Introducing Warm Inflation: Dynamics, Microphysics, and Observational Signatures

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Plan of the Lectures:

- **Lecture 1: Introduction and Motivation**
  - Inflation, what for ?
  - Cold versus warm inflation
  - Brief overview of applications
- **Lecture 2: Basics of nonequilibrium QFT - part 1**
  - Dissipation in Quantum Mechanics: the Caldeira-Leggett model
  - Schwinger-Keldysh formalism for non-equilibrium QFT
  - Deriving effective equations of motion
- **Lecture 3: Basics of nonequilibrium QFT - part 2**
  - Local approximations for non-Markovian dissipation terms
  - Some local dissipation coefficients
  - Applications to Cosmology: the case of warm inflation
- **Lecture 4: Warm inflation dynamics — background and perturbations**
  - Building warm inflation models
  - Background dynamics
  - Perturbations in warm inflation

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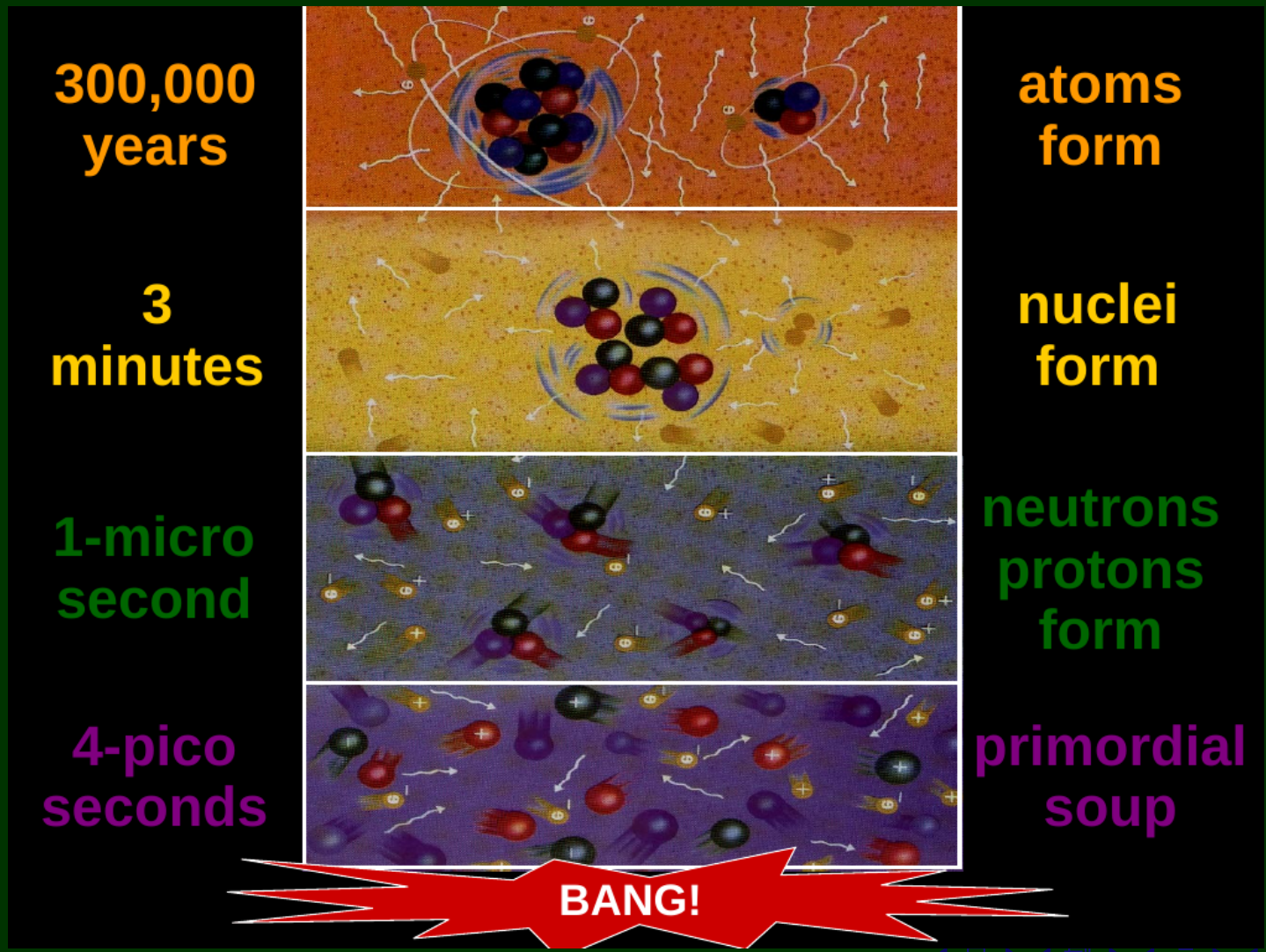
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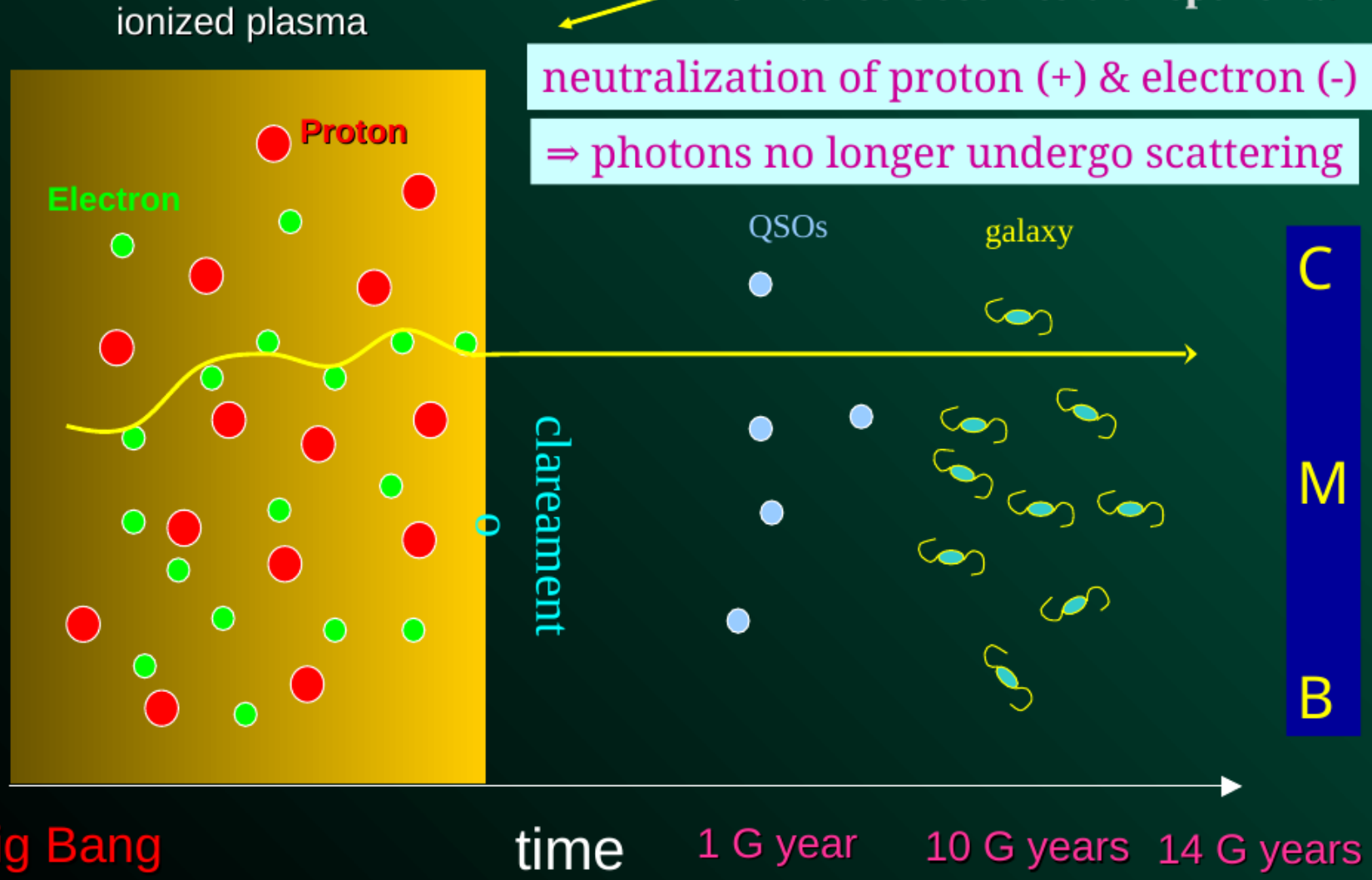
Inflation, what for ?

