# Introducing Warm Inflation: Dynamics, Microphysics, and Observational Signatures

## Lecture 4: Warm Inflation Dynamics
## Background, Perturbations, and Observational Signatures

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# Today: From microphysics to cosmology

- **Lecture 1: Introduction and Motivation**
  - Inflation, what for ?
  - Cold versus warm inflation
  - Brief overview of applications
- **Lecture 2: Basics of nonequilibrium QFT - part 1**
  - Dissipation in Quantum Mechanics: the Caldeira-Leggett model
  - Schwinger-Keldysh formalism for non-equilibrium QFT
  - Deriving effective equations of motion
- **Lecture 3: Basics of nonequilibrium QFT - part 2**
  - Local approximations for non-Markovian dissipation terms
  - Some local dissipation coefficients
  - Applications to Cosmology: the case of warm inflation
- **Lecture 4: Warm inflation dynamics — background and perturbations**
  - Building warm inflation models
  - Background dynamics
  - Perturbations in warm inflation

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# Specify the model before solving it

$$V(\phi), \qquad \Upsilon(\phi, T), \qquad \rho_r = C_R T^4, \quad C_R = \frac{\pi^2 g_*}{30}.$$

- Canonical scalar and an equilibrium relativistic bath.
- Local dissipative response.
- Microscopic hierarchy established in Lecture 3.

# Background equations: no slow-roll approximation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2,$$

$$3M_P^2H^2 = \tfrac{1}{2}\dot{\phi}^2 + V + \rho_r, \qquad Q = \frac{\Upsilon}{3H}.$$

$$\dot{H} = -\frac{\dot{\phi}^2 + 4\rho_r/3}{2M_P^2}, \qquad \epsilon_H = -\frac{\dot{H}}{H^2}.$$

**Inflation and its end**

$\epsilon_H < 1$ during accelerated expansion; solve for the first exit at $\epsilon_H = 1$.

# Slow roll is a controlled reduction

$$u = \dot{\phi}$$

$$u \simeq -\frac{V_{,\phi}}{3H(1+Q)}, \qquad \rho_r \simeq \frac{3}{4}Qu^2, \qquad H^2 \simeq \frac{V}{3M_P^2}.$$

- $|\dot{u}| \ll 3H(1+Q)|u|$.
- $|\dot{\rho}_r| \ll 4H\rho_r$.
- $u^2/2, \rho_r \ll V$.

# Slow-roll parameters and their use

$$\epsilon_V = \frac{M_P^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2,$$

$$\eta_V = M_P^2 \frac{V_{,\phi\phi}}{V}$$

Along a stable warm attractor

$\epsilon_V, |\eta_V| \ll 1 + Q$

# Radiation remains subdominant during inflation

$$\epsilon_H \simeq \frac{\epsilon_V}{1+Q}, \qquad \frac{\rho_r}{V} \simeq \frac{\epsilon_V Q}{2(1+Q)^2}.$$

$$T^4 \simeq \frac{\epsilon_V Q V}{2C_R(1+Q)^2}.$$

**Two different measures**

$Q$ measures friction; $T/H$ measures the thermal scale. $T > H$ need not require $Q > 1$.

# Perturb the whole interacting system

$$\delta\phi, \qquad \delta\dot{\phi},$$

$$\delta\rho_r \text{ (radiation perturbation)},$$

$$\Psi_r \text{ (momentum of radiation perturbation)}$$

- Radiation fluctuations $\Leftrightarrow$ temperature and friction.
- Momentum exchange couples the field to the fluid velocity.
- Metric constraints close the gravitational system.

# Metric conventions and gauge

$$\begin{aligned} ds^2 &= -(1+2\alpha)dt^2 - 2a\partial_i\beta dx^i dt \\ &+ a^2[\delta_{ij}(1+2\varphi) + 2\partial_i\partial_j\gamma]dx^i dx^j, \end{aligned}$$

metric equations

$$\chi = a(\beta + a\dot{\gamma}), \quad \kappa = 3(H\alpha - \dot{\varphi}) + \frac{k^2}{a^2}\chi,$$

$$-\frac{k^2}{a^2}\varphi + H\kappa = -\frac{1}{2M_{\rm Pl}^2}\delta\rho, \quad \kappa - \frac{k^2}{a^2}\chi = -\frac{3}{2M_{\rm Pl}^2}\Psi,$$

$$\dot{\chi} + H\chi - \alpha - \varphi = 0,$$

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = \frac{1}{2M_{\rm Pl}^2}\left(\delta\rho + 3\delta p\right),$$

# Thermal perturbations change the coefficient

For a general dissipation coefficient $\Upsilon \equiv \Upsilon(\phi, T)$:

$$\frac{\delta T}{T} = \frac{\delta \rho_r}{4\rho_r}, \qquad \delta\Upsilon = \Upsilon_{,\phi}\delta\phi + \Upsilon_{,T}\delta T,$$

$$\frac{\delta\Upsilon}{\Upsilon} = p\frac{\delta\phi}{\phi} + \frac{c}{4}\frac{\delta\rho_r}{\rho_r}.$$

where

$$c = \frac{\partial \ln \Upsilon(T,\phi)}{\partial \ln T},$$
$$p = \frac{\partial \ln \Upsilon(T,\phi)}{\partial \ln \phi},$$

