A BESIII Testcase for the DEMOS Project

Partial Wave Analysis of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ in AmpTools

DEMOS Hadron Models Meeting

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Institute of Nuclear Physics, JGU Mainz, 05.06.2026

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BESIII

DEMOS Democratizing Models

Analysis Basics

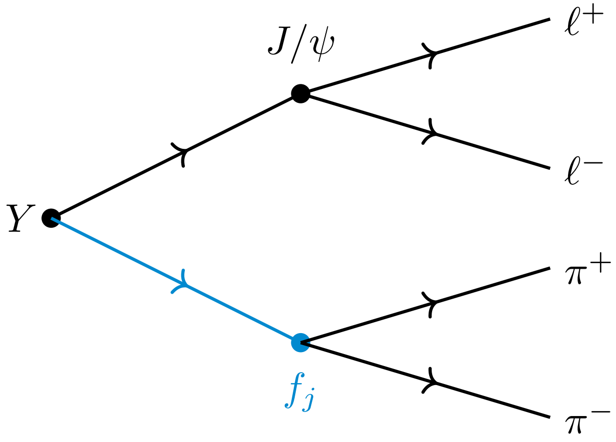
BESIII Paper:
Partial wave analysis of $e^+e^- \to \pi^+\pi^- J/\psi$ and cross section measurement of $e^+e^- \to \pi^\pm Z_c(3900)^\mp$ from 4.1271 to 4.3583 GeV
https://arxiv.org/abs/2505.13222

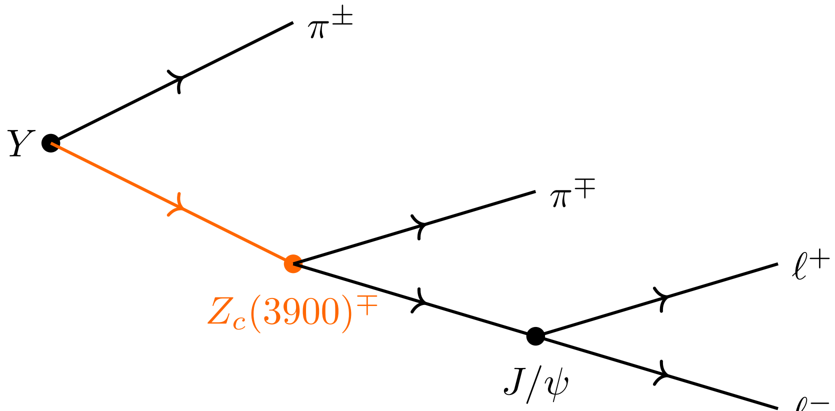
Our testcase: $e^+e^- \to \pi^+\pi^- J/\psi \to \pi^+\pi^- \ell^+\ell^-$