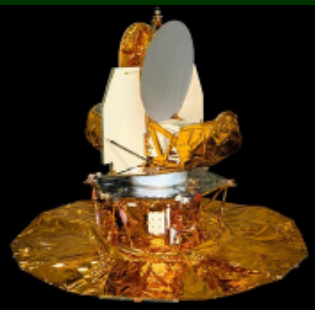
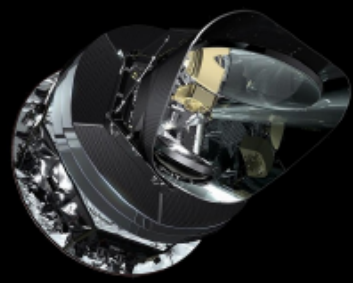
Hot Big Bang:

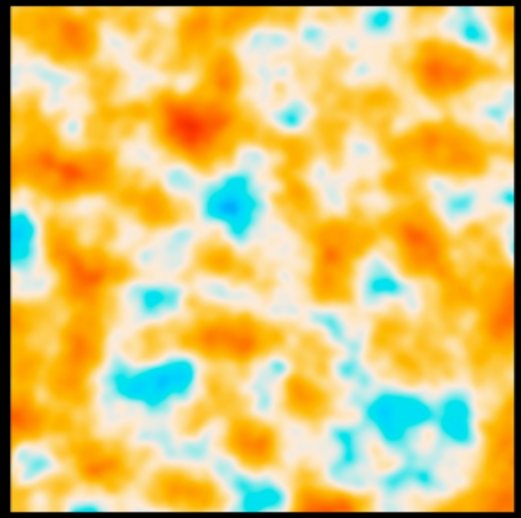
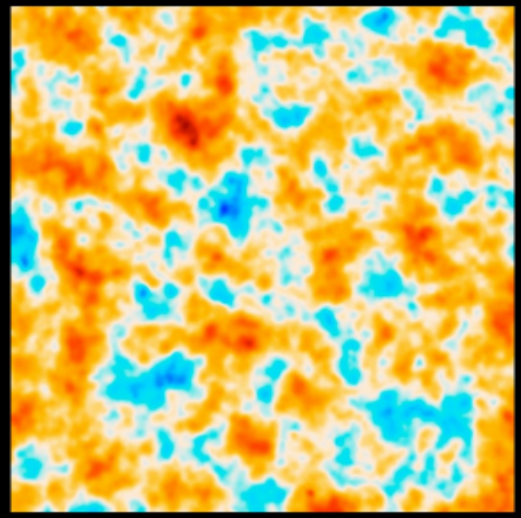


Universe becoming transparent

~380.000 years after big bang electrons bind to nucleons making atoms **universe becomes transparent..**



