Cosmological probes of Gravity

Lecture 1: Testing the background

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1st Arica School on Cosmology and Gravitation
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Universidad de Tarapacá

Cosmological probes of gravity

1. The cosmological background I
   - Brief theoretical review of modified gravity theories.
   - Cosmic Chronometers
   - Type Ia Supernovae and the Hubble tension
2. The cosmological background II
   - Primordial Nucleosynthesis
   - Baryon Acoustic Oscillations
   - Recombination
   - Cosmological probes of the Background and Modified Gravity theories
3. The perturbative level I
   - Cosmic Microwave Background
   - Introduction to Weak Lensing
   - CMB Lensing
4. The perturbative level II
   - Galaxy Clustering
   - Weak Lensing

Outline

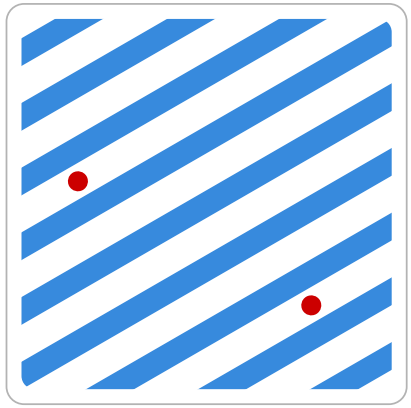
1. The cosmological background in ΛCDM
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4. History of the Universe
5. Cosmic Chronometers
6. Type Ia Supernovae
7. Local Determination of H_0
8. Relevant papers for this lecture

The cosmological background

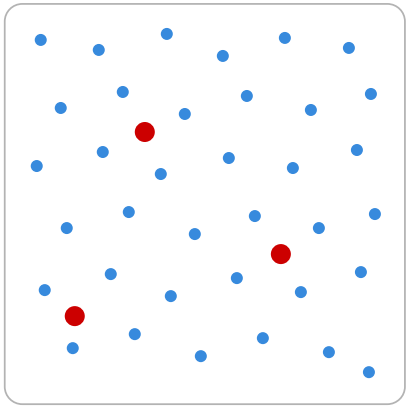
- Cosmological Principle: The universe is homogeneous and isotropic on large scales.



Isotropic, not homogeneous
Only from the center



Homogeneous, not isotropic
Preferred direction



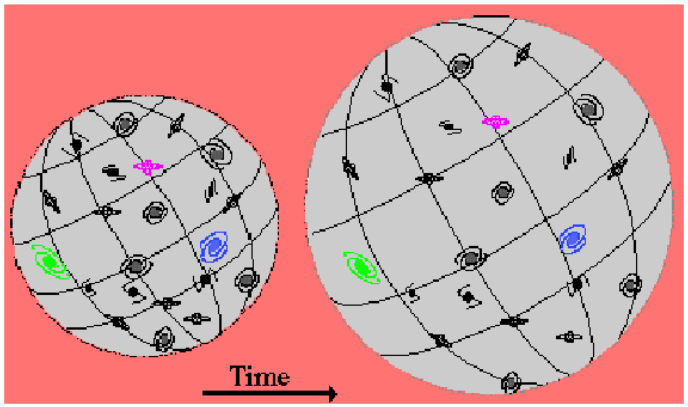
Homogeneous and isotropic
Cosmological principle

● = observer

The cosmological background in ΛCDM

- The metric that describes such a universe is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$\boxed{ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)} \tag{1}$$



- The scale factor $\boxed{a(t)}$ and the Hubble parameter $\boxed{H(t) = \frac{\dot{a}}{a}}$ describe the expansion of the universe.

Friedmann Equations in ΛCDM

- Let us recall Einstein's field equations with a cosmological constant:

$$\boxed{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}} \tag{2}$$

- Assuming the FRW metric, the Friedmann equations are obtained:

$$\boxed{H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}} \tag{3}$$

$$\boxed{2\dot{H} + 3H^2 = -8\pi G p + \Lambda} \tag{4}$$

Here we also assume that the content of $T_{\mu\nu}$ is a perfect fluid, where ρ and p refer to the density and pressure of the matter content of the universe (both relativistic and non-relativistic) and $H = \frac{\dot{a}}{a}$.

Friedmann equation and the matter-energy densities

- The first Friedmann equation can be rewritten in terms of the present density parameters:

$$\Omega_i^0 = \Omega_i = \frac{\rho_i^0}{\rho_c}, \quad \rho_c = \frac{3H_0^2}{8\pi G} \tag{5}$$

where ρ_c is the critical density today and Ω_i are the present-day density parameters for different components of the universe (matter, radiation, cosmological constant, and curvature). The critical density is defined as the density for which the universe is spatially flat.

- The Friedmann equation then reads:

$$H^2 = H_0^2 \left(\frac{\Omega_b}{a^3} + \frac{\Omega_{\rm cdm}}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_k}{a^2} \right) \tag{6}$$

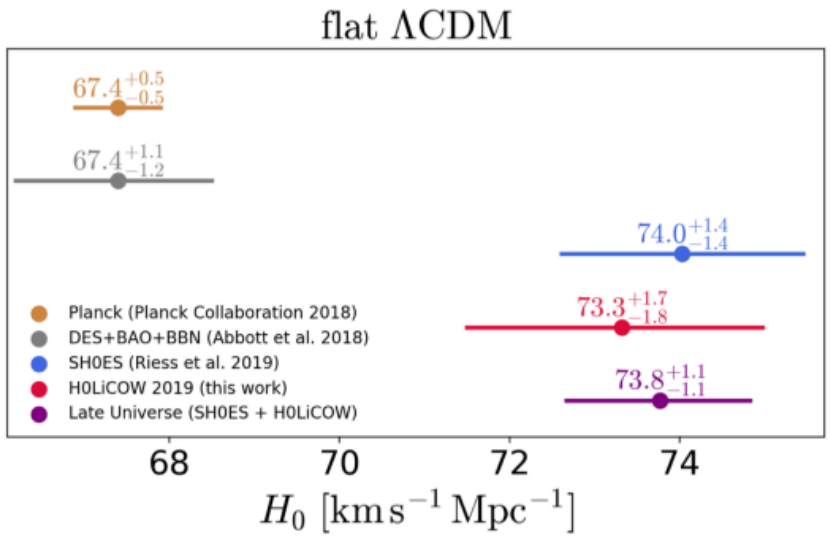
where Ω_b, $\Omega_{\rm cdm}$, Ω_r, Ω_Λ, and Ω_k are the present-day density parameters for baryons, cold dark matter, radiation, the cosmological constant, and curvature, respectively.

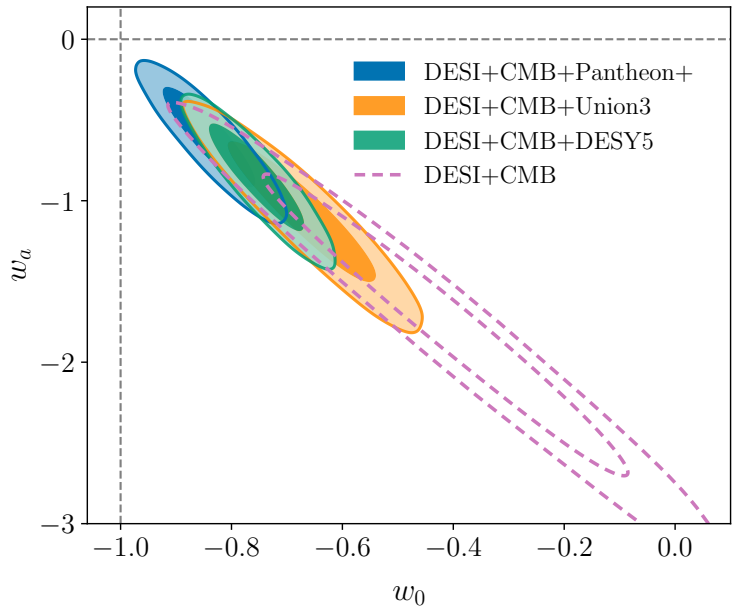
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Main motivations to study alternative cosmological models

- Explain the observed value of the cosmological constant.
- Solve or alleviate tensions in cosmological data (e.g., Hubble tension, growth tension).
- Explain recent release of the DESI data.





Alternative theories of gravity

- Lovelock's theorem states basic principles that a theory of gravity has to meet, concluding that the only theory that fulfills these principles is General Relativity (plus a cosmological constant).
- Modified gravity theories can be understood as those that violate one or more assumptions of this theorem.
  - Spacetime is a 4-dim. pseudo-Riemannian differential manifold $\longrightarrow$ Higher dimensions, f(T) (Weitzenböck geometry), f(Q) theories (non metricity)
  - The theory is locally Lorentz invariant, which means that it is locally Minkowskian and invariant against diffeomorphisms. $\longrightarrow$ unimodular gravity, Einstein-Aether, Hořava-Lifshitz
  - Gravity is completely described by a metric tensor $\longrightarrow$ Additional fields (Brans-Dicke, Horndeski, massive gravity,STVG, TEVES, bimetric theories)
  - The theory is local, meaning that the equations depend only on the values of the fields and their derivatives at a single point. $\longrightarrow$ Non-local theories
  - The field equations are at most 2nd order differential equations. $\longrightarrow$f(R), massive gravity

