_3} \lambda^2 \right) + \mathcal{O}(g^2)$$
$$\mu \frac{dg}{d\mu} = 2g - g^2 \left( \overset{\text{quad div}}{B_1} + \overset{\text{log div}}{B_2} \lambda \right) + \mathcal{O}(g^3)$$

# Running Couplings in Quantum Gravity

---

- $\beta$ -functions in 4d one-loop quantum gravity

The diagram illustrates the relationship between different types of divergences in the beta functions of the cosmological constant  $\lambda$  and the gravitational coupling  $g$ . Colored arrows point from descriptive labels to specific terms in the equations:

- A magenta arrow labeled "quart div" points to the  $A_1$  term in the  $\lambda$  beta function.
- An orange arrow labeled "quad div" points to the  $B_1$  term in the  $g$  beta function.
- A blue arrow labeled "log div" points to the  $B_2$  term in the  $g$  beta function.

$$\mu \frac{d\lambda}{d\mu} = -2\lambda - g \left( A_1 + A_2 \lambda + A_3 \lambda^2 \right) + \mathcal{O}(g^2)$$
$$\mu \frac{dg}{d\mu} = 2g - g^2 \left( B_1 + B_2 \lambda \right) + \mathcal{O}(g^3)$$

**Power-law divergences are necessary to obtain Fixed Point!**

# Power-Law Sensitive Dimensional Regularization

---

- How to identify power-law divergences in dimensional regularization?

$$\int \frac{d^d p}{\pi^{d/2}} \frac{1}{p^2 + m^2} = \int dp p^{d-1} \int d\Omega_d \frac{1}{p^2 + m^2} = m^{d-2} \Gamma \left( 1 - \frac{d}{2} \right)$$

# Power-Law Sensitive Dimensional Regularization

---

- How to identify power-law divergences in dimensional regularization?

$$\int \frac{d^d p}{\pi^{d/2}} \frac{1}{p^2 + m^2} = \int dp p^{d-1} \int d\Omega_d \frac{1}{p^2 + m^2} = m^{d-2} \Gamma \left( 1 - \frac{d}{2} \right)$$

- Integrand on the left-hand side is quadratically divergent in  $d = 4$ 
  - It also contains a logarithmic divergence (linear in  $m^2$ )

# Power-Law Sensitive Dimensional Regularization

---

- How to identify power-law divergences in dimensional regularization?

$$\int \frac{d^d p}{\pi^{d/2}} \frac{1}{p^2 + m^2} = \int dp p^{d-1} \int d\Omega_d \frac{1}{p^2 + m^2} = m^{d-2} \Gamma \left( 1 - \frac{d}{2} \right)$$

- Integrand on the left-hand side is quadratically divergent in  $d = 4$ 
  - It also contains a logarithmic divergence (linear in  $m^2$ )
- The right-hand side is divergent in  $d = 2, 4, 6, \dots$ 
  - **Power-law divergences lead to divergences in  $d < 4$**  [Veltman, 1981]

# **Power Divergence Subtraction:** *Renormalize bare couplings to remove divergences in $d \leq 4$*

[Weinberg, 1979]

[Jack and Jones, 1990]

[Al-Sarhi and Jack and Jones, 1990]

[Al-Sarhi and Jack and Jones, 1992]

[Kaplan et al, 1998]

[**YK**, 2024]

# Renormalization

- One-loop renormalization leads to  $\beta$ -functions [YK, 2024]

The diagram shows two beta function equations. The first equation,  $\beta_\lambda = \mu \frac{d\lambda}{d\mu} = -2\lambda + 0g - \frac{28}{3}g\lambda + \frac{4}{3\pi}g\lambda^2$ , has a pink arrow pointing to the '0' term labeled 'quart div ( $d = 0$ )'. The second equation,  $\beta_g = \mu \frac{dg}{d\mu} = 2g - \frac{2 \cdot 26}{3}g^2 - \frac{26}{3\pi}g^2\lambda$ , has a red arrow pointing to the '26' in the coefficient of  $g^2$  labeled 'quad div ( $d = 2$ )' and a blue arrow pointing to the '26' in the coefficient of  $g^2\lambda$  labeled 'log div ( $d = 4$ )'. The '2' in the coefficient of  $g^2$  is also red.

$$\beta_\lambda = \mu \frac{d\lambda}{d\mu} = -2\lambda + 0g - \frac{28}{3}g\lambda + \frac{4}{3\pi}g\lambda^2$$
$$\beta_g = \mu \frac{dg}{d\mu} = 2g - \frac{2 \cdot 26}{3}g^2 - \frac{26}{3\pi}g^2\lambda$$