The full evolution equations for the perturbations:

$$\begin{aligned}
\delta\ddot{\phi} &+ 3H(1+Q)\,\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V_{,\phi\phi} + \frac{3pHQ\dot{\phi}}{\phi}\right)\delta\phi \\
&+ \frac{cH}{\dot{\phi}}\delta\rho_r - \dot{\phi}(\kappa+\dot{\alpha}) - [2\ddot{\phi} + 3H(1+Q)\dot{\phi}]\alpha = (6HQT)^{1/2}\xi^{(\phi)}, \\
\delta\dot{\rho}_r &+ H\left(4 - \frac{3cQ\dot{\phi}^2}{4\rho_r}\right)\delta\rho_r - \frac{k^2}{a^2}\Psi_r - 6HQ\dot{\phi}\delta\dot{\phi} \\
&- \frac{3pHQ\dot{\phi}^2}{\phi}\delta\phi - \frac{4\rho_r}{3}\kappa + 3H\left(Q\dot{\phi}^2 + \frac{4\rho_r}{3}\right)\alpha = -(6HQT)^{1/2}\dot{\phi}\xi^{(\phi} \\
\dot{\Psi}_r &+ 3H\Psi_r + 3HQ\dot{\phi}\delta\phi + \frac{1}{3}\delta\rho_r + 4\rho_r\frac{\alpha}{3} = 0,
\end{aligned}$$

$$\langle \xi^{(\phi)}(\mathbf{k},t)\xi^{(\phi)}(\mathbf{k}',t')\rangle = \frac{1}{a^3}\delta(t-t')(2\pi)^3\delta(\mathbf{k}+\mathbf{k}').$$

# Metric constraints in the chosen gauge

Choice of gauge: uniform-curvature gauge $\varphi = 0$

$$
\begin{aligned}
H\kappa &= -\frac{1}{2M_p^2}\left(\dot\phi\delta\dot\phi - \dot\phi^2\alpha + V_{,\phi}\delta\phi + \delta\rho_r\right), \\
H\alpha &= \frac{1}{2M_p^2}\left(\dot\phi\delta\phi - \Psi_r\right), \\
H\dot\alpha &= -\dot H\alpha + \frac{1}{2M_p^2}\left\{\left[-3H(1+Q)\dot\phi - V_{,\phi}\right]\delta\phi + \dot\phi\delta\dot\phi - \dot\Psi_r\right\},
\end{aligned}
$$

A closed system

Substitute these into the three matter equations.

# Total curvature perturbation

$$\mathcal{R} = \frac{H(\dot{\phi}\delta\phi - \Psi_r)}{\dot{\phi}^2 + 4\rho_r/3}$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} S_{\mathcal{R}}(k).$$

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}^*_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') S_{\mathcal{R}}(k).$$

# Power spectrum: analytic approximation[1]

Including quantum fluctuations (A.A. Starobinsky 1988, Stochastich Inflation approach):

$$\Phi(\vec{x},t) \rightarrow \Phi_{>}(\vec{x},t) + \Phi_{<}(\vec{x},t)\ .$$

- $\Phi_{>}(\vec{x},t) \rightarrow$ long wavelenght (super horizon) part
- $\Phi_{<}(\vec{x},t) \equiv \phi_q(\vec{x},t) \rightarrow$ summarizes high frequency, short-wavelength (sub horizon) quantum fluctuations ($k > k_h \approx aH$):

$$\Phi_{<}(\mathbf{x},t) \equiv \phi_q(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} W(k,t) \left[\phi_{\mathbf{k}}(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \hat{a}_{\mathbf{k}} + \phi^*_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{a}^{\dagger}_{\mathbf{k}}\right]$$

$W(k,t)$ is a filter or window function that projects out the long wavelength modes.

[1] Ramos and da Silva, JCAP 03 (2013) 032, 1302.3544

Backreaction of $\Phi_<$ into the dynamics of local order parameter $\Phi_>$

$\Downarrow$

**modelled as a (quantum) noise term**

$\Downarrow$

combine with the derivation in the warm inflation context[2]

$$\left[\frac{\partial^2}{\partial t^2} + (3H + \Upsilon)\frac{\partial}{\partial t} - e^{-2Ht}\nabla^2\right]\Phi_>(\vec{x},t) + \frac{\partial V(\Phi_>)}{\partial \Phi_>} = \xi_q(\vec{x},t) + \xi_T(\vec{x},t)\ ,$$

quantum noise: $\xi_q = -\left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\nabla^2 + V_{,\phi\phi}(\Phi_>)\right]\phi_q$

$\Rightarrow$ Eq. for $\Phi_>$ is similar to a Langevin equation with quantum and thermal noise terms (stochastic process w/ gaussian noises).

[2] ROR and L. A. da Silva, JCAP 03 (2013) 032

Equation for the fluctuations:

$$\delta\ddot{\varphi}(\vec{k},t)+(3H+\Upsilon)\delta\dot{\varphi}(\vec{k},t)+V_{,\phi\phi}(\phi)\delta\varphi(\vec{k},t)+a^{-2}k^2\delta\varphi(\vec{k},t)=$$
$$\tilde{\xi_T}(\vec{k},t)+\tilde{\xi_q}(\vec{k},t)\ .$$