1) $e^+e^- \to \Upsilon(4230) \to f_j \; J/\psi$

with f_j: $f_0(500), f_0(980), f_0(1370), f_2(1270)$

2) $e^+e^- \to \Upsilon(4230) \to \pi^\pm Z_c(3900)^\mp$





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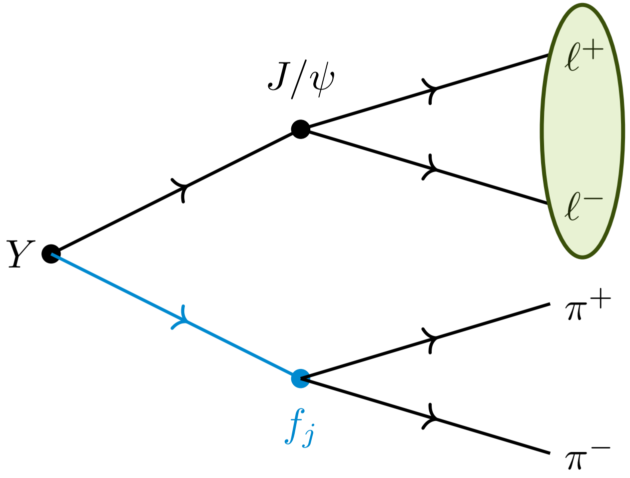
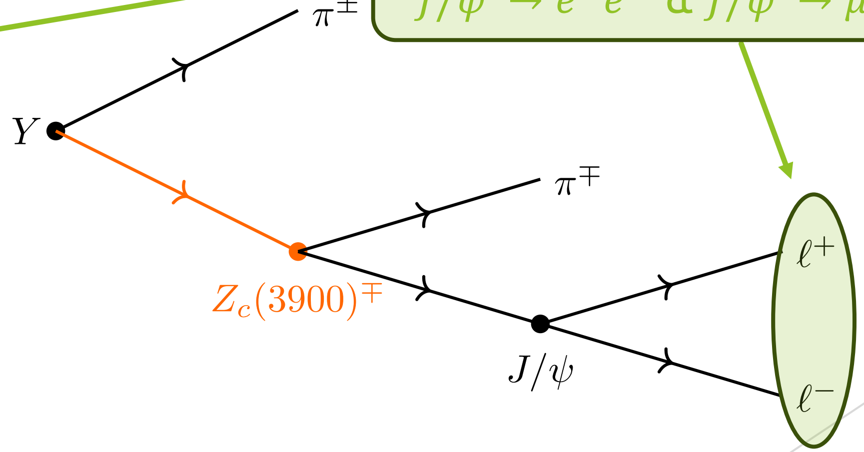
Lepton universality:
$J/\psi \to e^+e^-$ & $J/\psi \to \mu^+\mu^-$





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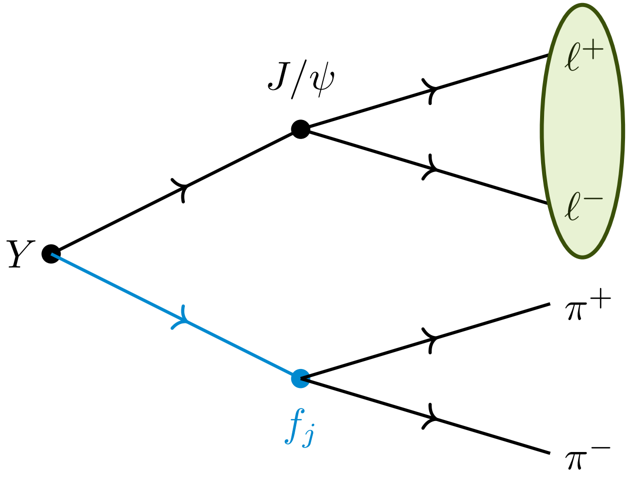
Our testcase: $e^+e^- \to \pi^+\pi^- J/\psi \to \pi^+\pi^- \ell^+\ell^-$

1) $e^+e^- \to \mathrm{Y}(4230) \to \boldsymbol{f_j}\ J/\psi$

with f_j: $f_0(500), f_0(980), f_0(1370), f_2(1270)$



2) $e^+e^- \to \mathrm{Y}(4230) \to \pi^\pm \boldsymbol{Z_c(3900)^\mp}$

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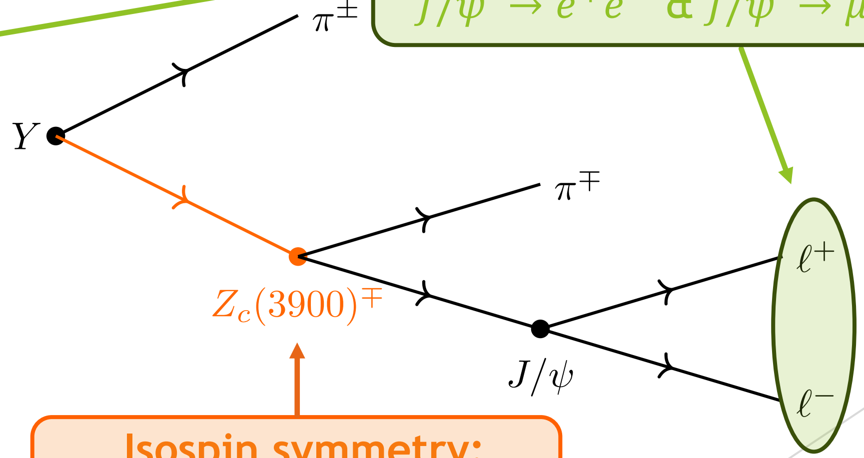

Isospin symmetry:
$Z_c(3900)^+$ & $Z_c(3900)^-$

Amplitude Construction (via Helicity Formalism)

