

1 Basics

Color conservation. There is **no other** color flowing in/out of the white ellipse, and it can be any contraction you can think of! (Recover the Lie algebra for $q\bar{q}g$ and the Jacobi identity for ggg)

$$R - \text{[diagram 1]} + R - \text{[diagram 2]} + \dots + R - \text{[diagram n]} = 0 \quad (1)$$

Fierz identity [rotate it and consider permutations (remember the generator sign convention)]

$$\text{Diagram 1} = \frac{1}{N_c} \left[\text{Diagram 2} \right] + 2 \left[\text{Diagram 3} \right] \quad (2)$$

Three gluon vertices (rhs $\pm$ is either the antisymmetric tensor or symmetric tensor)

$$\left\{ \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \end{array} \right\} = 2 \left(\begin{array}{c} \text{diagram 3} \\ \text{diagram 4} \end{array} \right) \pm \left(\begin{array}{c} \text{diagram 5} \\ \text{diagram 6} \end{array} \right) \quad (3)$$

2 Memo for simplifications

Dimensions - Zero point (Trace a Kronecker delta in either fundamental or adjoint irreps)

$$\bigcirc = N_c, \quad \bigcirc_{\text{wavy}} = N_c^2 - 1 \quad (4)$$

Two points

$$\begin{array}{c} \text{---}\rightarrow\text{---} \\ \text{---}\circlearrowleft\text{---} \end{array} = \frac{N_c^2 - 1}{2N_c} \text{---}\rightarrow\text{---}, \quad \begin{array}{c} \text{---}\circlearrowright\text{---} \\ \text{---}\circlearrowleft\text{---} \end{array} = \frac{1}{2} \text{---}\text{---}\text{---} \quad (5)$$

$$\text{gluon loop} = N_c \text{ ghost loop}, \quad \text{gluon tadpole} = \frac{N_c^2 - 4}{N_c} \text{ ghost tadpole} \quad (6)$$

Three points

$$\begin{array}{c} \text{diagram 1} \end{array} = \frac{-1}{2N_c} \begin{array}{c} \text{diagram 2} \end{array}, \quad \begin{array}{c} \text{diagram 3} \end{array} = \frac{N_c}{2} \begin{array}{c} \text{diagram 4} \end{array} \quad (7)$$

$$\begin{array}{c} \text{diagram with red triangle and star} \end{array} = \frac{N_c^2 - 4}{2N_c} \begin{array}{c} \text{diagram with triangle} \end{array} \quad (8)$$

$$\begin{array}{c} \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \end{array} = \frac{N_c}{2} \begin{array}{c} \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \end{array}, \quad \begin{array}{c} \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \end{array} = \frac{N_c}{2} \begin{array}{c} \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \\ | \\ \text{---}\text{---}\text{---} \end{array} \quad (9)$$

$$= \frac{N_c^2 - 4}{2N_c} \text{ (diagram with red wavy line)} , \quad = \frac{N_c^2 - 12}{2N_c} \text{ (diagram with red wavy line)} \quad (10)$$

A nice relation with four points

$$\begin{array}{c} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \quad \text{---} \end{array} = \frac{1}{2N_c} \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} \quad (11)$$