General solution can be expressed in terms of a Green function

$$\delta\varphi(\mathbf{k},z)=\int_z^\infty dz'\,G(z,z')\frac{(z')^{1-2\nu}}{z'^2H^2}\left[\tilde{\xi}_q(z')+\xi_T(z')\right],\quad (z\equiv\frac{k}{aH}),$$

$$G(z,z')=\frac{\pi}{2}z^\nu z'^\nu\left[J_\alpha(z)Y_\alpha(z')-J_\alpha(z')Y_\alpha(z)\right]\ ,z'>z$$

$$\nu=3(1+Q)/2\ ,\quad \alpha=\sqrt{\nu^2+\frac{3\beta Q}{1+Q}-3\eta}$$

$$\varepsilon=\frac{1}{16\pi G}\left[\frac{V_{,\phi}}{V}\right]^2\ ,\quad \eta=\frac{1}{8\pi G}\frac{V_{,\phi\phi}}{V}\ ,\quad \beta=\frac{1}{8\pi G}\frac{\Upsilon_{,\phi}V_{,\phi}}{\Upsilon V}$$

slow-roll coefficients: $\varepsilon,\eta,\beta\ll 1+Q,\ Q=\Upsilon/(3H)$

Thermal $\xi_T$ and quantal $\xi_q$ noises are uncorrelated (decoupled), so they give separated contributions to the power spectrum:

$$P_{\delta\varphi}(z) = \frac{k^3}{2\pi^2}\int \frac{d^3k'}{(2\pi)^3}\langle\delta\varphi(\mathbf{k},z)\delta\varphi(\mathbf{k}',z)\rangle = P^{(\mathrm{th})}_{\delta\varphi}(z) + P^{(\mathrm{qu})}_{\delta\varphi}(z)$$

Explicitly, we find:

$$P^{(\mathrm{qu})}_{\delta\varphi} \simeq \left[\frac{2}{\exp(H/T)-1}+1\right]\frac{H^2}{4\pi^2}$$

$$P^{(\mathrm{th})}_{\delta\varphi} \simeq \frac{H^2}{4\pi^2}\frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}\frac{T}{H}$$

Conventional inflation: $P_{\delta\varphi}(T/H\to 0)\to \frac{H^2}{4\pi^2}$

# Combine the quantum and thermal pieces

$$\mathcal{P}_{\mathcal{R}}^{\rm proxy} \simeq \left(\frac{H_*^2}{2\pi u_*}\right)^2 \left[1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3+4\pi Q_*}}\frac{T_*}{H_*}\right].$$

Checking the limiting behavior:

$$\mathcal{P}_{\delta\phi}^{T} \propto QHT \ (Q \ll 1), \qquad \propto \sqrt{Q}HT \ (Q \gg 1).$$

**Cold limit**

$Q \to 0$ and $n_* \to 0$ recover the vacuum result. A populated inflaton state is an additional assumption.

# Define the coupling correction operationally

$$G(Q_*) \equiv \frac{\mathcal{P}_{\mathcal{R}}^{\rm coupled}}{\mathcal{P}_{\mathcal{R}}^{\rm proxy}}.$$

$$\mathcal{P}_{\mathcal{R}} \simeq \left(\frac{H_*^2}{2\pi u_*}\right)^2 \left[1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3+4\pi Q_*}}\frac{T_*}{H_*}\right] G(Q_*).$$

**Specify the calibration**

$G(Q)$ is obtained by numerically solving the complete set of equations. $G(Q)$ is defined relative to a stated proxy. It can absorb more than temperature feedback, including noise prescription and proxy errors; it is not universally a function only of Q

# Spectral tilt, running, and tensors

$$n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}, \quad \alpha_s = \frac{dn_s}{d \ln k}, \quad \frac{d \ln k}{dN_*} = 1 - \epsilon_{H*}.$$

$$\mathcal{P}_t \simeq \frac{2H_*^2}{\pi^2 M_P^2}, \quad r = \frac{\mathcal{P}_t}{\mathcal{P}_{\mathcal{R}}}, \quad n_t \simeq -2\epsilon_H.$$

**Leading vacuum tensors**

Scalar enhancement can suppress $r$; thermal tensor states or additional tensor sources require separate physics.

# Computing a spectrum without sampling many noises

G. S. Rodrigues and R. O. R., "WI2easy: warm inflation dynamics made easy," JCAP **09**, 014 (2025)

linear Langevin system $\rightarrow$ Fokker–Planck equation

$\longrightarrow$ matrix ODE system.

**WI2easy's central idea**

For linear perturbations with Gaussian sources, second moments form a closed deterministic system.

# A first-order stochastic vector

$$\Phi = (\delta\phi, \frac{d\delta\phi}{dN}, \delta\rho_r, \Psi_r)^T,$$

$$d\Phi = -A(N,k)\Phi\, dN + B(N,k)\, dW, \qquad \langle dW_a dW_b \rangle = \delta_{ab}\, dN.$$

- Eliminate lapse and expansion perturbations using constraints.
- Read from the full perturbation equations the elements of the $(4 \times 4)$ matrix.
- Columns of $B$ label noise amplitudes; rows label variables.

# The Fokker–Planck implementation

$$\mathbf{\Phi} = \left(\delta\phi, \frac{d\delta\phi}{dN_e}, \delta\rho_r, \Psi_r\right)^T \Rightarrow \frac{d\mathbf{\Phi}}{dN_e} + \mathbf{A}\mathbf{\Phi} = \mathbf{B}\xi_{N_e},$$

The power spectrum in matrix form:

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\langle\left|\frac{H}{\rho_T + p_T}\left(\dot{\phi}\delta\phi - \Psi_r\right)\right|^2\rangle \Rightarrow \frac{k^3}{2\pi^2}\mathbf{C}^T.\langle\mathbf{\Phi}\mathbf{\Phi}^\dagger\rangle.\mathbf{C}\,,$$

$$\langle\mathbf{\Phi}\mathbf{\Phi}^\dagger\rangle = \int\prod_i d\Phi_i \int\prod_j d\Phi_j^* P(\mathbf{\Phi}, \mathbf{\Phi}^*, N_e)\mathbf{\Phi}\mathbf{\Phi}^\dagger,$$

with $P(\mathbf{\Phi}, \mathbf{\Phi}^*, N_e)$ is the probability density, which is the solution of the Fokker-Planck equation,

$$\frac{\partial P}{\partial N_e} = \sum_{i,j}\left[A_{ij}\frac{\partial}{\partial\Phi_i}\left(\Phi_j P\right) + A_{ij}\frac{\partial}{\partial\Phi_i^*}\left(\Phi_j^* P\right) + \left(\mathbf{B}\mathbf{B}^T\right)_{ij}\frac{\partial^2 P}{\partial\Phi_i\partial\Phi_j^*}\right].$$

