Cosmological probes of Gravity

Lecture 2: Testing the background

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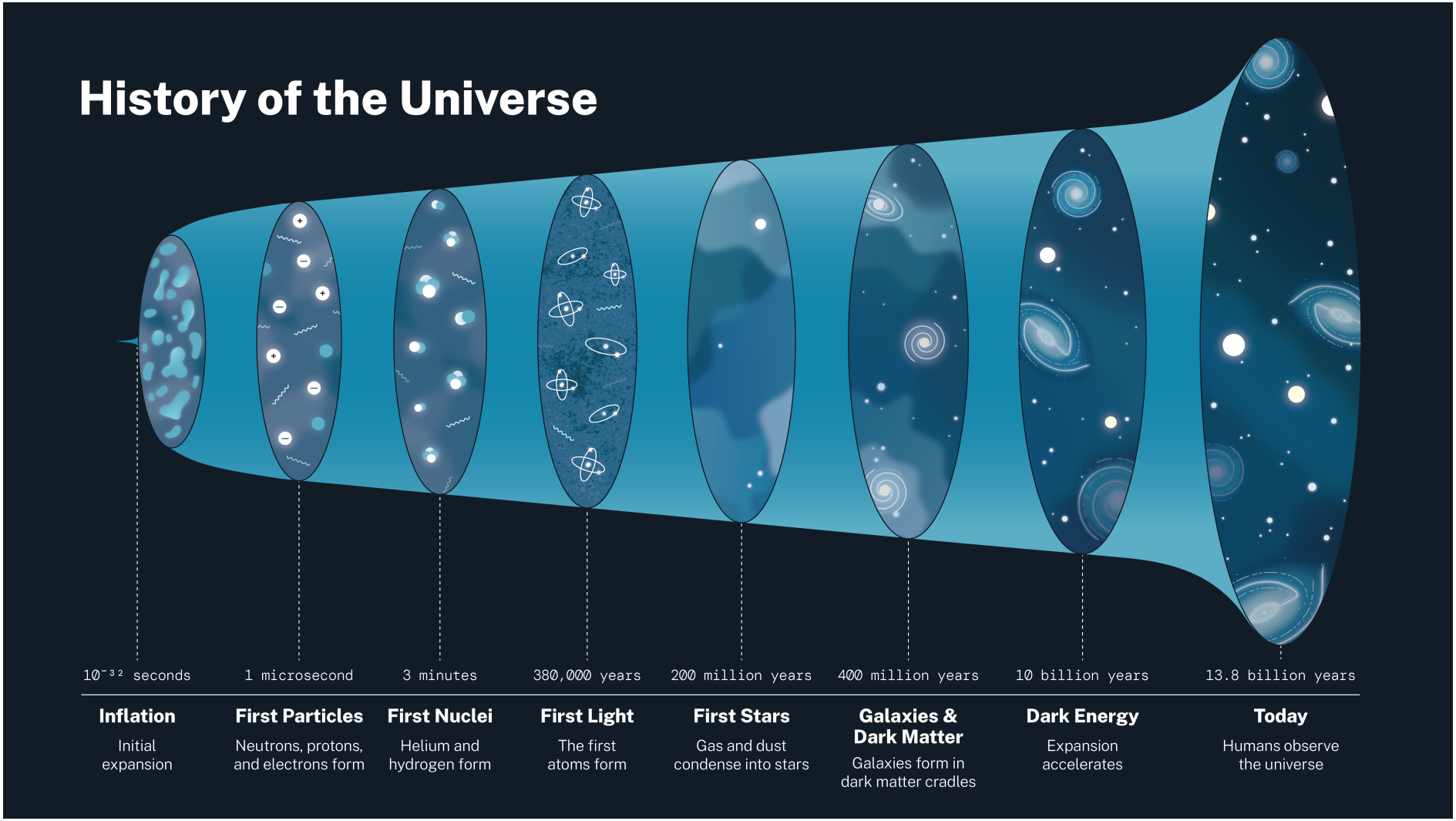
Outline

1. Primordial Nucleosynthesis
2. Baryon Acoustic Oscillations
3. Recombination
4. Cosmological probes of the Background and Modified Gravity theories
5. Relevant papers for this lecture

Evolution of the Universe



Primordial Nucleosynthesis

- The abundances of light elements (D, ^{3}He, ^{4}He, ^{7}Li) produced during Big Bang Nucleosynthesis (BBN) depend on the expansion rate of the universe at that time.
- Therefore, measurements of light element abundances can be used to constrain the expansion history of the universe and test modified gravity theories.
- The primordial abundances are sensitive to the baryon-to-photon ratio η_b, which is related to the baryon density $\Omega_b h^2$ (see discussion in this lecture).
- Observations of the Cosmic Microwave Background (CMB) provide independent measurements of $\Omega_b h^2$, allowing for consistency checks between BBN and CMB predictions (see next lecture).

Baryon to photon ratio

- The key quantity for BBN is the present baryon-to-photon ratio η_b, which relates to $\Omega_b h^2$:

$$\eta_b \equiv \frac{n_b}{n_\gamma} \approx (6.1 \pm 0.04) \times 10^{-10}$$

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- Now we define the mass fraction of species i as

$$X_i \equiv \frac{M_i}{M_b} = \frac{\rho_i}{\rho_b} \ ,$$

with M_i and M_b the total mass of species i and the total baryon mass, respectively and ρ_i and ρ_b the corresponding mass densities.

Mass fractions

- The mass fraction of ^{4}He is often noted as $X_{^4\mathrm{He}} = Y_p$
- At the end of BBN essentially all baryons are in H and ^{4}He. Therefore the mass fraction of hydrogen is then $X_{\mathrm{H}} = 1 - Y_p$

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- The total number of hydrogen nuclei is

$$N_{\mathrm{H}} = \frac{X_{\mathrm{H}} M_b}{m_{\mathrm{H}}} = \frac{(1 - Y_p) M_b}{m_{\mathrm{H}}} \,,$$

- The total number of helium nuclei is

$$N_{\mathrm{He}} = \frac{X_{\mathrm{He}} M_b}{m_{\mathrm{He}}} = \frac{Y_p M_b}{m_{\mathrm{He}}} \,,$$

- In this way, the total number of baryons is

$$N_b = N_{\mathrm{H}} + 4 N_{\mathrm{He}} = \frac{(1 - Y_p) M_b}{m_{\mathrm{H}}} + 4 \frac{Y_p M_b}{m_{\mathrm{He}}} \,,$$

Baryon to photon ratio

- The mean baryon mass can be expressed as

$$m_b = \frac{M_b}{N_b} \ ,$$

with M_b the total baryon mass and N_b the total number of baryons.

- Using the expression for N_b derived above, we can write

$$\begin{aligned} m_b &= \frac{M_b}{N_b} = \frac{M_b}{\frac{(1-Y_p)M_b}{m_{\rm H}} + 4\frac{Y_p M_b}{m_{\rm He}}} = \left(\frac{(1-Y_p)m_{\rm He} + 4Y_p m_{\rm H}}{m_{\rm H} m_{\rm He}} \right)^{-1} \ , \\ &= \frac{m_{\rm H}}{1+\epsilon Y_p} \ , \\ &\simeq m_{\rm H}\left(1-\epsilon Y_p\right) \ , \end{aligned}$$

where $\epsilon \equiv \frac{4m_{\rm H}}{m_{\rm He}} - 1$.

