

Jakub Gizbert-Studnicki

in collaboration with

J. Ambjørn, A. Görlich and D. Németh

Causal Dynamical Triangulations From the infrared to the ultraviolet

Ricci flow in quantum gravity

Aachen, 10th September 2026

Is lattice quantum gravity asymptotically safe?

Making contact between causal dynamical triangulations and the functional renormalization group.

Phys. Rev. D 110, 126006 [arXiv:2408.07808]

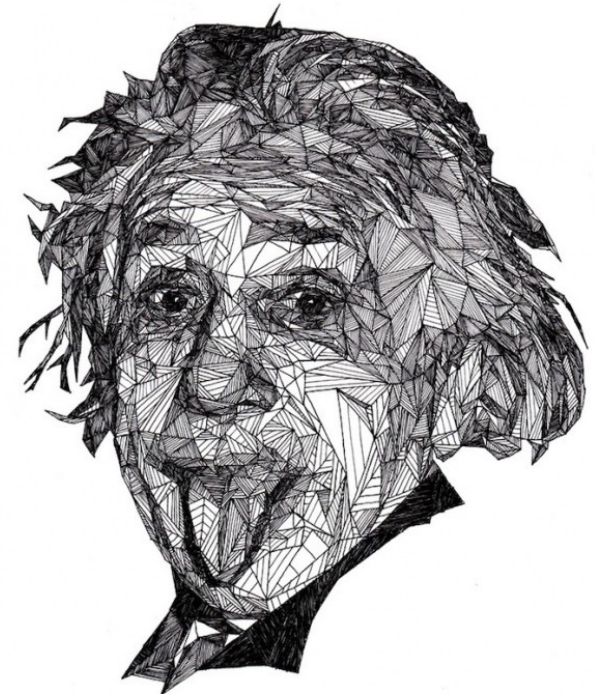
IR and UV limits of CDT and their relations to FRG

A. Phys. Pol. B 55, 12-A2 [arXiv:2411.02330]



Challenges of Quantum Gravity (QG)

- ✧ As early as in 1916 Einstein* pointed out that „quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation”
- ✧ After more than 100 years a complete, consistent quantum theory of gravity is still missing
- ✧ We have a number of interesting but incomplete research programs
 - ✧ string theory
 - ✧ loop quantum gravity
 - ✧ group field theory
 - ✧ causal set theory
 - ✧ noncommutative geometry
 - ✧ asymptotic safety (FRG)
 - ✧ Lattice QFT approaches (DT, quantum Regge calc., ...)
 - ✧ ...

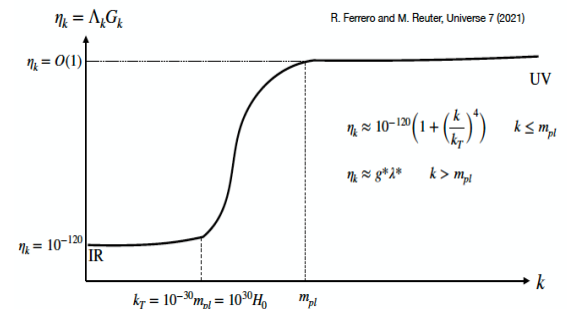
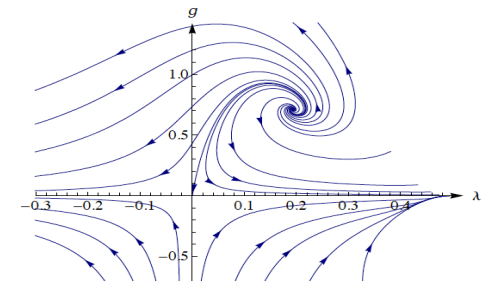
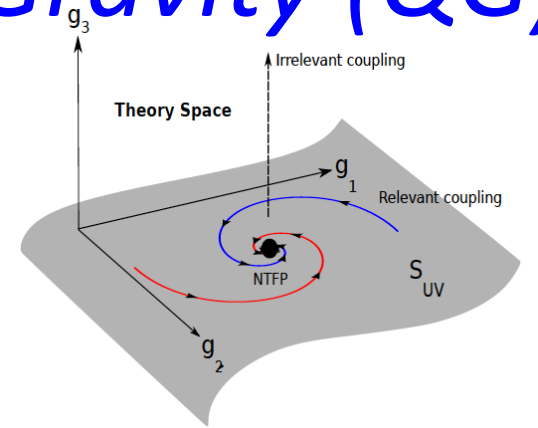


A. Einstein triangulation by J. Bryan

* Sitzungsber. Preuss. Akad. Wiss. Berlin (1916) 688

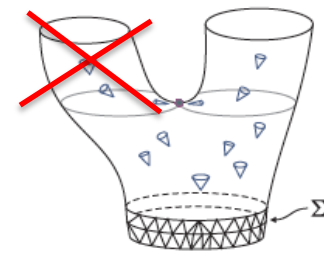
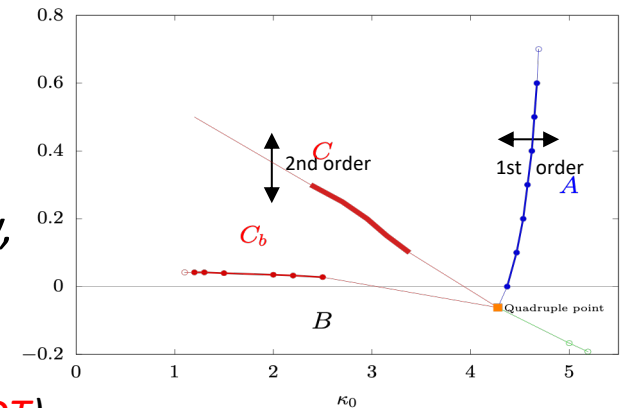
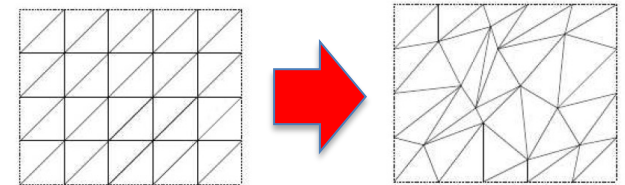
Challenges of Quantum Gravity (QG)

- ✧ Lack of experimental guidance
- ✧ Conceptual issues: QFT based on Einstein's GR is perturbatively non-renormalizable in $D > 2$ dim.*
- ✧ But it can be renormalizable in a non-perturbative regime: **asymptotic safety conjecture** S. Weinberg, 1980
 - ✧ renormalization group flow can lead to a **non-Gaussian UV fixed point** where QG becomes **scale invariant** (UV complete)
- ✧ Lattice formulation would allow to study a unitary, non-perturbative, background-independent and diffeomorphism-invariant quantum gravity
 - ✧ to encode geometry we need a dynamical lattice (DT)
 - ✧ UV fixed point should be associated with a phase transition
 - ✧ one should be able to reproduce semi-classical gravity (IR limit)
- ✧ Imposing causal structure may be important: CDT (J. Amjørn, J. Jurkiewicz, R. Loll)



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✧ **Imposing causal structure** may be important:

CDT (J. Ambjørn, J. Jurkiewicz, R. Loll) -2-

2D: J. Ambjorn, R. Loll, Nucl.Phys. B 536 (1998) 407

3D: J. Ambjorn, J. Jurkiewicz, R. Loll, Phys.Rev.Lett. 85 (2000) 924

4D: J. Ambjorn, J. Jurkiewicz, R. Loll, Nucl.Phys. B610 (2001) 347

*Renormalizable extensions have problems with **unitarity**

Outline

- ✧ *Causal Dynamical Triangulations*
- ✧ *Phase structure*
- ✧ *Semi-classical phase*
- ✧ *Functional Renormalization Group*
- ✧ *RG flow on the lattice (ϕ^4 example)*
- ✧ *RG flow in CDT*
- ✧ *Conclusions*

Causal Dynamical Triangulations

✧ CDT approach to QG is via a *lattice QFT*, using the *path integral* (PI) quantization

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i S_{\text{grav}}[g]) \quad \leftarrow \text{(Lorentzian) geometries}$$

✧ One has to give a precise meaning to:

- ✧ what class of *geometries* g should be included in the PI
- ✧ what (classical) *action* S_{grav} should be used
- ✧ which *symmetries* (GR diffeomorphisms ?) should be preserved and how to do that
- ✧ what is the integration *measure*
- ✧ *how to compute* the PI in practice
- ✧ how to deal with *unitarity* and (generic) *PI divergencies*
- ✧ (how to include matter fields)

Causal Dynamical Triangulations

✧ CDT approach to QG is via a *lattice QFT*, using the *path integral* (PI) quantization

✧ CDT (quantum) *geometries*:

✧ In classical GR one deals with smooth (pseudo-)Riemannian manifolds

✧ But in the PI one should also include non-smooth *continuous* (Lorenz.) geometries

✧ *Causality*: globally *hyperbolic* spacetimes which can be *foliated* into spacial slices of equal *cosmological proper time*

✧ We *fix the topology of the manifold* (in here we will use the topology: $S^3 \times S^1$)

✧ Geometries in the PI are approximated by *piecewise linear simplicial manifolds* (*triangulations*) built from two kinds of *identical* (internally flat) 4-simplices with *fixed edge lengths* (l_s)

