Introducing Warm Inflation: Dynamics, Microphysics, and Observational Signatures

Lecture 3: Nonequilibrium Quantum Field Theory – Part 2

Prof. Rudnei O. Ramos

Rio de Janeiro State University
Department of Theoretical Physics

1st Arica School of Cosmology and Gravitation
University of Tarapacá, Arica, Chile

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Where we are in the minicourse

- **Lecture 1: Introduction and Motivation**
  - Inflation, what for ?
  - Cold versus warm inflation
  - Brief overview of applications
- **Lecture 2: Basics of nonequilibrium QFT - part 1**
  - Dissipation in Quantum Mechanics: the Caldeira-Leggett model
  - Schwinger-Keldysh formalism for non-equilibrium QFT
  - Deriving effective equations of motion
- **Lecture 3: Basics of nonequilibrium QFT - part 2**
  - Local approximations for non-Markovian dissipation terms
  - Some local dissipation coefficients
  - Applications to Cosmology: the case of warm inflation

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Today: From microphysics to cosmology

Interactions $\longrightarrow$ bath correlations

$\longrightarrow \Sigma_R,\ N \longrightarrow \Upsilon,\ \xi$

$\longrightarrow$ inflaton–radiation evolution

Recall the nonlocal equation

$$\ddot{\phi} + V'_{\rm loc}(\phi) + \int_{t_0}^{t} dt' \, \Sigma_R(t,t')\phi(t') = \xi(t),$$

$$\langle \xi(t) \rangle = 0, \qquad \langle \xi(t)\xi(t') \rangle = N(t,t').$$

- Displayed for a homogeneous mode
- The kernels contain the bath response and fluctuations.

When can a memory integral become local ?

A controlled equation with $\Upsilon(\phi, T)$ and consistent noise

Conventions carried from Lecture 2

$$f(\omega) = \int dt\, e^{i\omega t} f(t), \qquad \int_p \equiv \int \frac{d^4p}{(2\pi)^4}.$$

$$G^{>}(x) = \langle \chi(x)\chi(0)\rangle, \quad G^{<}(x) = \langle \chi(0)\chi(x)\rangle.$$

$$A_\chi(p) = G^{>}(p) - G^{<}(p), \quad R_O(t) = i\theta(t)\langle [O(t), O(0)]\rangle.$$

$$\Sigma_R = -\lambda_B^2 R_O, \qquad \Sigma_R(\omega) = \Sigma_R(0) - i\omega\Upsilon + \cdots .$$

When does memory become friction?

A separation of scales

The background must change little during the time over which the bath remembers it.

$$\tau_B \ll \tau_{\rm system}.$$

- Memory: decay time of the relevant response kernel.
- Relaxation: return of bath distributions toward equilibrium.
- Evolution: $|\dot{\phi}/\phi|^{-1}$, $|\dot{T}/T|^{-1}$, H^{-1}.

Use the relevant rates

Expand the recent history

$$\phi(t-\tau) = \phi(t) - \tau\dot{\phi}(t) + \frac{\tau^2}{2}\ddot{\phi}(t) + \cdots .$$

$$I_n \equiv \int_0^\infty d\tau\, \tau^n \Sigma_R(\tau).$$

$$\int_0^\infty d\tau\, \Sigma_R(\tau)\phi(t-\tau) = I_0\phi - I_1\dot{\phi} + \frac{I_2}{2}\ddot{\phi} + \cdots .$$

$I_0\phi(t) \quad \longrightarrow \quad$ local mass or force correction.

- Combine it with the renormalized effective potential.
- A constant field does not dissipate energy through a friction term.

The first derivative gives friction

$$\Upsilon = -I_1 = -\int_0^\infty d\tau\, \tau \Sigma_R(\tau).$$

$$\ddot{\phi} + V'_{\rm eff}(\phi) + \Upsilon\dot{\phi} = \xi.$$

Energy transfer

For a passive thermal bath, $\Upsilon \geq 0$: the coherent field loses energy at rate $\Upsilon\dot{\phi}^2$.

What has been neglected?

$$\frac{I_2}{2}\dddot{\phi}, \quad -\frac{I_3}{6}\phi^{(3)}, \quad \dots$$

- Higher moments modify inertia and produce higher derivatives.

Check a ratio

Compare retained and omitted terms along the solution.