1) $e^+e^- \to \boldsymbol{f_j}\, J/\psi \to \boldsymbol{\pi^+ \pi^-} \ell^+\ell^-$ with $\boldsymbol{f_j}$: $f_0(500), f_0(980), f_0(1370), f_2(1270)$

2) $e^+e^- \to \pi^\pm \boldsymbol{Z_c(3900)^\mp} \to \pi^\pm \boldsymbol{\pi^\mp\, J/\psi} \to \pi^+\pi^-\ell^+\ell^-$

$$\mathrm{T} = \sum_{\lambda_Y, \Delta\lambda_\ell} | \sum_{\boldsymbol{f_j}, \lambda_{J/\psi}, \lambda_{f_j}} A^{Y \to \boldsymbol{f_j} J/\psi}_{\lambda_{f_j}, \lambda_{J/\psi}} (\theta_{f_j}, \phi_{f_j}) BW(m(\pi^+\pi^-)) A^{\boldsymbol{f_j \to \pi^+\pi^-}}_{0,0} (\theta_{\pi^+}, \phi_{\pi^+}) A^{J/\psi \to \ell^+\ell^-}_{\lambda_{\ell^+}, \lambda_{\ell^-}} (\theta_{\ell^+}, \phi_{\ell^+})$$

$$+ e^{i\Delta\lambda_\ell \alpha_\ell} \sum_{\boldsymbol{Z_c}, \lambda_{Z_c}, \lambda_{J/\psi}} A^{Y \to \pi \boldsymbol{Z_c}}_{\lambda_{Z_c}, 0} (\theta_{Z_c}, \phi_{Z_c}) BW(m(\pi J/\psi)) A^{\boldsymbol{Z_c \to \pi J/\psi}}_{\lambda_{J/\psi}, \lambda_\pi} (\theta_{J/\psi}, \phi_{J/\psi}) A^{J/\psi \to \ell^+\ell^-}_{\lambda_{\ell^+}, \lambda_{\ell^-}} (\theta_{\ell^+}, \phi_{\ell^+})|^2$$

According to BESIII Paper:
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Amplitude Construction (via Helicity Formalism)

1) $e^+e^- \to \boldsymbol{f_j} J/\psi \to \boldsymbol{\pi^+ \pi^-} \ell^+\ell^-$ with $\boldsymbol{f_j}$: $f_0(500), f_0(980), f_0(1370), f_2(1270)$

2) $e^+e^- \to \pi^\pm \boldsymbol{Z_c(3900)^\mp} \to \pi^\pm \boldsymbol{\pi^\mp J/\psi} \to \pi^+\pi^-\ell^+\ell^-$

Coherent sum over amplitudes

$$T = \sum_{\lambda_Y,\Delta\lambda_\ell} \Big| \sum_{f_j,\lambda_{J/\psi},\lambda_{f_j}} A^{Y\to f_j J/\psi}_{\lambda_{f_j},\lambda_{J/\psi}}(\theta_{f_j},\phi_{f_j}) BW(m(\pi^+\pi^-)) A^{f_j\to\pi^+\pi^-}_{0,0}(\theta_{\pi^+},\phi_{\pi^+}) A^{J/\psi\to\ell^+\ell^-}_{\lambda_{\ell^+},\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+})$$

$$+ e^{i\Delta\lambda_\ell \alpha_\ell} \sum_{Z_c,\lambda_{Z_c},\lambda_{J/\psi}} A^{Y\to\pi Z_c}_{\lambda_{Z_c},0}(\theta_{Z_c},\phi_{Z_c}) BW(m(\pi J/\psi)) A^{Z_c\to\pi J/\psi}_{\lambda_{J/\psi},\lambda_\pi}(\theta_{J/\psi},\phi_{J/\psi}) A^{J/\psi\to\ell^+\ell^-}_{\lambda_{\ell^+},\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+}) \Big|^2$$

Amplitude Construction (via Helicity Formalism)

1) $e^+e^- \rightarrow \boldsymbol{f_j}\, J/\psi \rightarrow \boldsymbol{\pi^+\pi^-}\ell^+\ell^-$ with $\boldsymbol{f_j}$: $f_0(500), f_0(980), f_0(1370), f_2(1270)$