# Project covariance onto curvature

$$\mathbf{J} \equiv \langle \mathbf{\Phi}\mathbf{\Phi}^{\dagger} \rangle \Rightarrow \frac{d\mathbf{J}}{dN_e} = -\mathbf{A}.\mathbf{J} - \mathbf{J}.\mathbf{A}^T + \mathbf{B}.\mathbf{B}^T.$$

$$C = \frac{H}{\dot{\phi}^2 + 4\rho_r/3}(\dot{\phi}, 0, 0, -1)^T, \qquad \mathcal{R} = C^T \mathbf{\Phi},$$

$$\boxed{\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} C^T.\mathbf{J}.C}$$

- Field–radiation cross-covariances enter automatically.
- The final-time curvature projection includes radiation momentum.

# Convergence checks that matter

- Begin sufficiently inside the horizon and test the starting time.
- Evolve far enough outside the horizon and test the final time.
- Increase precision

**A stable answer**

Convergence of the spectrum matters more than one apparently smooth trajectory.

# WI2easy: practical workflow

See arxiv:2504.17760 for code public repository github

1. Load the Mathematica package and define the model.
2. Generate consistent background initial conditions.
3. Solve covariance evolution and calibrate $G(Q)$.
4. Normalize the potential and compute observables.

# Load the package in a notebook

```
SetDirectory[NotebookDirectory[]];
Get["WI2easy.m"];
ParametersWI[]
```

- Place the package beside the notebook or on the Mathematica path.
- Use Mathematica 13.3 or later, as specified in the paper.
- Enter the potential with the normalization symbol `V0`.

# Worked setup: quartic plus linear temperature

| | |
|---|---|
| Potential | `V0*x^4/4` |
| Initial normalization | `10^(-14)` |
| Field guess | 25 |
| Potential shape | Large field; no hybrid endpoint |
| Dissipation function | `T` |
| Names | `quartic`, `linearT` |
| Bath degrees of freedom | `12.5` |
| Duration guess | 60 |
| Thermal occupation | Yes, then repeat with No |
| Radiation noise | Record choice and covariance prescription |

**Units**

$x = \phi/M_P$; code variables use reduced Planck units.

# Find consistent initial conditions

```
FindICs[Qi, Qf];
```

- Produces an initial-condition grid for $Q \in [Q_i, Q_f]$ and dissipation-slope data.
- Inspect the background before running perturbations.
- Choose a range that covers the eventual normalized trajectory.

**Quick background test**

For example: `FindICs[0.001,20]` tests one value; a single point is insufficient for a robust interpolated $G(Q)$.

# Inspect background functions

```
Plot[Qdiss[ne], {ne, 0, 60}]
Plot[TH[ne], {ne, 0, 60}]
Plot[epsilon0[ne], {ne, 0, 60}]
```

- Other functions: `phi`, `dphi`, `rad`, `Temp`, `Hubble`.
- Restrict plots to the actual integration domain.
- The initial normalization has not yet been fitted.

**Physics check**

$T/H$, bath relaxation, energy fractions, and end of acceleration.

