Squeezing as classical behavior

Density matrices

Decoherence

Quantum Origins of Cosmic Structure: From Vacuum Fluctuations to Primordial Inhomogeneities

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Lecture 3

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Something is not clear...

- We recall the angular spectrum,

$$C_l = 4\pi \int \frac{dk}{k} \Delta_l^2(k) \mathcal{P}_{\mathcal{R}}(k) \tag{1}$$

where

$$\mathbb{E}\{\mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'}\} = (2\pi^2/k^3) \mathcal{P}_{\mathcal{R}}(k) \delta^{(3)}(\mathbf{k} + \mathbf{k}') \tag{2}$$

and $\mathbb{E}\{...\}$ denotes an ensemble average of the **classical field** $\mathcal{R}_k$

Something is not clear...

- For the *primordial power spectrum* we ask: How does the identification

$$\mathbb{E}\{\mathcal{R}^{\rm cl}_{\mathbf{k}}\mathcal{R}^{\rm cl}_{\mathbf{k}'}\} = \langle {\rm BD}|\hat{\mathcal{R}}_{\mathbf{k}}\hat{\mathcal{R}}_{\mathbf{k}'}|{\rm BD}\rangle \tag{3}$$

  takes place ? Remember $\mathcal{R}_{\mathbf{k}} = v_{\mathbf{k}}/z$

- **Justification 1:** We present the most popular approaches:

(i) *Non-commutativity 'fades' during inflation*

(ii) **Quantum squeezing as feature of the emergence of classical behavior**

(iii) **Decoherence**

Outline

Squeezing as classical behavior

Density matrices

Decoherence

1. Squeezing as classical behavior
2. Density matrices
3. Decoherence

A Gaussian packet in an inverted potential

We begin with the Hamiltonian of an inverted oscillator with re-scaled canonical variables $Q = \sqrt{m\omega}\, x$ and $P = p/\sqrt{m\omega}$,

$$\hat{H} = \frac{\omega}{2}(\hat{P}^2 - \hat{Q}^2), \qquad [\hat{Q}, \hat{P}] = i. \tag{4}$$

Choose this Gaussian as the initial state of the inverted oscillator.

$$\psi_0(Q) = \pi^{-1/4} e^{-Q^2/2}, \qquad \hat{a}|0\rangle = 0, \qquad \hat{a} = \frac{\hat{Q}(0) + i\hat{P}(0)}{\sqrt{2}}. \tag{5}$$

with

$$(\Delta Q)^2 = (\Delta P)^2 = \frac{1}{2}, \qquad \frac{1}{2}\langle\{\hat{Q}, \hat{P}\}\rangle = 0. \tag{6}$$

Growing and squeezed quadratures

The equations $\dot{\hat{Q}} = \omega \hat{P}$ and $\dot{\hat{P}} = \omega \hat{Q}$ simplify in rotated quadratures:

$$\hat{U} = \frac{\hat{Q} + \hat{P}}{\sqrt{2}}, \qquad \hat{V} = \frac{\hat{P} - \hat{Q}}{\sqrt{2}}. \tag{7}$$

They give an amplified direction and a contracting direction:

$$\hat{U}(t) = e^{r}\hat{U}(0), \qquad \hat{V}(t) = e^{-r}\hat{V}(0), \qquad r = \omega t. \tag{8}$$