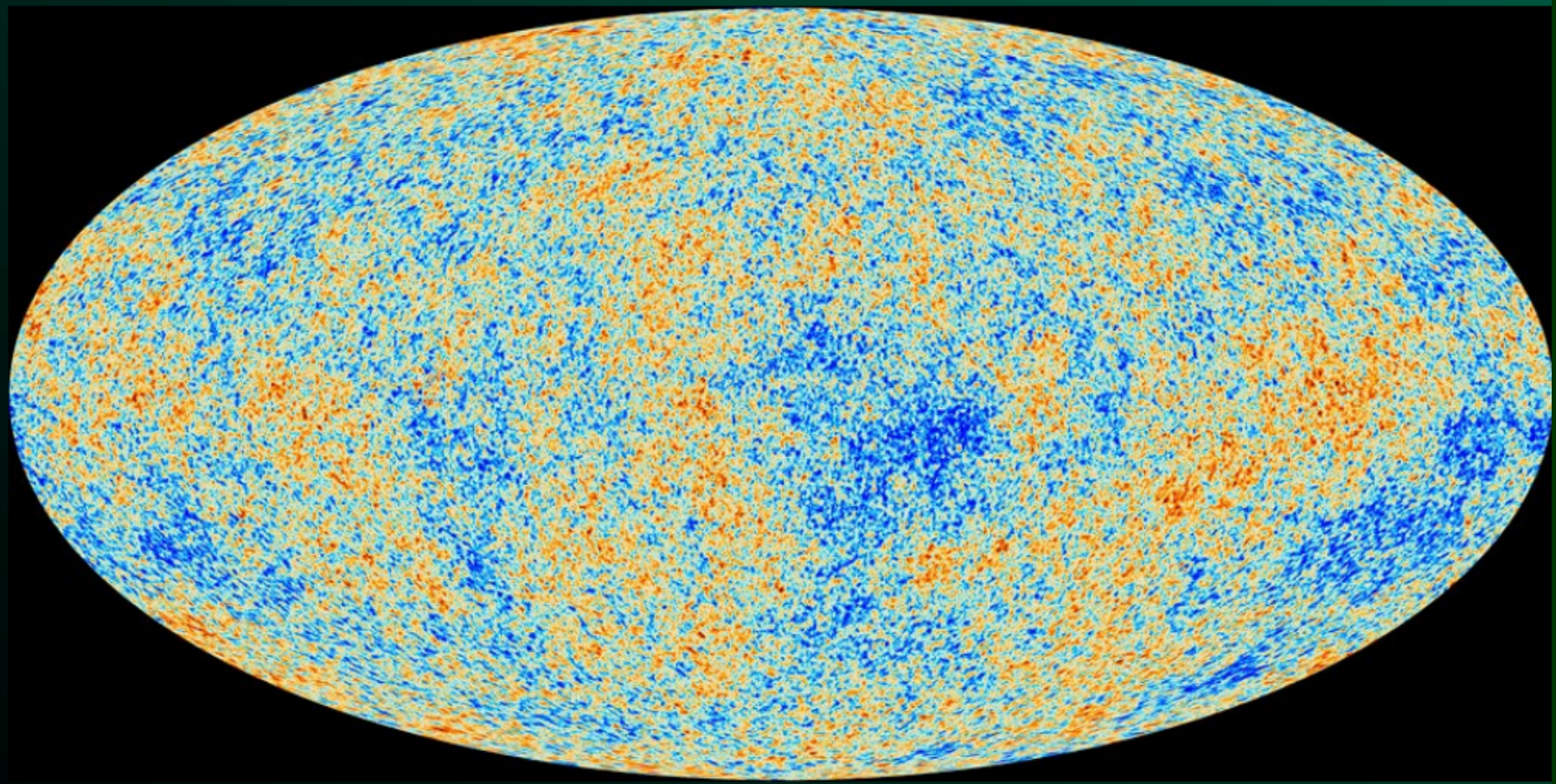




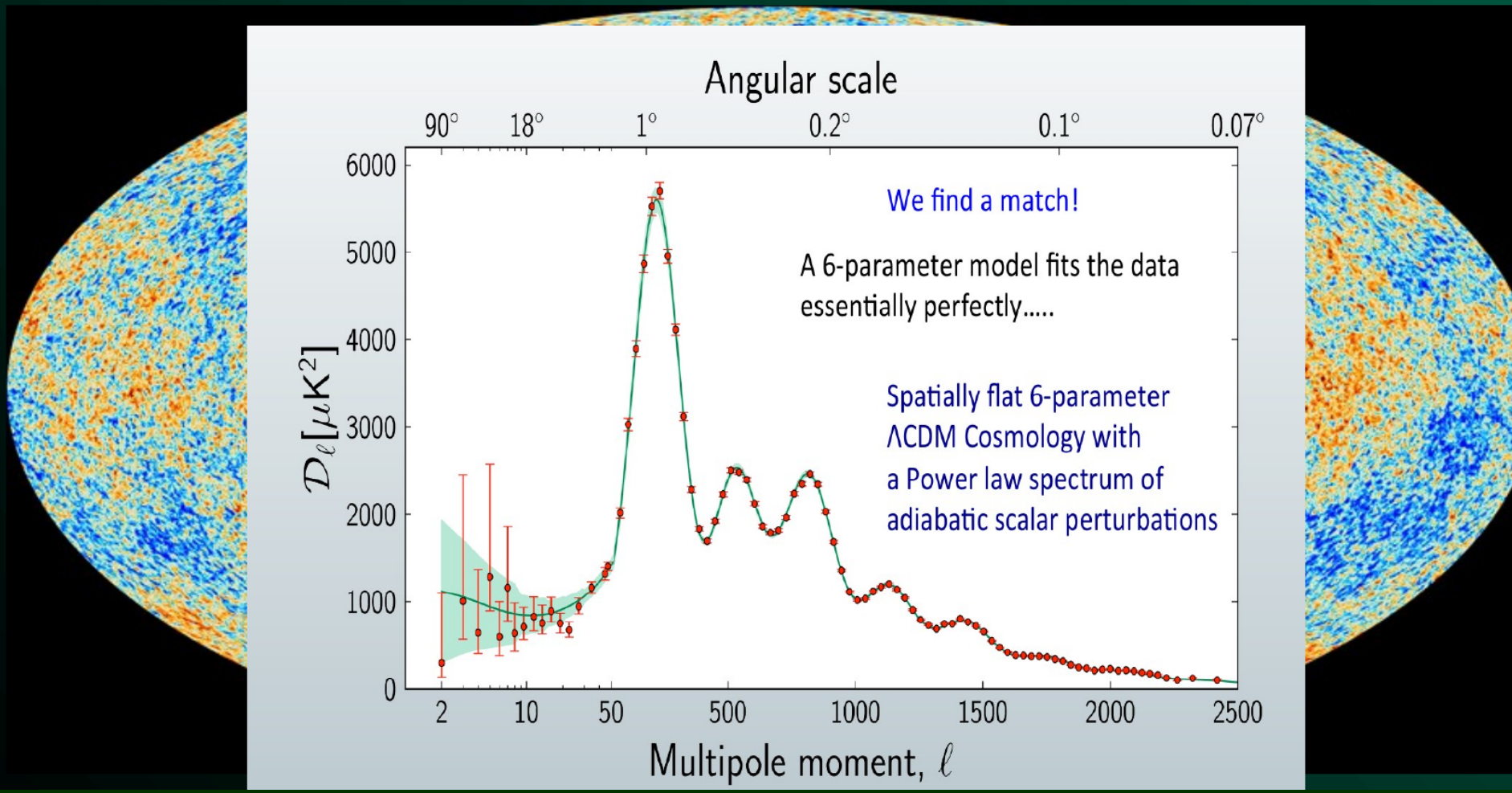
COBE WMAP Planck

1989 ***2001*** ***2009***

Accessing the composition and structure of the primordial Universe:
Measurements of the CMB spectrum, observation of galaxy clusters, etc.



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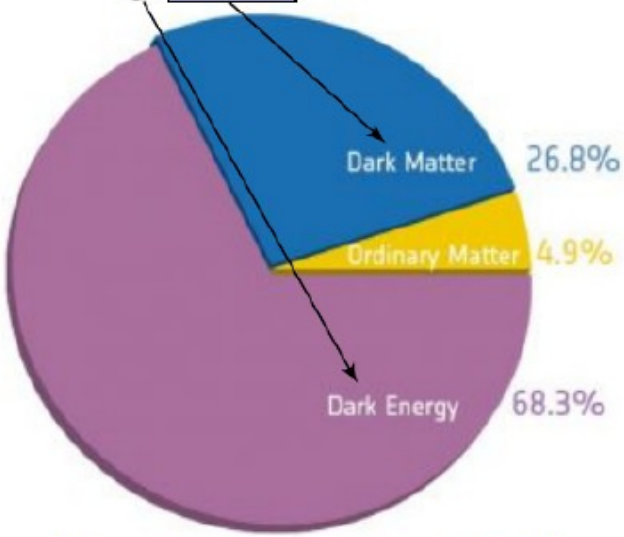


Cosmological parameters

General Relativity + Homogeneity isotropy

Cosmological evolution

ΛCDM model



[Planck collaboration, 2015]

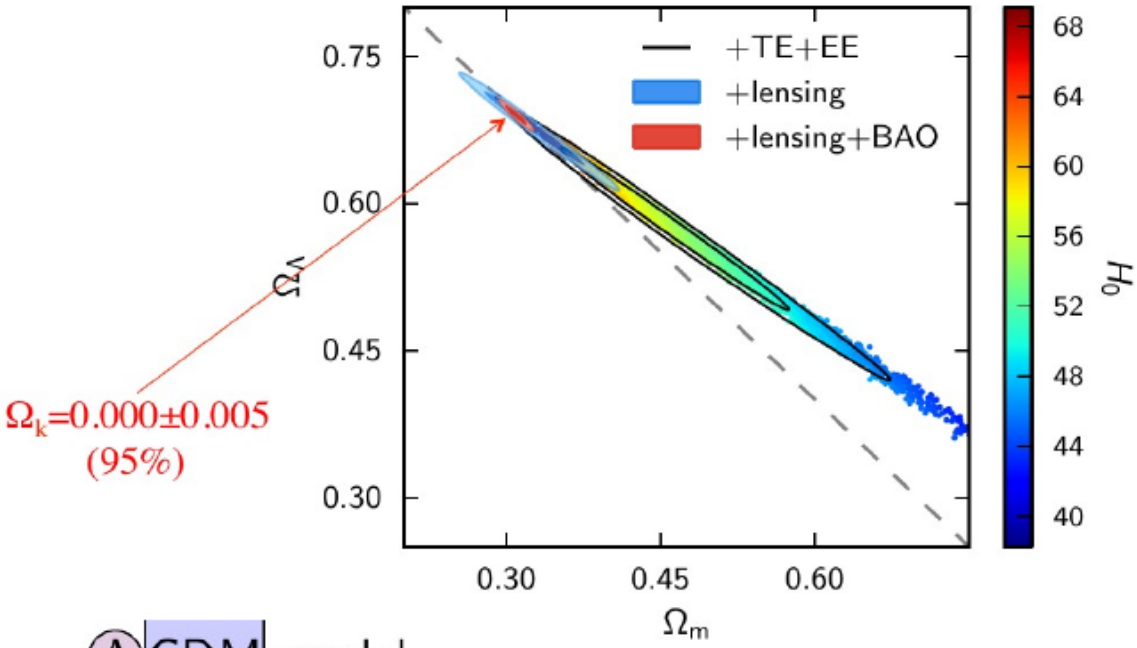
ΛCDM model described by 6 base parameters:

- $\omega_b = \Omega_b h^2$ baryon density today;
- $\omega_c = \Omega_c h^2$ CDM density today;
- τ optical depth to reionization;
- θ angular scale of acoustic peaks;
- n_s tilt and
- A_s amplitude of the power spectrum of initial curvature perturbations.

Other quantities can be derived:

- H_0 Hubble parameter today;
- σ_8 mean matter fluctuations at small scales;

Still flat after all these years ...