Baryon to photon ratio

- We want to calculate the baryon to photon ratio:

$$\begin{aligned}
\eta_b &= \frac{n_b}{n_\gamma} = \frac{\rho_b / m_b}{n_\gamma} \,, \\
&= \frac{\Omega_b \, \rho_c}{m_b \, n_\gamma} \,, \\
&= \frac{\Omega_b h^2 (1 + \epsilon Y_p)}{m_{\rm H} n_\gamma} \frac{3 \; 100^2}{8\pi G} \left(\mathrm{km/s/Mpc}\right)^2 \,, \\
&= 2.733 \times 10^{-8} \, \Omega_b h^2 \, (1 + 0.00717 \, Y_p) \left(\frac{T_0}{2.7255 \, \mathrm{K}}\right)^{-3} \,, \qquad (1)
\end{aligned}$$

- where in the last step we used the photon number density of a blackbody, with T_0 the present CMB temperature and $\zeta(3) \simeq 1.202$ the Riemann zeta function:

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(k_B T_0\right)^3 = 410.7 \left(\frac{T_0}{2.7255 \, \mathrm{K}}\right)^3 \mathrm{cm}^{-3}$$

The time–temperature relation

- To follow the nuclear reactions we need the temperature as a function of time. We recall the continuity equation:

$$\dot{\rho} + 3H\left(\rho + P\right) = 0$$

with ρ the total energy density of all species, P their total pressure and $H \equiv \dot{a}/a$ the Hubble parameter .

- From the continuity equation we can derive the temperature evolution:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}T}{\mathrm{d}\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -3H\left(\rho + P\right)\frac{\mathrm{d}T}{\mathrm{d}\rho} \tag{2}$$

which gives the cooling rate of the universe once $\rho(T)$ and $P(T)$ are known for every species.

The time temperature relation depends on the gravity theory through the Hubble parameter H

The main BBN reactions

- About 12 reactions determine the light-element abundances to $\sim 1\%$ (Smith, Kawano & Malaney, ApJS 85, 219 (1993)):

Weak interactions (n/p ratio):

$$n \leftrightarrows p + e^- + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrows p + e^-$$

$$n + e^+ \leftrightarrows p + \bar{\nu}_e$$

Deuterium formation (the bottleneck):

$$p + n \leftrightarrows \mathrm{D} + \gamma$$

Mass 3:

$$\mathrm{D} + p \leftrightarrows {}^3\mathrm{He} + \gamma$$

$$\mathrm{D} + \mathrm{D} \leftrightarrows {}^3\mathrm{He} + n$$

$$\mathrm{D} + \mathrm{D} \leftrightarrows {}^3\mathrm{H} + p$$

$${}^3\mathrm{He} + n \leftrightarrows {}^3\mathrm{H} + p$$

Mass 4 (^{4}He, the most bound light nucleus):

$${}^3\mathrm{H} + \mathrm{D} \leftrightarrows {}^4\mathrm{He} + n$$

$${}^3\mathrm{He} + \mathrm{D} \leftrightarrows {}^4\mathrm{He} + p$$

Mass 7:

$${}^3\mathrm{H} + {}^4\mathrm{He} \leftrightarrows {}^7\mathrm{Li} + \gamma$$

$${}^3\mathrm{He} + {}^4\mathrm{He} \leftrightarrows {}^7\mathrm{Be} + \gamma$$

$${}^7\mathrm{Be} + n \leftrightarrows {}^7\mathrm{Li} + p$$

$${}^7\mathrm{Li} + p \leftrightarrows {}^4\mathrm{He} + {}^4\mathrm{He}$$

There are no stable nuclei with $A = 5$ or 8: this gap stops the production of heavier elements.

Weak reactions and the n/p freeze-out

- At $T \gtrsim 1$ MeV the weak reactions of electron neutrinos keep n/p in equilibrium:

$$n + \nu_e \leftrightarrows p + e^-, \qquad n + e^+ \leftrightarrows p + \bar{\nu}_e, \qquad n \leftrightarrows p + e^- + \bar{\nu}_e$$

so that $n/p = e^{-Q/T}$, with $Q = m_n - m_p = 1.293$ MeV the neutron–proton mass difference.

- Freeze-out: since the leading term of the weak reaction rate $\lambda_{np} \approx 0.95\, T^5 \mathrm{s}^{-1}$ decreases faster than the expansion rate $H \approx 0.68\, T^2 \mathrm{s}^{-1}$ (T in MeV, $g_* = 10.75$), the weak rate eventually drops below the expansion rate:

$$\lambda_{np} \lesssim H$$

the reactions switch off and the neutron-to-proton ratio freezes out. Using the full rate, the freeze-out temperature is $T_f \simeq 0.8$ MeV ($t \sim 1$ s), so that $n/p = e^{-Q/T_f}$ gives

$$\frac{n}{p} \simeq \frac{1}{6} \quad \text{at freeze-out,} \qquad \frac{n}{p} \simeq \frac{1}{7} \quad \text{after neutron decay}$$

The freeze-out time is determined by H, and this eventually can change in a modified gravity theory. (see for example Asimakis et al. Phys.Rev.D 105, 084010 (2022))

Assembling the reaction network

- For nuclides i participating in reactions of the type $i + j \leftrightarrows k + l$, the evolution of their number density is governed by the competition between the forward and reverse reaction rates.
- The rate of change of the number density n_i is

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} + 3H\, n_i = -\, n_i n_j \langle \sigma v \rangle_{ij,kl} \;+\; n_k n_l \langle \sigma v \rangle_{kl,ij}$$

  where $\langle \sigma v \rangle_{ij,kl}$, $\langle \sigma v \rangle_{kl,ij}$ are the forward and reverse thermally averaged rates.
- In terms of the number of nuclei of species i per baryon, $Y_i \equiv n_i/n_b \simeq X_i/A_i$; since $n_b \propto a^{-3}$ ($\dot{n}_b = -3Hn_b$), the dilution term cancels:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = -\, Y_i Y_j\, \rho_b N_A \langle \sigma v \rangle_{ij,kl} \;+\; Y_k Y_l\, \rho_b N_A \langle \sigma v \rangle_{kl,ij}$$

  where $\rho_b N_A \simeq n_b$ and N_A Avogadro's number
- The thermally averaged cross section in $[ij]_{kl}$ is

$$\langle \sigma v \rangle_{ij,kl} = \left(\frac{8}{\pi \mu_{ij}}\right)^{1/2} T^{-3/2} \int_0^\infty \sigma_{ij}(E)\, e^{-E/T}\, E\, \mathrm{d}E$$