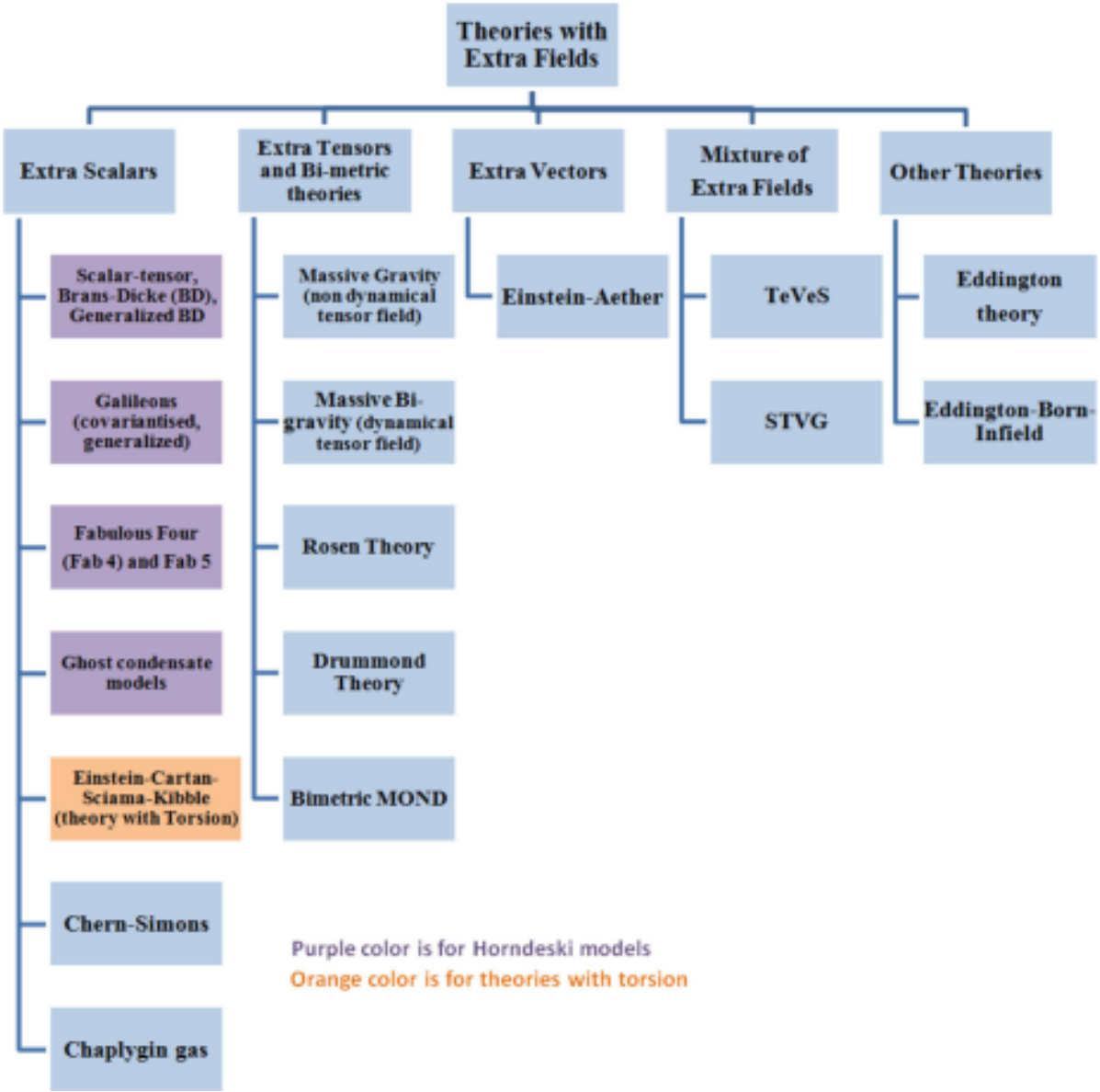


FIGURE: M. Ishak Living Reviews in Relativity 22, 1 (2019)

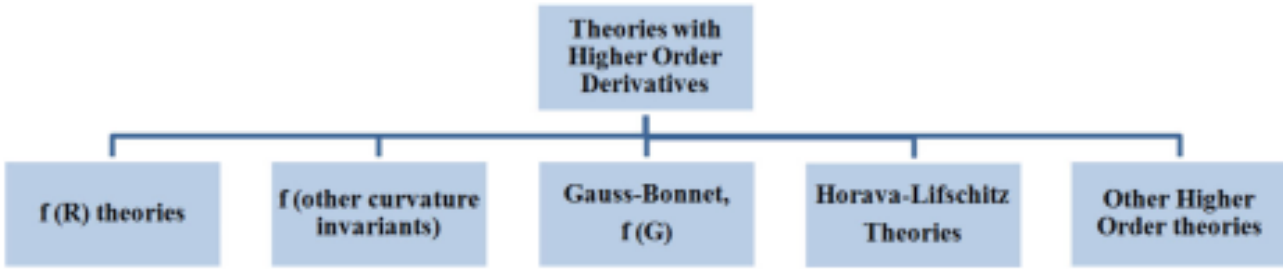


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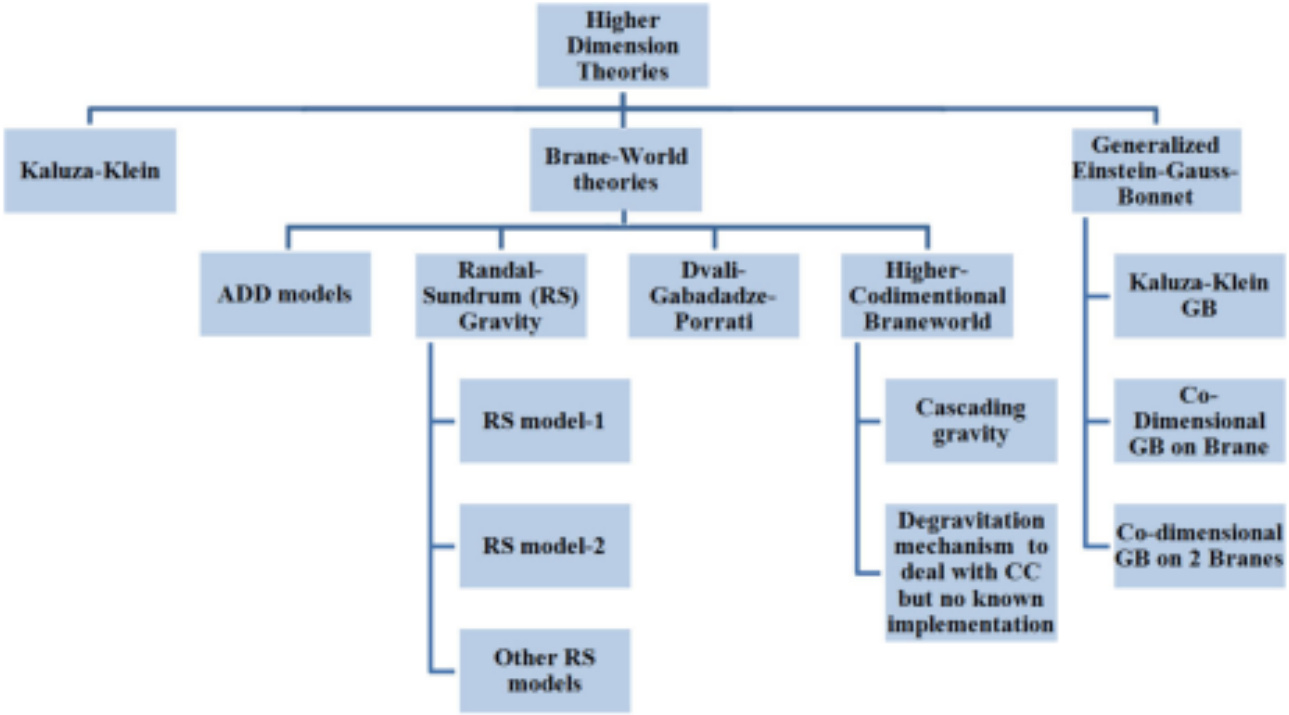


FIGURE: M. Ishak Living Reviews in Relativity 22, 1 (2019)

Friedmann Equations in $f(R)$

- The Einstein-Hilbert action in $f(R)$ theories is:

$$S = \int d^4x\sqrt{-g}\left[\frac{f(R)}{2\kappa} + \mathcal{L}_m\right] \tag{7}$$

where $\kappa = 8\pi G$ and $\mathcal{L}_m$ is the matter Lagrangian.

- Varying this action with respect to the metric, we obtain:

$$f_R R_{\mu\nu} - \frac{1}{2}f g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \kappa T_{\mu\nu} \tag{8}$$

where $f_R = df/dR$. Assuming the flat FRW metric, the modified Friedmann equations can be obtained:

$$\begin{aligned} H^2 &= \frac{1}{3f_R}\left[\kappa\rho_m + \frac{Rf_R - f}{2} - 3H\dot{R}f_{RR}\right], \\ 2\dot{H} + 3H^2 &= -\frac{1}{f_R}\left[-\frac{Rf_R - f}{2} + f_{RRR}\dot{R}^2\right. \\ &\quad \left. + \left(\ddot{R} + 2H\dot{R}\right)f_{RR}\right], \end{aligned} \tag{9}$$

Friedmann Equations in Horndeski Gravity

Let us consider four-dimensional Horndeski theories described by the Lagrangian (G. W. Horndeski Int.J.Theor. Phys. 10 (1974) 363-384)

$$\begin{aligned} L &= G_2(\chi, Y) + G_3(\chi, Y)\Box\chi + G_4(\chi, Y)R - \\ & 2G_{4Y}(\chi, Y)\left[(\Box\chi)^2 - \chi^{;\mu\nu}\chi_{;\mu\nu}\right] + G_5(\chi, Y)G_{\mu\nu}\chi^{;\mu\nu} \\ & + G_{5Y}(\chi, Y)\left[(\Box\chi)^3 - 3(\Box\chi)\chi_{;\mu\nu}\chi^{;\mu\nu} + 2\chi_{;\mu\nu}\chi^{;\mu\sigma}\chi^{;\nu}{}_{;\sigma}\right]/3\,, \end{aligned} \tag{10}$$

where G_i $(i = 2, 3, 4, 5)$ are generic functions of the scalar field χ and $Y = \partial_\mu\chi\partial^\mu\chi$ with $G_{iY} = \partial G_i/\partial Y$, R and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor in four dimensions.