A solvable velocity kernel

$$F_{\rm mem}(t) = \int_0^{t-t_0} d\tau\, K(\tau)\dot{\phi}(t-\tau),$$

$$K(\tau) = \frac{\Upsilon}{\tau_B} e^{-\tau/\tau_B}, \qquad \tau \geq 0.$$

The example isolates friction after static terms have been separated.

$$\int_0^\infty d\tau\, \tau^n K(\tau) = \Upsilon\, n! \tau_B^n.$$

$$F_{\rm mem} = \Upsilon\dot{\phi} - \Upsilon\tau_B\ddot{\phi} + \Upsilon\tau_B^2\phi^{(3)} + \cdots .$$

Accuracy estimate

$|\tau_B\ddot{\phi}/\dot{\phi}| \ll 1$ where the comparison is meaningful.

The same expansion in frequency space

$$\Sigma_R(\omega) = \int_0^\infty d\tau\, e^{i\omega\tau}\Sigma_R(\tau),$$

$$\Sigma_R(\omega) = I_0 + i\omega I_1 - \frac{\omega^2}{2} I_2 + \cdots .$$

$$\Sigma_R(\omega) = \Sigma_R(0) - i\omega\Upsilon + O(\omega^2).$$

Extracting the coefficient: a Kubo limit

$$\boxed{\Upsilon = -\lim_{\omega\to 0} \frac{\mathrm{Im}\,\Sigma_R(\omega,\mathbf{0})}{\omega}}$$

$$\Upsilon = \lambda_B^2 \lim_{\omega\to 0} \frac{\mathrm{Im}\,R_O(\omega,\mathbf{0})}{\omega}.$$

Thermal fluctuation–dissipation relation

$$N(\omega, \mathbf{k}) = -\coth\left(\frac{\omega}{2T}\right) \operatorname{Im}\Sigma_R(\omega, \mathbf{k}).$$

- The bath is stationary and thermal at the frozen background.
- The noise kernel is the symmetrized force covariance.
- The same dressed bath input must enter response and noise.

$$\operatorname{Im}\Sigma_R = -\Upsilon\omega \quad \Rightarrow \quad N(\omega) = \Upsilon\omega \coth\left(\frac{\omega}{2T}\right).$$

$$T \to 0: \qquad N(\omega) \to \Upsilon|\omega|.$$

Quantum fluctuations

The spectrum remains frequency dependent even for an Ohmic response.

The classical (thermal) case

$$|\omega| \ll T : \qquad \coth\left(\frac{\omega}{2T}\right) \simeq \frac{2T}{\omega}.$$

$$N(\omega) \simeq 2\Upsilon T.$$

Two approximations

$|\omega|\tau_B \ll 1$ for local damping; $|\omega| \ll T$ for classical thermal noise
Gleiser and Ramos, PRD **50**, 2441 (1994)

$$\langle \xi(t)\xi(t') \rangle = 2\Upsilon T\, \delta(t-t').$$

Field-dependent couplings: multiplicative noise

$$\ddot{\phi} + V'_{\rm eff} + \Upsilon(\phi, T)\dot{\phi} = \xi(\phi, T),$$

$$\langle \xi\xi \rangle \sim 2T\Upsilon(\phi, T)$$

- The amplitude changes with the field and bath state.
- A white-noise prescription comes from the limiting microscopic process.
- Overdamped reductions can generate additional drift terms Berera, Gleiser, Ramos, PRD **58**, 123508 (1998) .

Checkpoint: three distinct assumptions

- Short memory: response can be expanded locally.
- Classical occupation: relevant frequencies are small compared with temperature.
- Gaussian approximation: higher force cumulants are negligible.

Keep them separate

None of these conditions by itself proves the other two.

Calculating the bath response

A worked scalar example

Find the correlation function that supplies Υ.

Dissipation coefficient: choose an interaction

$$\mathcal{L}_{\rm int} = -\frac{g^2}{2}\phi^2\chi^2 + \mathcal{L}_{\chi\text{–bath}}, \qquad \phi = \phi + \delta\phi,$$

$$\mathcal{L}_{\rm linear} = -\lambda_B \delta\phi\, O, \qquad \lambda_B = g^2\phi, \quad O = \chi^2 - \langle\chi^2\rangle.$$

$$R_O(t) = i\theta(t)\langle[O(t), O(0)]\rangle, \qquad \Sigma_R = -\lambda_B^2 R_O.$$

$$\boxed{\Upsilon = -\lim_{\omega\to 0}\frac{\mathrm{Im}\,\Sigma_R(\omega,\mathbf{0})}{\omega} = \frac{\lambda_B^2}{2}\lim_{\omega\to 0}\frac{A_O(\omega,\mathbf{0})}{\omega}}$$

Instantaneous bath

Compute at fixed ϕ, T, with $A_O = C_O^{>} - C_O^{<}$ and $\mathrm{Im}\,R_O = A_O/2$.