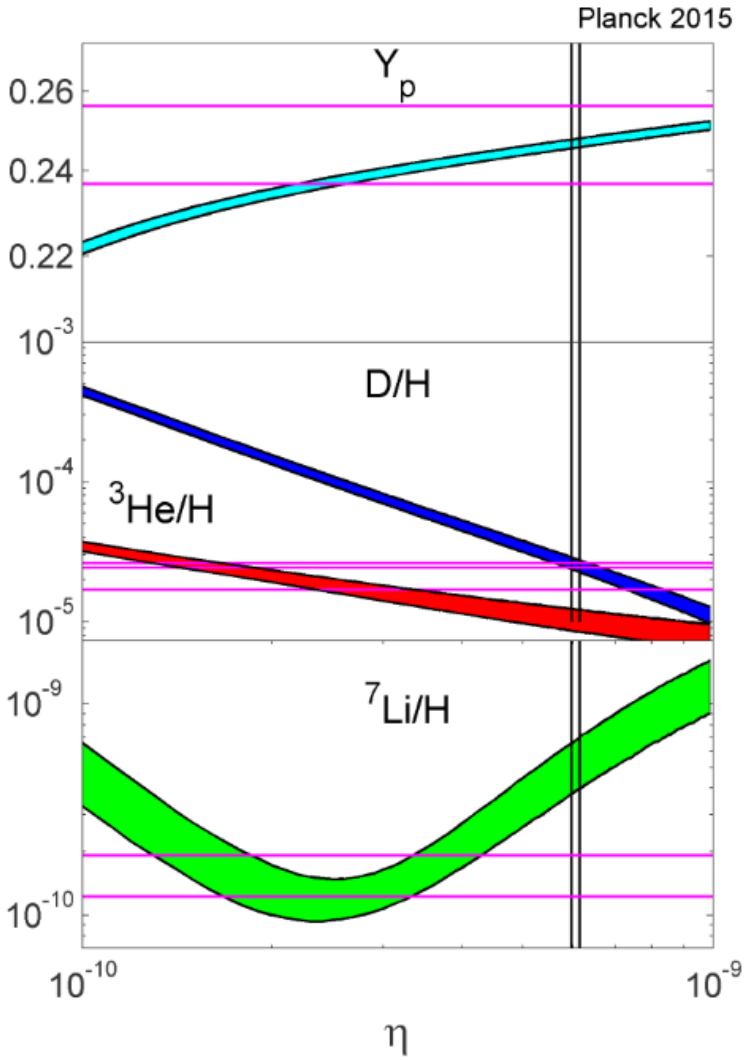
  with E the centre-of-mass energy, $\mu_{ij} = m_i m_j/(m_i + m_j)$ the reduced mass.

Observational constraints on BBN

- The network evolves the Y_i; what is observed are ratios of them:

observable	in terms of Y_i
Y_p (helium "mass fraction")	$4\, Y_{^4\mathrm{He}}$
D/H	$Y_{\mathrm{D}}/Y_{\mathrm{H}}$
^{3}He/H	$Y_{^3\mathrm{He}}/Y_{\mathrm{H}}$
^{7}Li/H	$[Y_{^7\mathrm{Li}} + Y_{^7\mathrm{Be}}]/Y_{\mathrm{H}}$, after ^{7}Be decays

  - $\mathrm{D/H} = (2.53 \pm 0.04) \times 10^{-5}$, from narrow-line Ly$\alpha$ absorbers towards high-z QSOs (only ~ 6 well resolved systems). The only post-BBN production channels are cosmic-ray spallation and photodisintegration in energetic environments (below 1% smaller than the primordial abundance). Each generation of star formation cycles gas through stars and removes deuterium from it. Therefore $(\mathrm{D/H})_{\mathrm{obs}} \leq (\mathrm{D/H})_{\mathrm{prim}}$
  - $Y_p = 0.2449 \pm 0.0040$, from metal-poor extragalactic HII regions extrapolated to zero metallicity.^{4}He is the ash of hydrogen burning and does get consumed in helium burning to carbon and oxygen, but that's a net conversion of a small fraction into heavier elements. The Galactic history of helium is monotonic increase
  - $^3\mathrm{He/H} = (0.7 \pm 0.5) \times 10^{-5}$, from the 8.665 GHz line of $^3\mathrm{He}^+$.
  - $^7\mathrm{Li/H} = (1.6 \pm 0.3) \times 10^{-10}$, from the Spite plateau of old metal-poor halo stars. There is both production and depletion of lithium in stars.



- Predicted primordial abundances of light elements as a function of the baryon-to-photon ratio η_b. The regions between violet lines represent observational constraints.

FIGURE: International Journal of Modern Physics E 26, 08 (2017) arXiv:1706.03138

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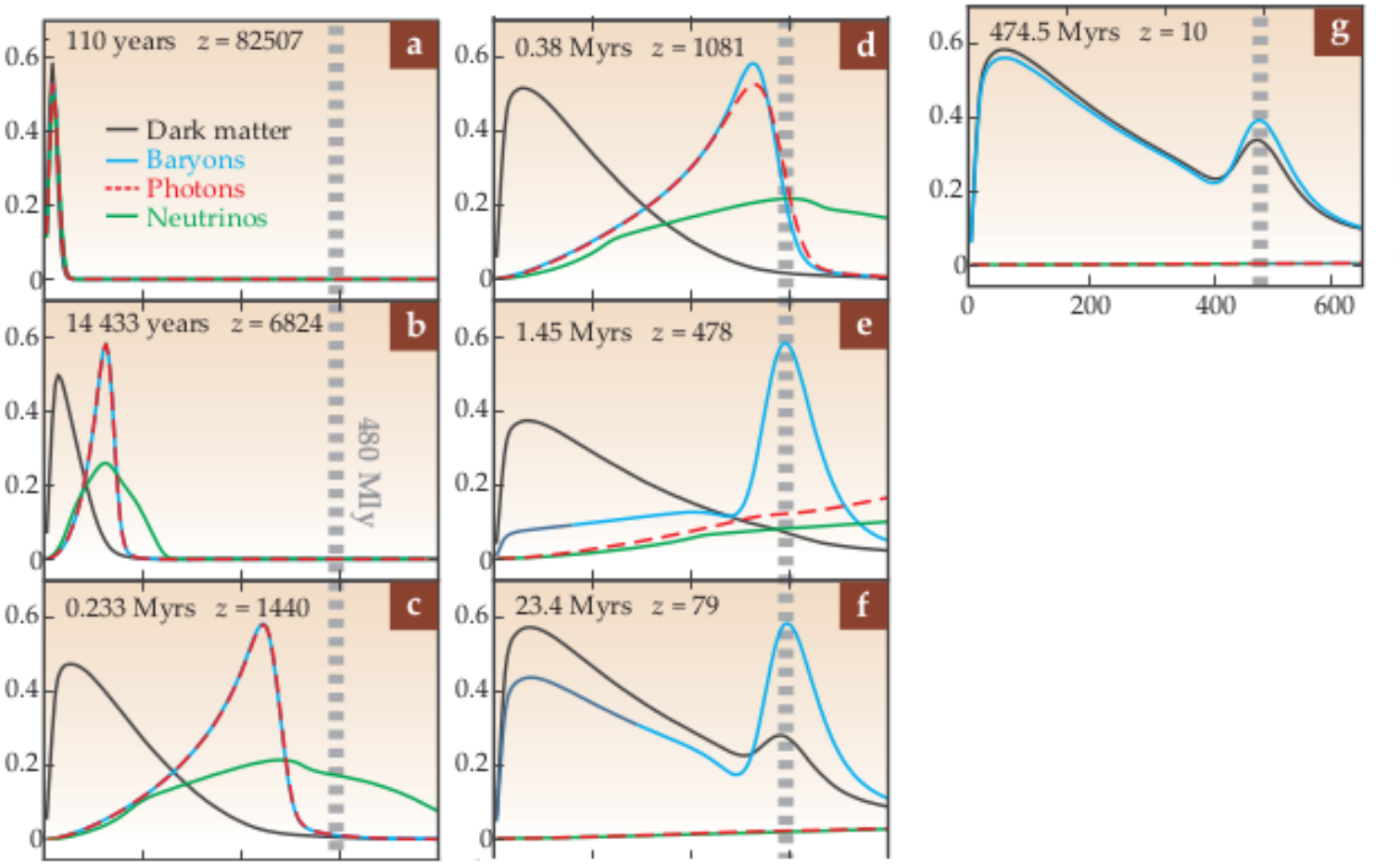
Baryon Acoustic Oscillations

- Oscilations in the baryon-photon plasma in the early universe, due to the competition between gravity and radiation pressure.
  - Electrons cluster together in gravitational potential wells and are compressed. This compression heats the plasma, resulting in an increase in radiation.
  - The increased radiation pressure pushes the plasma outwards, creating a rarefaction. In this way, due to the competition between gravity and radiation pressure, a sound wave is created.
  - The distance that the sound wave can travel from the Big Bang to the time of recombination is called the sound horizon at recombination (r_s). This sound horizon scale is imprinted in the Cosmic Microwave Background (CMB) as the characteristic angular scale of the acoustic peaks in the CMB power spectrum.
  - The sound horizon scale is also imprinted in the distribution of matter in the universe and can be observed in the large-scale structure of the universe as a preferred scale in the clustering of galaxies.