Friedmann Equations in Horndeski Gravity

Assuming a flat FRW metric, the modified Friedmann equations can be obtained:

$$A_2 - 6H^2 A_4 - 12H^3 A_5 + 2\dot{\chi}^2 \left(A_{2Y} + 3HA_{3Y} + 6H^2 A_{4Y} + 6H^3 A_{5Y}\right) = \rho_M \,, \tag{11}$$

$$A_2 - 6H^2 A_4 - 12H^3 A_5 - \dot{A}_3 - 4\dot{H}A_4 - 4H\dot{A}_4 - 12H\dot{H}A_5 - 6H^2 \dot{A}_5 = -P_M \,. \tag{12}$$

where

$$A_2 = G_2 - YF_{3\chi} \,, \qquad A_3 = 2(-Y)^{3/2} F_{3Y} - 2\sqrt{-Y} G_{4\chi} \,,$$

$$A_4 = 2YG_{4Y} - G_4 + YG_{5\chi}/2 \qquad B_4 = G_4 + Y(G_{5\chi} - F_{5\chi})/2 \,,$$

$$A_5 = -(-Y)^{3/2} G_{5Y}/3 \,, \qquad B_5 = -\sqrt{-Y} F_5$$

$$G_3 = F_3 + 2YF_{3Y} \qquad G_{5Y} = F_5/(2Y) + F_{5Y}$$

Cubic and Cuartic Galileon

Recalling the Horndenski Lagrangian:

$$
\begin{aligned}
L \;=\; & G_2(\chi, Y) + G_3(\chi, Y)\Box\chi + G_4(\chi, Y)R - \\
& 2G_{4Y}(\chi, Y)\left[(\Box\chi)^2 - \chi^{;\mu\nu}\chi_{;\mu\nu}\right] + G_5(\chi, Y)G_{\mu\nu}\chi^{;\mu\nu} \\
& + G_{5Y}(\chi, Y)\left[(\Box\chi)^3 - 3(\Box\chi)\chi_{;\mu\nu}\chi^{;\mu\nu} + 2\chi_{;\mu\nu}\chi^{;\mu\sigma}\chi^{;\nu}{}_{;\sigma}\right]/3\,,
\end{aligned}
$$

- Cubic Galileon: $G_2 = c_2 Y$, $G_3 = c_3 Y/M^3$, $G_4 = \frac{M_{\rm pl}^2}{2}$, $G_5 = 0$
- Quartic Galileon: $G_2 = c_2 Y$, $G_3 = c_3 Y/M^3$, $G_4 = \frac{M_{\rm pl}^2}{2} + c_4 Y^2/M^6$, $G_5 = 0$
- Quintic Galileon: $G_2 = c_2 Y$, $G_3 = c_3 Y/M^3$, $G_4 = \frac{M_{\rm pl}^2}{2} + c_4 Y^2/M^6$, $G_5 = 3c_5 Y^2/M^9$
- Here, c_i are dimensionless constants, M is a mass scale, and $M_{\rm pl}$ is the reduced Planck mass.
- These models introduce a scalar field with derivative self-interactions that can drive cosmic acceleration while satisfying local gravity constraints via the Vainshtein mechanism.

Hubble parameter in Galileon Gravity

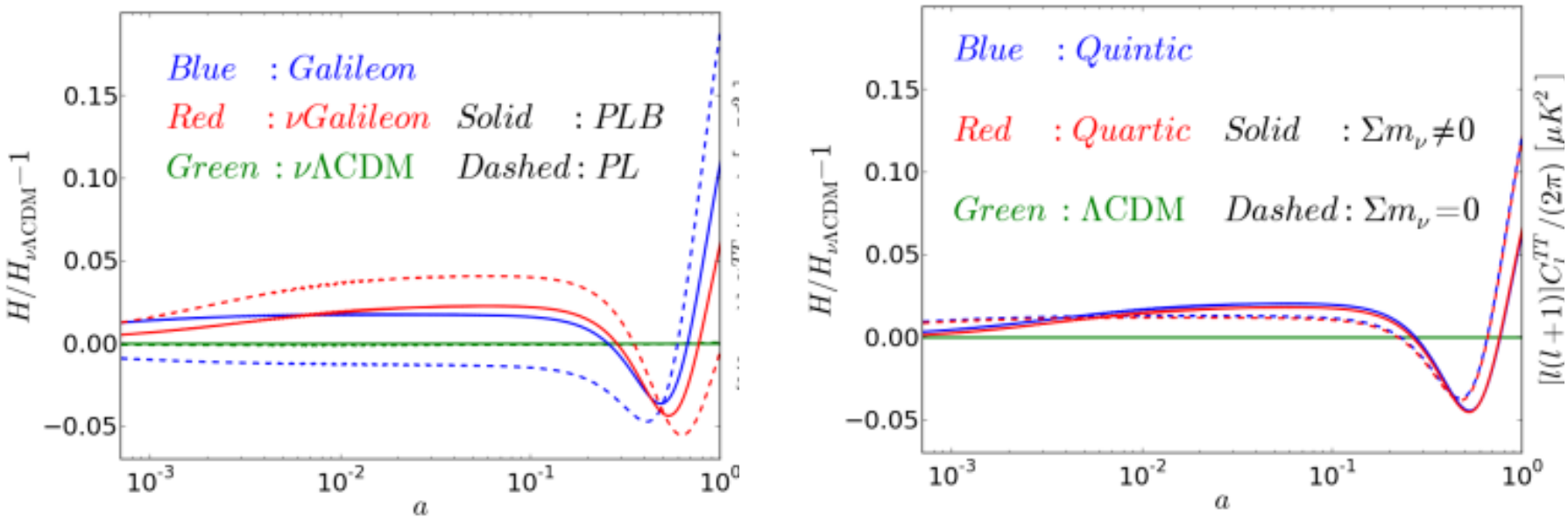


Figure: Best fits models to CMB + CMB lensing + BAO data (solid line). Only CMB + CMB lensing (dashed line)

Theories with torsion: f(T) gravity

- In $f(T)$ gravity, as in all torsion-based theories, gravity is described by torsion instead of curvature. The vierbein fields or tetrads e^{μ}_{A} provide an orthonormal basis for the tangent space at every point x^{μ} on the manifold.The metric then reads : $g_{\mu\nu} = \eta_{AB} e^{A}_{\mu} e^{B}_{\nu}$.
- One can write the tetrad field as

$$e^{A}_{\ \mu} = \partial_{\mu} x^{A} + \omega^{A}_{\ B\mu} x^{B} + B^{A}_{\ \mu}, \tag{13}$$

The spin connection of TG is given by

$$\omega^{A}_{\ B\mu} = \Lambda^{A}_{\ D}(x) \partial_{\mu} \Lambda_{B}^{\ D}(x), \tag{14}$$

with $\Lambda^{A}_{\ D}(x)$ a local (point-dependent) Lorentz transformation. In the presence of gravitation ($B^{A}_{\ \mu} \neq 0$) the tetrad field leads to the non-zero torsion tensor

$$T^{\rho}_{\mu\nu} \equiv e^{\ \rho}_{A} \left(\partial_{\mu} e^{A}_{\ \nu} - \partial_{\nu} e^{A}_{\ \mu} + \omega^{A}_{\ B\mu} e^{B}_{\ \nu} - \omega^{A}_{\ B\nu} e^{B}_{\ \mu} \right). \tag{15}$$

- The torsion scalar T represents the lagrangian of the theory is built from the contractions of the torsion tensor

$$T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\ \ \rho} T^{\nu\mu}_{\ \ \ \nu}. \tag{16}$$

Friedmann Equations in $f(T)$ gravity

- Motivated by the f(R) theories, simple torsion-modified theories of gravity have been developed:

$$S = \frac{1}{16\pi G}\int d^4x\, e\,[T + f(T)] + S_m \tag{17}$$

where $e = \det(e^A_\mu) = \sqrt{-g}$ and S_m is the matter action.

- Varying this action with respect to the tetrad and assuming a flat FRW metric, the modified Friedmann equations can be obtained:

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm m} + \rho_{\rm r}) + \frac{Tf_T}{3} - \frac{f}{6}, \tag{18}$$

$$\dot{H} = -\frac{4\pi G(\rho_{\rm m} + p_{\rm m} + \rho_{\rm r} + p_{\rm r})}{1 + 2Tf_{TT} + f_T}, \tag{19}$$

where $f_T = df/dT$ and $f_{TT} = d^2f/dT^2$ and $T = -6H^2$

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Cosmological Distances

- The comoving distance (χ) is given by:

$$\chi = \int_0^z \frac{dz'}{H(z')} \tag{20}$$

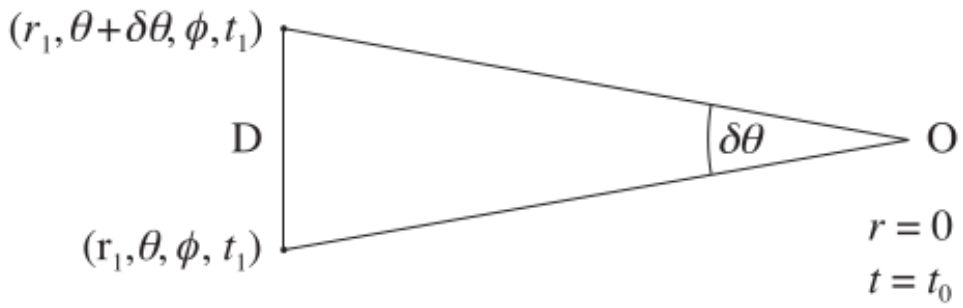
where $H(z)$ is the Hubble parameter at redshift z.

- The transverse comoving distance (D_M) is given by:

$$D_M = \begin{cases} \frac{1}{H_0\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k}\frac{H_0\chi}{c}\right) & \text{if } \Omega_k > 0 \\ \chi & \text{if } \Omega_k = 0 \\ \frac{1}{H_0\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|}\frac{H_0\chi}{c}\right) & \text{if } \Omega_k < 0 \end{cases} \tag{21}$$

This quantity will be important for BAO.

