What are the constituents of nature?

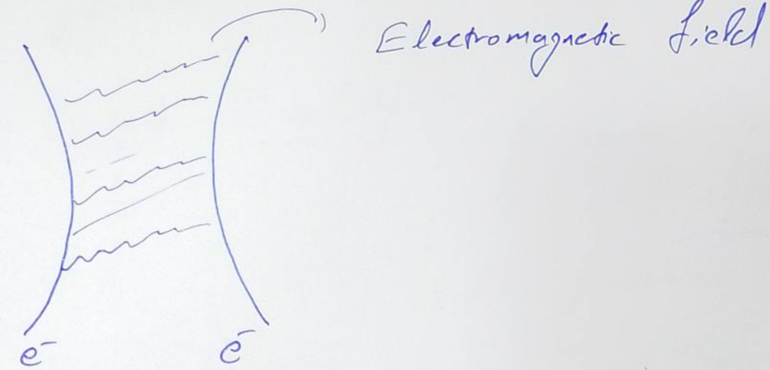
Democritus: Indivisible atoms.

→ Everything is made of e^-, P N.

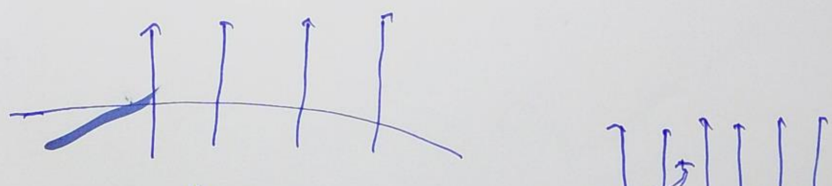
What about forces between them?

Electromagnetic Waves → forces!

All forces that you are familiar with are electric forces



When a charged particle wiggles these fields get disturbed and produce electromagnetic waves.



Molecules : Elec force
Chemistry : Elec force
Computer : Elec force
Materials : Electric force.

- Manifestation of this electric field is the electromagnetic ~~f~~wave: Light.

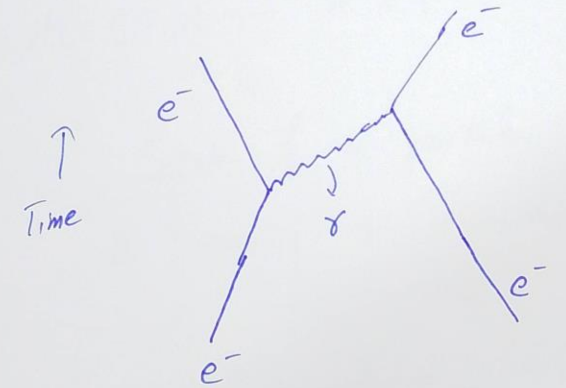
Packets of these waves.
: Particles!

~~Hav~~ There are particles: Forces between them are also particles:

<u>Wave</u> - <u>Particle</u> <u>Duality</u>

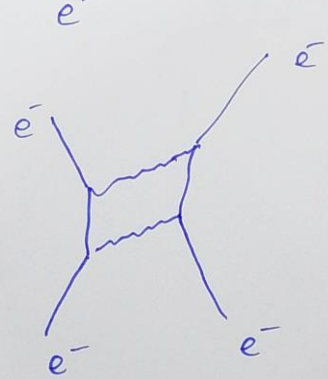
⇝ Continuous sublime substances themselves can be <u>considered</u> behave like particles:

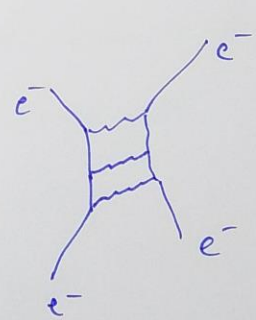
Feynman Diagram



Probability

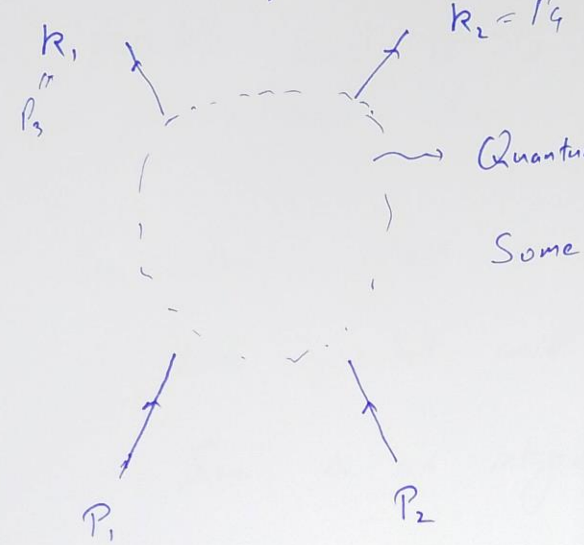
Repulsion between two electrons!