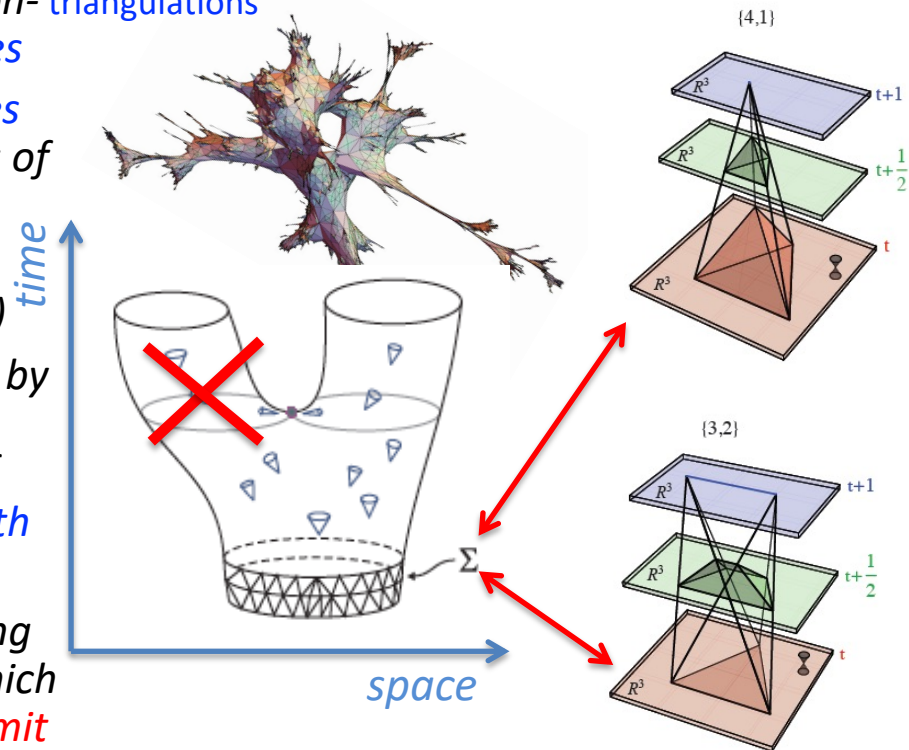
✧ As in ordinary lattice QFT: lattice spacing (l_s) plays a role of the *UV cutoff* l_s^{-1} which should be removed in the *continuum limit* ($l_s \rightarrow 0, N_4 \rightarrow \infty$)

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i S_{grav}[g])$$

← (Lorenzian) geometries

$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i S_R[T])$$

causal → ← # symmetries of T



Causal Dynamical Triangulations

✧ CDT approach to QG is via a *lattice QFT*, using the *path integral* (PI) quantization

✧ CDT action & diffeom. symmetry:

✧ We use the *Einstein-Hilbert action*

✧ For a *pieewise linear simplicial manifold* (triangulation) it takes the form of the *Regge action*
T. Regge,
Nuovo Cim. A19 (1961) 558

✧ Curvature is defined by deficit angles around $D-2$ dim. „hinges” (triangles in 4D)

✧ Regge's formulation uses only geometric invariants (geodesic edge lengths and deficit angles) making CDT *coordinate free* and therefore manifestly (at least *spatial**) *diffeomorphism invariant*

✧ In CDT one has only two types of building blocks with fixed edge lengths thus the Regge *action becomes very simple*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i S_{grav}[g])$$

← (Lorentzian) geometries

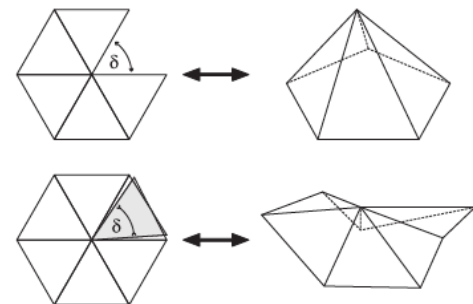
$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i S_R[T])$$

causal triangulations ← # symmetries of T

$$S_{grav} = \frac{1}{16\pi G} \int_M d^4x \sqrt{-\det g} (R - 2\Lambda)$$

4-simplices # {4,1} 4-simpl. # vertices

$$S_R = \underbrace{+k_0 N_0}_{l/G} + \underbrace{+K_4 N_4}_{\Lambda} + \underbrace{\Delta \left(N_4^{(4,1)} - 6N_0 \right)}_{\alpha \quad (l_t^2 = -\alpha l_s^2)}$$



Causal Dynamical Triangulations

✧ CDT approach to QG is via a *lattice QFT*, using the *path integral* (PI) quantization

✧ CDT computations:

- ✧ In order to investigate the 4D PI one has to use **Monte Carlo (MC)** simulations
- ✧ MC requires *Euclidean formulation* (*Wick's rotation*)
- ✧ Due to the imposed time foliation each CDT Lorentzian geometry can be *Wick-rotated to an Euclidean geom.*
- ✧ At the level of the Regge action Wick's rotation is achieved by an analytical continuation ($\alpha \rightarrow -\alpha$): *general form of the action S_R is the same in (L) and (E)*
- ✧ MC algorithm performs a **Markov chain** in the space of triangulations by **local moves*** which change the geometry (in 4 dim CDT: 4 moves & 4 anti-moves)

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causal triangulations ← # symmetries of T (trivial measure)

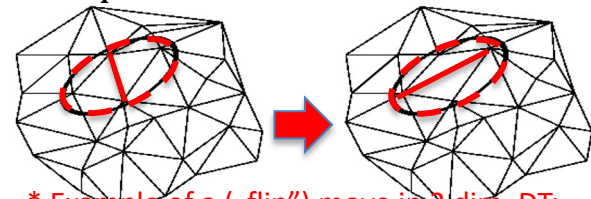
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$\alpha \rightarrow -\alpha$

$$Z_{CDT}^{(E)} = \sum_T \frac{1}{C_T} \exp\left(\frac{1}{2} S_R[T]\right)$$



* Example of a („flip”) move in 2 dim. DT:

Causal Dynamical Triangulations

✧ CDT approach to QG is via a *lattice QFT*, using the *path integral* (PI) quantization

✧ CDT unitarity and PI divergencies:

✧ CDT has a (time) *reflection positive**, *symmetric and bounded transfer matrix*. In standard lattice QFT it guarantees *unitarity* if continuum limit exists

✧ For *finite* N_4 there is no problem with the PI convergence as the *number of configurations is exponentially bounded*

✧ Lattice spacing (l_s) plays a role of the *UV cutoff* l_s^{-1} as in ordinary lattice QFT

✧ We want to investigate *continuum limit* ($l_s \rightarrow 0$, $N_4 \rightarrow \infty$), hopefully consistent with the UV fixed point of QG

✧ CDT does not assume that spacetime is discrete in any sense !

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i S_{grav}[g])$$

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4-simplices # {4,1} 4-simpl. # vertices

$$S_R = \underbrace{+k_0}_{I/G} N_0 + \underbrace{+K_4}_{\Lambda} N_4 + \underbrace{+\Delta}_{\alpha} (N_4^{(4,1)} - 6N_0)$$

I/G

Λ

α

$$(l_t^2 = \alpha l_s^2)$$

$\alpha \rightarrow -\alpha$

$$Z_{CDT}^{(E)} = \sum_T \frac{1}{C_T} \exp(\underbrace{+}_{\alpha \rightarrow -\alpha} S_R[T])$$

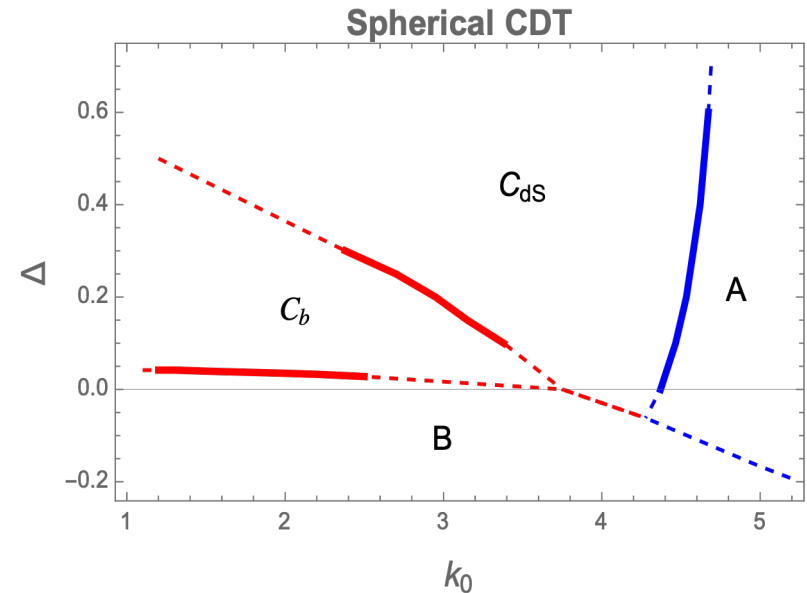
* TM^2 is positive-definite

Phase structure

Phase structure

✧ Phase structure:

- ✧ We perform MC simulations with *fixed* N_4 . The cosmological constant K_4 is tuned to N_4 and we effectively have *two coupling constants*: k_0 and Δ
- ✧ *Four phases* (A , B , C_{ds} , C_b) of different generic geometries were discovered
- ✧ The observable: physical 3-volume of spatial layers: $V_3(t_i) \propto N_3(i) \cdot l_s^3$
- ✧ The difference between phases C_{ds} and C_b is captured by effective dimensions
- ✧ One observes 1st order (blue lines) and 2nd order (red lines) phase transitions