Dissipation coefficient: reduce to a thermal loop

$$C_O^>(k) = \langle OO \rangle_k^> \simeq 2 \int_p G_\chi^>(p) G_\chi^>(k-p), \qquad \int_p \equiv \int \frac{d^4p}{(2\pi)^4}.$$

$$G_\chi^> = (1+n_B) A_\chi, \qquad G_\chi^< = n_B A_\chi, \qquad n_B(p_0) = \frac{1}{e^{p_0/T}-1}.$$

$$\underbrace{C_O^< = e^{-\omega/T} C_O^>}_{\text{KMS}} \quad \Longrightarrow \quad \lim_{\omega \to 0} \frac{A_O(\omega, \mathbf{0})}{\omega} = \frac{C_O^>(0,\mathbf{0})}{T}.$$

$$\boxed{\Upsilon = \frac{\lambda_B^2}{2T} C_O^>(0,\mathbf{0})}$$

Dissipation coefficient: the spectral integral

$$A_\chi(-p) = -A_\chi(p), \qquad n_B(-p_0) = -[1 + n_B(p_0)].$$

$$C_O^>(0, \mathbf{0}) = 2 \int_p A_\chi(p)^2 n_B(p_0)[1 + n_B(p_0)].$$

$$\boxed{\Upsilon = \frac{g^4 \phi^2}{T} \int \frac{d^4 p}{(2\pi)^4} A_\chi(p)^2 n_B(p_0)[1 + n_B(p_0)]}$$

Checks

$[\Upsilon]$ = mass and $\Upsilon \geq 0$ for the passive equilibrium bath.

Dissipation coefficient: supply the dressed spectrum

$$m_\chi^2(\phi, T) = m_0^2 + g^2\phi^2 + \Pi_{\rm th}, \qquad E_{\mathbf{p}}^2 = \mathbf{p}^2 + m_\chi^2,$$

$$A_\chi(p) \simeq \frac{4p_0\Gamma_\chi(p)}{[p_0^2 - E_{\mathbf{p}}^2]^2 + [2p_0\Gamma_\chi(p)]^2}.$$

- Bath interactions determine the thermal mass and width.
- Insert this spectral input into the integral for Υ.
- Compare pole and off-shell regions within their validity domains.

Width convention

$p_0 \simeq E_{\mathbf{p}} - i\Gamma_\chi$: Γ_χ denotes amplitude damping.

What produces the width?

- Decays and inverse decays when kinematically allowed.
- Scattering with particles in the bath.
- Thermal masses and occupation factors change the available channels.

Why not use free on-shell propagators?

$$A_{\rm free}(p) = 2\pi\ \mathrm{sgn}(p_0)\delta(p_0^2 - E_{\mathbf{p}}^2).$$

$$\Upsilon \text{ would contain } \quad \delta(p_0^2 - E_{\mathbf{p}}^2)^2.$$

A singular limit

The long-time response requires relaxation. Setting the width to zero before the response limit removes it.

- Dress the propagators before taking the low-frequency limit.

A width is necessary, but is it sufficient?

- Near on-shell denominators can enhance higher-order diagrams.
- Vertex or ladder corrections may contribute at the same order.
- Use power counting appropriate to the thermal and coupling regime.

Scope of our calculation

The dressed loop teaches the mechanism; a controlled leading coefficient may require more resummation.

Resummation discussion: M. Bastero-Gil, A. Berera and R.O.R., JCAP **09**, 033 (2011)

The microscopic calculation in four steps

1. Specify fields, vertices, masses and multiplicities.
2. Calculate thermal spectral functions and relaxation channels.
3. Insert them into the response integral.
4. Check limits, resummation and the derivative expansion.

What is not an input

Υ is a result of this calculation, not an independent decay width.