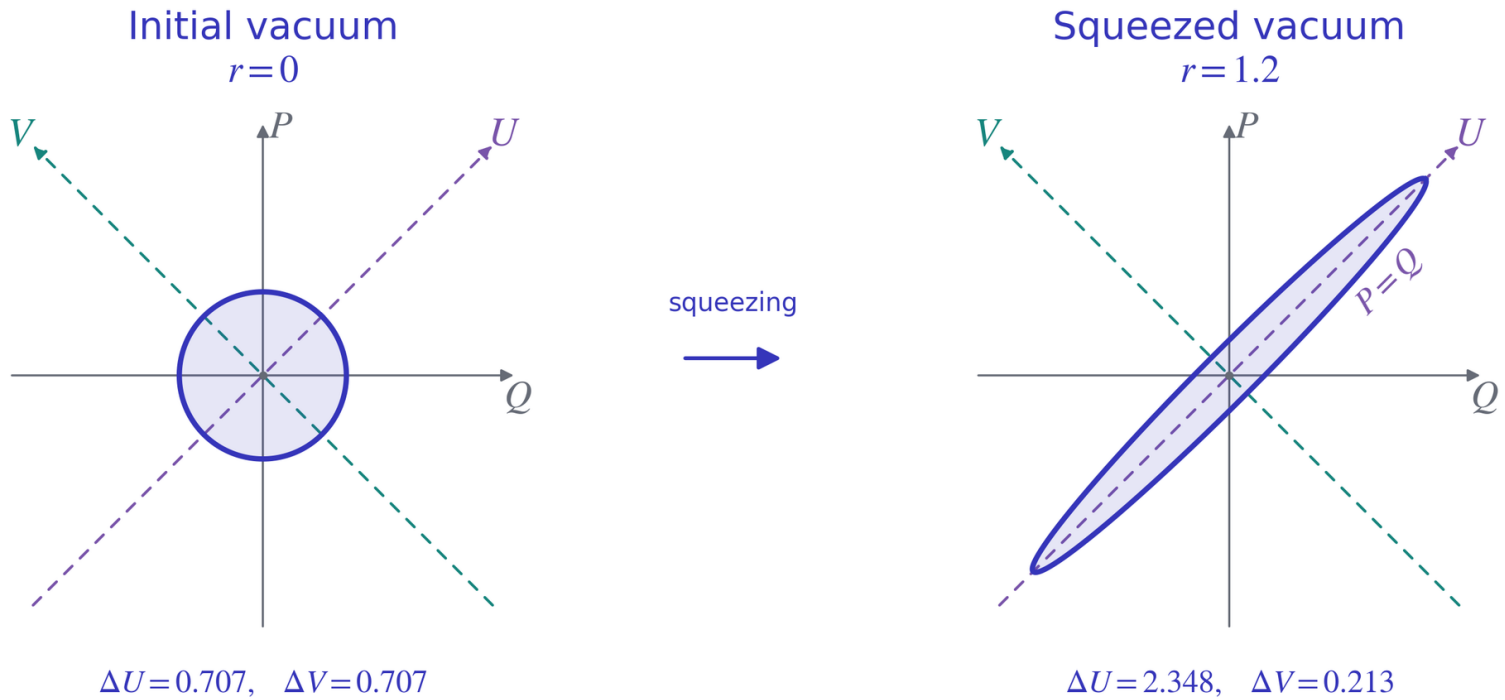
The vacuum uncertainties therefore become

$$(\Delta U)^2 = \frac{1}{2}e^{2r}, \qquad (\Delta V)^2 = \frac{1}{2}e^{-2r}. \tag{9}$$

Squeezing as classical behavior
Density matrices
Decoherence

The squeezing parameter

For $r \gg 1$, the uncertainty ellipse is broad along U and narrow along V. Since $V = (P - Q)/\sqrt{2}$, its long axis follows $P = Q$.



The growing amplitude and its probability

The exact solution contains two initial quadratures:

$$\hat{Q}(t) = \frac{e^r \hat{U}(0) - e^{-r}\hat{V}(0)}{\sqrt{2}},$$
$$\hat{P}(t) = \frac{e^r \hat{U}(0) + e^{-r}\hat{V}(0)}{\sqrt{2}}, \qquad r = \omega t. \tag{10}$$

For $r \gg 1$, the growing contribution determines both observables:

$$\hat{Q}(t) \simeq \hat{P}(t) \simeq \frac{e^r}{\sqrt{2}}\hat{U}(0). \tag{11}$$

Let $|u\rangle$ satisfy $\hat{U}(0)|u\rangle = u|u\rangle$.

The Born rule yields

$$\mathcal{P}(u) = |\langle u|0\rangle|^2 = \frac{e^{-u^2}}{\sqrt{\pi}}, \qquad \langle \hat{U}^2(0)\rangle = \int du\, u^2 \mathcal{P}(u) = \frac{1}{2}. \tag{12}$$

Thus $\mathcal{P}(u)\, du$ is the probability as determined by the initial quantum state.

One amplitude defines a classical history

Construct an ensemble by sampling a real number u from $\mathcal{P}(u)$. For each sample, the Eqs. of motion $\dot{Q} = \omega P$ and $\dot{P} = \omega Q$, have a classical growing solution to:

$$Q^{\rm cl}(t;u) = P^{\rm cl}(t;u) = \frac{e^{\omega t}}{\sqrt{2}} u. \tag{13}$$

The same u is used at every time.

The classical average over those trajectories gives:

$$\begin{aligned} \mathbb{E}[Q^{\rm cl}(t)Q^{\rm cl}(t')] &= \int du\, \mathcal{P}(u) Q^{\rm cl}(t;u) Q^{\rm cl}(t';u) \\ &= \frac{e^{\omega(t+t')}}{2} \int du\, \mathcal{P}(u) u^2 = \frac{e^{\omega(t+t')}}{4}. \end{aligned} \tag{14}$$

For $\omega t, \omega t' \gg 1$, the quantum correlation is

$$\langle \hat{Q}(t)\hat{Q}(t') \rangle \simeq \frac{e^{\omega(t+t')}}{2} \langle \hat{U}^2(0) \rangle = \frac{e^{\omega(t+t')}}{4}. \tag{15}$$

One amplitude defines a classical history

Therefore,

$$\langle \hat{Q}(t)\hat{Q}(t')\rangle \simeq \mathbb{E}[Q^{\rm cl}(t)Q^{\rm cl}(t')]. \tag{16}$$

an ensemble of classical growing trajectories reproduces the dominant quantum correlations considered in our example.