Baryon Acoustic Oscillations



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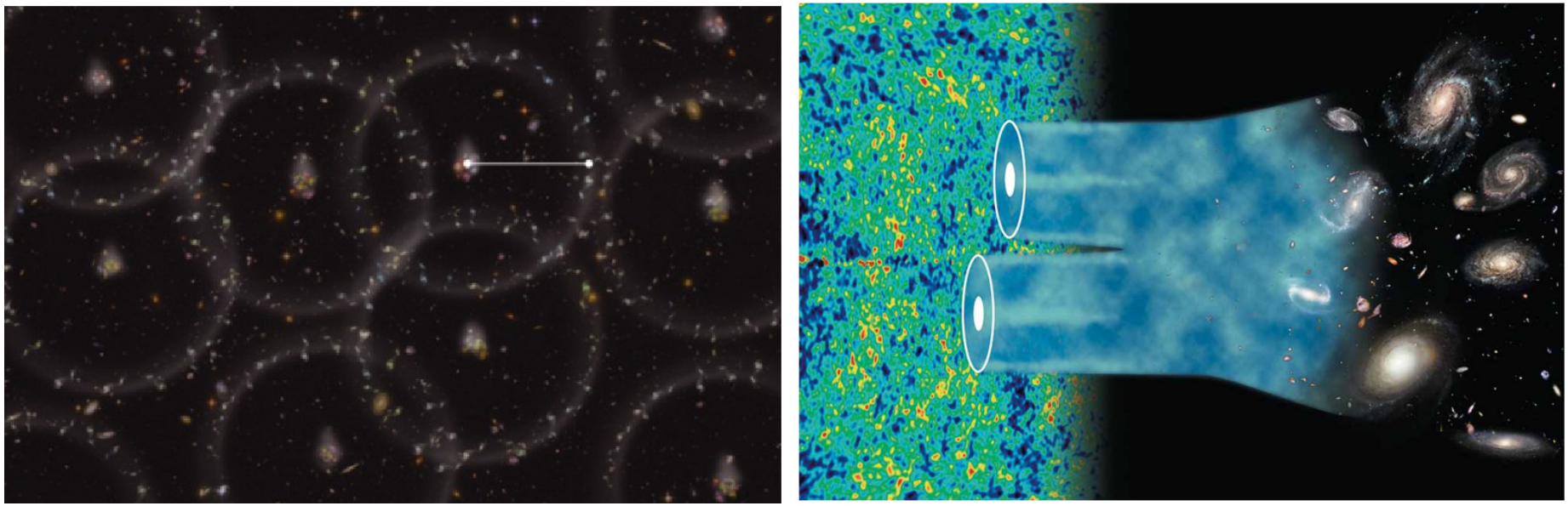


Figure: The large scale structure of the universe observed today shows a preferred scale in the clustering of galaxies, corresponding to the sound horizon at recombination. This is encoded in the two-point correlation function of galaxies as a peak at a separation corresponding to the sound horizon scale.

Baryon Acoustic Oscillations

- The sound horizon at recombination is the maximum distance that a sound wave could have traveled in the baryon-photon plasma before recombination.

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz \tag{3}$$

where z_* is the redshift at the time of recombination and $c_s(z)$ is the sound speed in the baryon-photon plasma:

$$c_s(z) = \frac{c}{\sqrt{3(1+R(z))}} \tag{4}$$

with $R(z) = \frac{3\rho_b}{4\rho_\gamma}$, the ratio of baryon to photon energy densities.

- The relevant scale for BAO is the sound horizon at the drag epoch (z_d), when baryons are released from photons, which is slightly different from that at recombination.

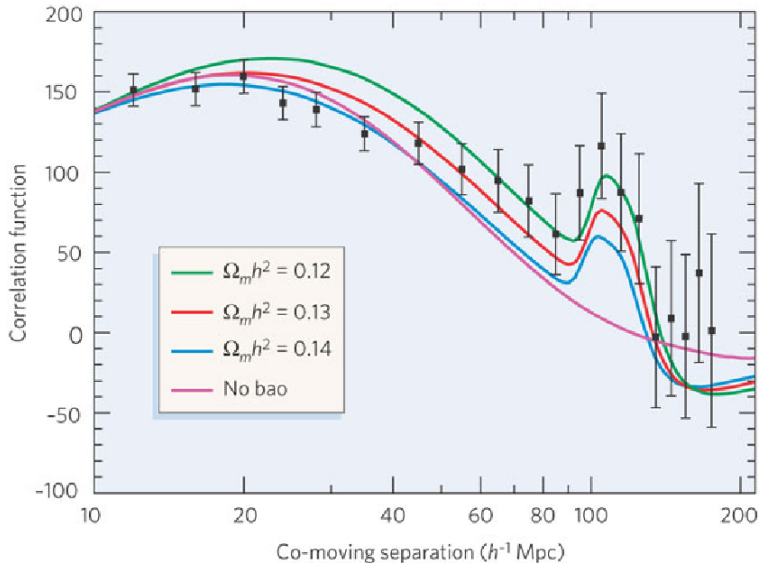
$$r_d(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz \tag{5}$$

Baryon Acoustic Oscillations

- The BAO feature (r_d) can be measured in the two-point correlation function of galaxies, which quantifies the excess probability of finding pairs of galaxies at a given separation compared to a random distribution.

$$\xi_{mm}(r) = \langle \delta_m(\mathbf{x}) \delta_m(\mathbf{x}+\mathbf{r}) \rangle \tag{6}$$

- The BAO feature appears as a peak in the correlation function at a separation corresponding to the sound horizon scale.



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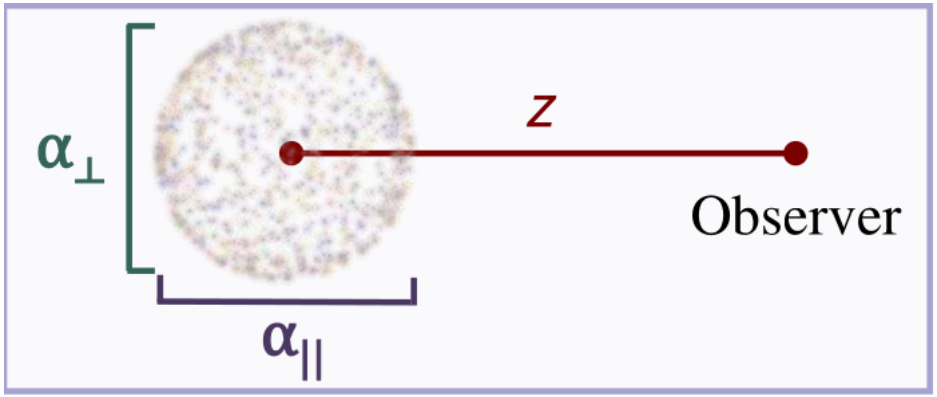