Cosmological distances



- The angular diameter distance (D_A) is given by:

$$D_A = \frac{D}{\delta\theta} = \frac{D_M}{1+z} = \frac{1}{1+z}\int_0^z \frac{dz'}{H(z')} \tag{22}$$

  This quantity is important for lensing observables.

- The luminosity distance (D_L) is given by:

$$D_L = (1+z)D_M = \ (1+z)\int_0^z \frac{dz'}{H(z')} \tag{23}$$

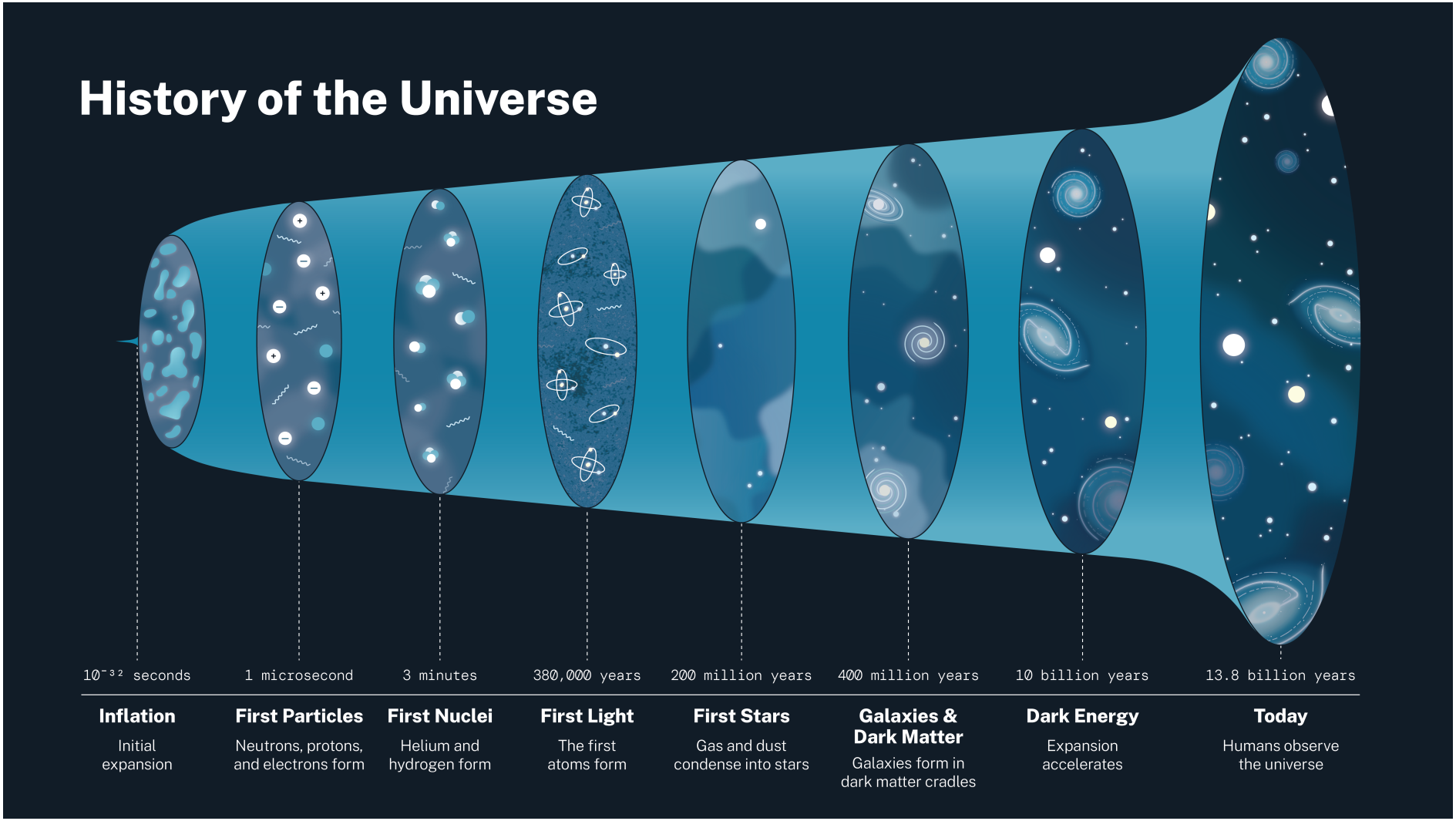
  This quantity is related to the observed intensity of type Ia supernovae explosions.

- In all cases the last equality holds for a flat universe $(\Omega_k = 0)$.

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Evolution of the Universe



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Cosmic Chronometers

- Cosmic chronometers are a method to measure the Hubble parameter $(H(z))$ directly from the differential age evolution of passively evolving galaxies.
- The Hubble parameter can be expressed as:

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} \tag{24}$$

By measuring dz/dt from the age difference of galaxies at different redshifts, we can obtain $H(z)$.

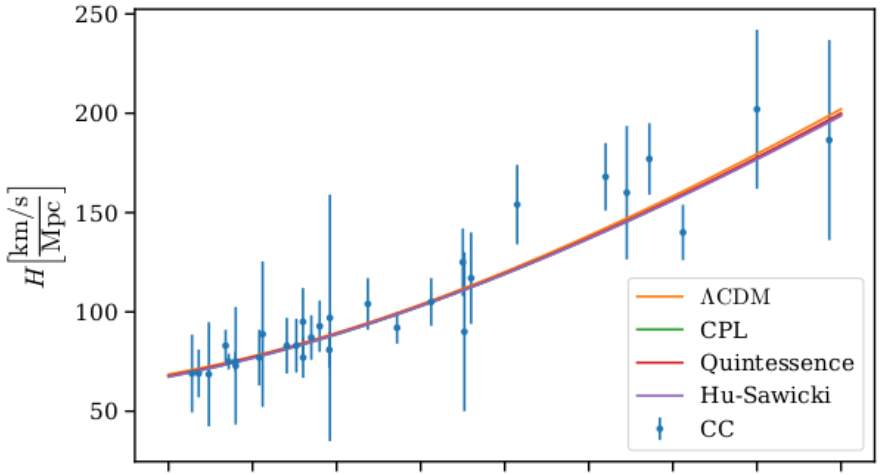


Figure: Compilation of Hubble parameter measurements from cosmic chronometers. Chantada et al. (2023)

Cosmic Chronometers: the likelihood

- Current compilations contain ~ 32 measurements of $H(z_i)$ in the range $0.07 < z < 1.97$. The likelihood is assumed to be Gaussian:

$$\mathcal{L}_{\rm CC} \propto e^{-\chi^2_{\rm CC}/2}\,, \qquad \chi^2_{\rm CC} = \Delta \boldsymbol{H}^T\, C^{-1}\, \Delta \boldsymbol{H} \tag{25}$$

  where $\Delta H_i = H_{\rm obs}(z_i) - H_{\rm th}(z_i;\boldsymbol{\theta})$, $\boldsymbol{\theta}$ are the cosmological parameters (e.g. H_0, Ω_m, or those of a modified gravity model) and $H_{\rm th}$ is obtained from the Friedmann equations.

- The covariance matrix has a statistical and a systematic contribution:

$$C_{ij} = C^{\rm stat}_{ij} + C^{\rm syst}_{ij} \tag{26}$$

  - $C^{\rm stat}$ is diagonal: $C^{\rm stat}_{ii} = \sigma^2_{H,i}$, the statistical error of each measurement.
  - $C^{\rm syst}$ is not diagonal: the ages of the galaxies at all redshifts are estimated with the same stellar population synthesis models, initial mass function, stellar library and metallicity, so their uncertainties are correlated between different redshifts.

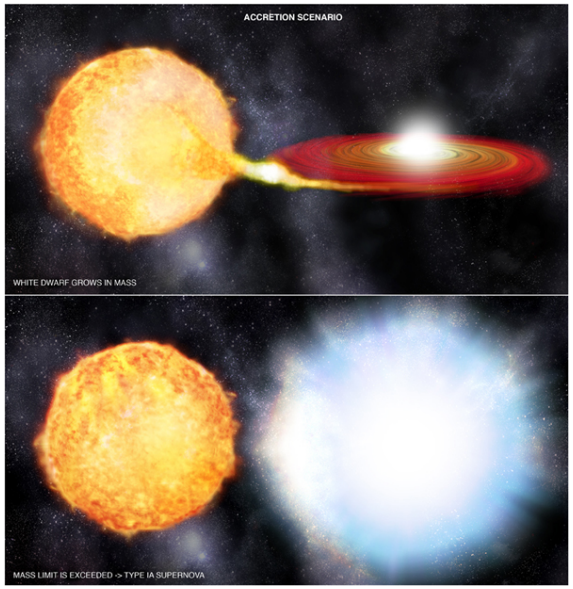
- Neglecting the off-diagonal terms (i.e. using only the error bars of the plot) underestimates the uncertainties of the cosmological parameters.

Moresco et al. ApJ 898, 82 (2020); Moresco et al. Living Rev. Relativ. 25, 6 (2022)

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Type Ia Supernovae



- Supernovae explosions are among the most energetic events in the universe.
- Type Ia supernovae are thought to occur in binary systems where a white dwarf accretes matter from a companion star until it reaches the Chandrasekhar value ($1.44 M_{\odot}$) and undergoes a thermonuclear explosion.
- Since the intensity of the explosion is always the same, they can be used as standard candles to measure distances in the universe.