# Generate the coupled correction

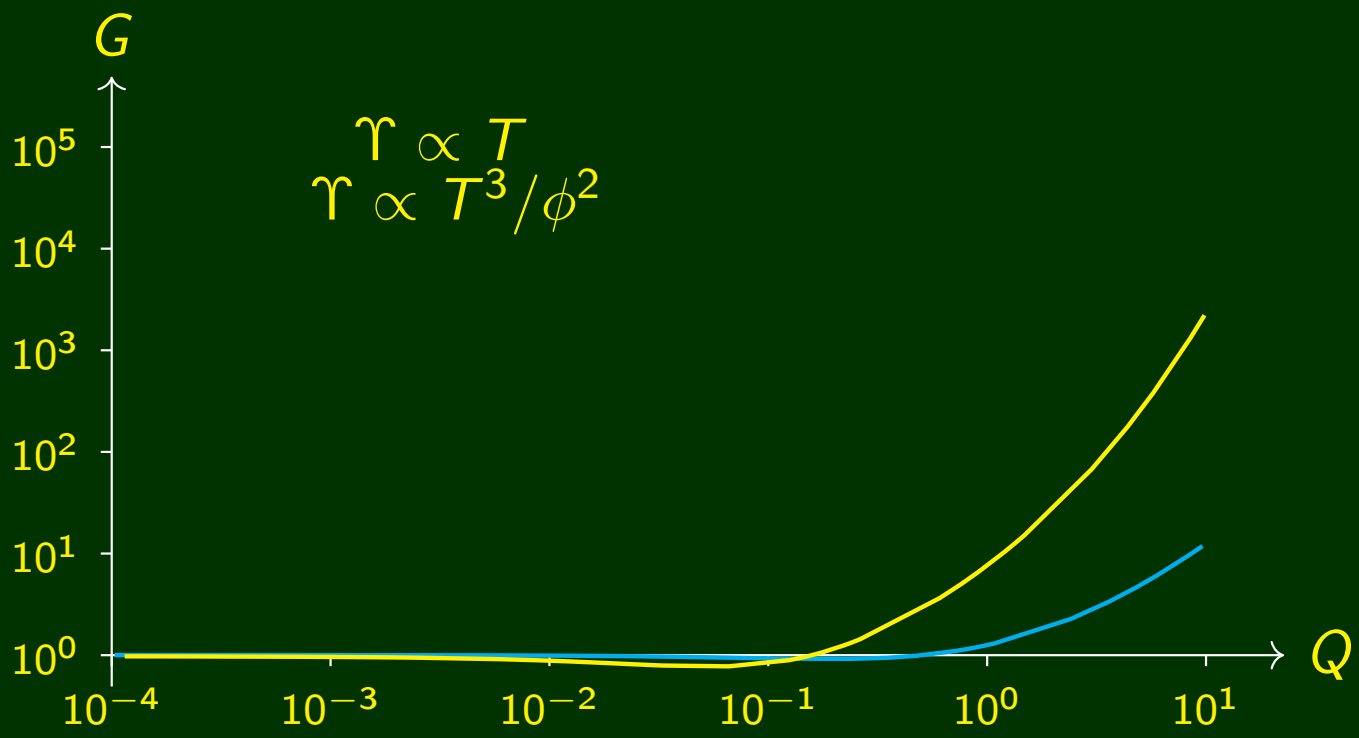
```
FindGQ[];
```

- Integrates perturbation covariance for the saved background grid.
- Compares the late-time spectrum with the chosen analytical proxy.
- Writes the $G(Q)$ data used in normalization.

**Default window in the paper**

Approximately seven e-folds before crossing and seven afterward; test whether this suffices for the model.

WI2easy output for a quartic potential with $T$ and $T^3/\phi^2$ dissipation




$G(Q)$ behavior illustration

Growth depends on the dissipation law.