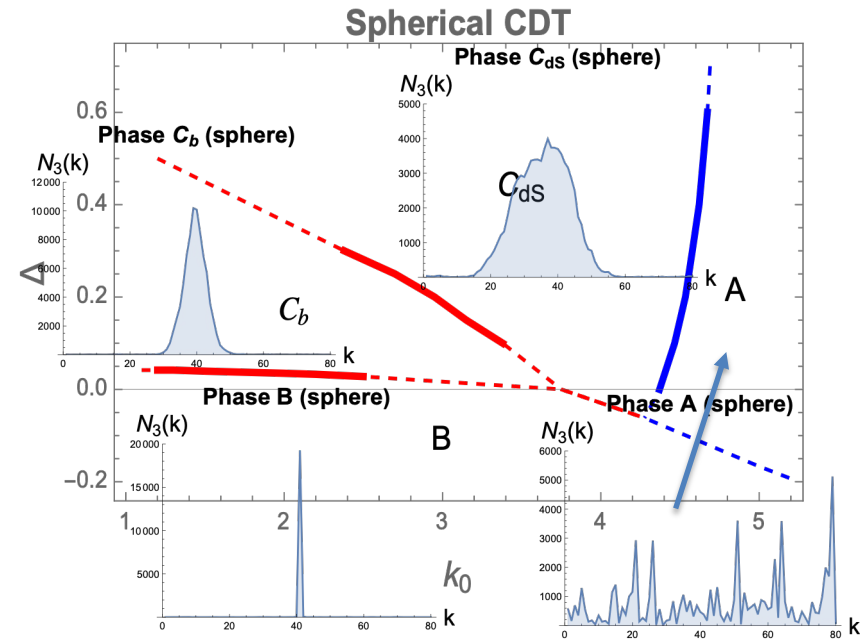


$$S_R = \cancel{k_0} N_0 + \cancel{K_4} N_4 + \cancel{\Delta} (N_4^{(4,1)} - 6N_0)$$

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lattice spacing in spatial directions

of tetrahedra at lattice time i

physical proper time $t_i = i \cdot l_t$

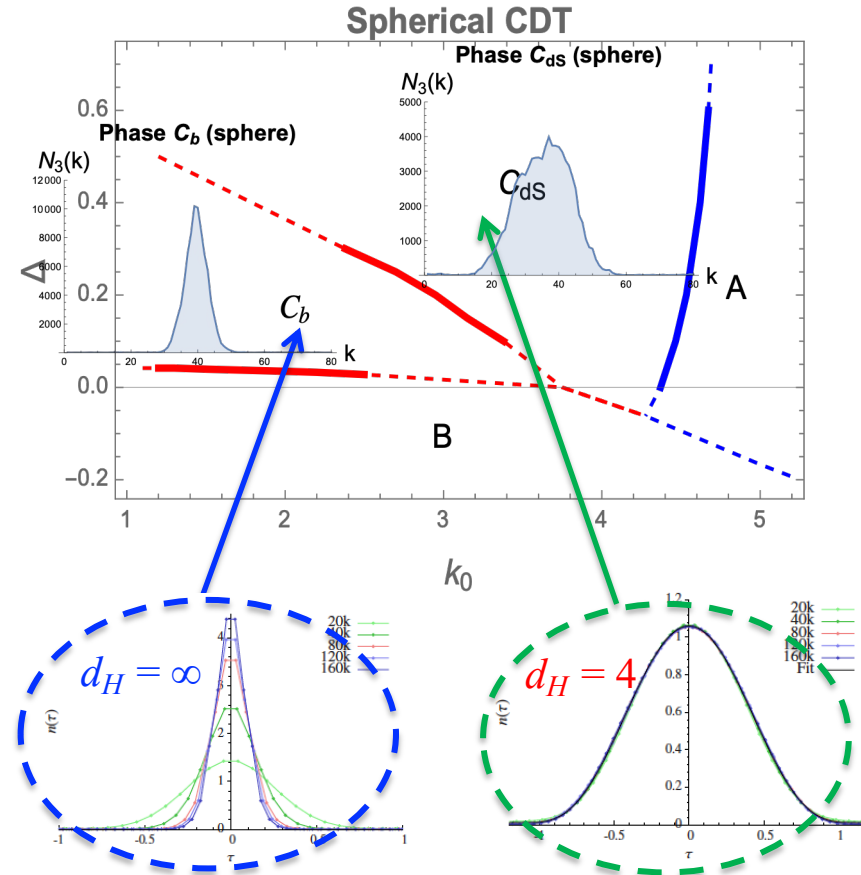
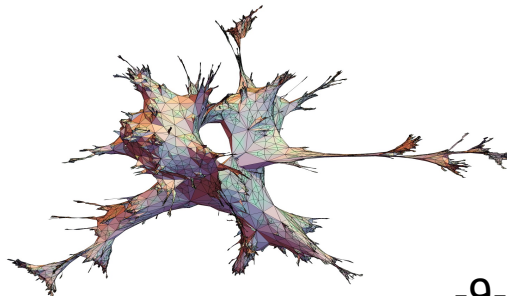
lattice time

lattice spacing in time direction

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Hausdorff dimension: $\langle N_3(i) \rangle \rightarrow \frac{\langle N_3(i) \rangle}{N_4^{1-1/d_H}}$

rescaled average volume profiles

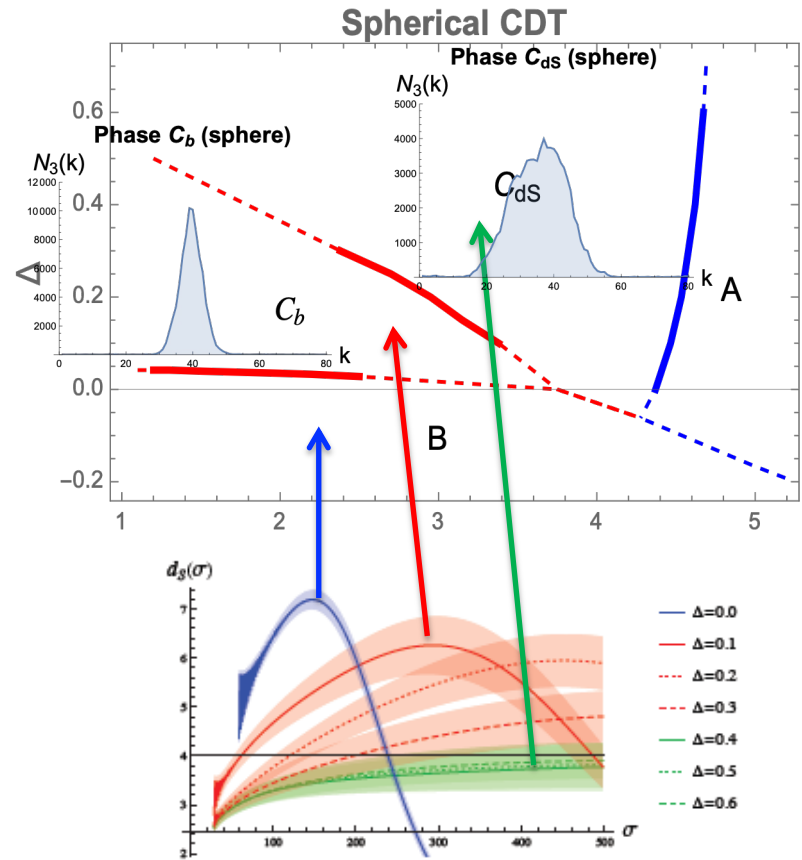
(scaling for $d_H = 4$)

$i \rightarrow \frac{i}{N_4^{1/d_H}}$

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Spectral dimension*:
related to heat kernel
of the Laplace operator

$$\frac{\partial}{\partial \sigma} K(x, x_0; \sigma) = \Delta_g K(x, x_0; \sigma)$$

$$P_R(\sigma) = \frac{1}{V} \int dx \sqrt{g} K(x, x; \sigma)$$

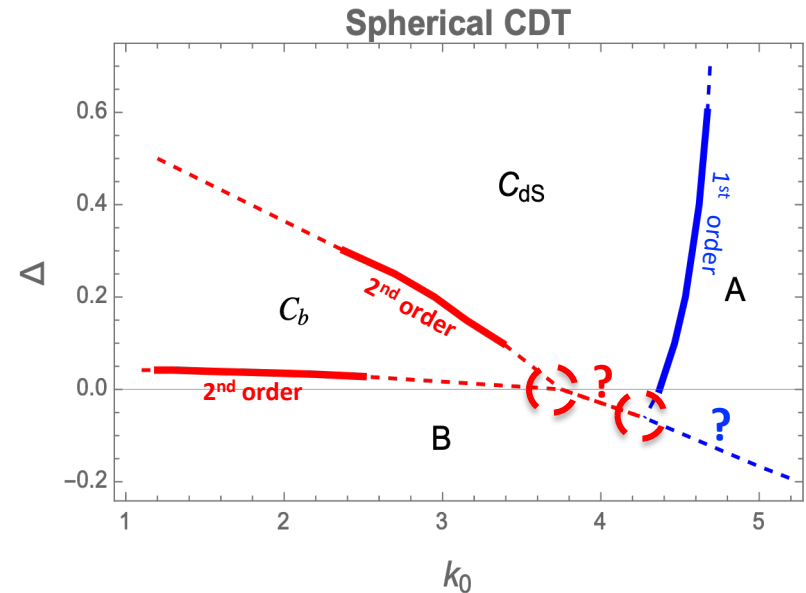
$$d_s(\sigma) = -2 \frac{d \log P_R(\sigma)}{d \log \sigma}$$

* Dimensional reduction observed
in many approaches to QG
(S. Carlip, CQG 34 (2017) 193001)