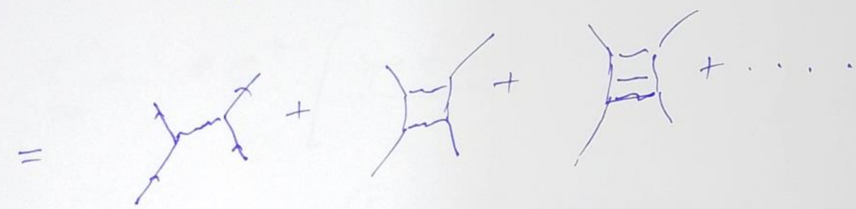




Probability Amplitudes are associated with each.

So a process:





$i\mathcal{M}\,(p_1, p_2 \longrightarrow k_1, k_2)$

∴ State of a particle is generally described by its energy & Momentum (given spatial homogeneity).

Various Interpretation: Did it come from one particle exchange, or two particle exchange, or three

In general, you can't tell.

→ One could try to pick one of ~~the~~ the processes but in general ~~the~~ it is not possible and it is the sum of all probabilities that matter.

Question: What is the probability that one photon will be exchanged?

There is an integral for that:

$$i\mathcal{M}^{(1)} \simeq -e^2 \Big[\underbrace{\text{spin factor}} \Big.$$

$$\sim -ie^2 \left[\frac{[\bar{u}(k_1)\gamma^\mu u(p_1)][\bar{u}(k_2)\gamma_\mu u(p_2)]}{(p_1-k_1)^2+i\varepsilon} - \frac{\bar{u}(k_2)\gamma^\mu u(p_1)\cdots}{(p_1-k_2)^2+i\varepsilon} \right.$$

How would you check that?

The ~~problem~~ point is that you can't!

We say there is a background: You have to trust the high loop theory to subtract that contribution!

~~However, there is simplification and~~

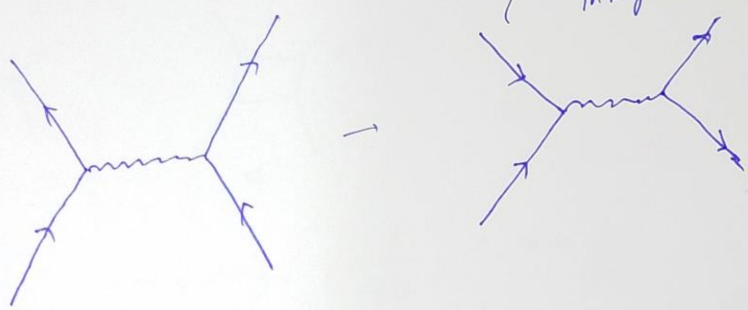
→ We in principle observe the combined process!

In this case there is a simplification:

Feynman Rules!

Forget about time direction:

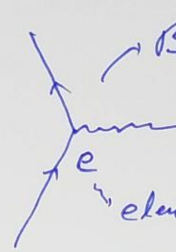
flip the arrow
interpret appropriately



= Pair Annihilation

= Pair Creation:

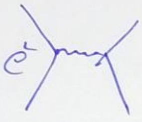
Alternatively: Photon Decay!



Basic Vertex

e electric charge

Photon couples to particles with electric charge.



e^2

α

Box

e^4

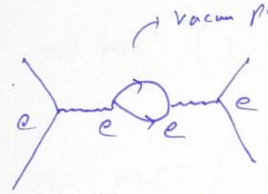
α^2

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

$$\frac{1}{137} = \alpha = \frac{e^2}{4\pi}$$

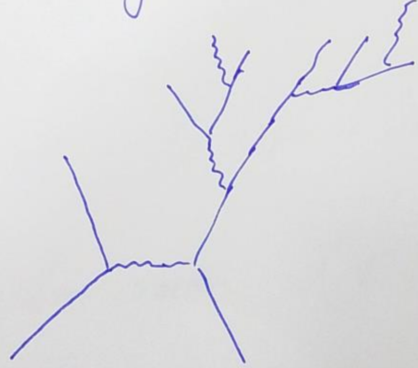
$$e^2 = 0.09$$

vacum polarization

$+$ 

α^2

$\therefore$ Tree Diagram dominates.