One integral, several momentum regions

$$\Upsilon \sim \frac{\lambda_B^2}{T} \int_p A_\chi^2 \, n_B(1 + n_B).$$

- Near poles: narrow peaks and thermal populations.
- Away from poles: virtual mediator response to light bath modes.
- Which region wins depends on couplings as well as temperature.

Thermally populated quasiparticles

$$p_0 \simeq \pm E_{\mathbf{p}}, \qquad \Gamma_\chi \ll E_{\mathbf{p}}.$$

$$\int \frac{dp_0}{2\pi} A_\chi^2 n_B(1+n_B) \simeq \frac{n_B(E_{\mathbf{p}})[1+n_B(E_{\mathbf{p}})]}{2E_{\mathbf{p}}^2\Gamma_\chi}.$$

Conditions

Occupation factors and the width must vary little over each pole.

The narrow-width contribution

$$\Upsilon_{\rm pole} \simeq \frac{g^4\phi^2}{2T}\int \frac{d^3p}{(2\pi)^3}\frac{n_B(E_{\mathbf{p}})[1+n_B(E_{\mathbf{p}})]}{E_{\mathbf{p}}^2\Gamma_\chi}.$$

- At fixed spectral parameters, the peak integral scales as $1/\Gamma_\chi$.
- Longer-lived excitations respond more strongly to slow driving.

Not a zero-coupling result

The adiabatic limit must remain valid.

A large coefficient can signal a slow bath

$$\Gamma_\chi \downarrow \quad \Rightarrow \quad \Upsilon_{\rm pole} \uparrow, \qquad \tau_B \uparrow .$$

- The background may then change within one bath memory time.
- The local response limit and the zero-width limit do not commute.
- A large formal coefficient is not proof of a valid warm regime.

Always accompany the coefficient with its hierarchy $\omega_{\rm system} \ll \Gamma_{\rm relevant}$.

High temperature: useful, but model dependent

$T \gtrsim m_\chi$: thermal poles can be populated.

- Thermal masses regulate infrared behavior.
- Widths depend on the available interactions and distributions.
- Near-pole enhancement can require ladder resummation.

How to quote an estimate

State the interaction, mass hierarchy, and resummation used.

Low temperature means heavy mediators

$$m_{\mathrm{light}} \ll T \ll m_\chi(\phi), \qquad m_\chi \simeq g|\phi|.$$

$$n_B(m_\chi) \sim e^{-m_\chi/T}.$$

- Real heavy particles are Boltzmann suppressed.
- Light bath fields can still carry energy and equilibrate.
- Off-shell heavy propagation can transmit the response.

The low-momentum off-shell region

$$|p_0|, |\mathbf{p}| \sim T \ll m_\chi : \qquad A_\chi(p) \simeq \frac{4 p_0 \Gamma_\chi(p)}{m_\chi^4}.$$

- Use the imaginary self-energy in this momentum region.
- An on-shell decay width is not generally valid off shell.

The mechanism

Virtual heavy mediators connect the inflaton to light radiation.

Dissipation coefficient in the low momentum approximation

$$A_\chi(p) \sim \frac{\kappa}{m_\chi^2} a(p/T) \quad \Longrightarrow \quad \int_p A_\chi^2 n_B(1+n_B) \sim \frac{\kappa^2 T^4}{m_\chi^4}.$$

$$\Upsilon_{\rm LM} \sim \frac{g^4 \phi^2 \kappa^2 T^3}{m_\chi^4} \sim \kappa^2 \frac{T^3}{\phi^2}.$$

A conditional scaling

In suitable two-stage models: $\Upsilon_{\rm LM} = C_\phi T^3/\phi^2$. C_ϕ depends on the model.

Pole versus low momentum: a crossover

$\Upsilon_{\rm pole}$ contains $\dfrac{e^{-m_\chi/T}}{\Gamma_\chi}$, $\Upsilon_{\rm LM}$ has power-law suppression.

- Boltzmann suppression favors the low-momentum contribution.
- A sufficiently narrow pole can counter that suppression.
- Compare both terms inside their common validity domain.

Temperature is not the whole answer

The low-temperature pole contribution can matter at small couplings.

Compare the approximations

Region	Useful approximation	Main check
Pole	Narrow quasiparticle peaks; inverse-width dependence	Relaxation faster than driving; transport resummation
Low momentum	Heavy virtual mediators; possible T^3/ϕ^2	Correct off-shell self-energy and light bath hierarchy
Crossover	Compare or integrate full spectral input	Avoid extrapolating either asymptotic formula

All regimes

Specify the couplings, multiplicities, and instantaneous background.