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$$+ e^{i\Delta\lambda_\ell \alpha_\ell} \sum_{Z_c, \lambda_{Z_c}, \lambda_{J/\psi}} A^{Y\rightarrow \pi Z_c}_{\lambda_{Z_c}, 0}(\theta_{Z_c}, \phi_{Z_c}) BW(m(\pi J/\psi)) A^{Z_c \rightarrow \pi J/\psi}_{\lambda_{J/\psi}, \lambda_\pi}(\theta_{J/\psi}, \phi_{J/\psi}) A^{J/\psi \rightarrow \ell^+\ell^-}_{\lambda_{\ell^+}, \lambda_{\ell^-}}(\theta_{\ell^+}, \phi_{\ell^+}) \Big|^2$$

Alignment angle

Amplitude Construction (via Helicity Formalism)

1) $e^+e^- \to \boldsymbol{f_j}\, J/\psi \to \boldsymbol{\pi^+\pi^-}\ell^+\ell^-$ with $\boldsymbol{f_j}$: $f_0(500), f_0(980), f_0(1370), f_2(1270)$

2) $e^+e^- \to \pi^\pm \boldsymbol{Z_c(3900)^\mp} \to \pi^\pm \boldsymbol{\pi^\mp J/\psi} \to \pi^+\pi^-\ell^+\ell^-$

Incoherent sum over unobserved helicities

$$T = \sum_{\lambda_Y,\Delta\lambda_\ell} \Big| \sum_{f_j,\lambda_{J/\psi},\lambda_{f_j}} A^{Y\to f_j J/\psi}_{\lambda_{f_j},\lambda_{J/\psi}}(\theta_{f_j},\phi_{f_j}) BW(m(\pi^+\pi^-)) A^{f_j\to\pi^+\pi^-}_{0,0}(\theta_{\pi^+},\phi_{\pi^+}) A^{J/\psi\to\ell^+\ell^-}_{\lambda_{\ell^+},\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+})$$

$$+\, e^{i\Delta\lambda_\ell \alpha_\ell} \sum_{Z_c,\lambda_{Z_c},\lambda_{J/\psi}} A^{Y\to\pi Z_c}_{\lambda_{Z_c},0}(\theta_{Z_c},\phi_{Z_c}) BW(m(\pi J/\psi)) A^{Z_c\to\pi J/\psi}_{\lambda_{J/\psi},\lambda_\pi}(\theta_{J/\psi},\phi_{J/\psi}) A^{J/\psi\to\ell^+\ell^-}_{\lambda_{\ell^+},\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+}) \Big|^2$$

Amplitude Parametrization

$$A^{J_a}_{\lambda_b,\lambda_c}(\theta,\phi) = N_{J_a} F^{J_a}_{\lambda_b,\lambda_c} D^{J_a *}_{M,\lambda}(\phi,\theta,0)$$

- N_{J_a}: normalization factor
- $\lambda = \lambda_b - \lambda_c$
- $D^{J_a *}_{M,\lambda}(\phi,\theta,0)$: Wigner-D function
- $F^{J_a}_{\lambda_b,\lambda_c}$: helicity amplitude given by

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{\mathrm{LS}} G^{J_a}_{LS} \sqrt{\frac{2L+1}{2J_a+1}} \langle L,0,S,\lambda | J_a,\lambda\rangle \langle S_b,\lambda_b,S_c,-\lambda_c|S,\lambda\rangle p^L B_L(p,r)$$

Line Shapes

- Constant-width relativistic Breit Wigner: $Z_c(3900)^{\pm}, f_0(1370)$ & $f_2(1270)$
- Energy-dependent width Breit Wigner: $f_0(500)$
- Flatté formula: $f_0(980)$
- $\mathcal{K}$ –matrix: $(\pi^+\pi^-)_{S-\mathrm{wave}}$ including $f_0(500)$, $f_0(980)$ & $f_0(1370)$