# Renormalization

- One-loop renormalization leads to  $\beta$ -functions [YK, 2024]

$$\beta_\lambda = \mu \frac{d\lambda}{d\mu} = -2\lambda + \overset{\text{quart div } (d=0)}{0}g - \frac{28}{3}g\lambda + \frac{4}{3\pi}g\lambda^2$$

$$\beta_g = \mu \frac{dg}{d\mu} = 2g - \frac{2 \cdot 26}{3}g^2 - \frac{26}{3\pi}g^2\lambda$$

Annotations for the equations above:

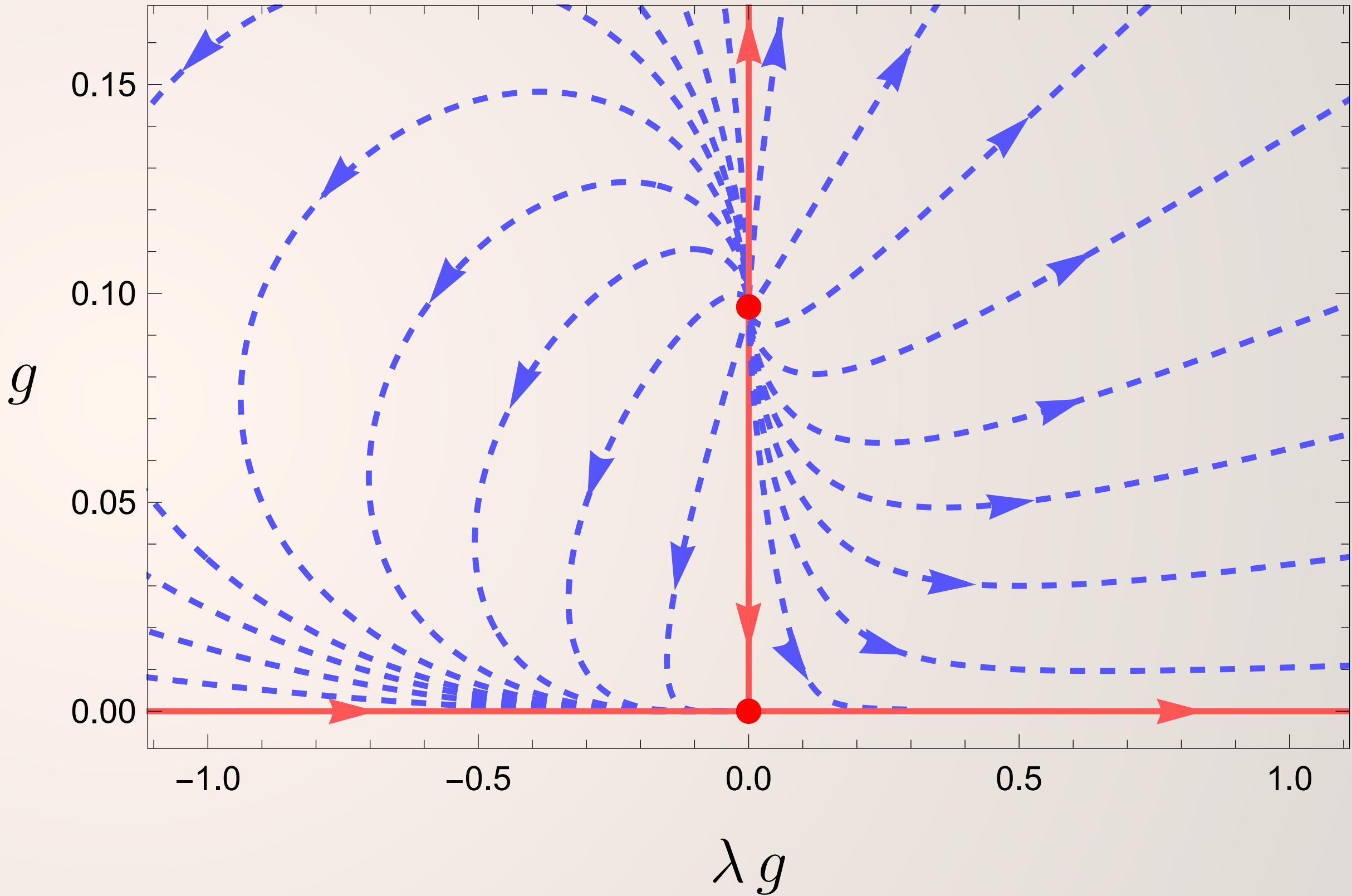
- quart div ( $d = 0$ )**: Points to the  $0g$  term in  $\beta_\lambda$ .
- quad div ( $d = 2$ )**: Points to the  $\frac{2 \cdot 26}{3}g^2$  term in  $\beta_g$ .
- log div ( $d = 4$ )**: Points to the  $\frac{4}{3\pi}g\lambda^2$  term in  $\beta_\lambda$  and the  $\frac{26}{3\pi}g^2\lambda$  term in  $\beta_g$ .