Whole bunch of particles starting from two: Very little probability!

Energy + Momentum are conserved at each ~~vertex~~ interaction vertex!

Energy + Momentum: $E = \frac{p^2}{2m} = \frac{m^2v^2}{2m} = \frac{1}{2}mv^2$

Relativity relates them

$$E^2 = p^2c^2 + m^2c^4$$

~~put c~~

$$E = \sqrt{p^2c^2 + m^2c^4}$$

$$E \sim mc^2\sqrt{\frac{p^2}{m^2c^2} + 1}$$

$$\sim mc^2\left(1 + \frac{p^2}{m^2c^2}\right)^{1/2}$$

$$\sim mc^2\left[1 + \frac{1}{2}\frac{p^2}{m^2c^2} + \cdots\right]$$

$$= mc^2 + \frac{p^2}{2m}$$

We have for $c = 1$

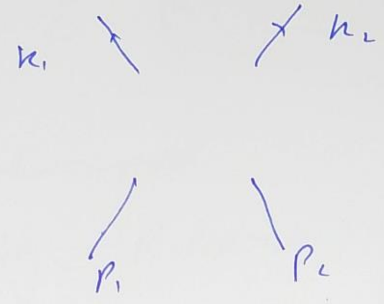
$$\cancel{E = p} \quad E^2 - |\vec{p}|^2 = m^2 \rightsquigarrow \text{Invariant Mass.}$$

For any four vector $(E, \vec{p})$: $E^2 - |\vec{p}|^2 = M^2$

same in all frames

$(E_1 + E_2, \ \vec{p}_1 + \vec{p}_2) =$

Consider two particles



$$(E_1, \vec{P}_1) + (E_2, \vec{P}_2) = (E_1 + E_2, \vec{P}_1 + \vec{P}_2)$$

compute length: $(E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2$

$$= E_1^2 + E_2^2 + 2E_1E_2 - |\vec{P}_1|^2 - |\vec{P}_2|^2 - 2\vec{P}_1 \cdot \vec{P}_2$$

$$= m_1^2 + m_2^2 + 2(E_1E_2 - \vec{P}_1 \cdot \vec{P}_2)$$

M^2 $$= m_1^2 + m_2^2 + 2(E_1E_2 - |\vec{P}_1||\vec{P}_2|\cos\theta)$$

$E_1 = \sqrt{|\vec{P}_1|^2 + m^2}$

Invariant Mass of the ingoing particles:

$\Rightarrow E_1 > |\vec{P}_1|, \quad E_2 > |\vec{P}_2|$

$E_1E_2 > |\vec{P}_1||\vec{P}_2|\cos\theta.$ Max $\cos\theta = 1$

$\Rightarrow (E_1E_2 - |\vec{P}_1||\vec{P}_2|\cos\theta) > 0.$

~~This is A~~ For Photon $m = 0$.

$E = P$, $E = h\nu$.

~~Important note~~

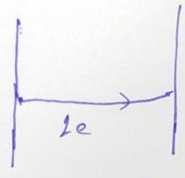
~~Highly Relativistic particle when E~~

Important note on units: with $c = 1$.

$E = mc^2 = m$ at rest : This is rest energy A particle has to carry this budget no matter what: Living Wage!

$m = E/c^2$: $\frac{GeV}{c^2}$ or GeV or MeV.

1eV :



1eV

$1eV = 1.6 \times 10^{-19}$ C.V $= 1.6 \times 10^{-19}$ J.

You have a battery : 3V

Current $\sim$ 120mA $\sim 120 \times 10^{-3}$ C per second

Energy $= 120 \times 10^{-3} \times 3$ J $= 3.6 \times 10^{2} \times 10^{-3}$ J

$= 0.36$ J

$= 2.25 \times 10^{18}$ eV

$= 2.25 \times 10^{9} \times$ GeV $\sim$ 2 billion GeV.

m_e : 0.5 MeV

m_p = 0.9 GeV ~ 1 GeV.

We also use units where $\hbar = 1$.

By Uncertainity Relation

$$\Delta p \Delta x \sim h$$

Minimum $p \sim eV$.

$$\Delta x \sim \frac{1}{\Delta p}$$

$$\Delta x \sim (GeV)^{-1}$$

$$\Delta x \sim \frac{h}{\Delta pc} \sim \frac{1}{\Delta E}$$

One should interpret it as to probe a distance of Δx we need energy of ΔE.