$\Omega_b h^2 = 0.02205 \pm 0.00028$

$\Omega_c h^2 = 0.1199 \pm 0.0027$

$n_s = 0.9603 \pm 0.0073$

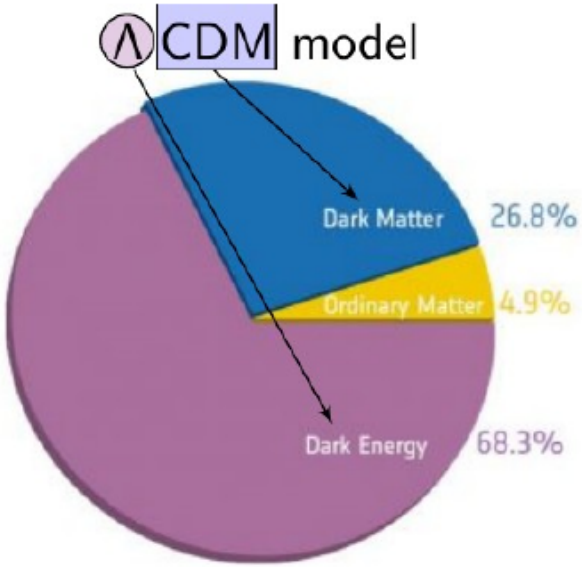
$\ln(10^{10} A_s) = 3.089 \pm 0.025$

$100\theta = 1.04131 \pm 0.00063$

$H_0 = 67.3 \pm 1.2$ km s^{-1} Mpc^{-1}

Age = 13.81± 0.05 billion years

Consistent with spatial flatness to % level



[Planck collaboration, 2015]

$\Omega \sim 1$

Problems with the Big Bang model

1- Why $\Omega=1$? The flatness problem

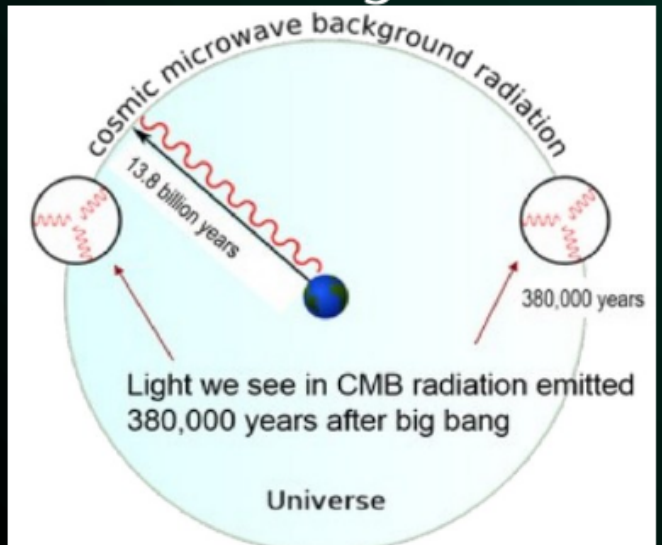
$$\Omega_T=1 \Rightarrow \Omega_T(t_{nucl})-1\approx 10^{-16}$$

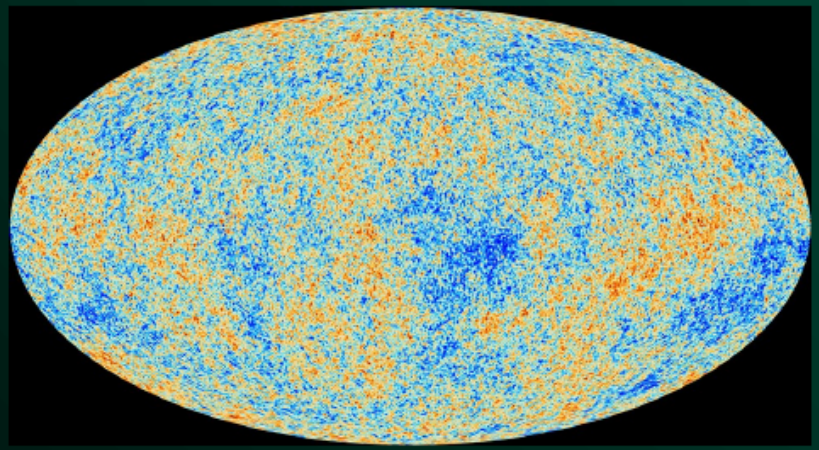
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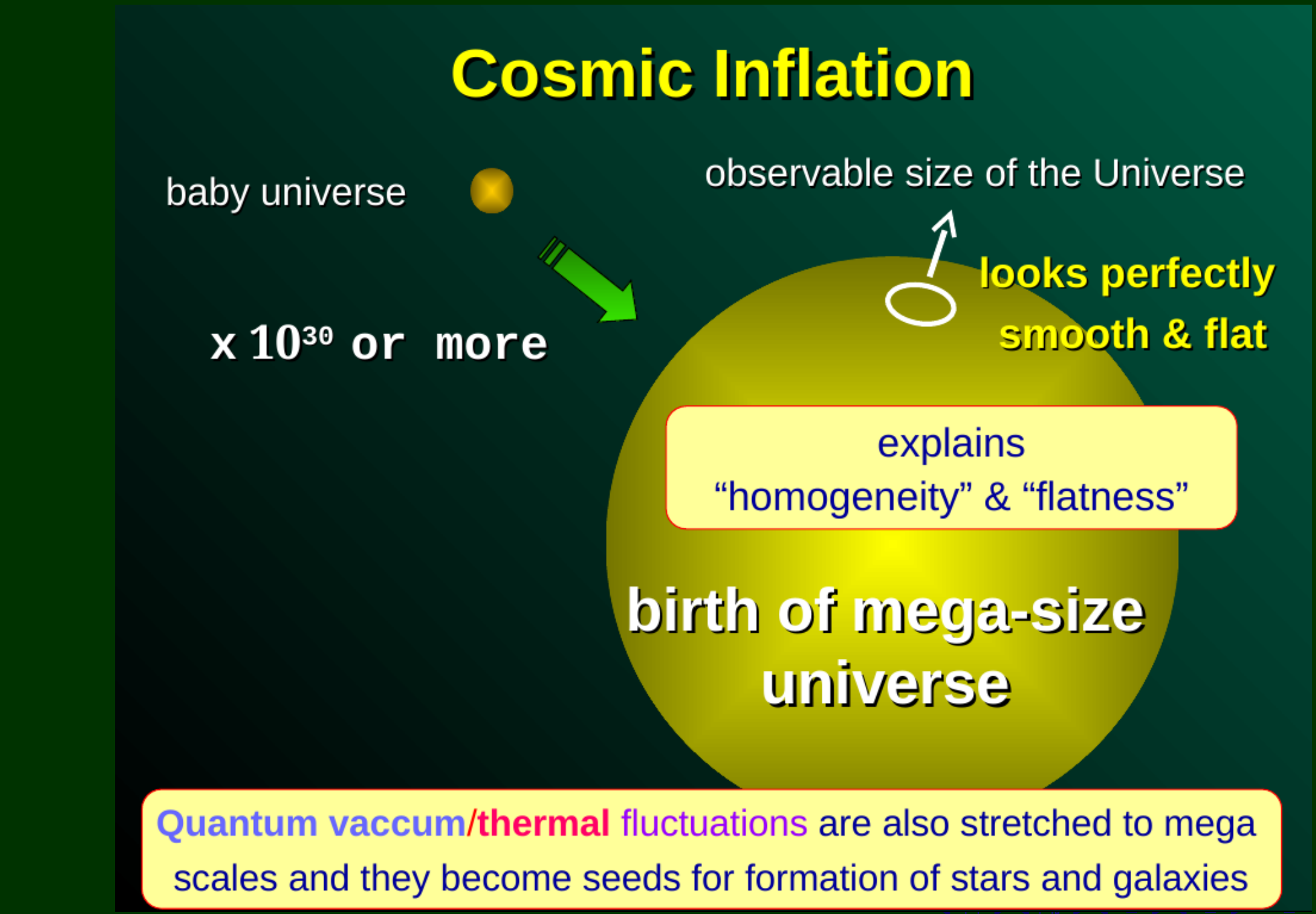
$$\Omega_T=1 \Rightarrow \Omega_T(t_{nucl})-1\approx 10^{-16}$$

2- The horizon problem: the Universe appears the same in all directions even though it arose from causally unconnected regions in the past.