Phase structure

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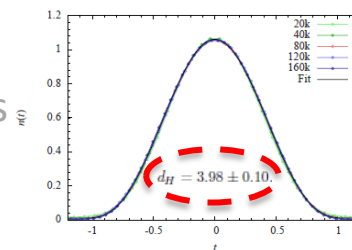
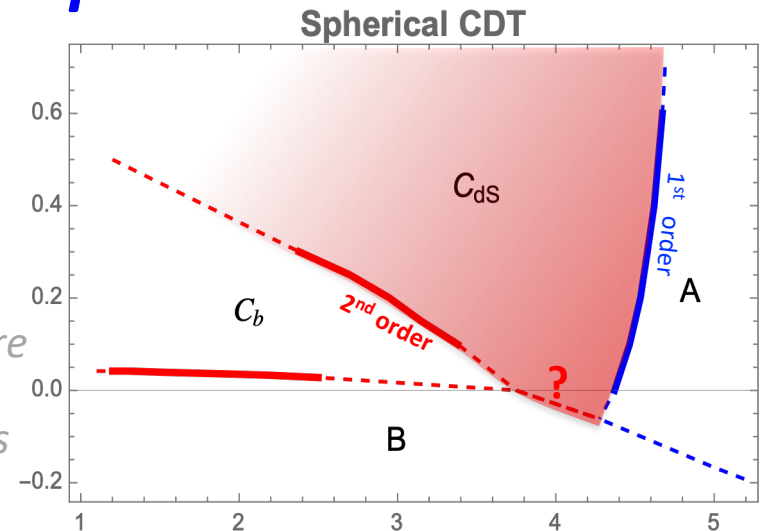
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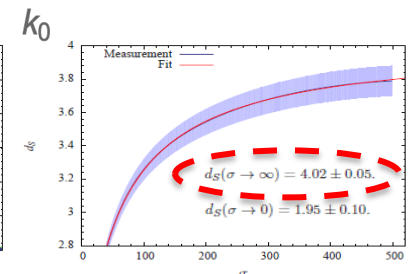
Semi-classical phase

Semi-classical phase

- ✧ Phase C_{dS} (*de Sitter phase*) has good *semi-classical properties* !
- ✧ *Effective dimensions* consistent with $d = 4$
- ✧ Dynamically emerging background geom. ◀
 - ✧ $\langle N_3(i) \rangle$ profile of elongated ($\tilde{\omega} \neq \omega_0$) 4-sphere
 - ✧ renormalizing $l_t \rightarrow l_t = l_s \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3}$ one obtains symmetric S^4 , i.e., classically: (Euclidean) de Sitter universe (max. sym. space with $\Lambda > 0$)
 - ✧ local (average) curvature* consistent with S^4
 - ✧ ~homogenous and isotropic** on large scales
- ✧ Minisuperspace behaviour of the scale factor
 - ✧ From quantum fluctuations of $N_3(i)$ one can recover the effective action of the scale factor
 - ✧ The effective action is consistent with the MS action (spatial homogeneity and isotropy)
- ✧ This was „derived“ from first principles !



$$\bar{n}_t \rightarrow \frac{\bar{n}_t}{N_4^{1-1/d_H}} \quad t \rightarrow \frac{t}{N_4^{1/d_H}}$$



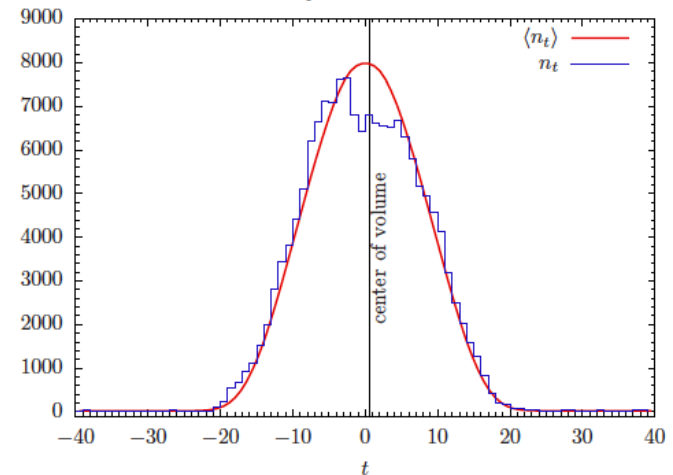
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J. Ambjorn, J. Jurkiewicz R. Loll,
PRL 95 (2005) 171301

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Spherical



$$\langle N_3(i) \rangle = \frac{3}{4} N_4 \frac{1}{\tilde{\omega} N_4^{1/4}} \cos^3 \left(\frac{i}{\tilde{\omega} N_4^{1/4}} \right)$$

$V_3(t_i) = N_3(i) l_s^3$
↓
↙ ↘

$$\langle V_3(t_i) \rangle = \frac{3}{4} V_4 \frac{1}{\omega_0 V_4^{1/4}} \cos^3 \left(\frac{t_i}{\omega_0 V_4^{1/4}} \right)$$

$\omega_0 = \left(\frac{3}{8\pi^2} \right)^{1/4}$
↙ ↘

$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$

$t = i l_t$
 $V_4 = l_s^3 l_t N_4$
 Λ is fixing V_4

* Def. by Quantum Ricci Curvature: N. Klitgaard, R. Loll, PRD 97 (2018) 046008

** Homogeneity measures in CDT: R. Loll, A. Silva, PRD 107 (2023) 086013

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✧ **Dynamically emerging background geom.**

✧ $\langle N_3(i) \rangle$ profile of elongated ($\tilde{\omega} \neq \omega_0$) 4-sphere

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✧ local (average) **curvature*** consistent with S^4

✧ **~homogenous** and **isotropic**** on large scales

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$$\omega_0 = \left(\frac{3}{8\pi^2} \right)^{1/4}$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

$$ds^2 = dt^2 + a^2(t) d\Omega_3^2$$

$$S_{HE} = - \frac{1}{16\pi G} \int d^4x \sqrt{\det g} (R - 2\Lambda)$$

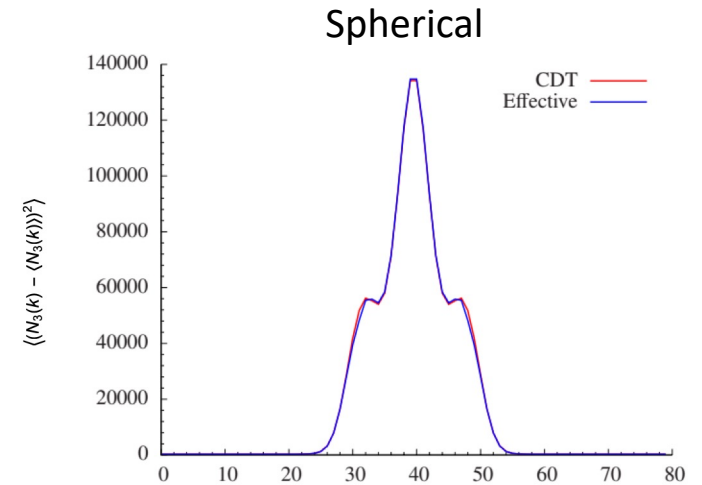
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- ✧ *Dynamically emerging background geom.*
 - ✧ $\langle N_3(i) \rangle$ profile of elongated ($\tilde{\omega} \neq \omega_0$) 4-sphere
 - ✧ renormalizing $l_t \rightarrow l_t = l_s \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3}$ one obtains symmetric S^4 , i.e., classically: (**Euclidean de Sitter universe** (max. sym. space with $\Lambda > 0$))
 - ✧ local (average) curvature* consistent with S^4
 - ✧ ~homogenous and isotropic** on large scales
- ✧ *Minisuperspace* behaviour of the *scale factor*
 - ✧ From quantum fluctuations of $N_3(i)$ one can recover the *effective action of the scale factor*
 - ✧ The effective action is consistent with the MS action (spatial homogeneity and isotropy)
- ✧ This was „derived“ from first principles !



$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

$$V_3(t_i) = N_3(i) l_s^3 \quad \mu_0 = 9(2\pi^2)^{2/3}$$

$$S = \frac{l_t}{l_s} \frac{1}{\tilde{\Gamma} l_s^2} \sum_i l_t \left(\frac{(V_3(t_i + l_t) - V_3(t_i))^2}{l_t^2 V_3(t_i)} + \frac{l_s^2}{l_t^2} \tilde{\mu} V_3(t_i)^{1/3} \right)$$

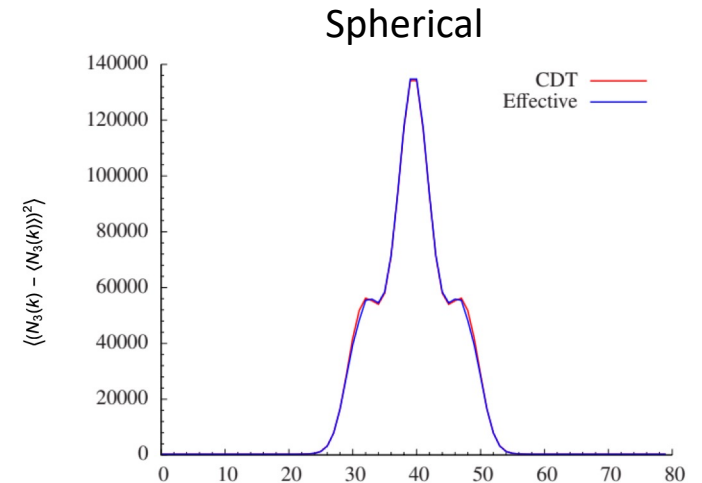
$1 / \left(\frac{\tilde{\omega}}{\omega_0} \right)^{4/3} \tilde{\Gamma} l_s^2$