- No quartic divergences [Akhmedov, 2002] [Ossola and Sirlin, 2003]

# Phase Diagram of One-Loop Gravity

|                            | $\lambda$           | $g$              | $\theta_0$       | $\theta_1$       |
|----------------------------|---------------------|------------------|------------------|------------------|
| <b>FP<sub>Gauss</sub></b>  | 0                   | 0                | 2                | -2               |
| <b>FP<sub>UV</sub></b>     | 0                   | $\frac{3}{26}$   | 3.077            | 2                |
| <b>FP<sub>unphys</sub></b> | $-\frac{40\pi}{11}$ | $-\frac{11}{78}$ | $0.658 + 2.662i$ | $0.658 - 2.662i$ |

- One-loop fixed point exists [YK, 2024]
- Critical exponents similar to FRG
- $\lambda = 0$  is a fixed point of the RG flow



# What about poles in higher Dimensions?

---

- Dimensional Regularization also gives rise to poles in  $d > 4$

# What about poles in higher Dimensions?

---

- Dimensional Regularization also gives rise to poles in  $d > 4$ 
  - Two classes of poles:
    - Higher dimensional operators
    - **Higher powers of mass parameters**

# What about poles in higher Dimensions?

---

- Dimensional Regularization also gives rise to poles in  $d > 4$ 
  - Two classes of poles:
    - Higher dimensional operators
    - **Higher powers of mass parameters**
- Consider QCD with massive quark fields

$$\mathcal{L} = -\frac{1}{4g_0^2} F^{a\mu\nu} F_{\mu\nu}^a + \sum_{r=1}^{n_F} [\bar{\Psi}^r i \not{D} \Psi^r - M_{0,r} \bar{\Psi}^r \Psi^r]$$

# What about poles in higher Dimensions?

---

- Dimensional Regularization also gives rise to poles in  $d > 4$ 
  - Two classes of poles:
    - Higher dimensional operators
    - **Higher powers of mass parameters**
- Consider QCD with massive quark fields

$$\mathcal{L} = -\frac{1}{4g_0^2} F^{a\mu\nu} F_{\mu\nu}^a + \sum_{r=1}^{n_F} [\bar{\Psi}^r i \not{D} \Psi^r - M_{0,r} \bar{\Psi}^r \Psi^r]$$

- **Renormalize coupling by subtracting poles in  $d \geq 4$**  [YK, 2026]

# Threshold Effects from Dimensional Regularization

---

- Subtracting all poles in  $d \geq 4$  that are proportional to the coupling results in

$$\delta Z_g^f = \frac{2}{3(4\pi)^2} \sum_{r=1}^{n_F} \sum_{n=0}^{\infty} \frac{(-m_r^2)^n}{n!(2n+4-d)}$$

# Threshold Effects from Dimensional Regularization

---

- Subtracting all poles in  $d \geq 4$  that are proportional to the coupling results in

$$\delta Z_g^f = \frac{2}{3(4\pi)^2} \sum_{r=1}^{n_F} \sum_{n=0}^{\infty} \frac{(-m_r^2)^n}{n!(2n+4-d)}$$

- This gives the  $\beta$ -function

$$\beta_\alpha = (d-4)\alpha - \frac{22}{3}C_A \frac{\alpha^2}{4\pi} + \sum_{r=1}^{n_F} \frac{4}{3} e^{-m_r^2} \frac{\alpha^2}{4\pi}$$

- **Mass-Dependent Damping Terms!**

[YK, 2026]

# Threshold Effects from Dimensional Regularization

- Subtracting all poles in  $d \geq 4$  that are proportional to the coupling results in