Cosmic Inflation



Quantum vaccum/thermal fluctuations are also stretched to mega scales and they become seeds for formation of stars and galaxies

Cosmological Inflation[1]

- Defined as an early accelerated expansion dominated by vacuum energy density and driven by a fluid with negative pressure:

$$\ddot{a} > 0, \quad p < -\rho/3 \quad \Rightarrow \quad \text{Inflation}$$

- Inflaton field ϕ:

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

- to have inflation the potential energy must dominate: $p \approx -\rho$
- the potential must be flat in order to inflation last long enough ($N_e \sim 60$ e-folds of inflation) and solve the BB problems

[1] (Starobinsky '80; Guth '81; Albrecht, Steinhardt '82; Linde '82)

Relevent cosmological parameters for inflation:

- Curvature perturbations: $\Phi = H\delta\phi/\dot{\phi}, \quad \delta\phi(\mathbf{x}, t) = \phi(\mathbf{x}, t) - \phi(t)$

$$P_R^2 = \frac{k^3}{2\pi^2}\int_{k'}\langle\Phi(k)\Phi(k')\rangle = A_s(k_0)\left(\frac{k}{k_0}\right)^{n_s-1+\frac{1}{2}dn_s/d\,lnk\,ln(k/k_0)+\ldots}, \quad \text{amplitude} = A_s(k_0$$

- Spectral index (n_s) and its running (n_s'):

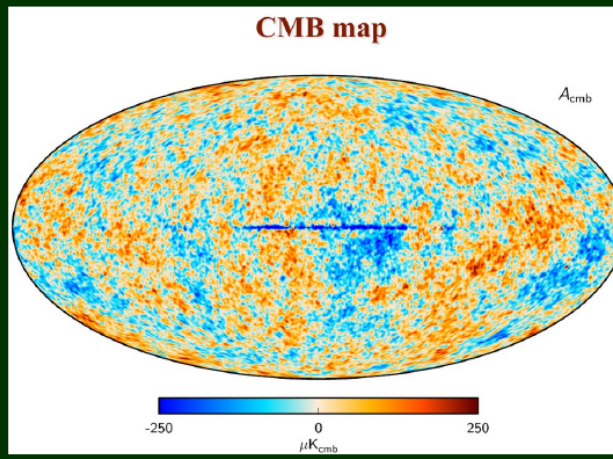
$$n_s - 1 = \frac{d\ln P_R^2}{dN_e} = \frac{d\ln P_R^2}{d\ln k}, \quad n_s' = dn_s/d\,lnk$$

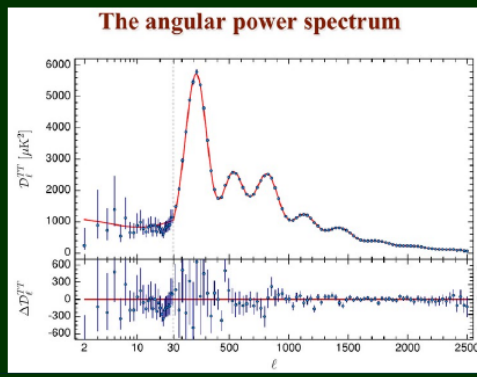
- Tensor to scalar curvature perturbation ratio:

$$r = \frac{P_T^2}{P_R^2}, \quad \langle h_{\mathbf{k}} h_{\mathbf{k}'}\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')P_T(k)$$

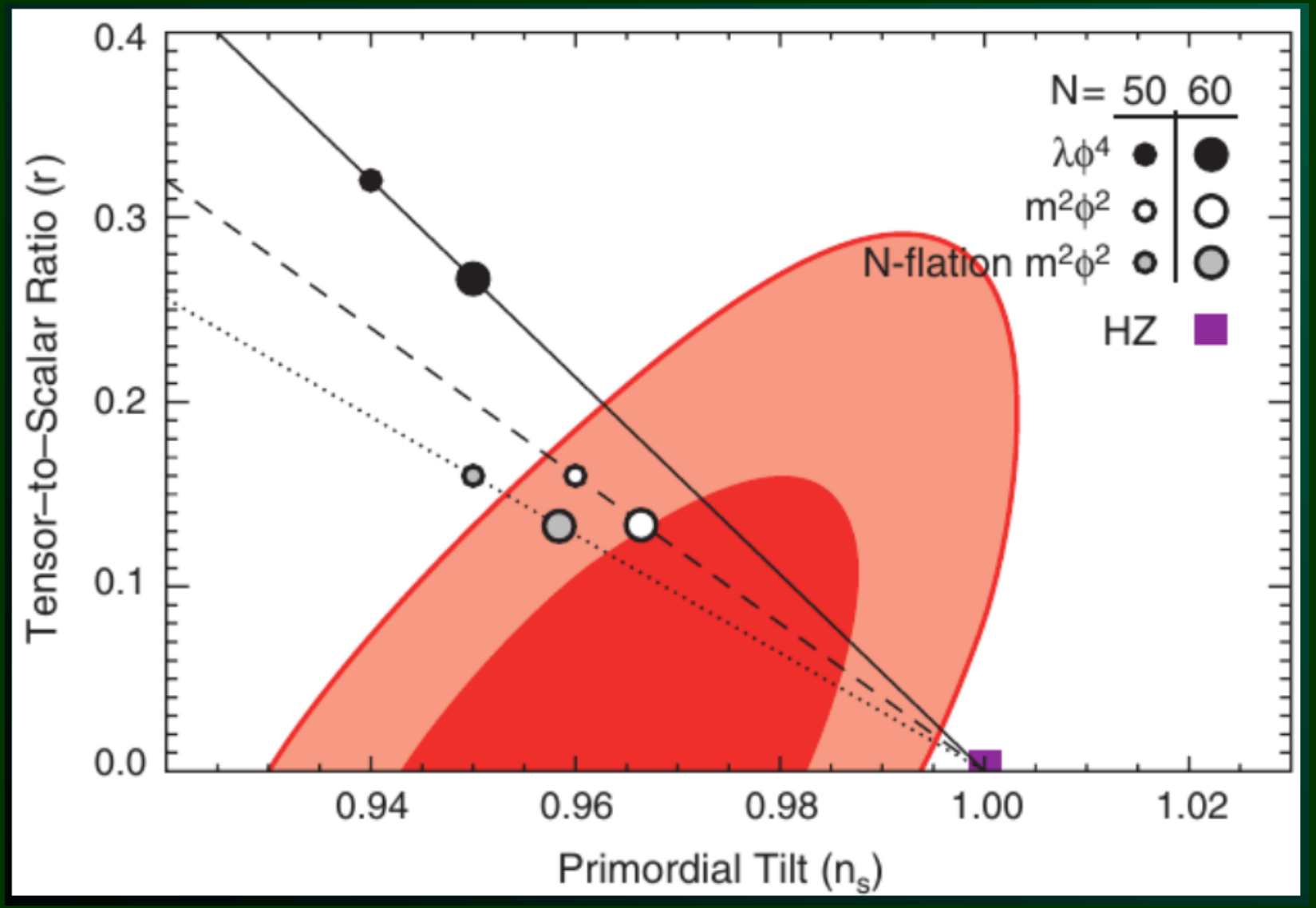
- NonGaussianity:

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$
$$B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = f_{NL}\ F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