~~Distances are~~

Mass : GeV

Time : $(GeV)^{-1}$

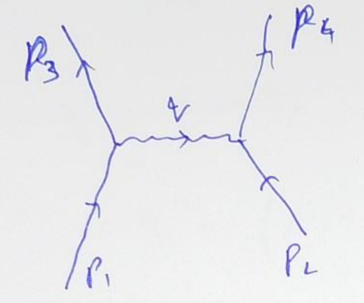
Distance : $(GeV)^{-1}$

All units in particle physics can be written in terms of units of energy

(11)

Energy-Momentum are conserved at each vertex!

$\Rightarrow$ Over all Energy & Momentum do not change



$P_1 = k_1 + q \Rightarrow$ $[P_1^0 = E_1$

$P_2 = P_1 + q$

$P_4 =$

$$P_1 = P_3 + q \quad \& \quad P_4 = P_2 + q.$$

$\Downarrow$ same.

$$\left(E_1 = E_3 + E_\gamma\ ,\ \vec{P}_1 = \vec{P}_3 + \vec{P}_\gamma\right)\ ,\ \left(E_4 = E_2 + E_\gamma\ ,\ \vec{P}_4 = \vec{P}_2 + \vec{P}_\gamma\right)$$

From (i) & (ii)

$$E_1 - E_3 = E_4 - E_2 \quad , \quad \vec{P}_1 - \vec{P}_3 = \vec{P}_4 + \vec{P}_2$$

$$\Rightarrow E_1 + E_2 = E_3 + E_4 \quad \Rightarrow \vec{P}_1 + \vec{P}_2 = \vec{P}_3 + \vec{P}_4$$

Overall energy is conserved

Photon Propagator

$$\frac{-i\,g_{\mu\nu}}{q^2} \rightarrow \frac{1}{q^2} = \frac{1}{E_\gamma^2 - |\vec{p}|_\gamma^2}$$

$$\cancel{q^2 = E_1 - E_2} \qquad q = (E_1 - E_3,\ \vec{P}_1 - \vec{P}_3)$$

$$q^2 = (E_1 - E_3)^2 - |\vec{P}_1 - \vec{P}_3|^2$$

For center of mass frame

$E_1 = E_3$

$\vec{P}_1 = -\vec{P}_2$ + $\vec{P}_3 = -\vec{P}_4$.

call $|\vec{P}_1| = p$ + $|\vec{P}_3| = p'$.

then

$E_{initial} = 2\sqrt{p^2 + m^2}$

$E_{final} = 2\sqrt{p'^2 + m^2}$

$\Rightarrow p^2 = p'^2 \rightarrow |\vec{p}| = |\vec{p}'|$

$\Rightarrow E' = E$.

$$\therefore q^2 = -|\vec{q}|^2$$

In center of mass frame incoming + outgoing particles equally share energy!

$$|\vec{q}|^2 = (\vec{p} - \vec{p}')\cdot(p - \vec{p}') = |\vec{p}|^2 + |\vec{p}'|^2 - 2\,\vec{p}\cdot\vec{p}'$$

$$= 2p^2(1 - \cos\theta$$

$$= 4p^2 \sin^2\frac{\theta}{2}$$

For Static limit:

$$J^{\mu}(t,\vec{x}) = J^{\mu}(x)$$

$$\int_{-\infty}^{\infty} dt\, e^{i\omega t} = 2\pi\delta(\omega) \rightarrow \boxed{\omega = 0}$$

or

$$e^{-iq\cdot x} = e^{-iq^0 t} e^{i\vec{q}\cdot\vec{x}} = e^{-i\frac{\Delta E t}{\hbar}} e^{i\vec{q}\cdot\vec{x}}$$

Non-relative limit.

In non-relative limit: $q^0 \approx \frac{\vec{p}^2 - \vec{p}'^2}{2mc}$

$$\vec{q} = \vec{p} - \vec{p}'$$

E, pc

$(E/c, p)$

$$\vec{p}^2 - \vec{p}'^2 = (\vec{p}-\vec{p}')\cdot(\vec{p}+\vec{p}') = \vec{q}\cdot(\vec{p}+\vec{p}')$$

$$q^0 \approx \frac{\vec{q}(\vec{p}+\vec{p}')}{2mc} \qquad |\vec{p}| + |\vec{p}'| \approx 2mv$$

$$|q^0| = \frac{|\vec{q}|\, mv}{mc} = \frac{v}{c}|\vec{q}|$$

But $\frac{v}{c} << 1$ $\therefore$ $q^{0\,2} << |q|^2$.