Cross check with Dalitz–plot decomposition of $e^+e^- \to \pi^+\pi^- J/\psi$

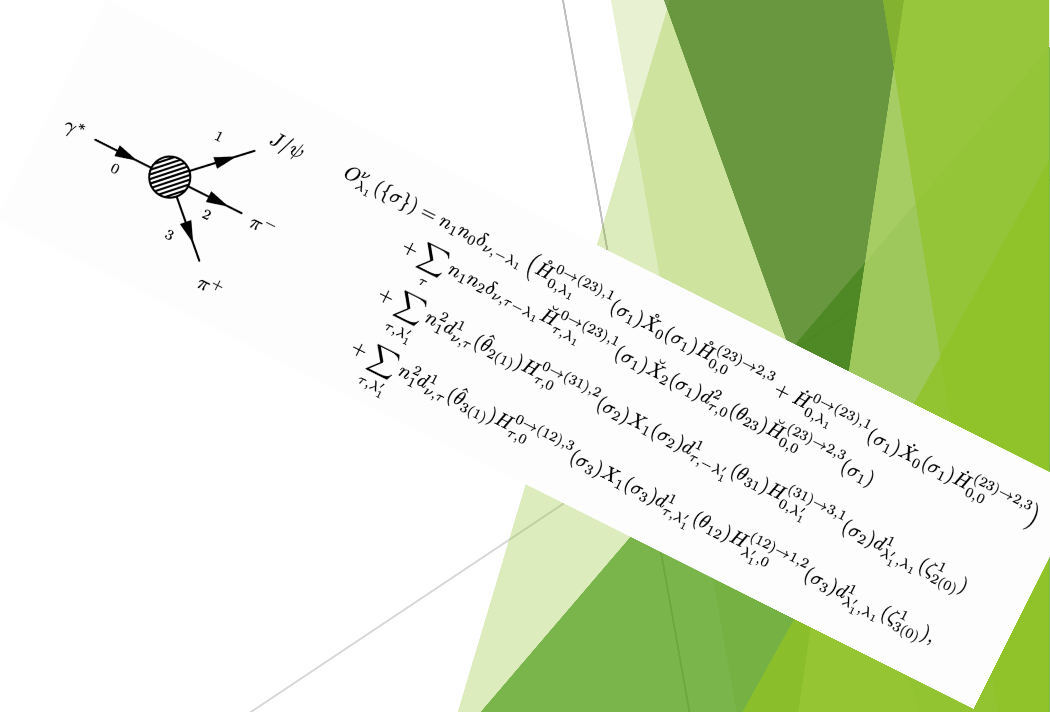
- Theoretical paper by V. Ermolina, Igor Danilkin & M. Vanderhaeghen (https://arxiv.org/pdf/2410.19946)

Dalitz-plot decomposition for the $e^+e^- \to J/\psi\,\pi\,\pi\,(K\bar{K})$ and $e^+e^- \to h_c\,\pi\,\pi$ processes

Viktoriia Ermolina[1,*] Igor Danilkin[1] and Marc Vanderhaeghen[1]

[1] *Institut für Kernphysik & PRISMA+ Cluster of Excellence, Johannes Gutenberg Universität, D-55099 Mainz, Germany*

We present an analysis of the $e^+e^- \to \gamma^* \to J/\psi\,\pi\,\pi\,(K\bar{K})$ and $e^+e^- \to \gamma^* \to h_c\,\pi\,\pi$ processes employing the recently proposed Dalitz-plot decomposition approach, which is based on the helicity formalism for three-body decays. For the above reactions, we validate the factorization of the overall rotation for all decay chains and spin alignments, along with the crossing symmetry between final states, using a Lagrangian-based toy model. For the model-dependent factors that describe the subchannel dynamics, we employ the dispersive treatment of the $\pi\,\pi\,(K\bar{K})$ final state interaction, which accurately reproduces pole positions and couplings of the $f_0(500)$ and $f_0(980)$ resonances. The constructed amplitudes serve as an essential framework to further constrain the properties of the charged exotic states $Z_c(3900)$ and $Z_c(4020)$, produced in these reactions.