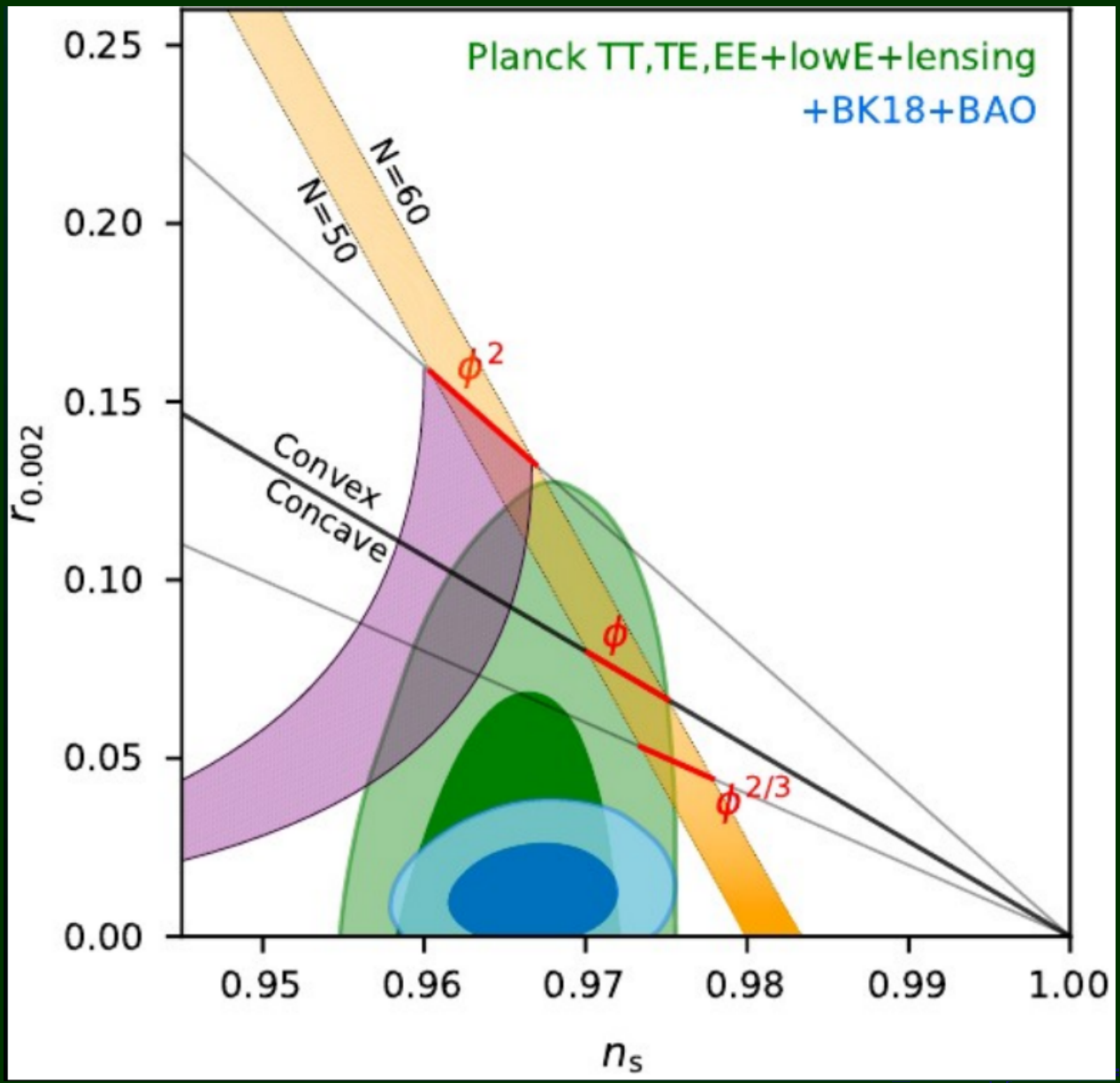




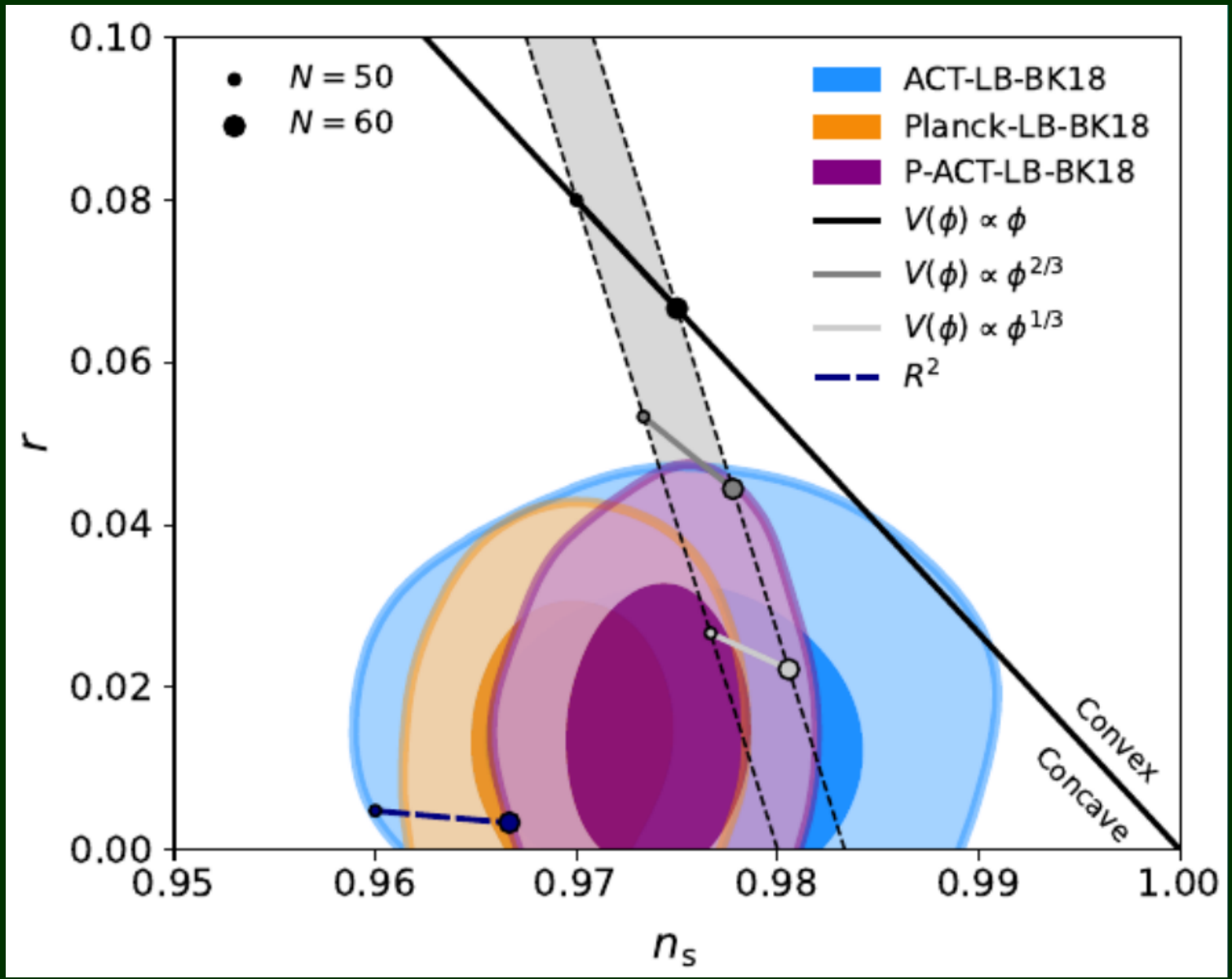
WMAP 2011 (7year)