Squeezing generates the common growing evolution; but **the Born rule** gives supplies the statistical distribution.

A Gaussian vacuum for each Fourier mode

We now consider linear perturbations in canonical single-field attractor inflation, with $\hbar = c = 1$:

$$v = z\mathcal{R}, \qquad z = a\sqrt{2\epsilon_1}\, M_{\rm P}, \qquad \epsilon_1 = -\frac{\dot{H}}{H^2}. \tag{17}$$

For simplicity let us put the field in finite box with periodic bound. cond. The **real mode component** is an oscillator:

$$\hat{q}_{\mathbf{k}} = \frac{\hat{v}_{\mathbf{k}} + \hat{v}_{-\mathbf{k}}}{\sqrt{2}}, \qquad \hat{b}_{\mathbf{k}} = \frac{\hat{a}_{\mathbf{k}} + \hat{a}_{-\mathbf{k}}}{\sqrt{2}}. \tag{18}$$

Its operator expansion and initial vacuum satisfy

$$\hat{q}_{\mathbf{k}} = f_k \hat{b}_{\mathbf{k}} + f_k^* \hat{b}_{\mathbf{k}}^\dagger, \qquad \hat{b}_{\mathbf{k}} |0_{\rm BD}\rangle = 0. \tag{19}$$

Deep inside the Hubble radius, the Bunch–Davies condition gives

$$f_k(\eta) \simeq \frac{e^{-ik\eta}}{\sqrt{2k}}, \qquad k \gg aH, \qquad k = |\mathbf{k}|. \tag{20}$$

This is the analogue of the initial vacuum of a simple oscillator.

Expansion produces squeezing

With $\hat{p}_{\mathbf{k}} = \hat{q}'_{\mathbf{k}}$ and $[\hat{q}_{\mathbf{k}}, \hat{p}_{\mathbf{k}}] = i$, the mode Hamiltonian is

$$\hat{H}_{\mathbf{k}} = \frac{1}{2}\left[\hat{p}_{\mathbf{k}}^2 + \Omega_k^2(\eta)\hat{q}_{\mathbf{k}}^2\right], \qquad \Omega_k^2 = k^2 - \frac{z''}{z}. \tag{21}$$

For $k^2 \ll z''/z$, the effective potential is inverted, as in the quantum-mechanical example.

We rescale the operators, $Q_{\mathbf{k}} = \sqrt{k}\, q_{\mathbf{k}}$, $P_{\mathbf{k}} = p_{\mathbf{k}}/\sqrt{k}$. And employ the following quadratures

$$\hat{X}_k(\eta) = \cos\theta_k(\eta)\hat{Q}_k(\eta) + \sin\theta_k(\eta)\hat{P}_k(\eta), \tag{22}$$

$$\hat{Y}_k(\eta) = -\sin\theta_k(\eta)\hat{Q}_k(\eta) + \cos\theta_k(\eta)\hat{P}_k(\eta), \tag{23}$$

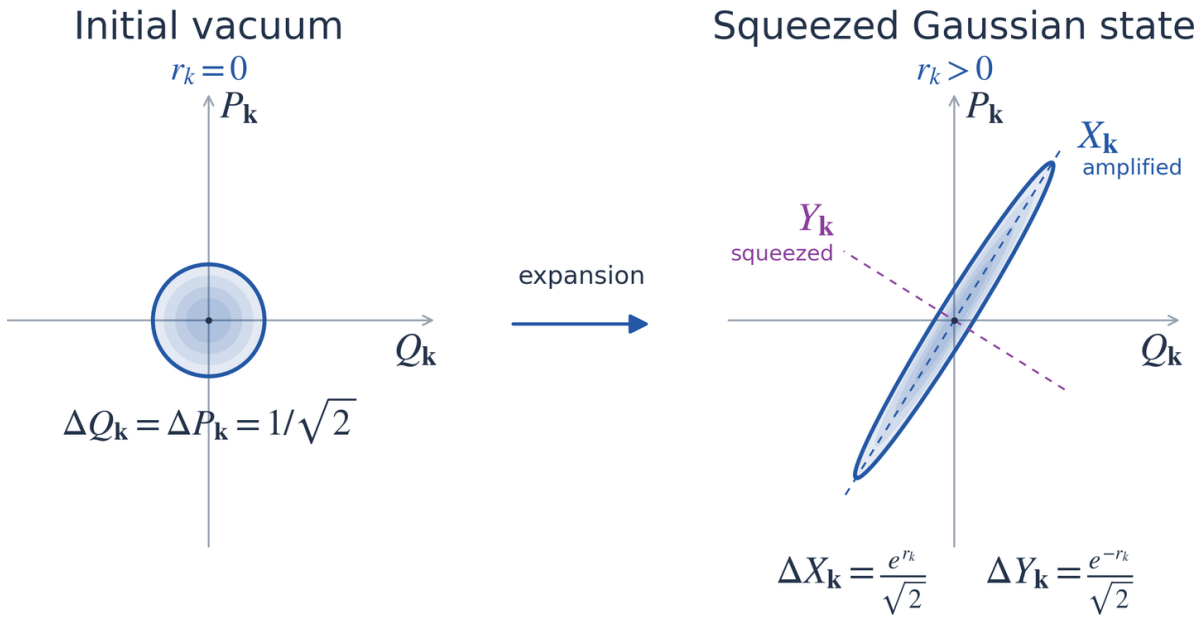
This rotation preserves $[\hat{X}_k, \hat{Y}_k] = i$
The initial uncertainty contour is circular. Along its evolving principal axes, the variances become