Type Ia Supernoave as standard candles

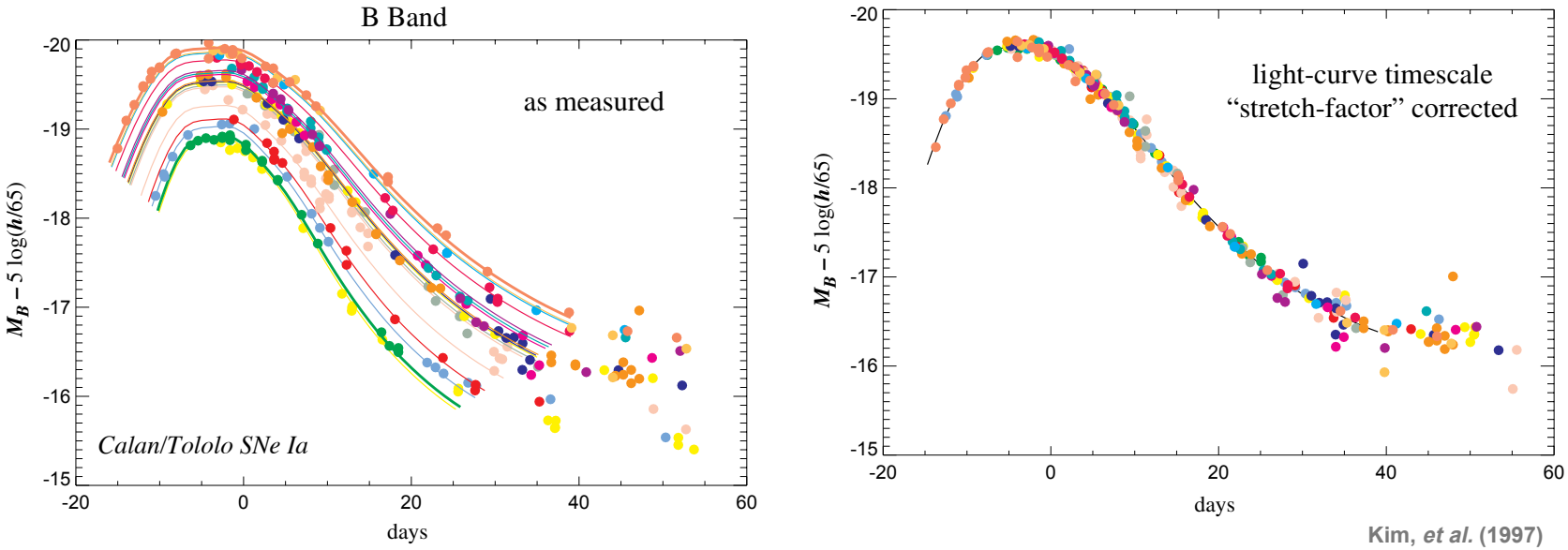


Figure: Left panel: Light curves of SNe Ia with different stretch factors. Right panel: After correcting for the stretch factor, the light curves align better, demonstrating the utility of the stretch correction in standardizing SNe Ia luminosities.

Distance Modulus

- The measured flux (F) from a source with luminosity (L) located at a luminosity distance (D_L) is given by:

$$F = \frac{L}{4\pi D_L^2} \tag{27}$$

- The apparent magnitude (m) of an object is related to its flux:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right) \tag{28}$$

  where $m = 0$ corresponds to a reference star (Vega).
- The absolute magnitude (M) is defined as the apparent magnitude the object would have if it were located at a distance of 10 parsecs.
- The distance modulus (μ) is defined as:

$$\boxed{\mu = m - M = 5 \log_{10} \left(\frac{D_L}{10 \text{ pc}} \right)} \tag{29}$$

SNIa and the Pantheon Plus compilation

- To include the corrections and standardization parameters for Pantheon Plus

$$\mu = m_B - M + \alpha x_1 + \beta c + \Delta_M + \Delta_B \,, \tag{30}$$

- m_B is the observed peak magnitude in the B band
- x_1 the deviation from the mean shape of the light curve,
- c the average B-V color index of the SNe Ia.
- M is the absolute magnitude in the B band of a fiducial SNe Ia with $x_1 = 0$ and $c = 0$,
- Δ_B is a distance correction based on biases predicted from simulations,
- Δ_M is a distance correction based on the mass of the host galaxy of the SNe Ia. For this compilation of SNe Ia, it is obtained from

$$\Delta_M = \gamma \times [1 + e^{(-(m - m_{\rm step})/\tau)}]^{-1} \,, \tag{31}$$

where $m_{\rm step}$ is a threshold mass for the split, γ is a relative offset in luminosity, and m is the mass of the host galaxy. Both $m_{\rm step}$ and τ are derived from different samples of host galaxies.

- α, β **and** γ **are the so called nuisance parameters**

Measurements from the Pantheon Plus compilation

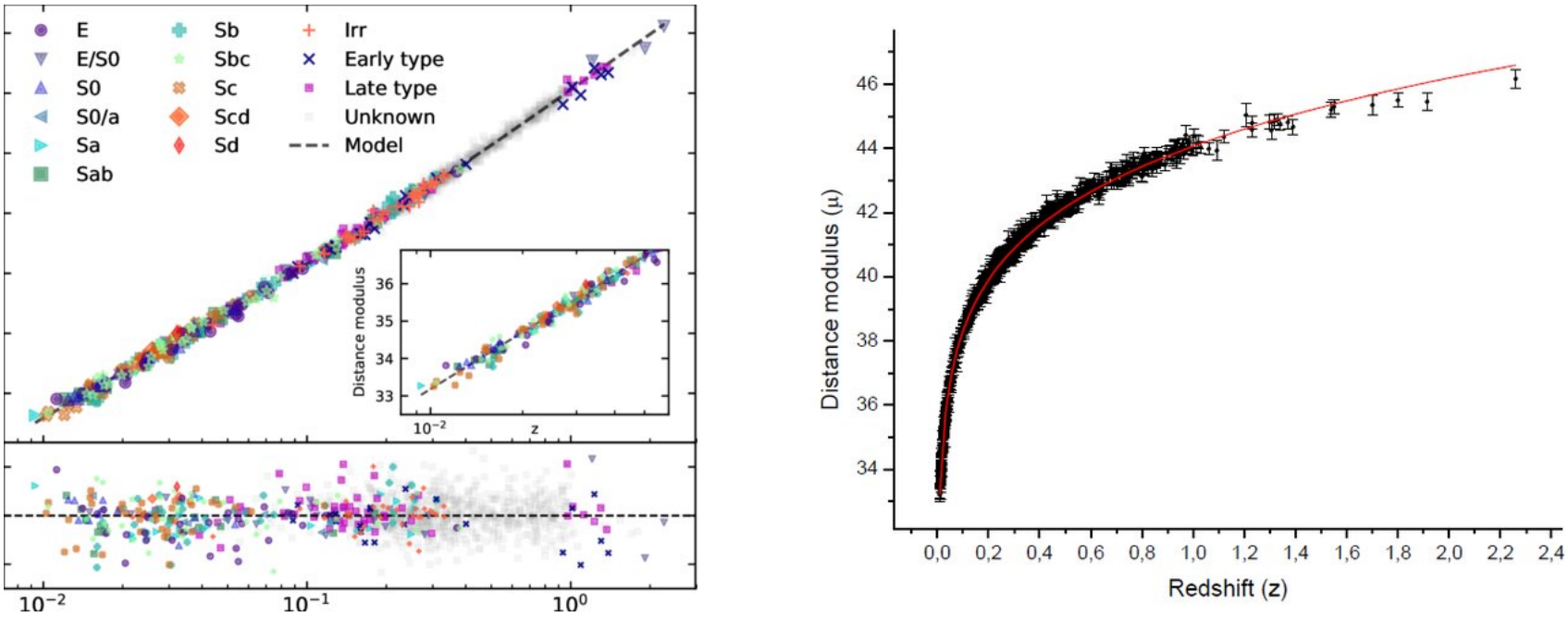


Figure: Left panel: Hubble diagram for the Pantheon Plus compilation of SNe Ia, showing the distance modulus (μ) versus redshift (z) on a logarithmic scale to highlight the low redshift behaviour. Right panel: Same data on a linear scale . The solid line represents the best-fit ΛCDM model.

Current SNIa compilations: overview

	Pantheon+	**Union3**	**DES-SN5YR / DES-Dovekie**
Size	$\sim$ 1550 SNe (1701 light curves)	$\sim$ 2087 SNe	$\sim$ 1635 DES + $\sim$ 194 low-z SNe
Redshift range	$0.001 < z < 2.26$	$0.01 \lesssim z < 2.26$	$0.025 \lesssim z < 1.13$
Data sources	$\sim$ 18 samples	$\sim$ 24 samples	One survey (DES) + external low-z
Classification	Spectroscopic	Spectroscopic	Photometric (SuperNNova)
Light-curve model	SALT2 (retrained)	SALT3	SALT3
Statistical method	BBC + simulation-based	UNITY1.5	BBC + simulations
	bias corrections	(Bayesian hierarchical)	+ non-Ia contamination
Ω_m (flat ΛCDM)	0.334 ± 0.018	$0.356^{+0.028}_{-0.026}$	0.352 ± 0.017 (original DES-SN5YR)

Scolnic et al. ApJ 938, 113 (2022); Brout et al. ApJ 938, 110 (2022); Rubin et al. arXiv:2311.12098; DES Collaboration ApJL 973, L14 (2024); Popovic et al. (2025)

Current SNIa compilations: key differences

- **Pantheon+:** successor of JLA and Pantheon. Includes SNe Ia in Cepheid hosts, designed to be used jointly with SH0ES for H_0. Cross-calibration with the *Fragilistic* system; host-mass step Δ_M modelled through dust.
- **Union3:** the main novelty is the statistical method. UNITY1.5 is a Bayesian hierarchical model that treats selection, calibration, SN populations and outliers in a single likelihood, instead of simulation-based bias corrections. Distances released as ~ 22 spline nodes in z. More conservative systematics $\longrightarrow$ larger error bars.
- **DES-SN5YR:** first large cosmological sample with photometric classification. Many homogeneous SNe at $0.1 < z < 1.1$ from one instrument, but non-Ia contamination must be modelled and the low-z anchor comes from external samples.
- **DES-Dovekie:** reanalysis of DES-SN5YR with a new cross-survey photometric calibration, motivated by a possible offset of ~ 0.04 mag between the low-z and DES samples. It reduces, but does not remove, the preference for evolving dark energy.