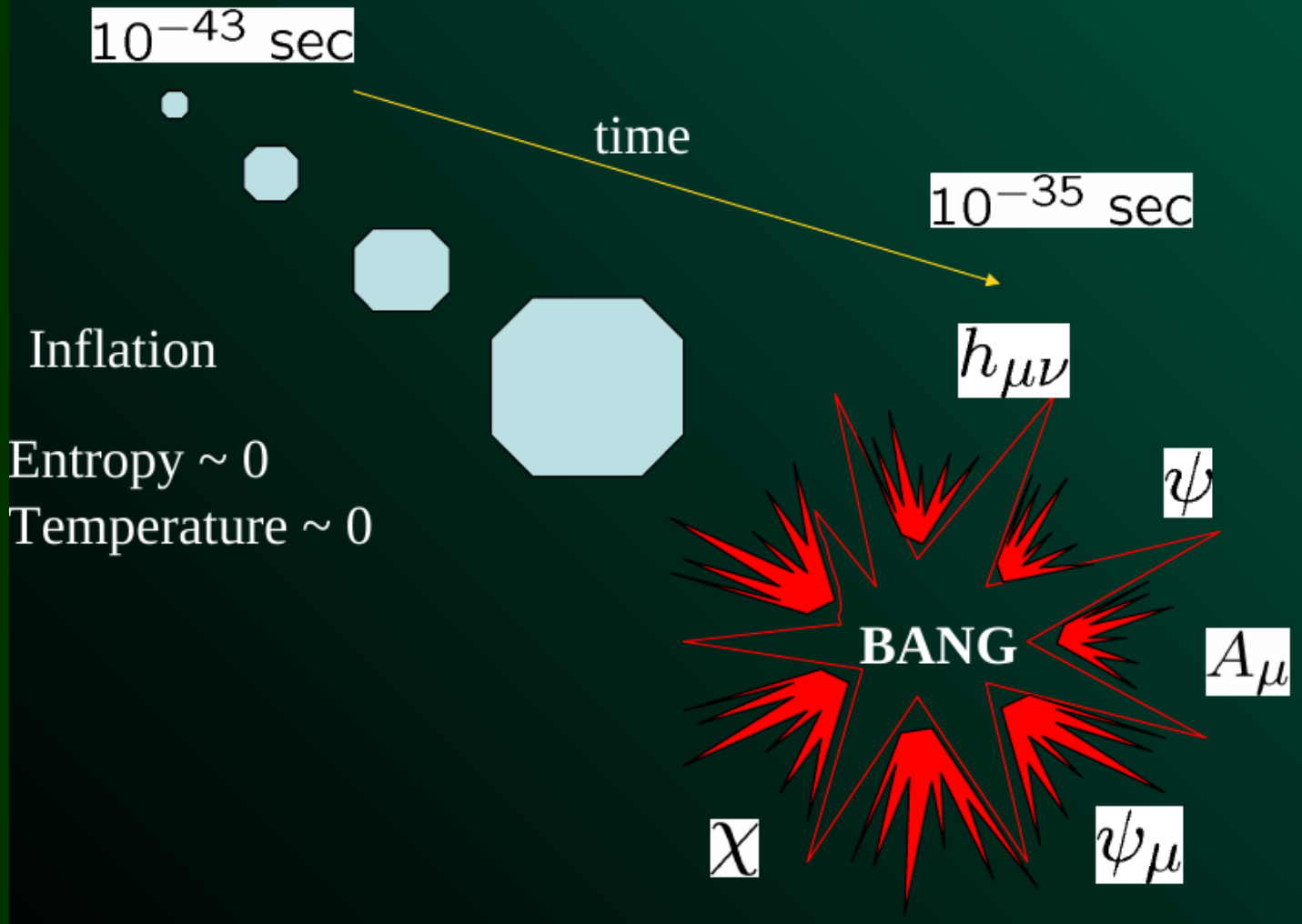
Planck+Bicep/Keck 2021



ACT 2025 DR6


0.10
0.08
0.06
0.04
0.02
0.00
r
0.95
0.96
0.97
0.98
0.99
1.00
n_s
$N = 50$
$N = 60$
ACT-LB-BK18
Planck-LB-BK18
P-ACT-LB-BK18
$V(\phi) \propto \phi$
$V(\phi) \propto \phi^{2/3}$
$V(\phi) \propto \phi^{1/3}$
R^2
Convex
Concave

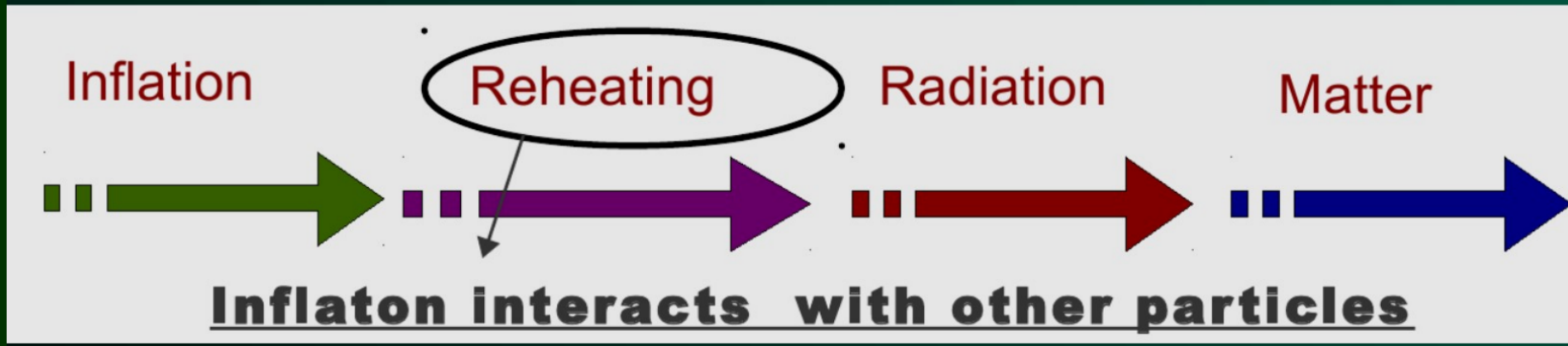
connecting inflation with the Big Bang model (radiation):



$$\mathcal{L}(\phi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu})$$

Birth of Big Bang Universe

From inflation to the Big Bang :



inflationary universe hot Big Bang universe

birth of matter (quarks, electrons, neutrinos, ...)
& radiation (photons)

~ 10^{-34} sec old universe

~ 10^{28} K



proton
= 3 quarks

From inflation to an interacting inflaton

- Inflation explains an early accelerated expansion and seeds for structure.
- A particle-physics inflaton interacts with other degrees of freedom.
- Can those interactions matter during inflation?

The question for this minicourse

When do interactions generate a sustained radiation bath, dissipation, and observable fluctuations?

Cold inflation: what is being neglected?

$$\hbar = c = k_B = 1, \qquad M_{\rm P} = (8\pi G)^{-1/2}, \qquad H = \dot{a}/a$$

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V, \quad p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V, \quad p_r = \rho_r/3$$

$$3M_{\rm P}^2 H^2 = \rho_\phi + \rho_r.$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \qquad \dot{\rho}_r + 4H\rho_r \simeq 0 \;\; \Rightarrow \;\; \rho_r \propto a^{-4}$$

- Energy transfer to a bath is negligible during the inflationary phase.
- Interactions become important in reheating or preheating.
- The usual scalar spectrum follows from vacuum fluctuations.

Negligible dissipation is a dynamical approximation.

Warm inflation: the coupled background

What if dissipation is not negligible during inflation ?

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2, \qquad Q \equiv \frac{\Upsilon}{3H}.$$

- The same positive coefficient damps the inflaton and sources radiation.
- In general, $\Upsilon = \Upsilon(\phi, T)$.
- These are local, near-equilibrium effective equations.

Physical interpretation

Energy leaves the inflaton sector and enters the bath. The combined system conserves energy in an expanding Universe.

Radiation without radiation domination

$$-\ddot{a} \propto \rho_{\rm tot} + 3p_{\rm tot} = 2\dot{\phi}^2 - 2V + 2\rho_r.$$

$$\ddot{a} > 0 \quad \Longleftrightarrow \quad V > \dot{\phi}^2 + \rho_r.$$

- Inflation does not require an empty Universe.
- A subdominant radiation bath can coexist with acceleration.
- The conditions for acceleration and for a thermal bath are distinct.

Two ratios, two different questions

$$Q = \frac{\Upsilon}{3H}, \qquad \frac{T}{H}.$$

- Q: how important is dissipative damping compared with Hubble damping?
- T/H: how hot is the bath compared with the Hubble scale?
- Weak dissipation: $Q \ll 1$; strong dissipation: $Q \gg 1$.
- The usual thermal warm regime has $T > H$ and a maintained bath.

The slow-roll, slowly varying bath approximation

$$|\ddot{\phi}| \ll 3H(1+Q)|\dot{\phi}|, \qquad |\dot{\rho}_r| \ll 4H\rho_r.$$

$$3H(1+Q)\dot{\phi} \simeq -V_{,\phi}, \qquad 4H\rho_r \simeq \Upsilon\dot{\phi}^2.$$

$$\rho_r \simeq \frac{3}{4}Q\dot{\phi}^2.$$

Slowly varying does not mean exactly constant. Lecture 4 will test when this approximation is an attractor.

One useful consequence: the radiation fraction

$$\epsilon_V = \frac{M_{\rm P}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad \eta_V = M_{\rm P}^2\frac{V_{,\phi\phi}}{V}, \qquad H^2 \simeq \frac{V}{3M_{\rm P}^2}.$$

$$\dot{\phi}^2 \simeq \frac{2\epsilon_V V}{3(1+Q)^2}, \qquad \frac{\rho_r}{V} \simeq \frac{\epsilon_V Q}{2(1+Q)^2}.$$

Take-home point

A bath can be sustained while its energy density remains small compared with the potential energy.

How can a small radiation fraction be hot?

$$\rho_r = C_R T^4, \qquad C_R = \frac{\pi^2 g_*}{30}.$$

$$\frac{T}{H} \simeq \left[\frac{3\epsilon_V Q}{2C_R(1+Q)^2}\left(\frac{M_{\rm P}}{H}\right)^2\right]^{1/4}.$$

- $H \ll M_{\rm P}$ permits $T > H$ even when $\rho_r \ll V$.
- This relation assumes a relativistic, equilibrated bath and slow roll.

Extra friction changes the rolling speed

$$\dot{\phi}_{\rm warm} \simeq -\frac{V_{,\phi}}{3H(1+Q)}.$$

At fixed V and H, the rolling speed is reduced by $1+Q$.