$$V(r) \propto \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2} = \frac{1}{4\pi r}.$$

for a particle with ~~Mon~~ Mass M.

$$E^2 = |\vec{q}|^2 + M^2$$

in static gauge

$$V(r) \propto \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{|\vec{q}|^2 + M^2} \propto \frac{e^{-Mr}}{r}.$$

$\Delta P \Delta x \sim \hbar$

$\hookrightarrow \frac{\Delta E}{c} \Delta x \sim \hbar$

$$\Delta x \sim \frac{\hbar c}{\Delta E} = \frac{\hbar c}{\Delta E} = \frac{1.06 \times 10^{-34} \text{ J}\cdot\text{s} \cdot 3\times 10^{8} \text{ m/s}}{90 \cdot \text{GeV} = 90 \times 10^{9} \times 1.6 \times 10^{-19} \text{ J}}$$

$$= \frac{1.06 \times 3}{90 \times 1.6} \times \frac{10^{-34} \times 10^{8} \times 10^{10}}{} \text{ m}$$

$$= \times 10^{-16}$$

$$\sim \frac{2 \times 10^{-16}}{90} \text{ m}$$

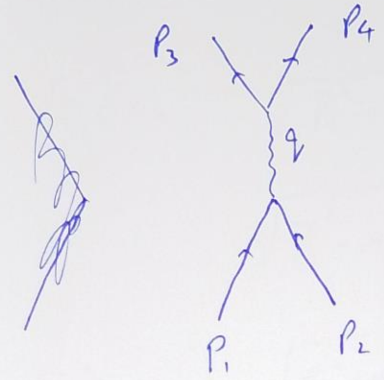
$$\sim 2.2 \times 10^{-18} \text{ m}.$$

$$t = \frac{d}{v} = \frac{d}{c} = \qquad v = \frac{d}{t}$$

$$0.73 \times 10^{-18} \times 10^{-8}$$

$$\sim 0.73 \times 10^{-26}$$

There is an issue:-



$$E_q = E_1 + E_2$$
$$E_q = E_3 + E_4$$

$$\vec{p}_q = \vec{p}_1 + \vec{p}_2$$

$$E_q^2 - \vec{p}_q^{\,2} = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)\cdot(\vec{p}_1 + \vec{p}_2)$$
$$= E_1^2 + E_2^2 + 2E_1E_2 - \vec{p}_1^{\,2} - \vec{p}_2^{\,2} - 2\,\vec{p}_1\cdot\vec{p}_2$$
$$= m_1^2 + m_2^2 + 2\left(E_1E_2 - \vec{p}_1\cdot\vec{p}_2\right) \stackrel{?}{=} 0$$

Never!

So what happens! E_q^2

$$E_1 = \sqrt{\vec{P}_1^2 + m_1^2} \qquad E_2^* = \sqrt{P_2^2 + m^2}$$

$$E_1 E_2 = \sqrt{\vec{P}_1^2 + m^2}\sqrt{P_2^2 + m^2} - \vec{P}_1 \cdot \vec{P}_2$$

$$\sqrt{\vec{P}_1^2 + m^2}\sqrt{\vec{P}_2^2 + m^2} - |\vec{P}_1||\vec{P}_2| \cos\theta$$

$$\sqrt{P_1^2 + m^2}\sqrt{P_2^2 + m^2} + |\vec{P}_1||\vec{P}_2|$$

$$|\vec{P}_1||\vec{P}_2|\left(\sqrt{1 + \frac{m^2}{P_1^2}}\sqrt{1 + \frac{m^2}{P_1^2}} + 1\right)$$

$$\left(\left(1 + \frac{1}{2}\frac{m^2}{P_1^2}\right)\left(1 + \frac{1}{2}\frac{m^2}{P_2^2} + 1\right)\right.$$

$$|P_1|^2 + m^2 + |P_1|^2$$

$$2\left(m^2 + 2|P_1|^2\right)$$

$$E_\gamma^2 - \bar{P}_\gamma^2 \sim m_1^2 + m_2^2 + 2E_1^2 + 2|\vec{P}_1|^2$$

$$2|P_1|^2 + 2|\vec{P}_1|^2 + 2m^2$$

$$\sim 4m^2 + 4|P_1|^2$$

$$E_\gamma^2 - \vec{P}_\gamma^2 = 0$$

$$\Delta E_\gamma^2 = 4(m^2 + |\vec{P}|^2)$$

$$\Delta E_\gamma = 2E^* \qquad \Delta t \sim \frac{1}{2E^*} \quad \boxed{\Delta t = \frac{1}{E_{cm}}}$$