$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$

Λ is fixing V_4

Semi-classical phase

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agrees with
Hartle–Hawking
„noboundary“ proposal

$$\mu_0 = 9(2\pi^2)^{2/3}$$

$$S_{MS} = \frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3} \right)$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0} \right)^{4/3} \tilde{\Gamma} l_s^2 \quad V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

$$ds^2 = dt^2 + a^2(t) d\Omega_3^2$$

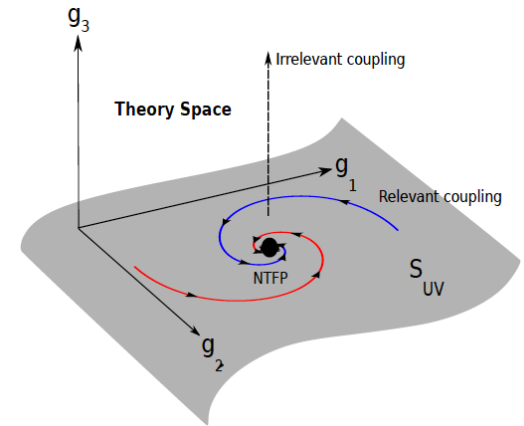
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$$S_{HE} = \frac{1}{16\pi G} \int d^4x \sqrt{\det g} (R - 2\Lambda)$$

Functional Renormalization Group

Functional Renormalization Group

- ✧ As CDT the **FRG** is also based on non-perturbative **QFT framework** to quantize gravity
 - ✧ Consider a (potentially ∞ dim.) **space of all effective actions** of QG (or in practice their truncations)
 - ✧ Alternatively one has a **space of scale-dependent dimensionless couplings** related to operators appearing in the effective actions
 - ✧ Solve **RG flow equations** (based on β -functions) of the couplings with the **cutoff scale k**
 - ✧ Find RG trajectories linking **IR** ($k \rightarrow 0$) and **UV** ($k \rightarrow \infty$) **fixed points** ($\beta = 0$) of the RG flow
- ✧ **Asymptotic Safety conjecture** (S. Weinberg)
 - ✧ **Scale invariance of the UVFP** imposes strong constraints on most operators (couplings)
 - ✧ On RG flow trajectories leading from IR to UV fixed points there is only a **finite number of relevant operators** (finite dim. subspace of **relevant couplings**)
 - ✧ Even though the values of the couplings in the UV limit are not small **one can get a predictive theory of QG at all scales** (**nonperturbative renormalizability**)
 - ✧ There is growing **evidence from FRG** in favour of AS

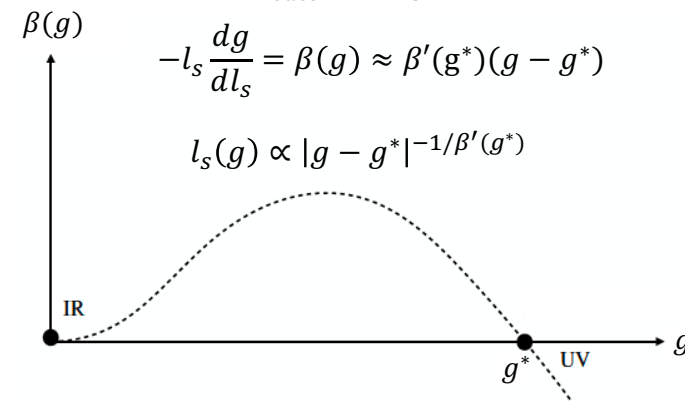


Close to UVFP:

$$k_{\text{cutoff}} \sim 1/l_s$$

$$-l_s \frac{dg}{dl_s} = \beta(g) \approx \beta'(g^*)(g - g^*)$$

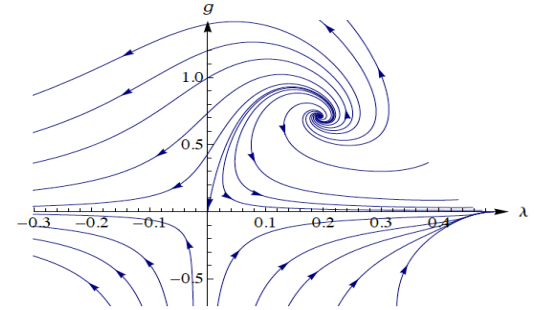
$$l_s(g) \propto |g - g^*|^{-1/\beta'(g^*)}$$



Functional Renormalization Group

✧ **Making contact** between **FRG** and **CDT**: $S_k = \cancel{X} \frac{1}{16\pi G_k} \int d^4x \sqrt{\det g} (R - 2\Lambda_k) + \text{gauge} + \text{ghost}$

- ✧ In **CDT** one measures the (minisuperspace) **Einstein-Hilbert effective action**
- ✧ Therefore in **FRG** we take the simplest **Einstein-Hilbert truncation** of the (Euclidean) effective actions with two scale-dependent couplings: G_k, Λ_k



- ✧ An extremum of the E-H effective action is a de Sitter universe (the four-sphere S^4) with a 4-volume given by the cosmological constant $V_4 \propto \Lambda_k^{-2}$
- ✧ As in CDT we measure only a behaviour of the scale factor $a(t)$ (or the 3-volume $V_3(t)$) we will also consider only minisuperspace fluctuations
- ✧ The (relative) fluctuations are governed by a dimensionless effective coupling $g_{ef}^2 \propto G_k \Lambda_k$

✧ In FRG one has both the IR and the UV fixed points

- ✧ In the IR ($k \rightarrow 0$): $G_k \Lambda_k \rightarrow 0$ as $G_k \rightarrow G_0 \approx G_N, \Lambda_k \rightarrow 0$ so one recovers semiclassical universe with $V_4 \rightarrow \infty$
- ✧ In the UV ($k \rightarrow \infty$): $G_k \Lambda_k \rightarrow g^* \lambda^* \sim 1$ as $G_k \rightarrow g^* k^{-2} \rightarrow 0, \Lambda_k \rightarrow \lambda^* k^2 \rightarrow \infty$ so $V_4 \rightarrow 0$

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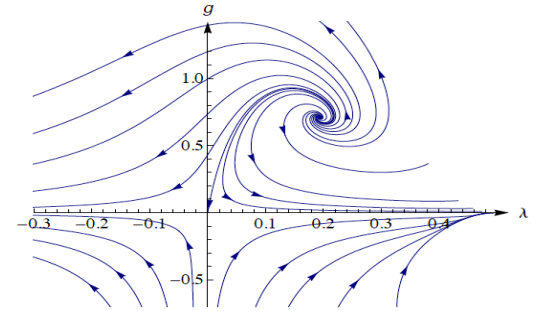
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$$V_3(t) = \frac{3}{4} V_4 \frac{1}{\omega_0 V_4^{1/4}} \cos^3 \left(\frac{t}{\omega_0 V_4^{1/4}} \right)$$

$$S_{MS} = \frac{1}{24\pi G_k} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3} \right)$$

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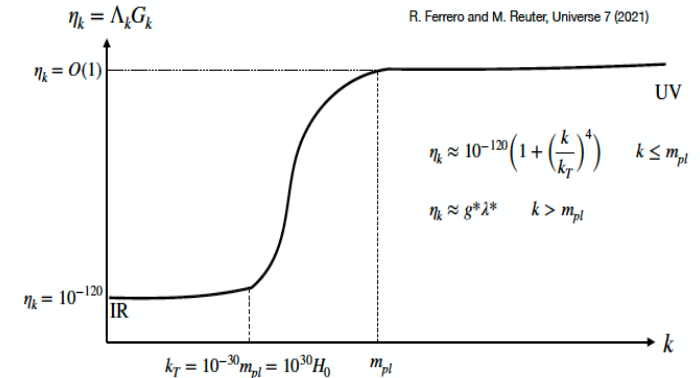
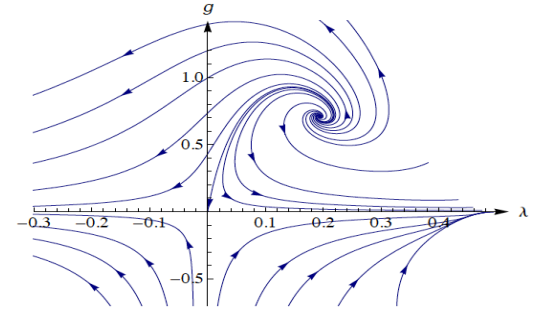
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$$\int ds v(s) = 1$$

Functional Renormalization Group

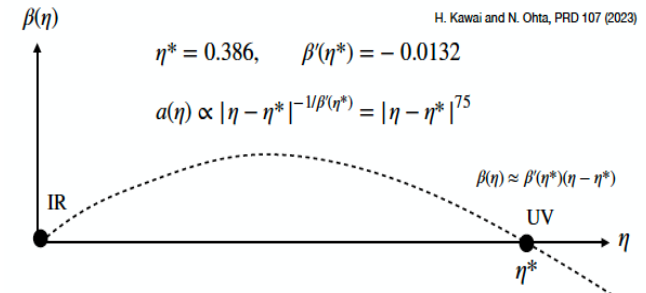
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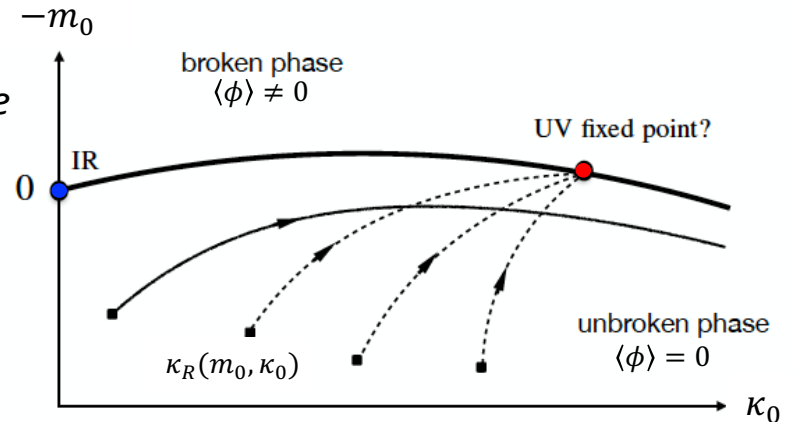