Why it matters: with DESI DR2 BAO + CMB, the preference for $w_0 w_a$ over ΛCDM is 2.8σ (Pantheon+), 3.8σ (Union3) and 4.2σ (original DES-SN5YR). The compilations share many SNe, so they cannot be combined: their spread is a proxy for SN systematics.

Efstathiou, MNRAS 538, 875 (2025); DESI Collaboration arXiv:2503.14738; Popovic et al. (2025)

Type Ia Supernovae: the likelihood

- For each SNIa the compilation provides the standardized magnitude $m_{B,i}^{\rm corr}$ (the nuisance parameters α, β, γ are already fitted). The residuals with respect to the model are:

$$\Delta\mu_i = m_{B,i}^{\rm corr} - M - \mu_{\rm th}(z_i;\boldsymbol{\theta})\,, \qquad \mu_{\rm th} = 5\log_{10}\left(\frac{D_L(z_i)}{10\,\mathrm{pc}}\right) \tag{32}$$

- The likelihood is Gaussian, $\mathcal{L}_{\rm SN} \propto e^{-\chi^2_{\rm SN}/2}$, with

$$\chi^2_{\rm SN} = \Delta\boldsymbol{\mu}^T\, C^{-1}\, \Delta\boldsymbol{\mu}\,, \qquad C = C^{\rm stat} + C^{\rm syst} \tag{33}$$

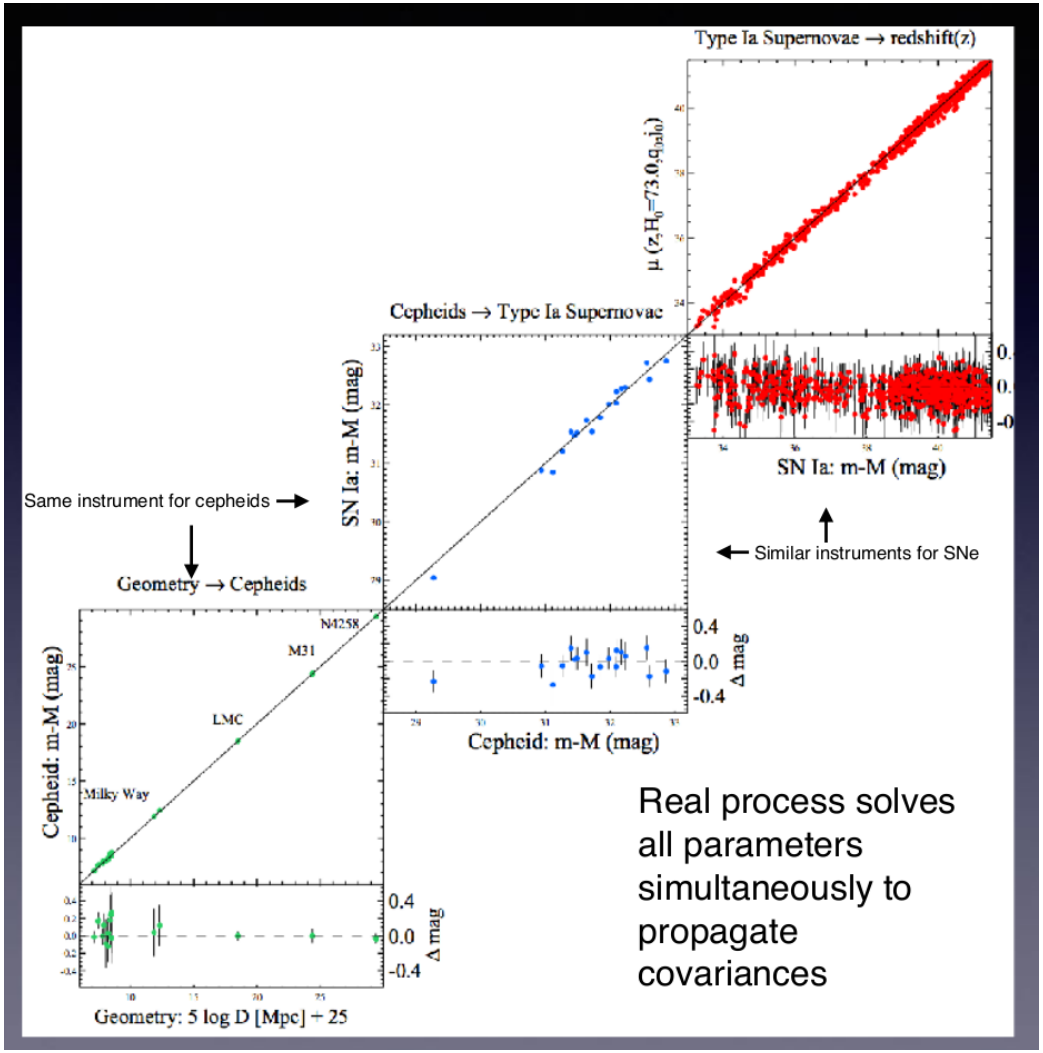
- The covariance matrix is a 1701×1701 matrix (Pantheon+). $C^{\rm stat}$ contains the light-curve fit uncertainties, peculiar velocities, lensing and intrinsic scatter. $C^{\rm syst}$ is dense: photometric calibration, light-curve model, bias corrections, dust and host-mass corrections affect many SNe at once. Systematics dominate the error budget, so the off-diagonal terms cannot be neglected.
- M is completely degenerate with H_0 (only the combination $M + 5\log_{10}(c/H_0)$ enters). Therefore SNIa alone do not constrain H_0: either M is marginalized analytically, or it is calibrated with Cepheids in the SNIa host galaxies (SH0ES), replacing $\mu_{\rm th} \to \mu_i^{\rm Ceph}$ for those SNe.

Conley et al. ApJS 192, 1 (2011); Brout et al. ApJ 938, 110 (2022)

Outline

1. The cosmological background in ΛCDM
2. Alternative Theories of Gravity
3. Cosmological Distances
4. History of the Universe
5. Cosmic Chronometers
6. Type Ia Supernovae
7. Local Determination of H_0
8. Relevant papers for this lecture

The distance ladder



- Cepheids are calibrated using distances that are obtained with geometrical methods (i.e., higher precision MW parallaxes, detached eclipsing binaries in the Magellanic Clouds,the megamaser distance to NGC 4258).
- SNIa luminosity is calibrated using distances to their host galaxies that are obtained using Cepheids
- Finally, SNe Ia luminosity distances in the Hubble flow and their redshifts are used to determine H_0.

Cepheid Period-Luminosity Relation

- For each Cepheid in band X, the intrinsic period-luminosity (PL) relation reads:

$$M_X = zp_X + b_X \log P \tag{34}$$

where M_X is the absolute magnitude in band X, zp_X is the (universal) PL zero point and b_X is the PL slope.

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- Combining both expressions, the apparent magnitude of a Cepheid in host galaxy i is directly related to its period and the galaxy's distance modulus $\mu_{0,i}$. Defining the host-galaxy intercept $z_{P_{X,i}} \equiv zp_X + \mu_{0,i}$:

$$m_X = z_{P_{X,i}} + b_X \log P \tag{36}$$

Period-Luminosity Relation: Relative Distance Modulus

- Adding and subtracting the intercept of the anchor galaxy NGC 4258, $Z_{X,4258}$:

$$m_X = \left(z_{P_{X,i}} - Z_{X,4258}\right) + Z_{X,4258} + b_X \log P \tag{37}$$

- Also, we can write

$$\left(z_{P_{X,i}} - Z_{X,4258}\right) \simeq \mu_{0,i} - \mu_{0,4258} \tag{38}$$

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- This is only approximate ($\simeq$), not exact, because it relies on the assumption that the PL zero point zp_X is universal: small differences can arise from metallicity dependence of the PL relation, a slope b_X not perfectly shared between galaxies, and sample/selection effects in each host.
- Replacing the intercepts by the distance moduli:

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Reddening and the Wesenheit Magnitude

- Interstellar dust dims and reddens Cepheids, biasing the PL relation unless corrected for extinction.

Reddening and the Wesenheit Magnitude

- Interstellar dust dims and reddens Cepheids, biasing the PL relation unless corrected for extinction.
- The use of two or more passbands allows for the measurement of reddening and the associated correction for extinction. Following Madore (1982), we define the reddening-free **Wesenheit magnitude**:

$$m_W = m_H - R(m_V - m_I) \tag{40}$$

  where m_H is the apparent magnitude in the near-infrared H band, $R \equiv A_H/(A_V - A_I)$, with A_H, A_V, A_I the extinctions (in magnitudes) due to dust in bands H, V, I; for a Cardelli et al. (1989) reddening law with a Galactic-like value $R_V = 3.1$, $R = 0.479$.
- R depends on the reddening law, i.e. how dust absorption varies with wavelength, which in turn depends on the size distribution of the dust grains; this dependence is parametrized by $R_V \equiv A_V/(A_B - A_V)$, with larger grains giving larger R_V and a flatter extinction curve.
- The same PL formalism above applies with $X \to W$, so that m_W replaces m_H throughout the distance-ladder analysis, removing the need for an independent extinction correction.

Metallicity Dependence of the Wesenheit Magnitude

- To account for the possible dependence of Cepheid magnitude on metallicity, even over the narrow range of metallicity in the Cepheid sample, we express m_W as:

$$\boxed{m_{W,i,j} = (\mu_{0,i} - \mu_{0,4258}) + zp_{W,4258} + b_W \log P_{i,j} + Z_W \Delta \log[\mathrm{O/H}]_{i,j}} \tag{41}$$

  where $m_{W,i,j}$ is the Wesenheit apparent magnitude of Cepheid j in host galaxy i, $[\mathrm{O/H}]_{i,j}$ is the oxygen abundance (a proxy for metallicity) at the position of that Cepheid, and Z_W is the metallicity dependence (coefficient) of the Wesenheit PL relation.