$$(\Delta X_{\mathbf{k}})^2 = \frac{1}{2}e^{2r_k}, \qquad (\Delta Y_{\mathbf{k}})^2 = \frac{1}{2}e^{-2r_k}. \tag{24}$$

Expansion produces squeezing

For this Gaussian state, the squeezing follows from the mode functions:

$$\begin{aligned} \cosh(2r_k(\eta)) &= k|f_k|^2 + \frac{|f_k'|^2}{k}, \\ &\simeq -2\ln|k\eta|, \qquad \text{for } -k\eta \ll 1 \qquad (25) \\ \theta_k(\eta) &= \frac{1}{2}\arccos\left[\frac{2k^2\eta^2-1}{\sqrt{1+4k^4\eta^4}}\right] \\ &\simeq \frac{\pi}{2} + k\eta \qquad \text{for } -k\eta \ll 1 \qquad (26) \end{aligned}$$



The growing solution

The equation $f_k'' + (k^2 - z''/z) f_k = 0$ gives, for $k \ll aH$,

$$f_k \simeq z(C_k + D_k I), \qquad I' = \frac{1}{z^2}, \tag{27}$$

where C_k and D_k are complex numbers, fixed by the BD solution. Define **the Hermitian amplitude operators**

$$\begin{aligned} \hat{\mathcal{A}}_{\mathbf{k}} &= C_k \hat{b}_{\mathbf{k}} + C_k^* \hat{b}_{\mathbf{k}}^\dagger, \\ \hat{\mathcal{B}}_{\mathbf{k}} &= D_k \hat{b}_{\mathbf{k}} + D_k^* \hat{b}_{\mathbf{k}}^\dagger. \end{aligned} \tag{28}$$

As the growing solution dominates,

$$\hat{q}_{\mathbf{k}} \simeq z(\hat{\mathcal{A}}_{\mathbf{k}} + I\hat{\mathcal{B}}_{\mathbf{k}}) \simeq z\hat{\mathcal{A}}_{\mathbf{k}}, \qquad \hat{p}_{\mathbf{k}} \simeq z'\hat{\mathcal{A}}_{\mathbf{k}}. \tag{29}$$

The analogue of the growing quadrature

$\hat{\mathcal{A}}_{\mathbf{k}}$ plays the role of $\hat{U}(0)$. The elongated ellipse follows $p_{\mathbf{k}} \simeq (z'/z) q_{\mathbf{k}}$, and the real curvature component freezes: $\hat{\mathcal{R}}_{\mathbf{k}}^R \equiv \hat{q}_{\mathbf{k}}/z \simeq \hat{\mathcal{A}}_{\mathbf{k}}$.

The Born distribution of the curvature amplitude

For this real mode, let

$$\hat{\mathcal{A}}_{\mathbf{k}}|A\rangle = A|A\rangle, \qquad A \in \mathbb{R}. \tag{30}$$

Born's rule yields

$$\mathcal{P}_k(A) = |\langle A|0_{\rm BD}\rangle|^2 = \frac{e^{-A^2/(2s_k^2)}}{\sqrt{2\pi s_k^2}}, \qquad s_k^2 = |C_k|^2. \tag{31}$$

Therefore, the amplitude moments satisfy

$$\langle \hat{\mathcal{A}}_{\mathbf{k}}\rangle = 0, \qquad \langle \hat{\mathcal{A}}_{\mathbf{k}}^2\rangle = \int dA\, \mathcal{P}_k(A) A^2 = s_k^2. \tag{32}$$

The same probability construction

The real number A now labels a growing curvature amplitude, just as u labeled the growing amplitude of the inverted oscillator. The initial quantum state (and Born's rule) gives its statistical distribution.