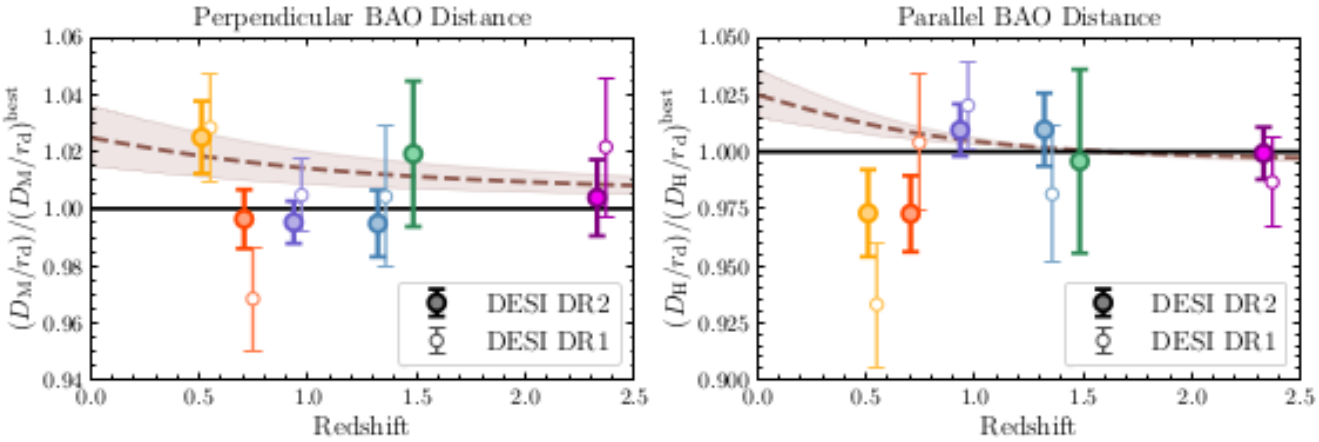
- From the two point correlation function, the following quantities can be measured:

$$\alpha_{||} = \frac{H^{\rm fid}(z) r_s(z_d)}{H(z) r_s^{\rm fid}(z_d)}, \quad \alpha_\perp = \frac{D_M(z) r_s^{\rm fid}(z_d)}{D_M^{\rm fid}(z) r_s(z_d)} \tag{7}$$

where $r_s^{\rm fid}$ refers to the values of the fiducial cosmological model.

Baryon Acoustic Oscillations

- Recent BAO measurements of D_H and D_M from the DESI survey:



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Recombination: the equilibrium approach

- Before recombination, photons, electrons and protons are tightly coupled through $e^- + p \longleftrightarrow \mathrm{H} + \gamma$.
- As long as the interaction rate is much larger than the expansion rate ($\Gamma \gg H(z)$), the reaction is in equilibrium and the chemical potentials satisfy $\mu_e + \mu_p = \mu_{\mathrm{H}}$ (since $\mu_\gamma = 0$).
- For a non relativistic species in thermal equilibrium ($\hbar = c = k_B = 1$):

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{(\mu_i - m_i)/T}$$

where g_i is the statistical weight (internal degrees of freedom) of species i: $g_e = g_p = 2$, $g_{\mathrm{H}} = 4$, so $g_e g_p / g_{\mathrm{H}} = 1$.

- Using the above expressions one gets the Saha equation:

$$\boxed{\frac{n_e\, n_p}{n_{\mathrm{HI}}} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B_1/T}} \tag{8}$$

where n_{HI} is the number density of neutral hydrogen atoms and $B_1 = m_e + m_p - m_{\mathrm{H}} = 13.6$ eV is the binding energy of the hydrogen ground state.

The Saha equation as a function of redshift

- Defining the ionization fraction $x_e \equiv n_e/n_{\rm H} = n_p/n_{\rm H}$, with $n_{\rm H} = n_{\rm HI} + n_p = (1 - Y_p)\, n_b$ the total number density of hydrogen nuclei and $n_e = n_p$:

$$\frac{x_e^2}{1 - x_e} = \frac{1}{n_{\rm H}} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_1/T} \tag{9}$$

- Both scale simply with redshift, $T(z) = T_0(1 + z)$ and $n_b(z) = \eta_b\, n_{\gamma,0}(1 + z)^3$, with $T_0 = 2.7255$ K, $n_{\gamma,0} \simeq 411\ {\rm cm}^{-3}$, $\eta_b \simeq 6.1 \times 10^{-10}$:

$$\boxed{\frac{x_e^2(z)}{1 - x_e(z)} = \frac{1}{(1 - Y_p)\, \eta_b\, n_{\gamma,0}(1 + z)^3} \left(\frac{m_e T_0 (1 + z)}{2\pi} \right)^{3/2} \exp\left[- \frac{B_1}{T_0(1 + z)} \right]} \tag{10}$$

- Recombination happens at $T \sim 0.3$ eV $\ll B_1 = 13.6$ eV: there are $\sim 10^9$ photons per baryon, so the high energy tail of the Planck distribution keeps the gas ionized well below $T = B_1$. Saha with $x_e = 1/2$ gives $z \simeq 1370$.

Why equilibrium is not enough

- The recombination rate is proportional to n_e: as x_e drops, $\Gamma_{\rm rec} \lesssim H(z)$, the reaction freezes out and Saha overestimates how fast recombination proceeds.
- Moreover, recombination directly to the ground state is ineffective: the emitted photon has $h\nu > 13.6$ eV and immediately ionizes a neighbouring atom.
- Peebles (1968) and Zel'dovich, Kurt & Sunyaev (1968): the hydrogen atom is modelled as an effective three level atom: ground state $1s$, first excited level $n = 2$ ($2s$, $2p$) and the continuum.
- Only recombinations to levels $n \geq 2$ contribute, described by the case B coefficient $\alpha^{(2)}(T) = \sum_{n\geq 2} \langle \sigma_{{\rm rec},n}\, v \rangle$ ($\sim 10^{-13}$ cm^{3}s^{-1}, fitted by Pequignot et al. 1991): $\alpha^{(2)} n_e n_p$ is the number of recombinations to $n \geq 2$ per unit volume and time.
- The atom must then reach the ground state without producing an ionizing photon. There are only two channels: the two photon decay $2s \to 1s$, with rate $\Lambda_{2s,1s} = 8.22\ {\rm s}^{-1}$, and the Ly$\alpha$ photons ($2p \to 1s$, $\lambda_{{\rm Ly}\alpha} = 1216$ Å) that redshift out of the line because of the expansion:

$$\Lambda_\alpha(z) = \frac{8\pi H(z)}{\lambda_{{\rm Ly}\alpha}^3\, n_{1s}}, \qquad n_{1s} \simeq (1 - x_e)\, n_{\rm H}$$

The recombination equation (Peebles equation)

- The Boltzmann equation for the free electron fraction becomes

$$\boxed{\frac{dx_e}{dt} = -C(z)\left[\alpha^{(2)}(T)\, n_{\rm H}\, x_e^2 \;-\; \beta^{(2)}(T)\,(1-x_e)\, e^{-(B_1-B_2)/T}\right]} \tag{11}$$

  with $n_{\rm H} = (1 - Y_p)\, n_b$ the number density of hydrogen nuclei (Y_p is the helium mass fraction).