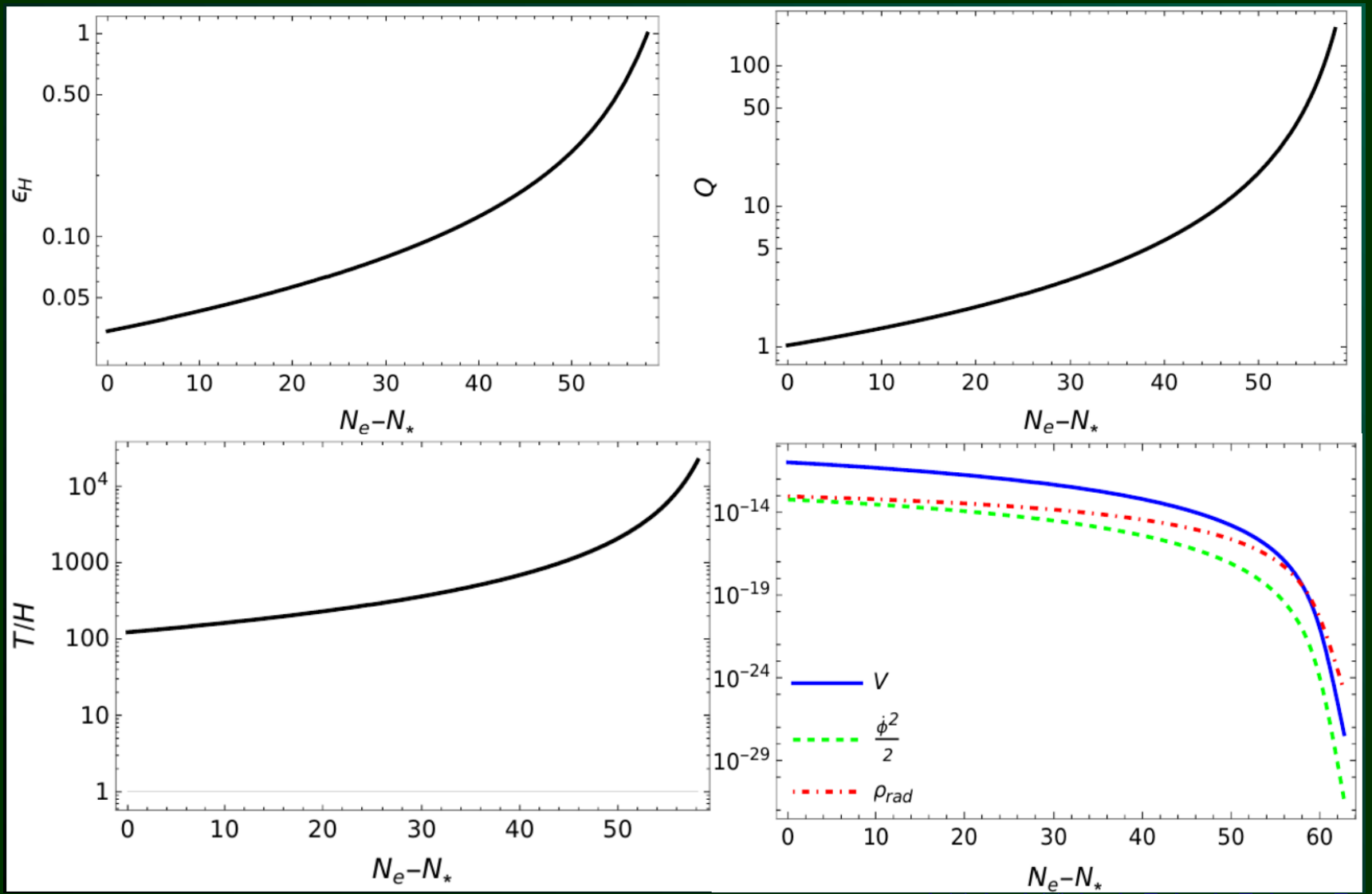
# Normalize and compute observables

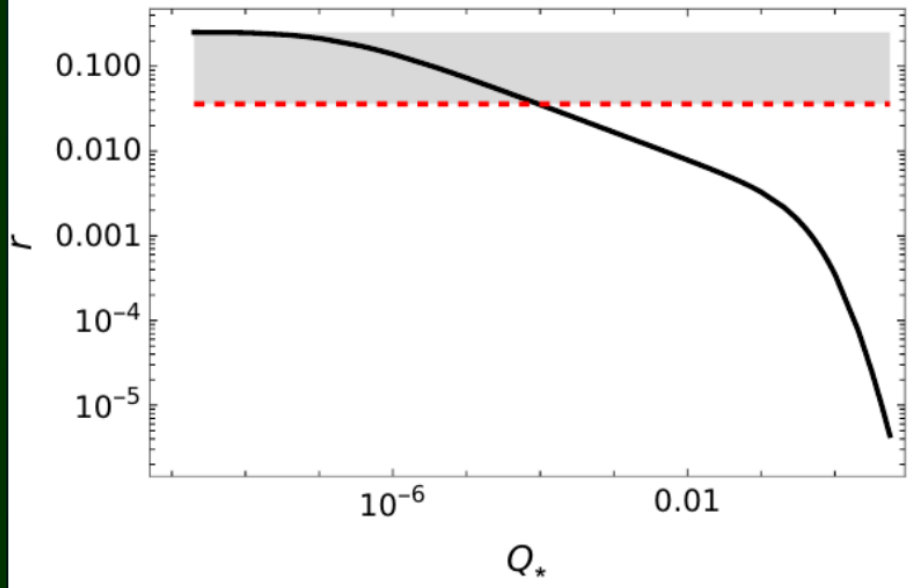
```
ObservationsWI[0.1, 25];
QrangeWI[0.1];
```

- The first command normalizes `V0` and finds the selected trajectory.
- The second scans normalized solutions in $Q$.

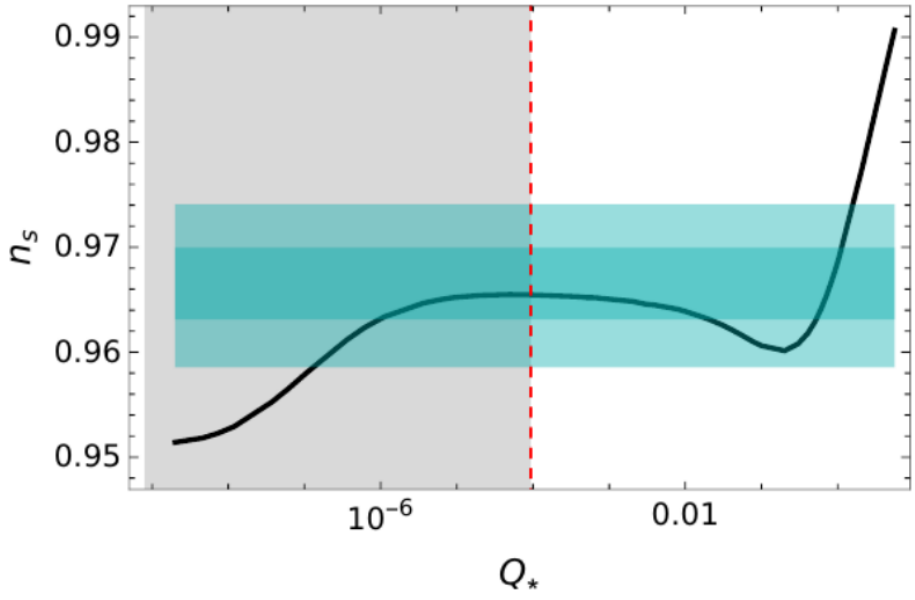
Defaults used in the code: $\ln(10^{10} A_s) = 3.047$ and $k_{\mathrm{pivot}} = 0.05 Mpc^{-1}$