Bring the calculation into an expanding universe

A coefficient has a domain of validity

The cosmological solution must remain inside the microscopic hierarchy used to derive it.

$\Upsilon(\phi, T)$ + validity tests along the trajectory.

Hubble friction and bath friction

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2, \qquad H = \dot{a}/a.$$

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V'_{\rm eff} = \xi.$$

- $3H\dot{\phi}$: dilution associated with expansion.
- $\Upsilon\dot{\phi}$: energy transfer to interacting bath fields.

Flat-space bath input is useful when microscopic relaxation is fast compared with expansion: $\Gamma_{\rm rel} \gg H, \quad \left|\frac{\dot{m}_\chi}{m_\chi}\right|, \left|\frac{\dot{T}}{T}\right|, \omega_{\rm bg}$

- For induced masses, field motion changes the bath spectrum.
- Near equilibrium is an approximation to be maintained dynamically.

Perturbations introduce additional scales

$$k_{\mathrm{phys}}(t) = \frac{k}{a(t)}$$

- Physical momenta redshift as the universe expands.

Bridge to Lecture 4

A background coefficient alone does not specify the perturbation spectrum.

Thermal noise in comoving coordinates

$$\langle \xi(t,\mathbf{x})\xi(t',\mathbf{x}')\rangle = 2\Upsilon T\, a^{-3}(t)\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}').$$

$$\xi_{\mathbf{k}} = \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\xi(t,\mathbf{x}),$$

$$\langle \xi_{\mathbf{k}}(t)\xi^*_{\mathbf{k}'}(t')\rangle = 2\Upsilon T a^{-3}(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\delta(t-t').$$

Physical volume

$\delta^3(\mathbf{r}-\mathbf{r}') = a^{-3}\delta^3(\mathbf{x}-\mathbf{x}')$.

Coherent energy loss feeds radiation

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V.$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\Upsilon\dot{\phi}^2,$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2.$$

The approximation

Coherent deterministic background, relativistic radiation, and a specified effective potential.

The coupled background system

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0,$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2,$$

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V + \rho_R, \qquad \rho_R = \frac{\pi^2}{30} g_* T^4.$$

A useful measure

$Q \equiv \Upsilon/(3H)$ compares bath and expansion damping.

Check more than the size of Q

- Radiative and thermal corrections to the inflaton force.
- Heavy/light mass hierarchy and perturbative control.
- Effective-theory range

A two-stage example ready for dynamics

$$\phi \longleftrightarrow \chi_{\mathrm{heavy}} \longleftrightarrow \sigma_{\mathrm{light}}, \qquad m_\sigma \ll T \ll g|\phi|.$$

$\Upsilon \simeq C_\phi \dfrac{T^3}{\phi^2}$ when the calculated low-momentum term dominates.

- Input: potential, interactions, coefficient and hierarchy.
- Next: solve for temperature, field motion and radiation.
- Then: analyze coupled inflaton–radiation perturbations.

The full workflow

- Derive causal response and noise from the microscopic theory.
- Separate conservative corrections from dissipative response.
- Calculate the spectral input in a controlled approximation.
- Check memory, thermal occupation and bath relaxation.
- Evolve the cosmology and monitor the hierarchy.

A feedback loop

The cosmological solution tests the microscopic assumptions.

Next in Lecture 4

- Local friction is a low-frequency approximation to causal memory.
- Thermal FDT fixes the matching fluctuations.
- Widths and spectral tails determine microscopic coefficients.
- Warm cosmology must preserve the bath hierarchy.

Next lecture

Build models, solve the background, and derive observational signatures.

Reading and source material

- Bastero-Gil, Berera and Ramos, *Dissipation coefficients from scalar and fermion quantum field interactions*, JCAP 09 (2011) 033. arXiv:1008.1929.
- Bastero-Gil, Berera, Ramos and Rosa, *General dissipation coefficient in low-temperature warm inflation*, JCAP 01 (2013) 016. arXiv:1207.0445.
- Berera, Moss and Ramos, *Warm Inflation and its Microphysical Basis*, Rept. Prog. Phys. 72 (2009) 026901. arXiv:0808.1855.