- The photoionization rate from $n = 2$ is related to $\alpha^{(2)}$ by detailed balance (Milne relation):

$$\beta^{(2)}(T) = \alpha^{(2)}(T)\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B_2/T}, \qquad B_2 = \frac{B_1}{4} = 3.4\ \text{eV}$$

  where B_2 is the binding energy of the $n = 2$ level, and the extra factor $e^{-(B_1-B_2)/T}$ (with $B_1 - B_2 = 10.2$ eV, the Lyα energy) converts the ground state population into the population of the $n = 2$ level.

- Consistency check: setting $dx_e/dt = 0$ and $C = 1$, and using $\beta^{(2)} e^{-(B_1-B_2)/T} = \alpha^{(2)}\,(m_e T/2\pi)^{3/2}\, e^{-B_1/T}$, the Saha equation is recovered.

The Peebles factor

- $C(z)$ is the probability that an atom in the $n = 2$ level reaches the ground state before being photoionized again, i.e. the competition between the two escape channels and photoionization:

$$\boxed{C(z) = \frac{\Lambda_{2s,1s} + \Lambda_\alpha(z)}{\Lambda_{2s,1s} + \Lambda_\alpha(z) + \beta^{(2)}(T)} = \frac{1 + K\Lambda_{2s,1s}\, n_{1s}}{1 + K\left(\Lambda_{2s,1s} + \beta^{(2)}\right) n_{1s}}}, \qquad K \equiv \frac{\lambda^3_{\mathrm{Ly}\alpha}}{8\pi H(z)} \tag{12}$$

- Limits:
  - $\beta^{(2)} \gg \Lambda$ (early times): $C \ll 1$, every recombination is undone by photoionization and x_e follows Saha;
  - $\beta^{(2)} \ll \Lambda$ (late times): $C \to 1$, every recombination to $n \geq 2$ is effective and the process is limited only by $\alpha^{(2)} n_{\mathrm{H}} x_e^2$.
- The same set of equations, with the corresponding energy levels and coefficients, must be solved for HeIII $\to$ HeII $\to$ HeI, which recombine earlier than hydrogen.

Note that C depends on $H(z)$ through Λ_α: the recombination history is sensitive to the expansion rate, and therefore to modified gravity or to any extra energy density at that epoch.

Recombination as a function of redshift and conformal time

- Using $\mathrm{d}z/\mathrm{d}t = -(1+z)H(z)$, the recombination equation reads

$$\boxed{\frac{\mathrm{d}x_e}{\mathrm{d}z} = \frac{C(z)}{H(z)\,(1+z)}\left[\alpha^{(2)}(T)\,n_{\mathrm{H}}\,x_e^2 - \beta^{(2)}(T)\,(1-x_e)\,e^{-(B_1-B_2)/T}\right]} \tag{13}$$

- In terms of the conformal time η, with $\mathrm{d}t = a\,\mathrm{d}\eta$ and $a = 1/(1+z)$, so that $\mathrm{d}/\mathrm{d}t = a^{-1}\mathrm{d}/\mathrm{d}\eta$:

$$\boxed{\frac{\mathrm{d}x_e}{\mathrm{d}\eta} = -\frac{C(z)}{1+z}\left[\alpha^{(2)}(T)\,n_{\mathrm{H}}\,x_e^2 - \beta^{(2)}(T)\,(1-x_e)\,e^{-(B_1-B_2)/T}\right]} \tag{14}$$

  with all the coefficients evaluated at $T(z) = T_0(1+z)$ and $n_{\mathrm{H}}(z) \propto (1+z)^3$.

- This is the form integrated, together with the Boltzmann hierarchy for the perturbations, in the numerical codes (CAMB, CLASS), where η is the time variable. The recombination modules are RECFAST (Seager et al. 2000), CosmoRec and HyRec.
- Recombination turns out to be delayed with respect to Saha, not instantaneous, and leaves a residual ionization $x_e \simeq 2 \times 10^{-4}$.

The recombination redshift z_*

- Once $x_e(\eta)$ is known, the Thomson optical depth and the visibility function are:

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e(\eta')\,\sigma_T\,a(\eta')\,\mathrm{d}\eta', \qquad g(\eta) = -\frac{\mathrm{d}\tau}{\mathrm{d}\eta}\,e^{-\tau(\eta)} \tag{15}$$

  $g(\eta)$ is the probability that a photon observed today last scattered at η: its peak defines the last scattering surface, and its width ($\Delta z \simeq 80$) the thickness of the surface.

- The recombination (or last scattering) redshift is defined by $\tau(z_*) = 1$, and there is no shortcut: it must be obtained by numerical integration,
  - integrating the recombination equation to get $x_e(\eta)$;
  - integrating $\tau(\eta)$ and solving $\tau(z_*) = 1$ (equivalently, locating the peak of g).

- Saha with $x_e = 1/2$ gives $z \simeq 1370$: only a rough estimate. The full numerical calculation gives $z_* \simeq 1090$, to be compared with Planck 2018: $z_* = 1089.80 \pm 0.21$.

- The data now determine z_* to $\sim 0.02\%$, so the complete numerical treatment is mandatory: helium recombination, a multilevel atom, two photon transitions from higher levels and Lyα radiative transfer are all needed at that accuracy.

The drag redshift z_d

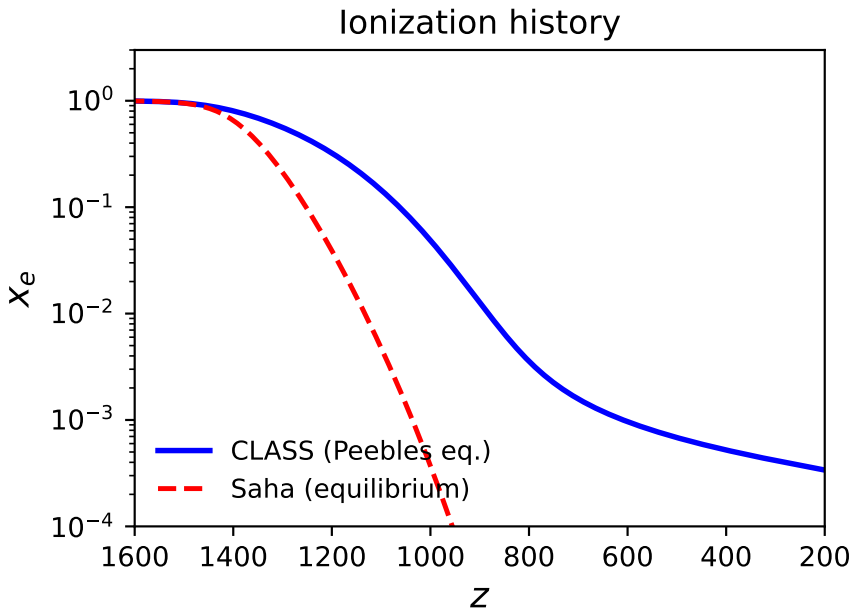
- Photons decouple from baryons at z_*, but the baryons remain dragged by the photons somewhat longer: there are $\sim 10^9$ photons per baryon, so the few residual electrons are still able to transfer momentum to the plasma.
- The optical depth relevant for the baryons is weighted by the ratio of baryon to photon momentum density, the same $R(z)$ that enters the sound speed c_s:

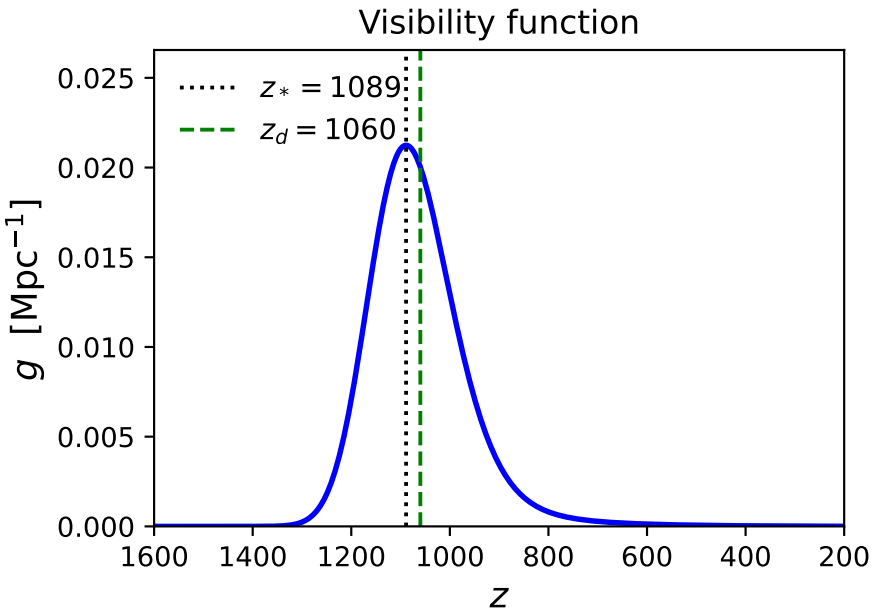
$$\tau_d(\eta) = \int_{\eta}^{\eta_0} \frac{n_e(\eta')\,\sigma_T\,a(\eta')}{R(\eta')}\,\mathrm{d}\eta', \qquad R(z) = \frac{3\rho_b}{4\rho_\gamma} = \frac{3\Omega_{b,0}}{4\Omega_{\gamma,0}}\,a \tag{16}$$

- The drag epoch is defined by $\tau_d(z_d) = 1$ and is again computed by numerical integration, from the same $x_e(\eta)$: it gives $z_d \simeq 1060 < z_*$.
- This is why the BAO standard ruler is $r_d = r_s(z_d)$ and not $r_s(z_*)$: the baryons keep travelling with the acoustic wave until z_d,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\,\mathrm{d}z = \int_0^{\eta_d} c_s(\eta)\,\mathrm{d}\eta, \qquad c_s = \frac{c}{\sqrt{3\,(1+R)}}$$

Ionization history and visibility function





- Left: the equilibrium (Saha) solution recombines too early; the numerical solution of the recombination equation delays recombination and leaves the residual ionization $x_e \simeq 3 \times 10^{-4}$.
- Right: the visibility function peaks at $z_* \simeq 1089$ with a width $\Delta z \simeq 80$ (the last scattering surface has a finite thickness), and the baryons are released slightly later, at $z_d \simeq 1060$.
- CLASS with Planck 2018 parameters gives $r_s(z_*) = 144.5$ Mpc and $r_d = 147.1$ Mpc: a 1.8% difference, well above the DESI precision.

Recombination: summary

- Equilibrium (Saha): an algebraic equation, exact while $\Gamma \gg H$, but it overestimates the speed of recombination and gives $z \simeq 1370$.
- Out of equilibrium (Peebles): a differential equation in t, z or η, with the effective three level atom, the case B coefficient $\alpha^{(2)}$, the Milne relation for $\beta^{(2)}$ and the Peebles factor $C(z)$ that encodes the two photon decay and the Lyα escape. It gives $z_* \simeq 1090$.
- Both z_* (CMB acoustic peaks) and z_d (BAO standard ruler) are derived quantities, obtained by numerical integration of the recombination equation and of the corresponding optical depths: they depend on $\Omega_b h^2$, $\Omega_m h^2$ and, through $\Lambda_\alpha \propto H(z)$, on the expansion rate at that epoch.
- Therefore any modification of gravity, or any extra component (e.g. early dark energy, extra relativistic species), z_*, z_d and r_d will be affected.

Peebles ApJ 153, 1 (1968); Zel'dovich, Kurt & Sunyaev Sov. Phys. JETP 28, 146 (1969); Seager, Sasselov & Scott ApJS 128, 407 (2000); Chluba & Thomas MNRAS 412, 748 (2011); Ali-Haïmoud & Hirata Phys. Rev. D 83, 043513 (2011).

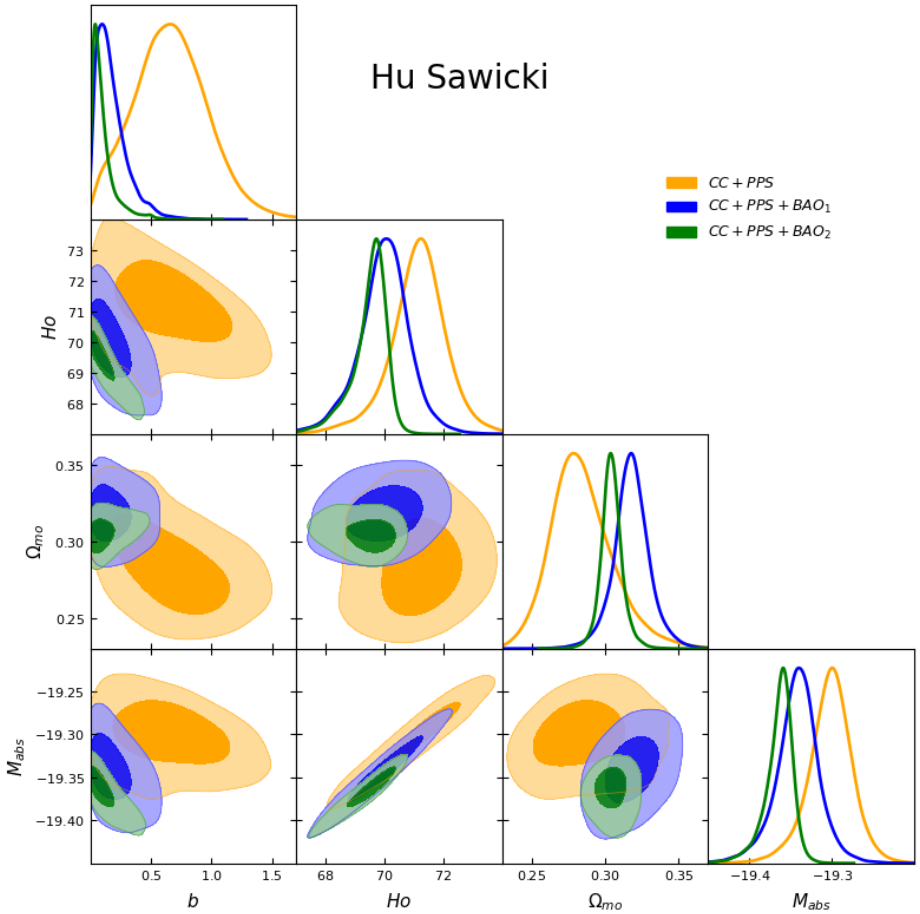
Outline

1. Primordial Nucleosynthesis
2. Baryon Acoustic Oscillations
3. Recombination
4. Cosmological probes of the Background and Modified Gravity theories
5. Relevant papers for this lecture

Cosmological probes of the Background and Modified Gravity theories

- Type Ia supernovae, Cosmic Chronometers and Baryon Acoustic Oscillations are probes that measure the background evolution of the universe.
- These probes can be used to constrain cosmological parameters and test modified gravity theories.
- This probes test the background evolution of the universe and are note sensitive enough to detect deviations from GR.
- Therefore they can be used to constrain the free parameters of MG theories but not to distinguish between GR and MG theories.
- Some papers where these probes are used to constrain MG theories:
  - Plaza & Kraiselburd Phys. Rev. D 112, 023554 (2025)
  - Verma et al. arXiv:2508.20107
  - Varela & Bertolami Phys.Dark Univ. 48 (2025) 101861