Identification of expectation values

The classical MS equation in the super-Hubble regime $q_k'' - (z''/z)q_k = 0$ Hence

$$q_{\mathbf{k}}^{\rm cl}(\eta; A) = z(\eta)A, \qquad p_{\mathbf{k}}^{\rm cl}(\eta; A) = z'(\eta)A. \tag{33}$$

Weight these classical solutions with $\mathcal{P}_k(A)$ and use the same A at every time. The classical ensemble gives

$$\mathbb{E}[q_{\mathbf{k}}^{\rm cl}(\eta)q_{\mathbf{k}}^{\rm cl}(\eta')] = z(\eta)z(\eta')\int dA\,\mathcal{P}_k(A)A^2 = z(\eta)z(\eta')s_k^2. \tag{34}$$

The quantum field correlation is

$$\langle\hat{q}_{\mathbf{k}}(\eta)\hat{q}_{\mathbf{k}}(\eta')\rangle \simeq z(\eta)z(\eta')\langle\hat{\mathcal{A}}_{\mathbf{k}}^2\rangle = z(\eta)z(\eta')s_k^2. \tag{35}$$

It seems that

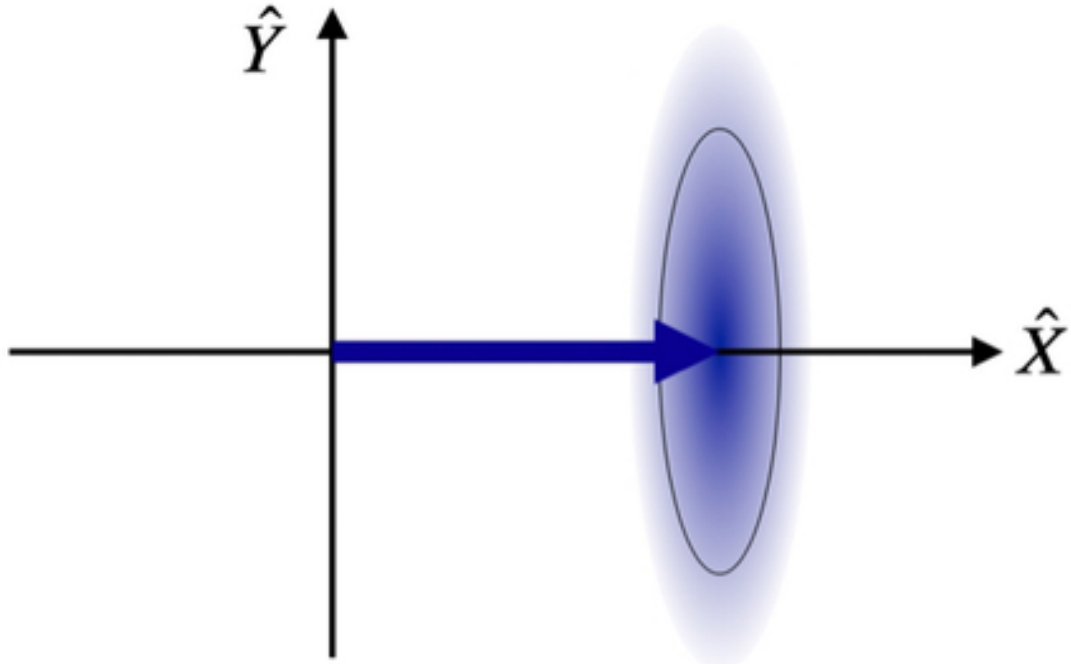
squeezing justifies the identification $\langle\hat{q}_{\mathbf{k}}(\eta)\hat{q}_{\mathbf{k}}(\eta')\rangle \simeq \mathbb{E}[q_{\mathbf{k}}^{\rm cl}(\eta)q_{\mathbf{k}}^{\rm cl}(\eta')]$

C. Kiefer and D. Polarski, arXiv:0810.0087, Sec. III.

Squeezing preserves quantum coherence

Squeezing as classical behavior
Density matrices
Decoherence

- The Born rule is a postulate of quantum mechanics that gives the probability **that a measurement of a quantum system** will yield a given result.
- Squeezing alone does not account for a **definite outcome**.



1. Density matrix: pure state

For a normalized pure state $|\psi\rangle$, define

$$\rho = |\psi\rangle\langle\psi|.$$

In an orthonormal basis, $|\psi\rangle = \sum_i c_i|i\rangle$,

$$\rho_{ij} = \langle i|\rho|j\rangle = c_i c_j^*.$$

- Diagonal entries give probabilities: $\rho_{ii} = P(i)$ **for a measurement** in this basis.
- Off-diagonal entries $\langle i|\rho|j\rangle$ with $i \neq j$ describe *quantum interference relative to this basis*.