PWA: AmpTools package (C++)

- Software written by Matthew Shepherd, Ryan Mitchell, and Hrayr Matevosyan at Indiana University, Bloomington and Lawrence Gibbons at Cornell University
- Publicly available since January 2011, latest release version from September 2025 (https://github.com/mashephe/AmpTools)
- Actively used with the BESIII collaboration
- General framework for unbinned likelihood fits
  - Independent of experiment and physics model
  - MPI and CUDA support for fast fits
  - Easy integration of custom amplitudes

Example: Config file

```
fit jpsipipi
reaction pipimumu p1 p2 p3 p4

genmc pipimumu jpsipipiDataReader genmc.root
accmc pipimumu jpsipipiDataReader accmc.root
data    pipimumu jpsipipiDataReader data.root

sum pipimumu sum_hel

amplitude pipimumu::sum_hel::ZcpPi_L0S1L0S1 angularPart indices
amplitude pipimumu::sum_hel::ZcpPi_L0S1L0S1 mass-dependentPart indices
…

constrain pipimumu::sum_hel::ZcpPi_L0S1L0S1 pipimumu::sum_hel::ZcmPi_L0S1L0S1
initialize pipimumu::sum_hel::ZcpPi_L0S1L0S1 cartesian 1.0 0.0

parameter mZc 3.9
```

- Define one or more reactions

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reaction pipimumu p1 p2 p3 p4

genmc pipimumu jpsipipiDataReader genmc.root
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- Every amplitude declared with the same reaction & sum is added coherently

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parameter mZc 3.9
```

Incoherent sum over unobserved helicities:

```
loop mY   -1  1 -1  1
loop ml   -1 -1  1  1
loop sum_hel helmm helpm helmp helpp
```

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genmc pipimumu jpsipipiDataReader genmc.root
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```

- Amplitudes with the same **reaction::sum::name** are multiplied
- Mass-dependent part: BW, Flatté, …

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```

- Constrain amplitudes, e.g. using isospin symmetry for the $Z_c(3900)^{\mp}$

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```

- Declare internal parameters
- Can be fixed, free, bounded or gaussian-bounded

Summary

- **Our testcase: $e^+e^- \rightarrow \pi^+\pi^- J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-$ at BESIII**
  - $e^+e^- \rightarrow \Upsilon(4230) \rightarrow f_j\ J/\psi$
  - $e^+e^- \rightarrow \Upsilon(4230) \rightarrow \pi^{\pm} Z_c(3900)^{\mp}$

  → **Symmetry-based constraints**

- **Many different line shapes**: Breit Wigner, Flatté, and $\mathcal{K}$-matrix parametrizations
- Amplitudes are constructed using the **helicity formalism** (https://arxiv.org/abs/2505.13222)
- Cross check with **Dalitz-plot decomposition** of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ (https://arxiv.org/pdf/2410.19946)
- **AmpTools** framework used for partial wave analysis

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Thank you for your attention!

Backup

Amplitude Construction

$$T = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{f_j,\lambda_{J/\psi},\lambda_{f_j}} A^{Y\to f_j J/\psi}_{\lambda_{f_j},\lambda_{J/\psi}}(\theta_{f_j},\phi_{f_j}) BW(m(\pi^+\pi^-)) A^{f_j\to\pi^+\pi^-}_{0,0}(\theta_{\pi^+},\phi_{\pi^+}) A^{J/\psi\to l^+l^-}_{\lambda_{l^+},\lambda_{l^-}}(\theta_{l^+},\phi_{l^+}) \right.$$

$$\left. + e^{i\Delta\lambda_l\alpha_l} \sum_{Z_c,\lambda_{Z_c},\lambda_{J/\psi}} A^{Y\to\pi Z_c}_{\lambda_{Z_c},0}(\theta_{Z_c},\phi_{Z_c}) BW(m(\pi J/\psi)) A^{Z_c\to\pi J/\psi}_{\lambda_{J/\psi},\lambda_\pi}(\theta_{J/\psi},\phi_{J/\psi}) A^{J/\psi\to l^+l^-}_{\lambda_{l^+},\lambda_{l^-}}(\theta_{l^+},\phi_{l^+}) \right|^2$$

- λ_P: helicity of the particle P $\left(l^\pm, Z_c(3900)^\pm, J/\psi, f_j\right)$
- (θ_P, ϕ_P): polar and azimuthal angles of P in the helicity frame of the cascading process
- Summation for virtual photon (λ_Y) and lepton pairs in the final state $(\Delta\lambda_l)$
- $m(\pi^+\pi^-), m(\pi J/\psi)$: invariant masses of $\pi^+\pi^-$ and $\pi J/\psi$
- $e^{i\Delta\lambda_l\alpha_l}$: needed to align helicity frames in two different decay chains