Constraints on f(R) models

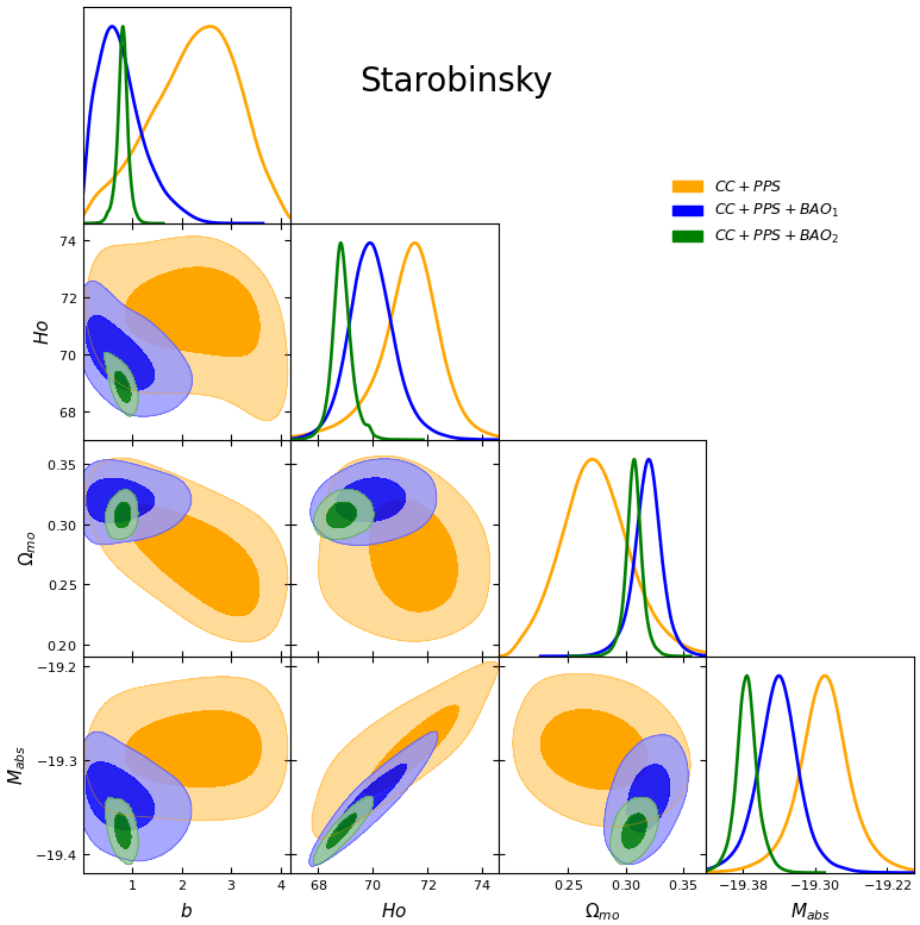


- Hu-Sawicki model:

$$f_{\rm HS}(R) = R - 2\Lambda\left[1 - \frac{1}{1+\left(\frac{R}{\Lambda b}\right)^n}\right]$$

Figure: Plaza & Kraiselburd Phys. Rev. D 112, 023554 (2025)

Constraints on f(R) models



- Starobinsky model:

$$f_{\rm ST}(R) = R - 2\Lambda\left[1 - \frac{1}{\left[1+\left(\frac{R}{\Lambda b}\right)^2\right]^n}\right]$$

Figure: Plaza & Kraiselburd Phys. Rev. D 112, 023554 (2025)

Constraints on f(T) models

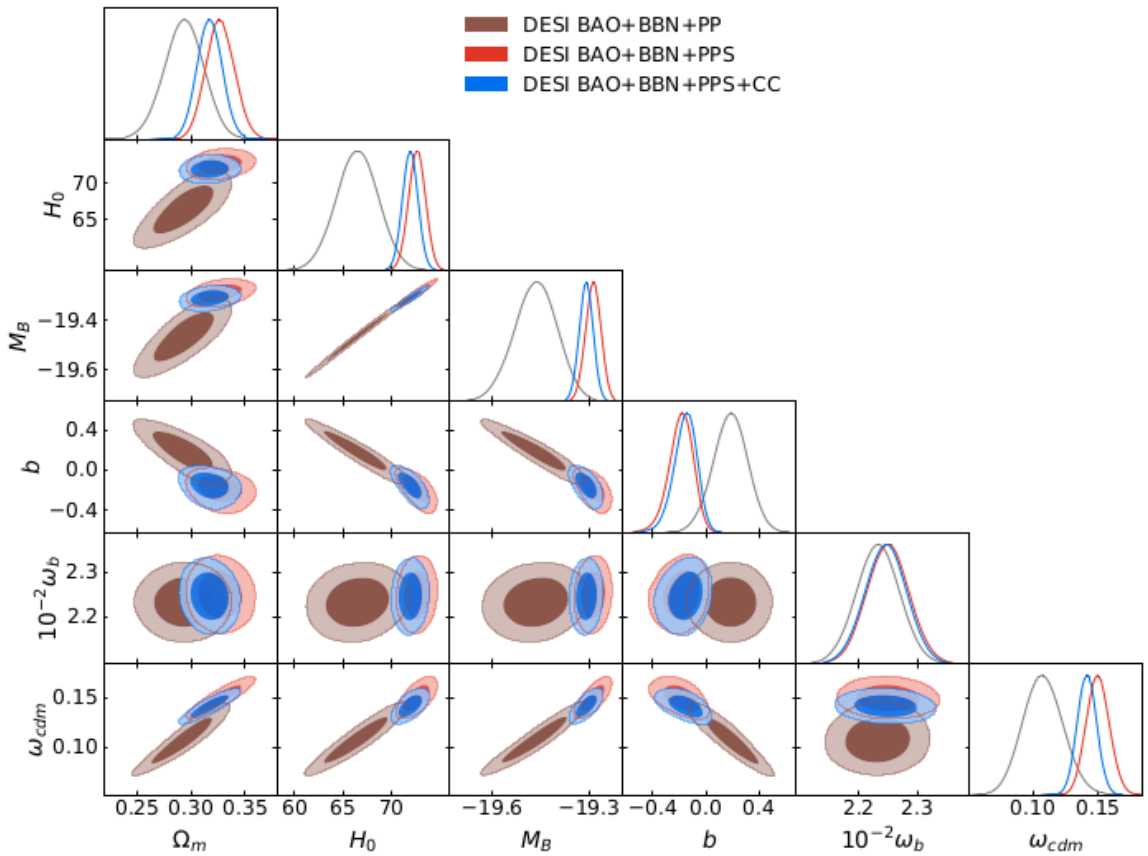


Figure: Verma et al. arXiv:2508.20107

$$f(T) = \alpha(-T)^b$$

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Relevant papers for this lecture

- DESI Collaboration DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints, Physical Review D112, 083515, (2025).
- DESI collaboration Journal of Cosmology and Astroparticle Physics, 04, 012, (2025).
- Asimakis et al. Phys.Rev.D 105, 084010 (2022)
- Seager, Sasselov & Scott ApJS 128, 407 (2000)
- Chluba & Thomas MNRAS 412, 748 (2011)
- Ali-Haïmoud & Hirata Phys. Rev. D 83, 043513 (2011)
- Plaza & Kraiselburd Phys. Rev. D 112, 023554 (2025)
- Verma et al. arXiv:2508.20107