Squeezing as classical behavior

Density matrices

Decoherence

Expectation values

We can compute expectation values,

$$\mathrm{Tr}(\rho A) = \sum_n \langle n|\psi\rangle\langle\psi|A|n\rangle = \langle\psi|A|\psi\rangle,$$

for some ON basis $\{|n\rangle\}$. If the **outcome** corresponds to a normalized state $|a\rangle$, its probability is

$$P(a) = \langle a|\rho|a\rangle = \mathrm{Tr}(\rho|a\rangle\langle a|)$$

Every density matrix satisfies $\rho^\dagger = \rho$, and $\mathrm{Tr}\,\rho = 1$. For a pure state

$$\mathrm{Tr}\,\rho^2 = 1 \quad \text{(pure)},$$

2. Density matrix: mixed state

- A **mixed density matrix** describes a quantum state that cannot be represented by a single state vector $|\psi\rangle$.
- For example a system prepared in the state $|0\rangle$ with probability 3/4, and in the state $|1\rangle$ with probability 1/4. **No superposition**
- However, the rules for computing probabilities and expectation values are exactly the same as for a pure density matrix.
- More generally, a mixed state can be written as

$$\rho = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|, \qquad p_\alpha \geq 0, \qquad \sum_\alpha p_\alpha = 1. \tag{36}$$

The states in this decomp. need not be orthogonal, and the decomp. is not unique.

Density matrix: mixed state

Mixture density matrix

For a normalized state $|a\rangle$, the probability to obtain such a state is

$$P(a) = \langle a|\rho|a\rangle = \sum_\alpha p_\alpha |\langle a|\psi_\alpha\rangle|^2 \tag{37}$$

and

$$\langle A\rangle = \sum_a aP(a) = \mathrm{Tr}(\rho A) = \sum_\alpha p_\alpha \langle\psi_\alpha|A|\psi_\alpha\rangle \tag{38}$$

For a mixed states one has $\mathrm{Tr}\,(\rho^2) < 1$ (mixed) . This is a **proper mixture.**
Note however that, in this case, there are **two types** of probabilities: the **quantum** ones but also the **classical** ones p_α.

Density matrix: mixed state

- We also have an **improper mixture.**
- The system A is part of a larger system AB in a **pure entangled state**, for example

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

- The full density matrix is pure $\rho_{AB} = |\psi\rangle\langle\psi|$. Tracing out B gives the **reduced density matrix**

$$\rho_A = \mathrm{Tr}_B \rho_{AB} = \left(\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|\right)_A$$

- This is the same matrix as the previous proper mixture. But here A **does not have** a definite pure state that one is ignorant of.

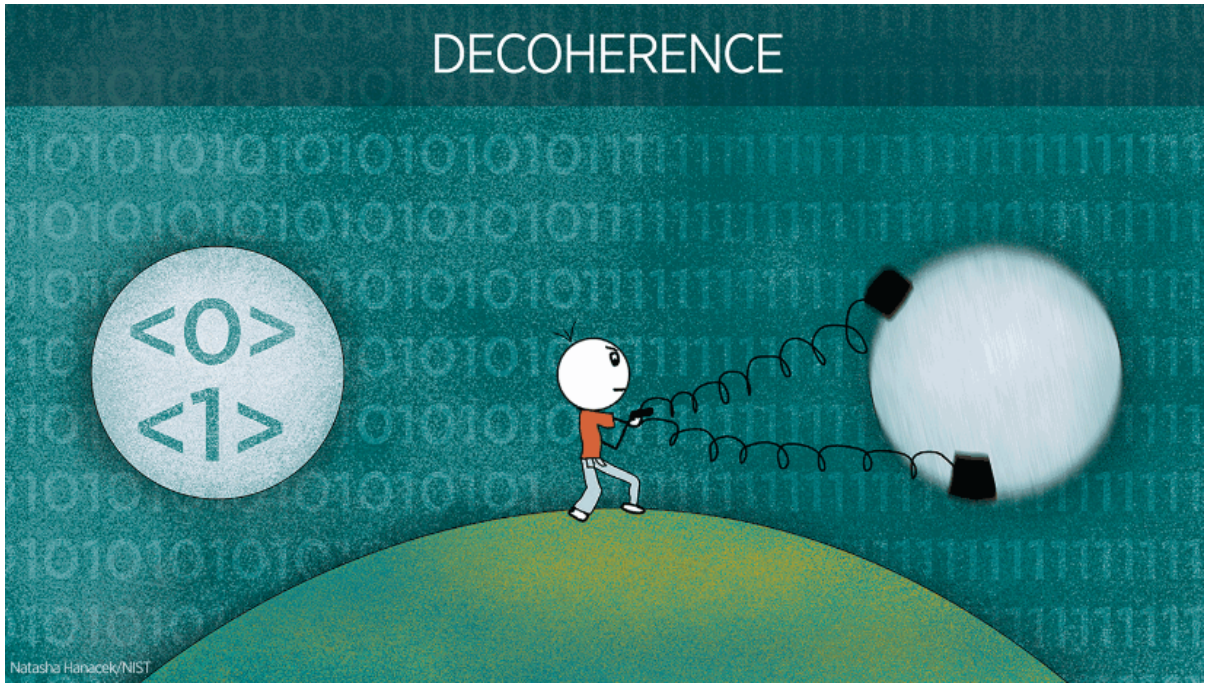
Squeezing as classical behavior

Density matrices

Decoherence

Basic idea

Decoherence is the suppression of interference between alternatives of a quantum system through its interaction with an environment.