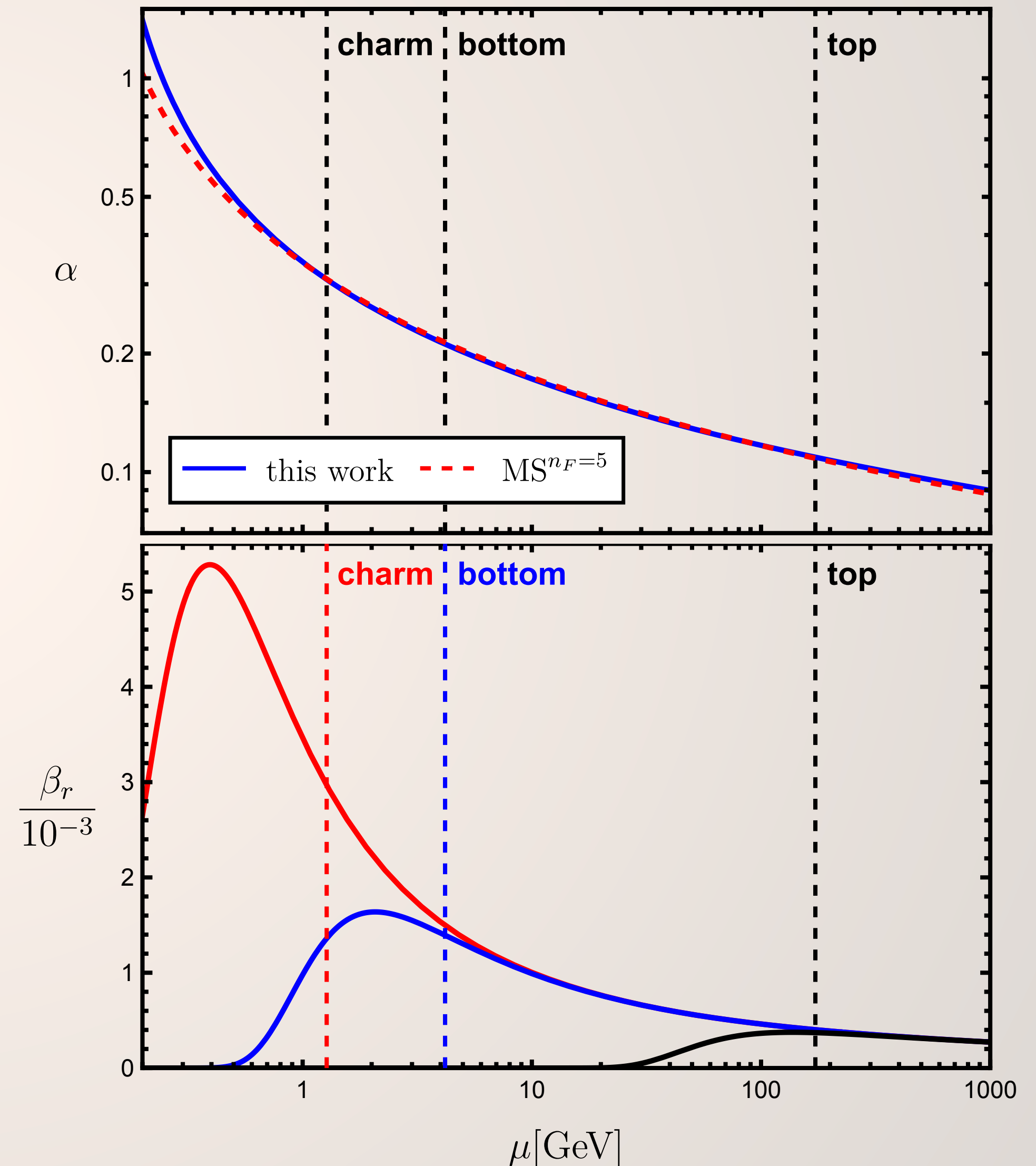
$$\delta Z_g^f = \frac{2}{3(4\pi)^2} \sum_{r=1}^{n_F} \sum_{n=0}^{\infty} \frac{(-m_r^2)^n}{n!(2n+4-d)}$$

- This gives the  $\beta$ -function

$$\beta_\alpha = (d-4)\alpha - \frac{22}{3}C_A \frac{\alpha^2}{4\pi} + \sum_{r=1}^{n_F} \overbrace{\frac{4}{3}e^{-m_r^2} \frac{\alpha^2}{4\pi}}^{\beta_r}$$

- Mass-Dependent Damping Terms!**

[YK, 2026]



# Conclusions

---

- Non-Perturbative techniques lead to indications for UV fixed point in quantum gravity
  - Persistent in various approximation schemes
  - Near-perturbative properties!
- Asymptotic Safety can be seen in perturbation theory
  - Dimensional regularization expresses power-law divergences as divergences in lower dimensions
  - **Are there other schemes?**