RG flow on the lattice (ϕ^4 example)

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✧ 4D ϕ^4 (lattice) QFT example*

- ✧ 2 dimensionless *bare couplings*: m_0, κ_0
- ✧ for each choice of m_0, κ_0 one can compute the *renormalized* m_R, κ_R and the *correl. length* ξ
- ✧ *physical correl. length* $\xi_{ph} = m_R^{-1} = \xi l_s$
- ✧ one can find *RG flow* where $\kappa_R, m_R = \text{const.}$
- ✧ there is a *phase transition* (where $\xi \rightarrow \infty$ so following the RG flow trajectory $l_s \rightarrow 0$)



✧ The UV limit

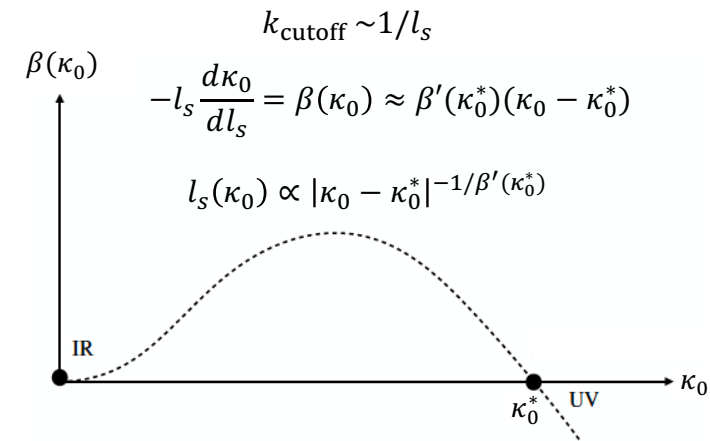
- ✧ we approach the phase transition ($\xi \rightarrow \infty$) keeping the renormalized coupling κ_R fixed
- ✧ in order to do that we have to tune the bare coupling κ_0

$$L = (\partial_\mu \phi)^2 + m_0 \phi^2 + \kappa_0 \phi^4$$

$$\kappa_R \propto \Gamma_4(p_i = 0; m_0, \kappa_0)$$

✧ The IR limit

- ✧ we approach the phase transition ($\xi \rightarrow \infty$) keeping the bare coupling κ_0 fixed
- ✧ we cross the $\kappa_R = \text{const}$ RG trajectories in the direction of $\kappa_R \rightarrow \kappa_R^{ir}$

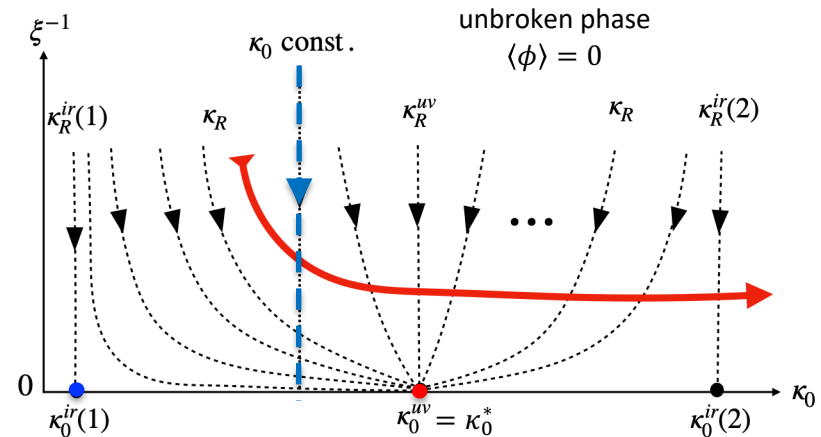


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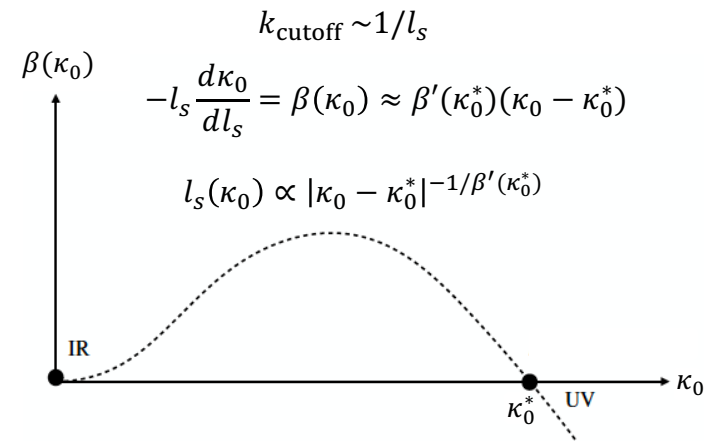
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✧ 4D CDT (lattice) QFT

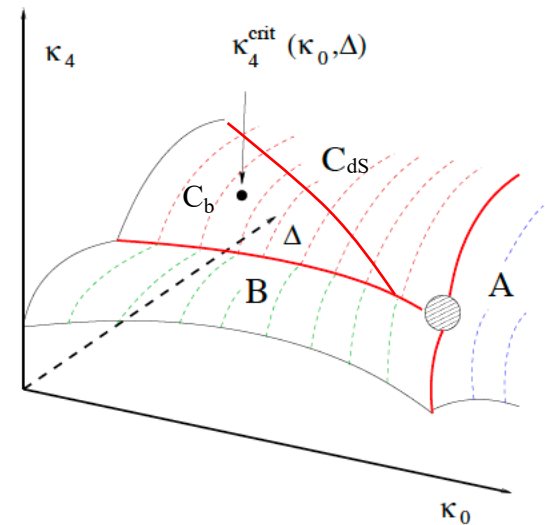
- ✧ 3 dimensionless *bare couplings*: K_4, k_0, Δ
- ✧ The bare cosmol. const. K_4 is related to *lattice volume* N_4 : $K_4 \rightarrow K_4^{crit}(k_0, \Delta)$ when $N_4 \rightarrow \infty$
- ✧ One can argue that inside phase C_{ds} the correl. length: $\xi \propto N_4^{1/4}$
- ✧ We assume that the CDT MS effective action is consistent with the E-H truncation in FRG
- ✧ This implies relations between the effective couplings

✧ The UV limit

- ✧ we will approach the $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \rightarrow \infty$) tuning the bare couplings k_0, Δ such that the effective coupling $G\Lambda$ stays fixed
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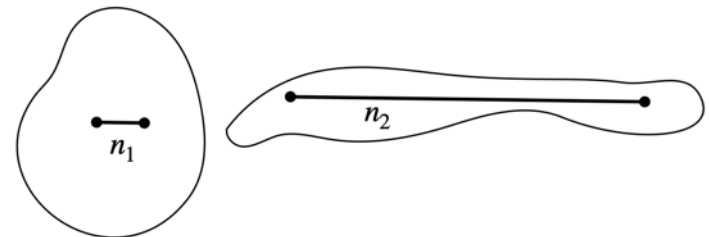
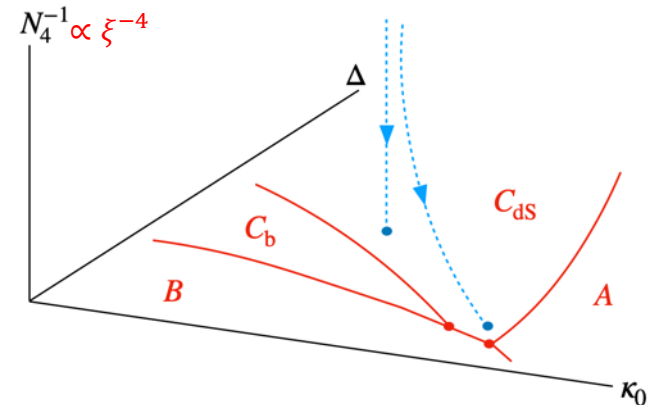
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n is the geodesic distance between the two points in the case of DT

$$G_2(n) \propto \exp \left[- \frac{n}{c \langle N_4 \rangle^{1/d_H}} \right], \quad n \gg \langle N_4 \rangle^{1/d_H}$$