- The individual values of $\Delta \log[\mathrm{O/H}]_{i,j}$ were derived from the metallicity values and gradients for the Cepheid hosts given by Riess et al. (2009).

SNIa: Relative Distance Modulus

- As with Cepheids, the distance modulus of a SNIa host galaxy relates its apparent and absolute magnitude:

$$\mu_{0,i} = m_{V,i} - M_V \tag{42}$$

where $m_{V,i}$ is the apparent magnitude of the SNIa in host galaxy i, and M_V is its absolute magnitude, assumed to be universal (i.e. the same for all SNIa, making them standard candles).

- Writing the same relation for the anchor galaxy NGC 4258 and subtracting, the (universal) absolute magnitude M_V cancels out:

$$\mu_{0,i} - \mu_{0,4258} = (m_{V,i} - M_V) - (m_{V,4258} - M_V) = m_{V,i} - m_{V,4258} \tag{43}$$

where $m_{V,4258}$ is the apparent magnitude of a SNIa in NGC 4258.

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where $m_{V,4258}$ is the apparent magnitude of a SNIa in NGC 4258.

- Rearranging, the apparent magnitude of the SNIa in the host galaxy i can be written as:

$$\boxed{m_V = \mu_{0,i} - \mu_{0,4258} + m_{V,4258}} \tag{44}$$

Determination of H_0 with SNIa and Cepheids

- The distance modulus for SNe Ia is given by:

$$\mu = m_V - M_V = 5\log_{10}(D_L) + 25 \tag{45}$$

where m_V is the apparent magnitude in the V band, M_V is the absolute magnitude, and D_L is the luminosity distance in megaparsecs (Mpc).

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- The luminosity distance at low redshift can be approximated as:

$$D_L(z) = \frac{cz}{H_0} \underbrace{\left[1 + \frac{1}{2}(1-q_0)z - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2 + O(z^3)\right]}_{f(z;q_0,j_0)} \tag{46}$$

where $q_0 = \left[\frac{\ddot{a}}{a}(\frac{a}{\dot{a}})^2\right]_{t_0}$ is the deceleration parameter and $j_0 = \left[\frac{\dddot{a}}{a}(\frac{a}{\dot{a}})^3\right]_{t_0}$ is the jerk parameter.

- After a little algebra with the previous equation, we obtain:

$$\log_{10}\left(\frac{cz}{H_0}f(z;q_0,j_0)\right) - 0.2m_V = \log_{10} H_0 - 0.2M_V - 5 \qquad (47)$$

where $f(z;q_0,j_0)$ is the function defined in the previous slide.

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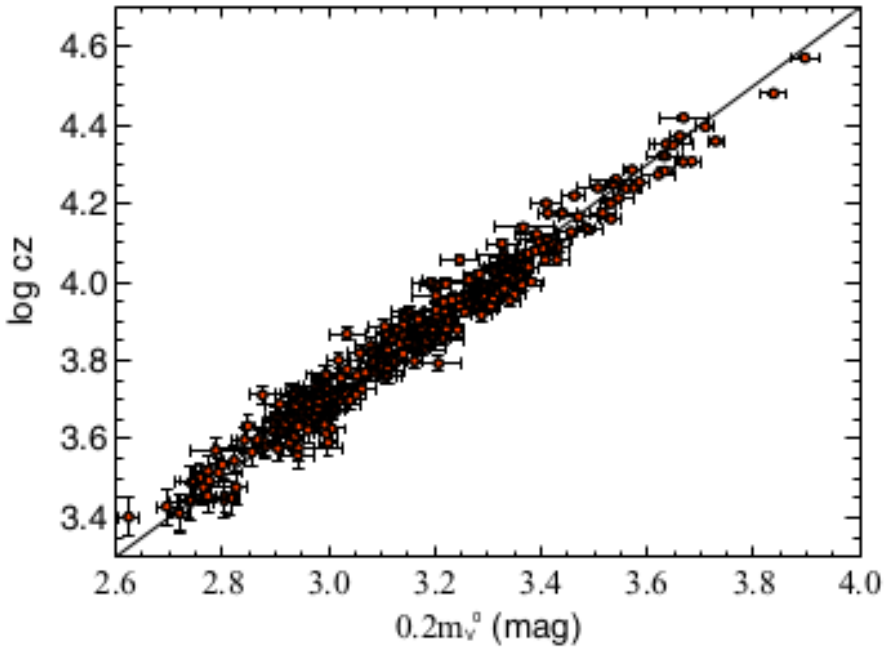
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where $f(z;q_0,j_0)$ is the function defined in the previous slide.

- Defining $a_V = \log_{10}\left(\frac{cz}{f(z;q_0,j_0)}\right) - 0.2m_V$, we have:

$$\log_{10} H_0 = 0.2M_V + 5 + a_V \qquad (48)$$

where a_V is measured from low-redshift SNIa ($z < 0.1$). Riess et al 2016 estimate $a_V = 0.71273 \pm 0.00176$



Determination of H_0 with SNIa and Cepheids

- We can write: $M_V = m_v - m_v + M_V = m_v - \mu_{0,\mathrm{Ceph}}$, where m_v is the apparent magnitude of SNIa in the host galaxy and $\mu_{0,\mathrm{Ceph}}$ is the distance modulus obtained from Cepheid variables in the same galaxy.

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- Therefore, we have:

$$\log_{10} H_0 = \frac{(m_v - \mu_{\mathrm{Ceph}}) + 25 + 5a_V}{5} \tag{49}$$

This equation shows how the Hubble constant (H_0) can be determined using the apparent magnitude of SNIa and the distance modulus from Cepheid variables.

Determination of H_0 with SNIa and Cepheids

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- We can also write:

$$m_v = m_v - M_V + M_V + m_{v,4258} - m_{v,4258} = \mu_{\mathrm{Ceph}} + m_{v,4258} - \mu_{4258} \tag{50}$$

where $m_{v,4258}$ is the apparent magnitude of SNIa in NGC 4258 and μ_{4258} is the distance modulus from water masers in NGC 4258 and μ_{Ceph} is the distance modulus of SNIa in the host galaxy, which can be obtained from Cepheids.

Determination of H_0 with SNIa and Cepheids

- Finally, we have:

$$\log_{10} H_0 = \frac{(m_{v,4258} - \mu_{4258}) + 25 + 5a_V}{5} \tag{51}$$

- In such way, the determination of H_0 depends on:
  - The distance modulus of NGC 4258 from water masers (μ_{4258}).
  - The Hubble diagram of low-redshift supernovae (a_v)
  - The apparent magnitude of SNIa in NGC 4258, which proceeds from high-redshift supernovae and Cepheids data (see next slide).
  - This is the method employed by Riess et. al 2009, this method has been updated by the local distance network by Casertano et al 2026

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Even though the cosmographic expansion that was used to derive a_V depends on q_0 and j_0, the final determination of H_0 has no strong dependence on these parameters and in consequence is independent from ΛCDM (S. Casertano, private communication, see also arXiv:2510.23823).

$$
\begin{pmatrix}
m_{w,1,1} \\
m_{w,1,2} \\
\dots \\
m_{w,1,r_1} \\
m_{w,2,1} \\
\dots \\
m_{w,2,r_2} \\
\dots \\
m_{w,n,1} \\
\dots \\
m_{w,n,r_n} \\
m_{w,4258,1} \\
\dots \\
m_{w,4258,r_0} \\
m^0_{v,1} \\
m^0_{v,2} \\
\dots \\
m^0_{v,n}
\end{pmatrix} =
$$

$$
\begin{pmatrix}
1 & 0 & \dots & 0 & 1 & \log P_{1,1} & 0 & \Delta\log[O/H]_{1,1} \\
1 & 0 & \dots & 0 & 1 & \log P_{1,2} & 0 & \Delta\log[O/H]_{1,2} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
1 & 0 & \dots & 0 & 1 & \log P_{1,r_1} & 0 & \Delta\log[O/H]_{1,r_1} \\
0 & 1 & \dots & 0 & 1 & \log P_{2,1} & 0 & \Delta\log[O/H]_{2,1} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 1 & \dots & 0 & 1 & \log P_{2,r_2} & 0 & \Delta\log[O/H]_{2,r_2} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & 1 & 1 & \log P_{n,1} & 0 & \Delta\log[O/H]_{n,1} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & 1 & 1 & \log P_{n,r_n} & 0 & \Delta\log[O/H]_{n,r_n} \\
0 & 0 & \dots & 0 & 1 & \log P_{4258,1} & 0 & \Delta\log[O/H]_{4258,1} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & 0 & 1 & \log P_{4258,r_0} & 0 & \Delta\log[O/H]_{4258,r_0} \\
1 & 0 & \dots & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & \dots & 0 & 0 & 0 & 1 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & 1 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

$$
* \begin{pmatrix}
\mu_{0,1} - \mu_{0,4258} \\
\mu_{0,2} - \mu_{0,4258} \\
\dots \\
\mu_{0,n} - \mu_{0,4258} \\
zp_{w,4258} \\
b_W \\
m^0_{v,4258} \\
Z_W
\end{pmatrix} + \textbf{noise} \qquad (18)
$$

Distance network

- Geometric distances to anchors
  - Milky Way: Paralaxes
  - LMC,SMC: detached Eclipsing Binaries
  - NGC 4258: Water masers orbiting the central supermassive black hole
- Primary distance indicators
  - Cepheid variables: Period-luminosity relation
  - J-region of the Asymptotic Giant Branch
  - Tip of the Red Giant Branch (TRGB): Luminosity of the TRGB
  - Mira variables: Period-luminosity relation
- Secondary distance indicators
  - Type Ia supernovae: Standard candles for measuring distances to galaxies
  - Tully-Fisher relation: Correlation between the luminosity and rotation velocity of spiral galaxies
  - Fundamental plane of elliptical galaxies: Relation between the effective radius, surface brightness, and velocity dispersion
  - Surface Brightness Fluctuations: Variation in the brightness of a galaxy due to the distribution of stars within it