Photon Decay

Photon creation

$$A \sim \frac{1}{E_{cm}^2}$$

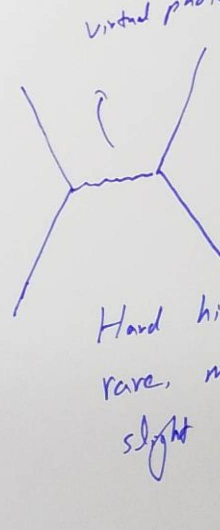
What

Low probability

$$E_2^2 - |\vec{P}_2|^2 = m_1^2 + m_2^2 + 2(E_1E_2 - \vec{p}_1\cdot\vec{p}_2)$$
$$= 2E \sim E_{cm}$$

Virtual Photon!

virtual photon

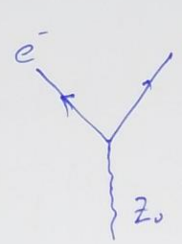


Hard hits are rare, most are slight glances!

$q^2 \sim M_Z^2$

falls on both sides

$M = M_Z$



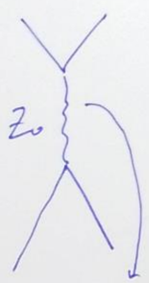
N

$\vec{p}$

$E^2 - |\vec{p}|^2 > 0$

You need some third particle nearby to take the momentum away

Even then you must have a photon energetic enough to start with to provide the Rest mass Energy!



$\cancel{\frac{1}{|\vec{p}|^2 - M^2}}$ $\frac{1}{q^2 - M_Z^2}$

$\cancel{\frac{1}{|\vec{p}|^2 - M^2}}$ $i\mathcal{M} \sim$ It is peaked

at invariant mass

$q^2 \sim M_Z^2$

falls on both sides

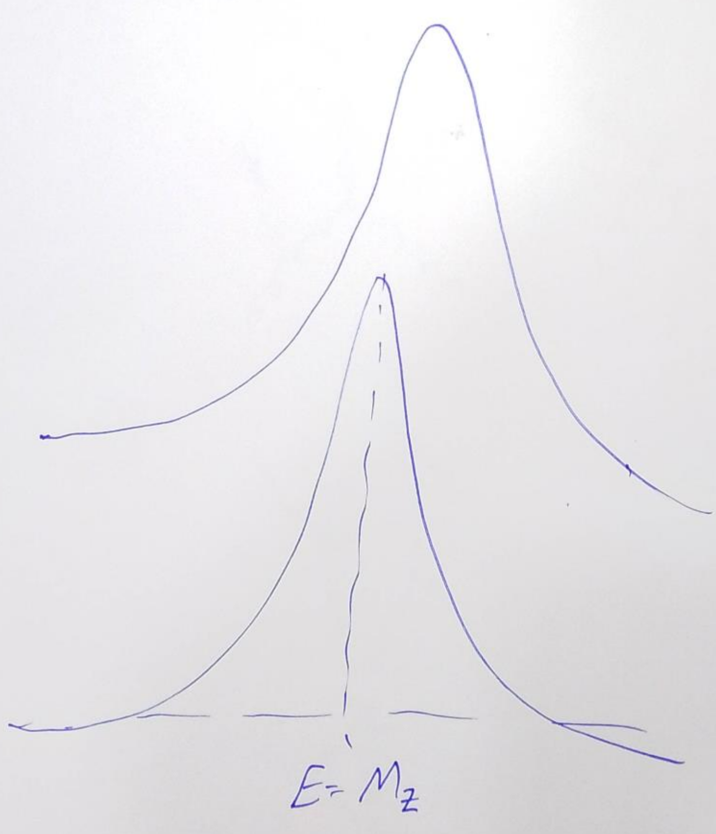
Breit-Wigner formula

$$\cancel{\sigma_f(M_Z^2) = 12}$$

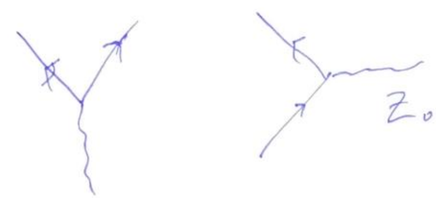
$$\sigma_f(s) = \frac{12\pi\, \Gamma_{Z\to q\bar{q}}\, \Gamma_{Z\to \ell\ell}}{(\underbrace{s}_{=E^2} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