ξ

RG flow in CDT

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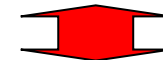
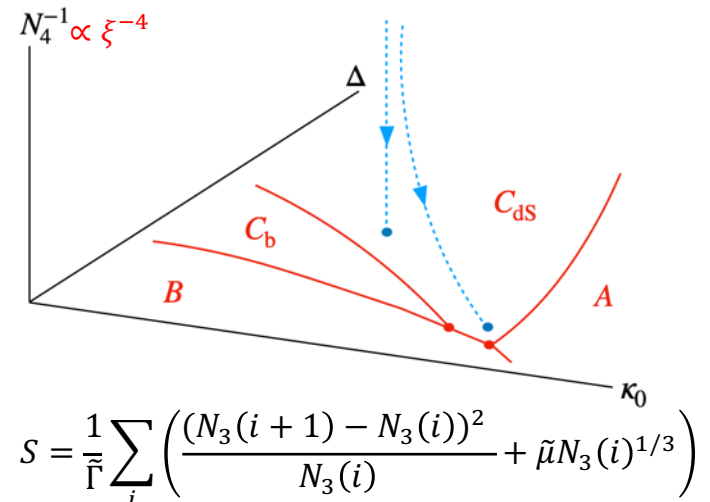
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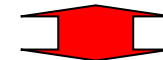
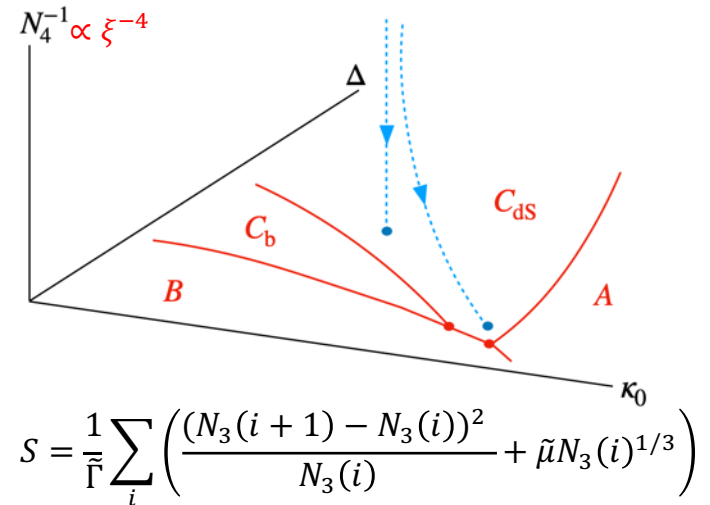
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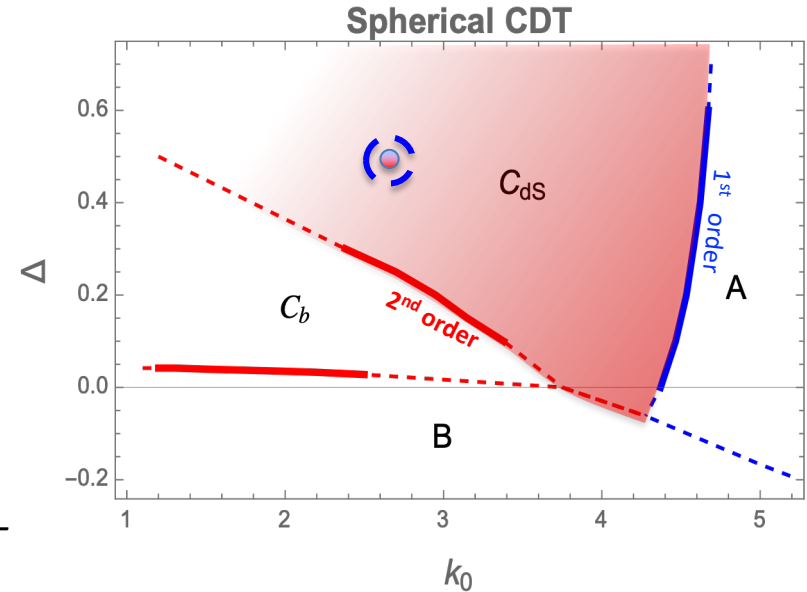
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RG flow in CDT

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- ✧ we approach the $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \rightarrow \infty$, i.e. $N_4 \rightarrow \infty$) keeping the bare couplings k_0, Δ fixed
- ✧ for fixed k_0, Δ we have $\tilde{\Gamma}, \tilde{\omega} = \text{const.} > 0$
- ✧ from FRG for $k \rightarrow 0$: $G_k \rightarrow G_0 \approx G_N = \ell_{Pl}^2$
- ✧ therefore in CDT lattice spacing remains constant: $l_s \sim \ell_{Pl}$
- ✧ as $N_4 \rightarrow \infty$ and $l_s > 0$ the volume of the CDT universe $V_4 \rightarrow \infty$
- ✧ this is consistent with FRG as for $k \rightarrow 0$: $\Lambda_k \rightarrow 0$ so $V_4 \propto \Lambda_k^{-2} \rightarrow \infty$
- ✧ CDT (relative) fluctuations vanish and one reproduces (semi) classical spacetime
- ✧ this is also consistent with FRG where $G_k \Lambda_k \rightarrow 0$



$$\langle N_3(i) \rangle = N_4^{3/4} \frac{3}{4\tilde{\omega}} \cos^3 \left(\frac{i}{\tilde{\omega} N_4^{1/4}} \right)$$

$$\langle |\delta N_3(i)| \rangle = \tilde{\Gamma}^{1/2} N_4^{1/2} F \left(\frac{i}{\tilde{\omega} N_4^{1/4}} \right)$$

$$\frac{\langle |\delta V_3(t_i)| \rangle}{\langle V_3(t_i) \rangle} = \frac{\langle |\delta N_3(i)| \rangle}{\langle N_3(i) \rangle} \propto \frac{\sqrt{\tilde{\Gamma} \tilde{\omega}^2}}{N_4^{1/4}} \rightarrow 0$$

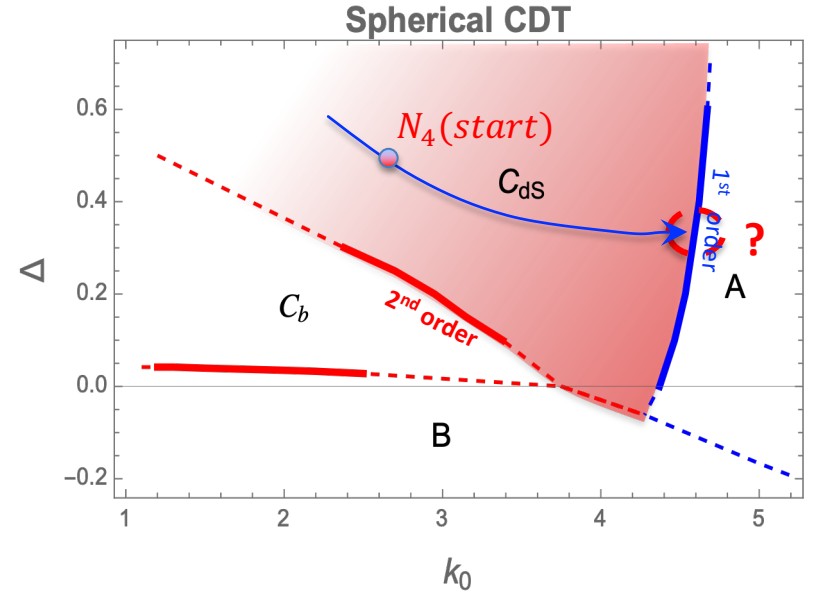
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$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

RG flow in CDT

✧ The UV limit

- ✧ we approach the $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \rightarrow \infty$, i.e. $N_4 \rightarrow \infty$) tuning the bare couplings k_0, Δ such that the effective $G_k \Lambda_k = g^* \lambda^* = const$
- ✧ from FRG: $G_k \rightarrow g^* k^{-2} \rightarrow 0$
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 - ✧ $\gamma/4\nu_{uv} = 0.54 \pm 0.04 \geq 1/2$ but the existence of UV limit is not conclusive within error bars



$$\tilde{\Gamma}(k_0(N_4), \Delta(N_4)) \tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4}$$

$$\frac{\tilde{\Gamma}}{\sqrt{N_4}} \left(\frac{\tilde{\omega}}{\omega_0} \right)^2 \propto G_k \Lambda_k = g^* \lambda^* = const$$

$$\frac{\langle |\delta V_3(t_i)| \rangle}{\langle V_3(t_i) \rangle} = \frac{\langle |\delta N_3(i)| \rangle}{\langle N_3(i) \rangle} \propto \frac{\sqrt{\tilde{\Gamma} \tilde{\omega}^2}}{N_4^{1/4}} \rightarrow const \sim 1$$

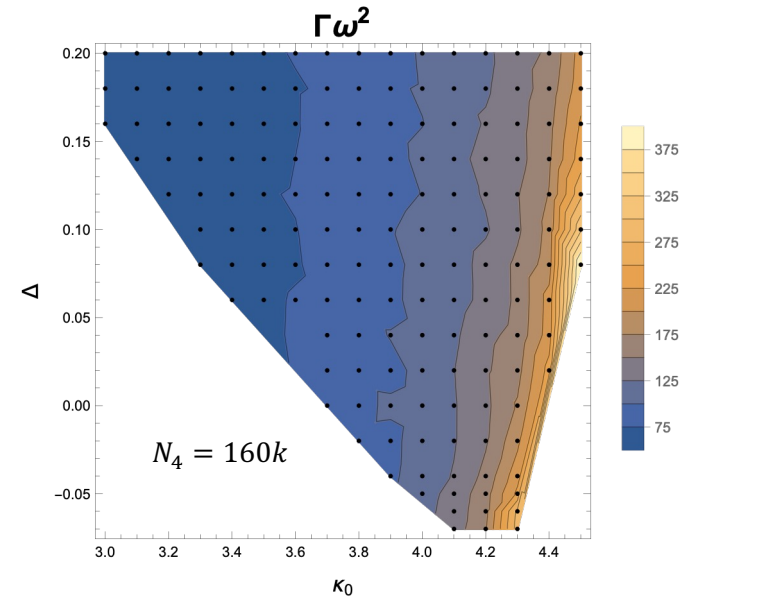
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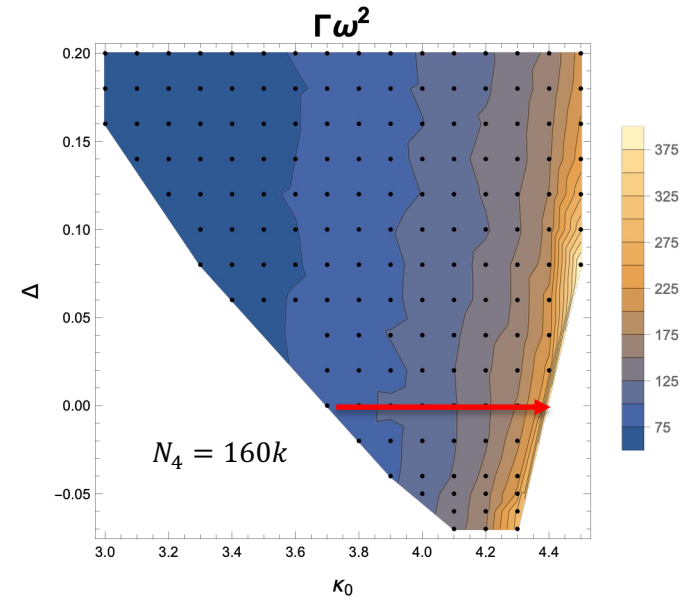


$$\left[\tilde{\Gamma}(k_0(N_4), \Delta(N_4)) \tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4} \right]$$