Decoherence: the role of the environment

- An **open system interacts** with **degrees of freedom** that we do not observe or control.
- This interaction **entangles the system with its environment**: different system alternatives become correlated with different environmental states.
- **Interference becomes negligible** in observations restricted to the system.

Decoherence follows from ordinary quantum dynamics.

The basic procedure

1. **Identify the system and its environment.**
   Separate the degrees of freedom of interest from those that remain unobserved: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$.

2. **Compute the reduced density matrix.**
   Evolve the joint state and trace over the environment:

$$\rho_S(t) = \mathrm{Tr}_E\, \rho_{SE}(t).$$

The basic procedure

3 **Determine the suppression of coherence.**
Study how the interaction suppresses off-diagonal elements in the basis it dynamically selects:

$$\rho_S(t) \simeq \sum_i p_i(t) \, |s_i\rangle\langle s_i|.$$

4 **Extract the effective statistical description.**
Once the off-diagonal contributions are negligible, system observables obey

$$\langle A_S \rangle = \mathrm{Tr}_S(\rho_S A_S) \simeq \sum_i p_i \, \langle s_i|A_S|s_i\rangle.$$

Example: Schrödinger's cat



A simple model of the cat and its environment

Step 1: identify the system and the environment.
We represent the cat by two orthogonal, idealized states:

$$|A\rangle \quad \text{(alive)}, \qquad |D\rangle \quad \text{(dead)}.$$

We model the environment with the following basis: $\{|0_E\rangle, |1_E\rangle\}$.
We choose the initial state

$$|\Psi(0)\rangle = \frac{|A\rangle + |D\rangle}{\sqrt{2}} \otimes |0_E\rangle.$$

This is a **product state**: both cat alternatives accompany the same environmental state $|0_E\rangle$.

Unitary evolution gives

$$|\Psi(t)\rangle = \frac{|A\rangle|e_A(t)\rangle + |D\rangle|e_D(t)\rangle}{\sqrt{2}},$$

The interaction preserves the cat alternatives and makes the environment evolve differently in each component.

Environmental correlations and the reduced state

Step 2: compute the reduced density matrix.
Tracing over the environment introduces its state overlaps:

$$\rho_{\mathrm{cat}}(t) = \mathrm{Tr}_E\, |\Psi(t)\rangle\langle\Psi(t)| = \frac{1}{2}\begin{pmatrix} 1 & \langle e_D(t)|e_A(t)\rangle \\ \langle e_A(t)|e_D(t)\rangle & 1 \end{pmatrix}_{\{|A\rangle,|D\rangle\}}$$

The environmental overlap controls the cat's off-diagonal matrix elements.

Suppression of interference

Step 3: identify when coherence is suppressed.
After some time t_D the environmental states become orthogonal

$$\langle e_A(t_D)|e_D(t_D)\rangle \simeq 0 \qquad \rightarrow \qquad \rho_{\text{cat}}(t_D) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Step 4: extract the effective statistical description. The interference terms have vanished and there are **only two possible outcomes** for the cat with probability $1/2$.

Decoherence during inflation

For a fixed k, the "amplitude state" $|\mathcal{R}_k\rangle$ labels definite values of both $\mathcal{R}_k^{\rm R}$ and $\mathcal{R}_k^{\rm I}$, associated to the quantum field $\hat{\mathcal{R}}_k$. A superposition of amplitudes has the form

$$|\psi_k(\eta)\rangle = \int d^2\mathcal{R}_k\, \psi_k(\mathcal{R}_k^{\rm R}, \mathcal{R}_k^{\rm I}, \eta)|\mathcal{R}_k\rangle. \tag{39}$$

Here $d^2\mathcal{R}_k \equiv d\mathcal{R}_k^{\rm R}\, d\mathcal{R}_k^{\rm I}$. This is analog to a wavefunction $\psi(x,t)$ in a superposition of x in QM.

- The initial vacuum evolves into a highly **squeezed state**.
- Inflation stretches the mode beyond the Hubble radius $\frac{k}{aH} \ll 1$, and $\mathcal{R}_k$ becomes nearly constant in the super-Hubble regime.

An environment within the Universe

System: The long-wavelength mode, described by $|\mathcal{R}_k\rangle$ with Fourier coefficient $\mathcal{R}_k$.