# Examples of results for a quartic potential and dissipation coefficient $\Upsilon \propto T$

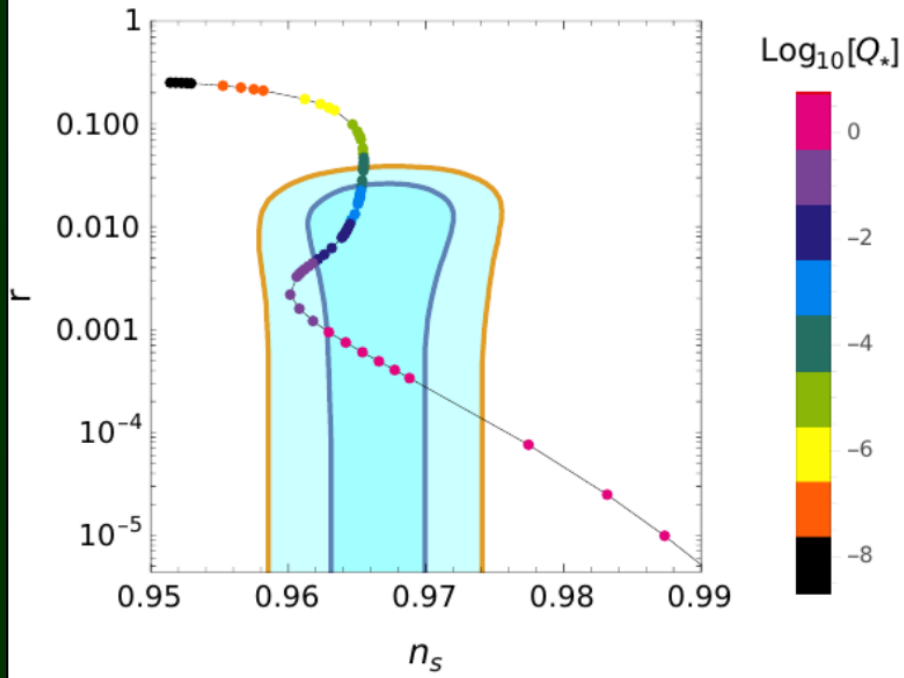


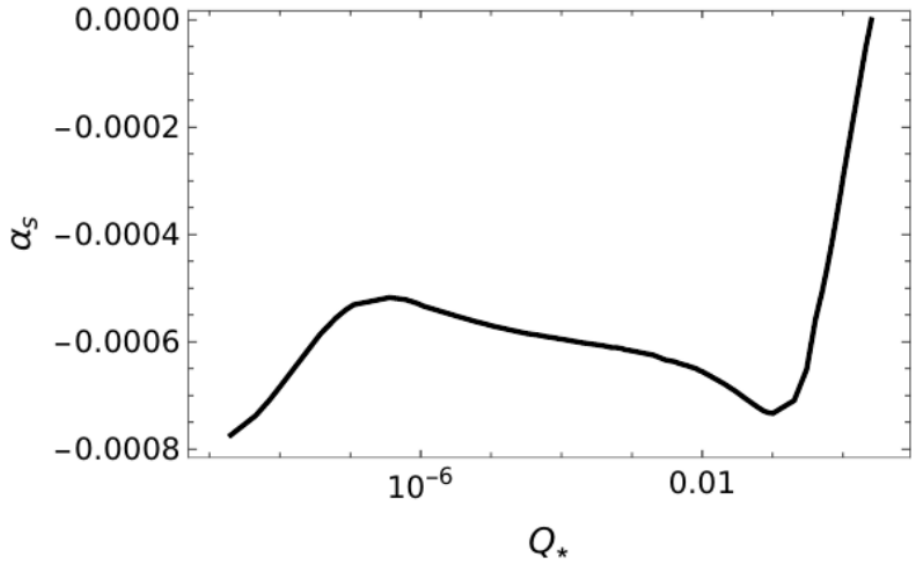

0.100
0.010
0.001
$10^{-4}$
$10^{-5}$
$r$
$10^{-6}$
0.01
$Q_*$

(a)


0.99
0.98
0.97
0.96
0.95
$n_s$
$10^{-6}$
0.01
$Q_*$

(b)

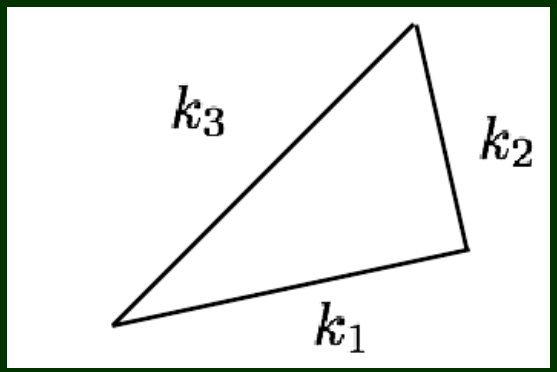

1
0.100
0.010
0.001
$10^{-4}$
$10^{-5}$
$r$
0.95
0.96
0.97
0.98
0.99
$n_s$
$\mathrm{Log}_{10}[Q_*]$
0
−2
−4
−6
−8


0.0000
−0.0002
−0.0004
−0.0006
−0.0008
$\alpha_s$
$10^{-6}$
0.01
$Q_*$

# Beyond two-point statistics: Non-Gaussianities

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(k_1,k_2,k_3)$$

$\Phi \equiv (3/5)\zeta$, $\zeta =$ comoving curvature perturbation



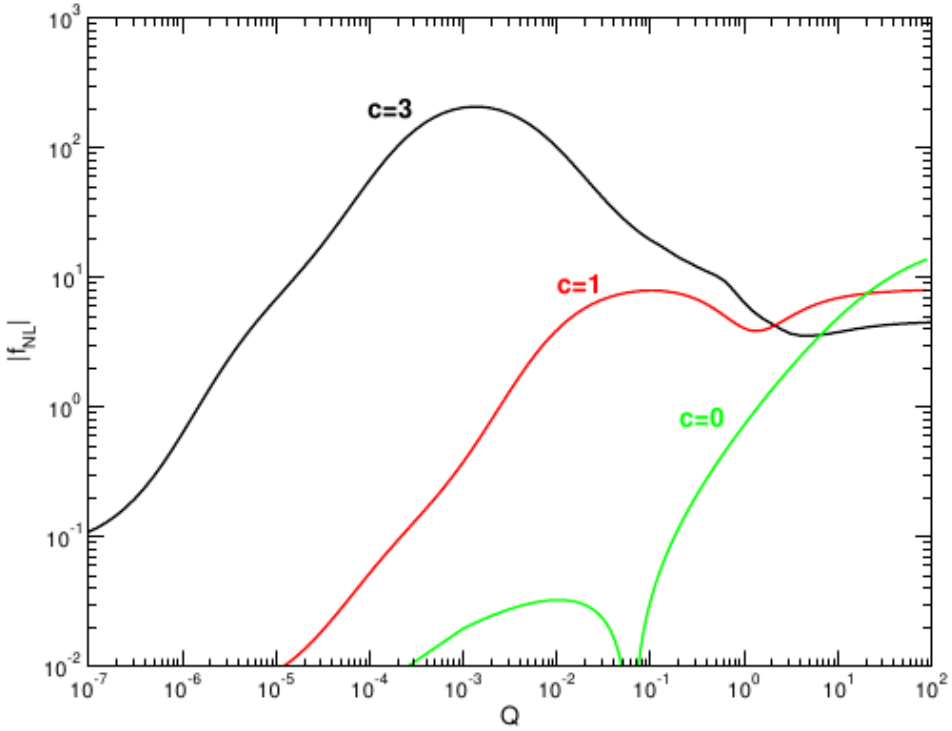

Nonlinearity parameter $f_{NL}$:

$$B_\Phi(k_1,k_2,k_3) = f_{NL}F(k_1,k_2,k_3)$$

function $F(k_1,k_2,k_3)$ depends on the shape (type of triangle)