RG flow in CDT

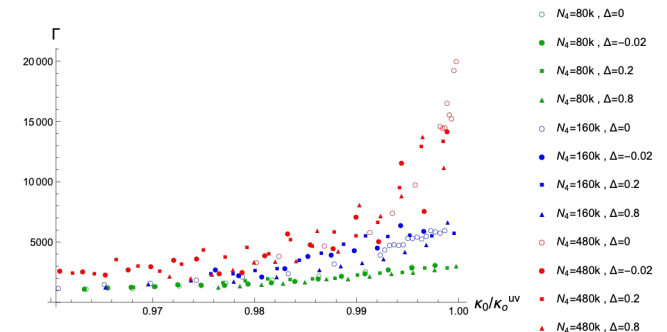
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$$\left[\tilde{\Gamma}(k_0(N_4), \Delta(N_4)) \tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4} \right]$$

*Results seem independent of Δ choice !

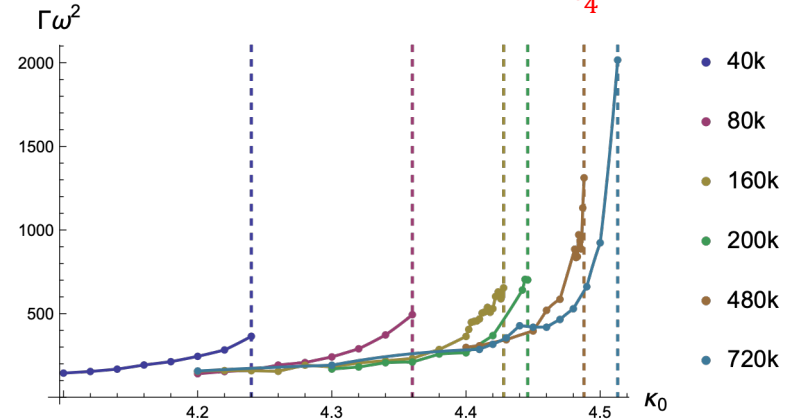


RG flow in CDT

$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}}$$

✧ The UV limit

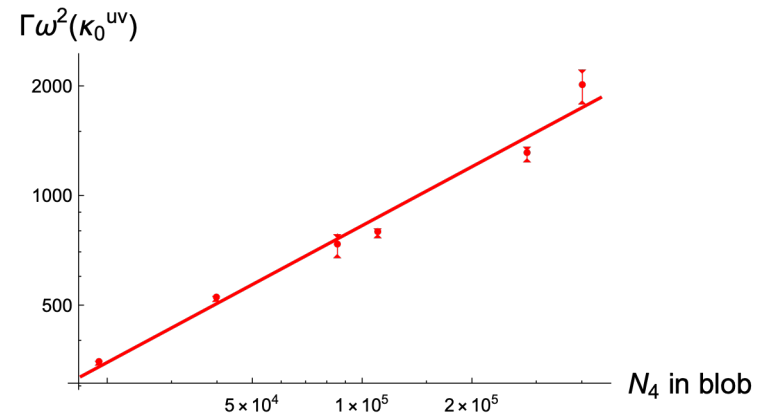
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$$\tilde{\Gamma} \tilde{\omega}^2(k_0^{uv}(N_4)) \propto N_4^{\gamma/4\nu_{uv}}$$

$$\left[\tilde{\Gamma}(k_0(N_4), \Delta(N_4)) \tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4} \right]$$

— fit: $(\alpha-2\beta)/4\nu_{uv}=0.54\pm0.04$

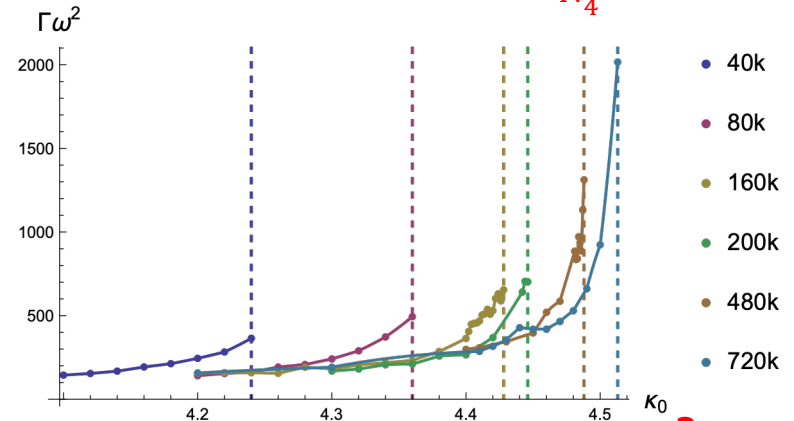


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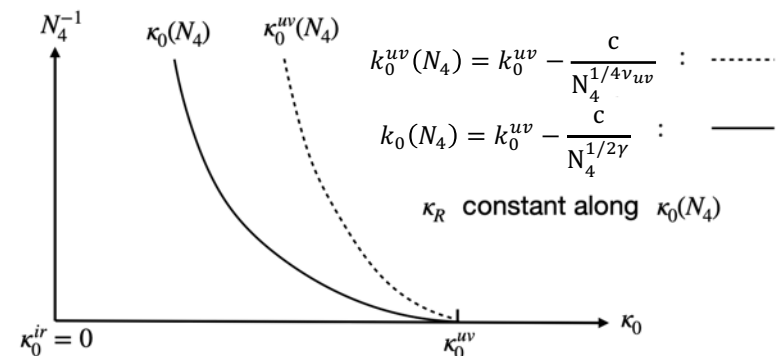
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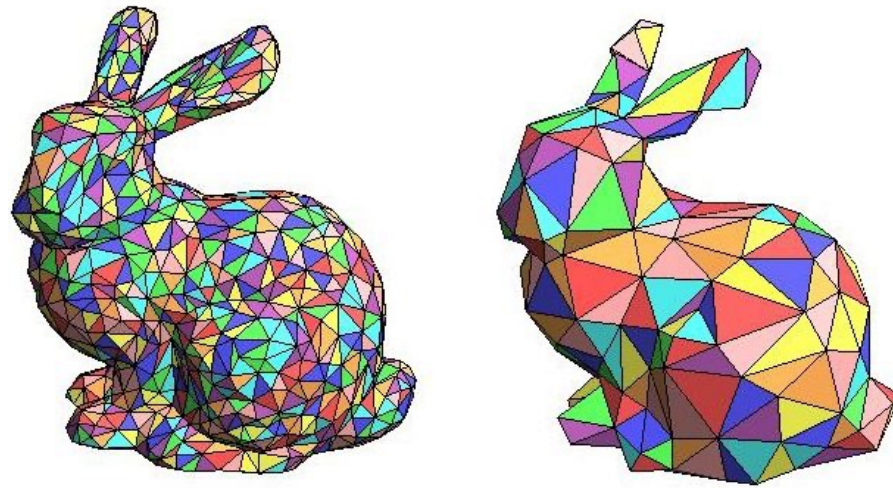
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$$\tilde{\Gamma}(k_0(N_4), \Delta(N_4)) \tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4}$$



Conclusions

- ✧ *CDT is a **lattice QFT** and a promising candidate for a **UV complete** theory of **QG** formulated in a fully **non-perturbative** and **background independent** way*
- ✧ *One can study **dynamically emerging background geometry** and quantum **fluctuations***
- ✧ *CDT has a rich phase structure including the **semi-classical phase** C_{ds}*
 - ✧ ***correct IR limit** of the **scale factor** (spatial volume) **consistent with (Eucl.) de Sitter space***
 - ✧ *quantum **fluctuations** of the scale factor are **described by the minisuperspace action***
- ✧ *CDT can provide independent **tests of asymptotic safety** in a setting not dependent on FRG truncations*
- ✧ ***One can make contact with FRG by defining RG flow in CDT** and searching for the **IR** and **UV** fixed points*
- ✧ *The results for the **UV continuum limit** seem promising but **not conclusive***
- ✧ ***Open problems and questions:***
 - ✧ *the (potential) UV limit of CDT is obtained at the **1st order phase transition** (non-standard)*
 - ✧ *this is possible because we observe **finite-size scaling** and thus **the correl. length** $\xi \propto N_4^{1/4}$*
 - ✧ *this is actually a new (generic ?) **feature of quantum gravity**, where one can define correlations between fluctuating space-time points separated by a geodesic distance (not as in ordinary lattice QFTs, where space-time is fixed)*



Thank You !