Environment: "Other degrees of freedom," e.g. shorter-wavelength perturbations, tensor modes, other fields, thermal bath...

The environment is part of the Universe.

Different amplitudes leave different environmental records

For illustration, consider the toy model

$$|\Psi(\eta_0)\rangle = \left[\int d^2\mathcal{R}_k\, \psi_k(\mathcal{R}_k^{\rm R}, \mathcal{R}_k^{\rm I}, \eta_0)|\mathcal{R}_k\rangle\right] \otimes |E_{\rm i}\rangle,$$

$$|\Psi(\eta)\rangle \simeq \int d^2\mathcal{R}_k\, \psi_k(\mathcal{R}_k^{\rm R}, \mathcal{R}_k^{\rm I}, \eta)|\mathcal{R}_k\rangle \otimes |E_{\mathcal{R}_k}(\eta)\rangle. \quad (40)$$

- $|E_{\rm i}\rangle$ is the common initial environmental state.
- $|E_{\mathcal{R}_k}(\eta)\rangle$ is the environmental state correlated with the Fourier coefficient $\mathcal{R}_k$.
- Distinguishable environmental states carry information about different perturbation "coefficients" $\{\mathcal{R}_k, \tilde{\mathcal{R}}_k, \ldots\}$.

The reduced state of the perturbation

Tracing over the environment gives the state relevant to the mode pair:

$$\hat{\rho}_S(\eta) = \mathrm{Tr}_E[|\Psi(\eta)\rangle\langle\Psi(\eta)|]\,. \tag{41}$$

Let $\mathcal{R}_k$ and $\widetilde{\mathcal{R}}_k$ denote two possible realizations for a fixed k also called **pointer states**. Then

$$\rho_S(\mathcal{R}_k, \widetilde{\mathcal{R}}_k; \eta) = \psi_k(\mathcal{R}_k, \eta)\psi_k^*(\widetilde{\mathcal{R}}_k, \eta)\,\underbrace{\langle E_{\widetilde{\mathcal{R}}_k}(\eta) \mid E_{\mathcal{R}_k}(\eta)\rangle}_{\text{environmental overlap}}\,. \tag{42}$$

- Small overlap suppresses interference between the two amplitudes in the reduced state.

Suppression of interference between amplitudes

A simple Gaussian damping model gives

$$\rho_S(\mathcal{R}_k, \widetilde{\mathcal{R}}_k; \eta) \simeq \rho_0(\mathcal{R}_k, \widetilde{\mathcal{R}}_k; \eta) e^{-\Gamma_k(\eta)|\mathcal{R}_k - \widetilde{\mathcal{R}}_k|^2}. \tag{43}$$

Here ρ_0 is the state without this damping, and $\Gamma_k \geq 0$ measures its strength.

The separation includes both real components:

$$|\mathcal{R}_k - \widetilde{\mathcal{R}}_k|^2 = (\mathcal{R}_k^{\mathrm{R}} - \widetilde{\mathcal{R}}_k^{\mathrm{R}})^2 + (\mathcal{R}_k^{\mathrm{I}} - \widetilde{\mathcal{R}}_k^{\mathrm{I}})^2. \tag{44}$$

- Interference is suppressed when $\Gamma_k |\mathcal{R}_k - \widetilde{\mathcal{R}}_k|^2 \ll 1$.
- **On the diagonal**, the damping factor is unity. This approximation preserves amplitude probabilities.

An effective probability distribution for the mode

After strong suppression of interference, the reduced state behaves approximately as a mixture of narrow amplitude packets.

In the usual homogeneous Gaussian approximation,

$$P_k(\mathcal{R}_k) = \rho_S(\mathcal{R}_k, \mathcal{R}_k) \simeq \frac{1}{\pi\sigma_k^2} \exp\left[-\frac{|\mathcal{R}_k|^2}{\sigma_k^2}\right]. \qquad (45)$$

- $P_k(\mathcal{R}_k)\, d^2\mathcal{R}_k$ is a probability. The variance is $\sigma_k^2 = \langle|\mathcal{R}_k|^2\rangle$. **The power spectrum !**.

Connection with cosmological observations

- Inflationary quantum evolution determines the variance/power spectrum.
- Environmental correlations suppress observable interference between different perturbation amplitudes.
- Tracing over the “environmental DoF”, we get an ensemble of different classical universes, with different classical realizations of the perturbations $\mathcal{R}_{\mathbf{k}}$ (the same idea in S.cat)
- Each member of the ensemble corresponds to a particular realization of the CMB.

Open issues of the decoherence program

- **Who or what** selects the **environment** ?
- The total state remains a quantum superposition. **Decoherence alone does not select one particular realization** of the primordial pattern.
- At the end one still needs to invoke **Born's postulate**.