# Warm Inflation and Non-Gaussianity[3]

$f_{NL}$ relative to the equilateral shape: $f_{NL} = \frac{18}{5}\frac{B(k,k,k)}{P(k)^2}$, for $\Upsilon \propto T^c$

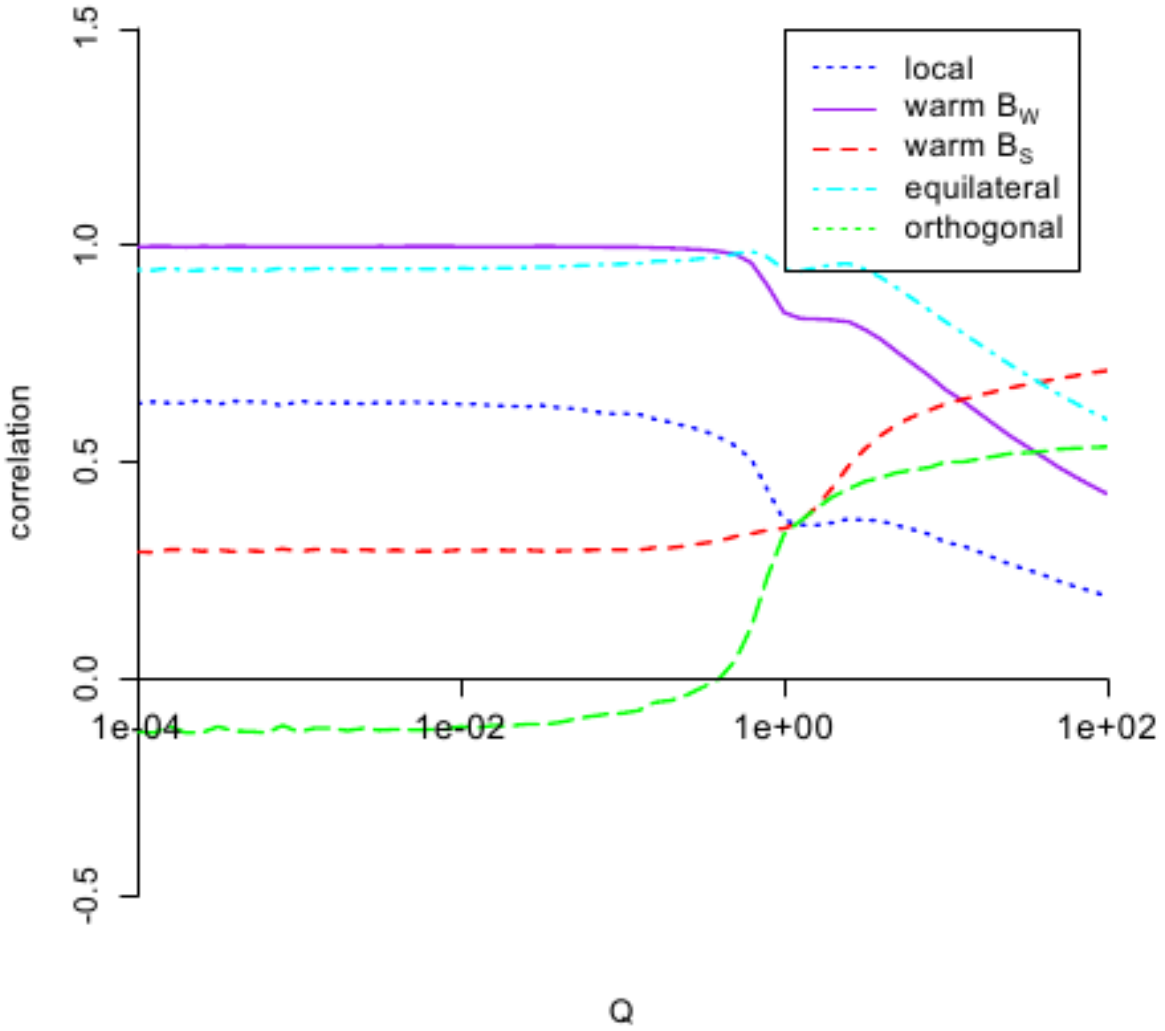



Non-linearity parameter $|f_{NL}|$ versus $Q$ for different values of $c$ ($\Upsilon \propto T^c$) as indicated in the plot.

Planck constraints:

$f_{NL}^{\rm local} = 2.7 \pm 5.8\,,\ f_{NL}^{\rm equi} = -42 \pm 75\,,\ f_{NL}^{\rm warm} = 4 \pm 33$

[3] M. Basteiro-Gil, A. Berera, I. G. Moss, R.O.R., JCAP12, 008 (2014)




(a)The correlation between the numerical bispectrum and the template shapes plotted as a function of the dissipation strength parameter $Q$.

# The lectures in one chain

microphysics $\to$ response and noise $\to \Upsilon \to$ background

$\to$ coupled perturbations $\to \mathcal{P}_{\mathcal{R}},\ B_{\mathcal{R}} \to$ tests.

## Reading

Ramos–da Silva: 1302.3544; Graham–Moss: 0905.3500.
Rodrigues–Ramos, WI2easy: 2504.17760; code repository.
Bastero-Gil et al., non-Gaussianity: 1408.4391.