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} \simeq \frac{\epsilon_V}{1+Q}.$$

Potential slopes ruled out in a cold realization may work in a warm realization. This is a possibility, not a guarantee of a consistent model.

Warm inflation to radiation domination

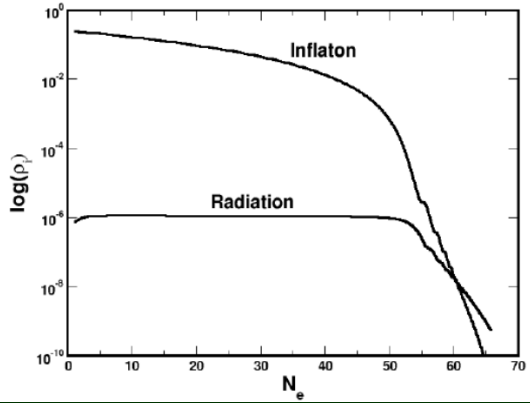
Ending inflation: a dynamical question

$\epsilon_H \longrightarrow 1$ ends accelerated expansion.

- Radiation is already present in the warm regime.
- Suitable evolution can lead smoothly to radiation domination.
- The evolution of V, Q, and ρ_r determines whether this occurs.

A (small) fraction of the vacuum energy is converted into radiation during inflation

$\rho = \rho(\phi) + \rho_R, \quad \rho_R \ll \rho(\varphi)$



$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_\phi = 0$$

$$\dot{\rho_R} + 4H\rho_R = \Upsilon \dot{\phi}^2 \quad \text{“Source term”}$$

“Decay” into light dof= extra friction

$\Rightarrow$ Warm inflation permits a smooth exit.

Microscopic justification: open-system description

We have introduced the effective equations. We now need their microscopic justification: system+environment

$$\underbrace{\phi}_{\text{system}} + \underbrace{\chi, \psi, \ldots}_{\text{environment}} + \text{interactions.}$$

- Follow the inflaton rather than every microscopic bath degree of freedom.
- Tracing over the environment gives response, memory, and fluctuations.
- Dissipation emerges in the effective system dynamics.

Lecture 2

Start with a quantum-mechanical system plus an oscillator bath, then move to QFT.

A preview of the effective equation

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V_{,\phi} + \int_{t_0}^{t} dt'\, K(t,t')\dot{\phi}(t') = \xi.$$

- K: a causal response kernel; spatial dependence suppressed here.
- ξ: stochastic environmental force, with $\langle\xi\rangle = 0$.
- Local conservative corrections are included in V.

This schematic equation organizes what we must derive.

Dissipation comes with fluctuations

In a local, classical thermal limit, a familiar schematic form is

$$\langle \xi(\mathbf{x}, t)\xi(\mathbf{x}', t')\rangle = 2\Upsilon T\, a^{-3}(t)\delta^3(\mathbf{x}-\mathbf{x}')\delta(t-t').$$

- Coordinates are comoving; the normalization refers to a local force density.
- The noise and damping are related by bath correlations.
- Quantum and finite-memory regimes require a more general relation.

Increasing damping also changes the fluctuation problem.

Why nonequilibrium QFT?

- We need time-dependent expectation values for a specified initial state.
- Retarded response must preserve causality.
- Bath correlations determine both noise and dissipation.

initial density matrix $\longrightarrow$ closed time path $\longrightarrow$ effective equations.

Lecture 2

The Schwinger–Keldysh formalism supplies this framework.

When can memory be replaced by friction?

$$\int_{t_0}^{t} dt'\, K(t,t')\dot{\phi}(t') \simeq \Upsilon\dot{\phi}(t) \quad \Rightarrow \quad \Upsilon = \Upsilon(\phi, T; \text{couplings, masses, widths})$$

Local damping and white noise are approximations to be justified

- Bath memory must be short compared with the relevant background evolution (e.g. $\tau_{\rm bath} H \ll 1$ and $\tau_{\rm bath}|\dot{T}/T| \ll 1$)
- Thermal bath: relevant equilibration rates exceed cosmological evolution rates.
- Adiabatic response: parameters vary slowly over the bath memory time.

Lecture 3

Derive the local approximation and representative dissipation coefficients, with their domains of validity.

The model-building tension

For a simple Yukawa interaction,

$$\mathcal{L}_{\rm int} = -g\phi\bar{\psi}\psi, \qquad m_\psi(\phi) = g\phi.$$

- Large $g\phi$ can remove the fermion from the light thermal bath.
- If thermally populated, it can give $\Delta m_\phi^2 \sim g^2 T^2$.
- The same interactions must provide enough dissipation and acceptable potential corrections.

A phenomenological friction term is not yet a microscopic model.

Three routes to microscopic realizations

- Heavy mediators: the inflaton couples indirectly to light bath fields.
- Symmetry protection: arrange small potential corrections while retaining dissipation.
- Axion–gauge interactions: use a different microscopic response channel.

Each route has its own thermal hierarchy, effective-theory limits, and relaxation constraints.

Examples of temperature dependence

$$\Upsilon = C_\phi \frac{T^3}{\phi^2} \qquad \text{or} \qquad \Upsilon = C_T T.$$

- These forms arise in particular models and parameter regimes.
- The coefficients and allowed ranges come from microphysics.
- Different temperature dependence changes bath feedback and fluctuations.

Choose a mechanism before treating a coefficient as a model prediction.

From a warm bath to primordial perturbations

- Vacuum fluctuations remain part of the fluctuation problem.
- Bath noise adds a thermal source to inflaton fluctuations.
- Inflaton, radiation, and metric perturbations evolve together.

microscopic noise $\longrightarrow$ $\delta\phi, \delta\rho_r$ $\longrightarrow$ $\mathcal{R}$ $\longrightarrow$ CMB and structure.

Thermal dominance is regime dependent.

The scalar spectrum: an organizing formula (spoilling)

For many canonical warm-inflation calculations,

$$\Delta_{\mathcal{R}}(k_*) \simeq \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2 \mathcal{F}_*.$$

- $\mathcal{F}_*$ collects occupation, thermal noise, and coupled evolution.
- Its value depends on Q_*, T_*/H_*, and the dissipation law.
- The familiar cold result has $\mathcal{F}_* = 1$ in its usual limit.

Lecture 4 will unpack the enhancement rather than assume a universal fit.

Tensors and the tensor-to-scalar ratio

For vacuum tensor modes in standard Einstein gravity,

$$\Delta_T = \frac{2H_*^2}{\pi^2 M_{\rm P}^2}, \qquad r = \frac{\Delta_T}{\Delta_{\mathcal{R}}}.$$

Using the preceding slow-roll expressions,

$$r \simeq \frac{16\epsilon_{V*}}{(1+Q_*)^2\mathcal{F}_*}.$$

$\Rightarrow$ Enhanced scalar fluctuations and slower rolling can suppress r.

This formula assumes no additional tensor sources.

What observations can distinguish?

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}}{d \ln k}, \qquad r = \frac{\Delta_T}{\Delta_{\mathcal{R}}}.$$

- The tilt depends on the evolution of the background and dissipative factors.
- Scalar amplitude and r constrain parameter combinations.
- Higher-point functions can carry additional information about bath coupling.

Applications: possibilities with extra ingredients

- Particle production and relics: a bath during inflation changes production and dilution.
- Baryogenesis: needs a specified source of baryon or lepton number and CP violation.
- Small-scale power: dissipative evolution can change the spectrum beyond CMB scales.

Dark-matter abundances, baryon asymmetry, or primordial black holes are other possible applications (require dedicated models and calculations).

Five messages to take away

1. Acceleration permits a subdominant radiation bath.
2. Dissipative damping and radiation production are linked by energy transfer.
3. Q, T/H, and equilibration conditions answer different questions.
4. Dissipation, noise, and memory have a common microscopic origin.
5. Predictions depend on both the potential and the interactions.

Next time: Lecture 2

A system plus a bath: deriving response and fluctuations.

Suggested reading

- Berera, Moss and Ramos, *Warm Inflation and its Microphysical Basis*, Rep. Prog. Phys. 72 (2009) 026901. arXiv:0808.1855
- Kamali, Motaharfar and Ramos, *Recent developments in warm inflation*, Universe 9 (2023) 124. arXiv:2302.02827

Read the background discussion first; use the microscopic sections alongside Lectures 2 and 3.