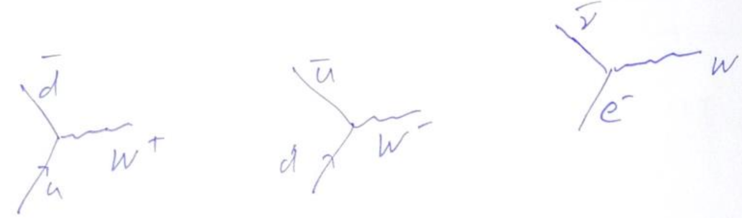
$$FWHM \equiv \Gamma$$

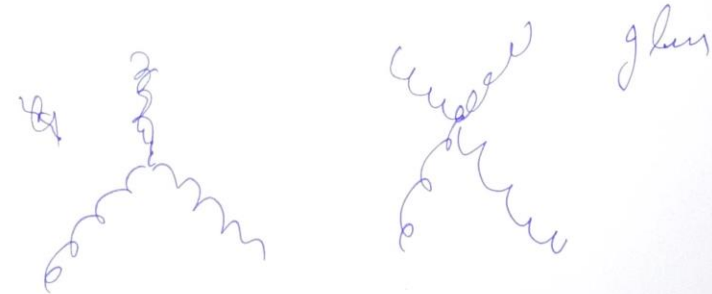
$$f(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E-M)^2 - (\Gamma/2)^2}$$

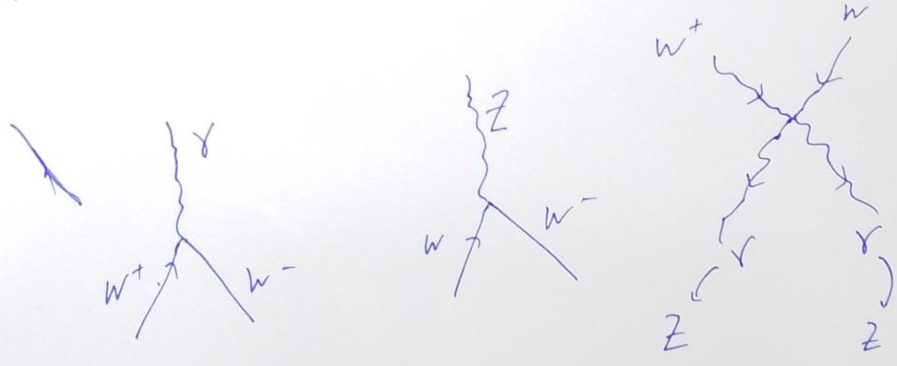


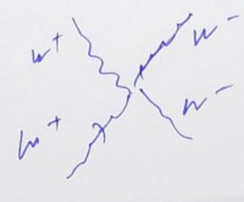
$\begin{pmatrix} e \\ \nu_e \end{pmatrix}$ $\begin{pmatrix} u \\ d \end{pmatrix}$; $\begin{pmatrix} c \\ s \end{pmatrix}$, $\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$, $\begin{pmatrix} t \\ t \end{pmatrix}$, $\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$.

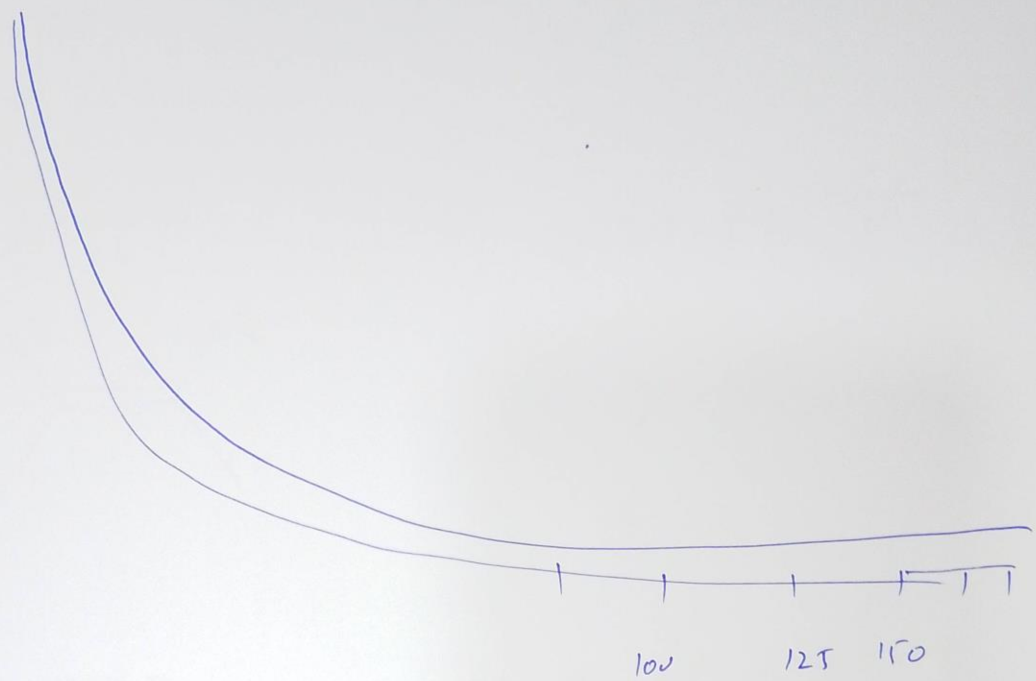


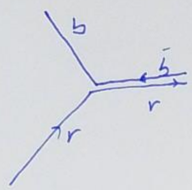


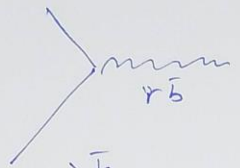




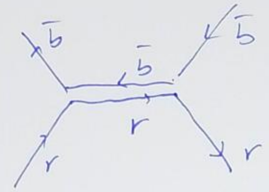



100
125
150





$3 \times 3 = 8 + 1$ singlet



$g \sim 0.37$

$g_c \sim$

Q.C.D is perturbative at this scale!

Decay Channels of Z!

$Z \to e^- e^+ , \mu^- \mu^+ , \tau^- \tau^+$ $\quad 3 \times 3.36\% \sim 9\%$

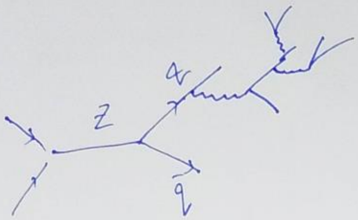
Partial width 84 MeV.

Ate $Z \to \nu_e \bar{\nu}_e , \nu_\mu \bar{\nu}_\mu , \nu_\tau \bar{\nu}_\tau$ $\quad 20\%$

$Z \to u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$ $\quad 70\%$

22

Jets:-



π, K, P, η, Ω

There are all kinds of Hadrons.

~~Atlas Main Components.~~

Z - back ground.

1- $q\bar{q} \longrightarrow \gamma^*, Z \longrightarrow \mu^+\mu^-$.

Z/γ^*

2- $Z \rightarrow \tau^+\tau^-$

$\tau^+ \longrightarrow \mu^-\bar{\nu}_\mu \nu_\tau$ $\qquad$ $\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$.

below M_Z peak

3- Top quark events:-

$pp \rightarrow t\bar{t}$ $\qquad$ $t \rightarrow W^+b$, $\bar{t} \longrightarrow W^-\bar{b}$

$W^+ \longrightarrow \mu^+\nu_\mu$, $W^- \longrightarrow \mu^-\bar{\nu}_\mu$.

b-jets + missing transverse momentum

W + Jets

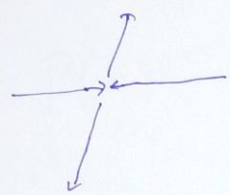
$pp \to W + \text{jets} \quad W \to \mu\nu$

jet → pion, π, K, mis identification

Diboson Production:-

Monte-Carlo:- Simulation of what you will observe.

Transverse Momentum:-



Width: $2.35 \times$ rms

$$P(r;\lambda) = e^{-\lambda}\frac{\lambda^r}{r!}$$

$\mu = \lambda$; Varince $= \lambda$

$\sigma = \sqrt{\lambda} = \sqrt{\mu}$.

$np = \lambda =$ constant.

$$P(\text{Theory} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Theory}) \times P(\text{Theory})}{P(\text{Data})}$$

$$P(K \mid \text{no signal}) = \frac{P(\text{no signal} \mid K) \times P(K)}{P(\text{no signal})}$$

$$P(A \mid B) = \frac{P(B \mid A)}{P(B)} \cdot P(A)$$

Beys theom.

$N = \sigma \mathcal{L}_{int}$

$\sigma = fb$

$N = \underbrace{\sigma}_{fb} \underbrace{\mathcal{L}_{int}}_{(fb)^{-1}}$

1 barn : $10^{-28}\ m^{-2}$

(fb^{-1})