Methods that bypass or alter the local distance network

- Direct distances to megamaser systems.
- SNeII with expanding photosfere method.
- Coma distance via SNIa (to calibrate the fundamental plane)

Local Distance Network

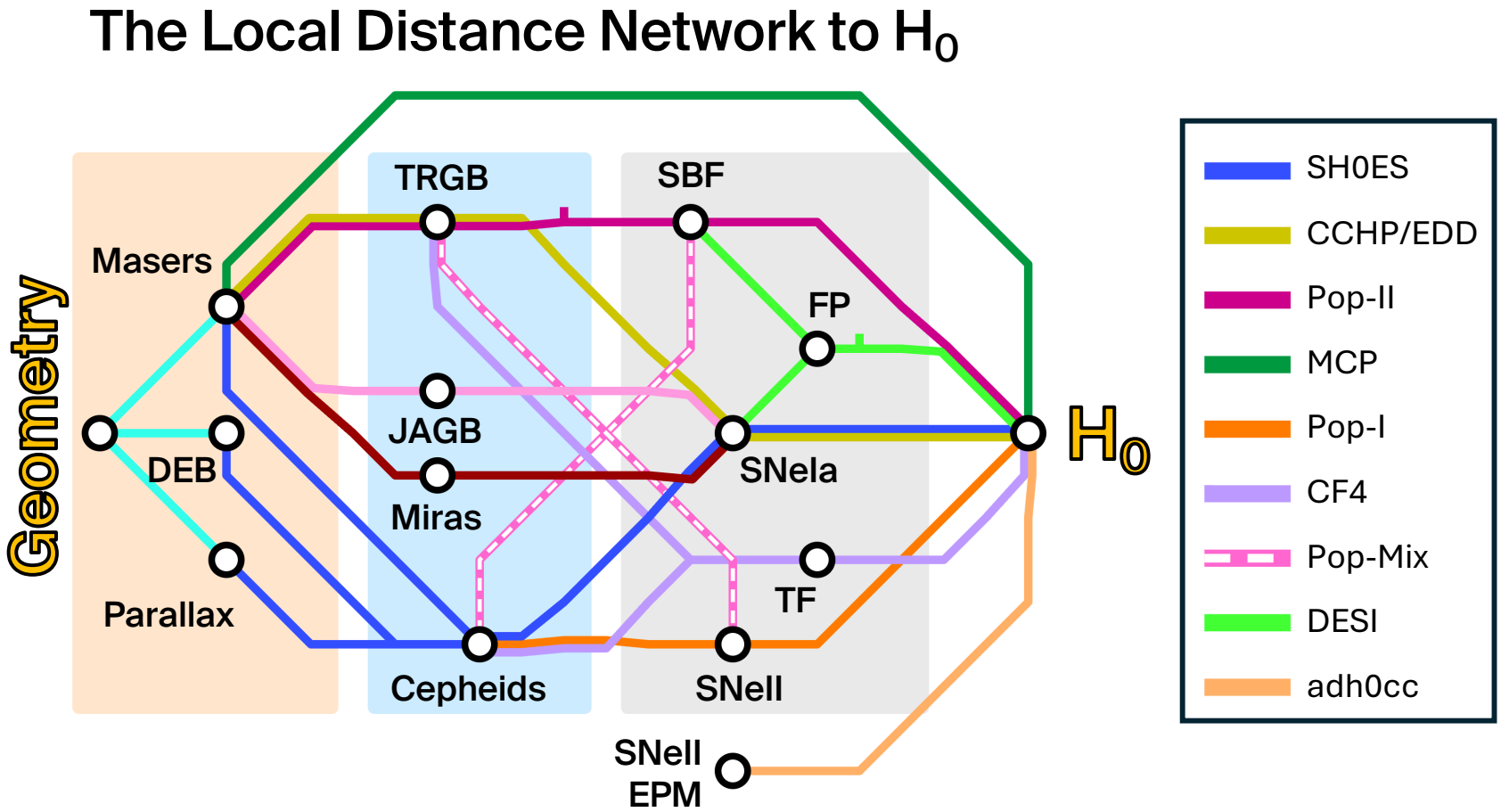
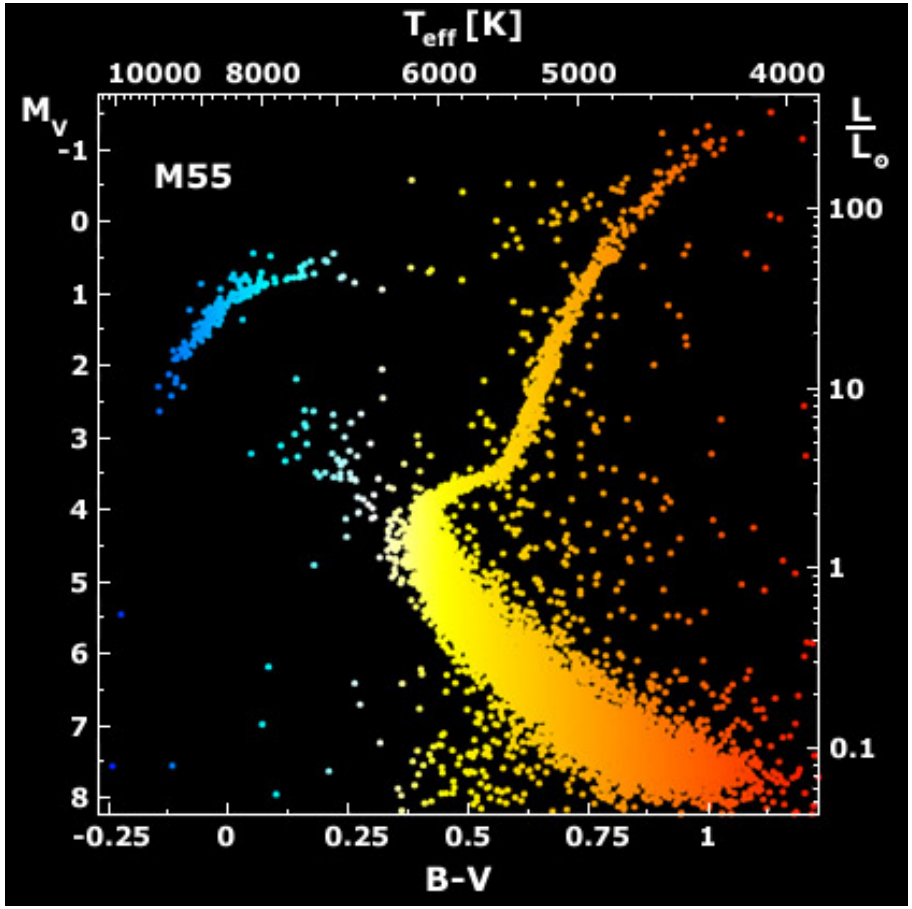


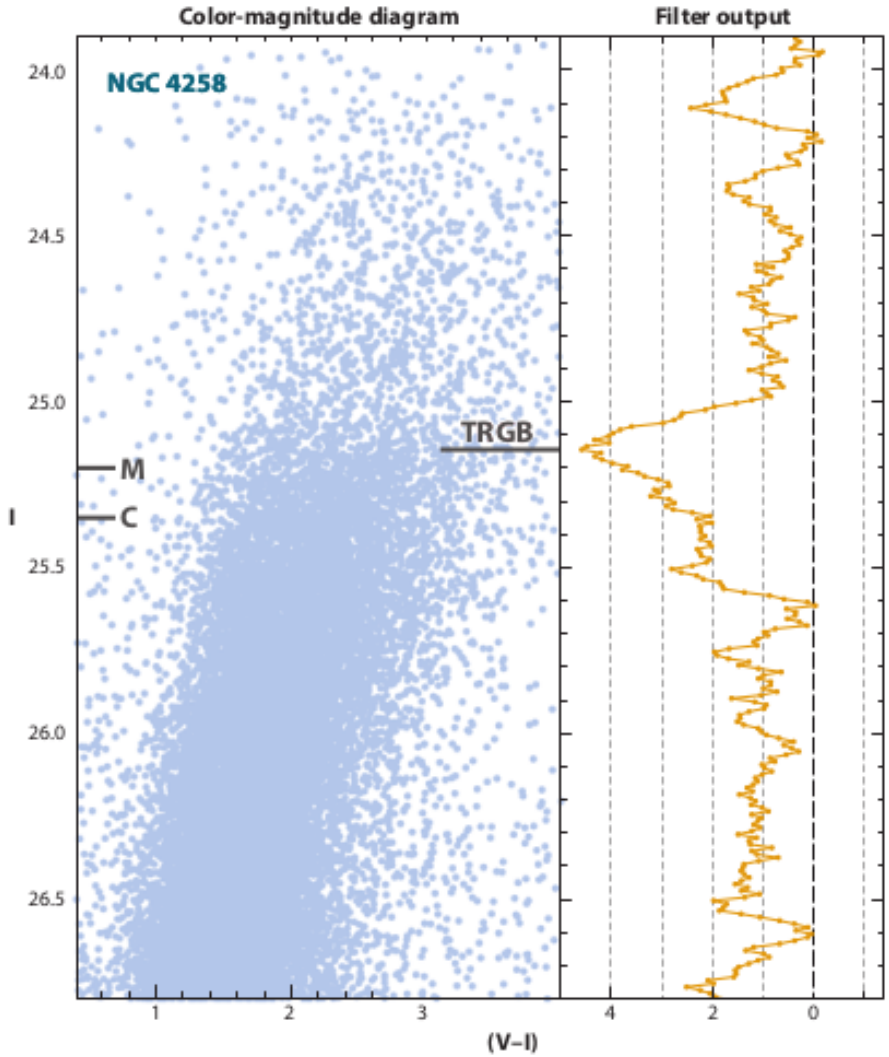
Figure: Casertano et al. A&A 708, A166 (2026) arXiv:2510.23823

Tip of the Red Giant Branch (TRGB)

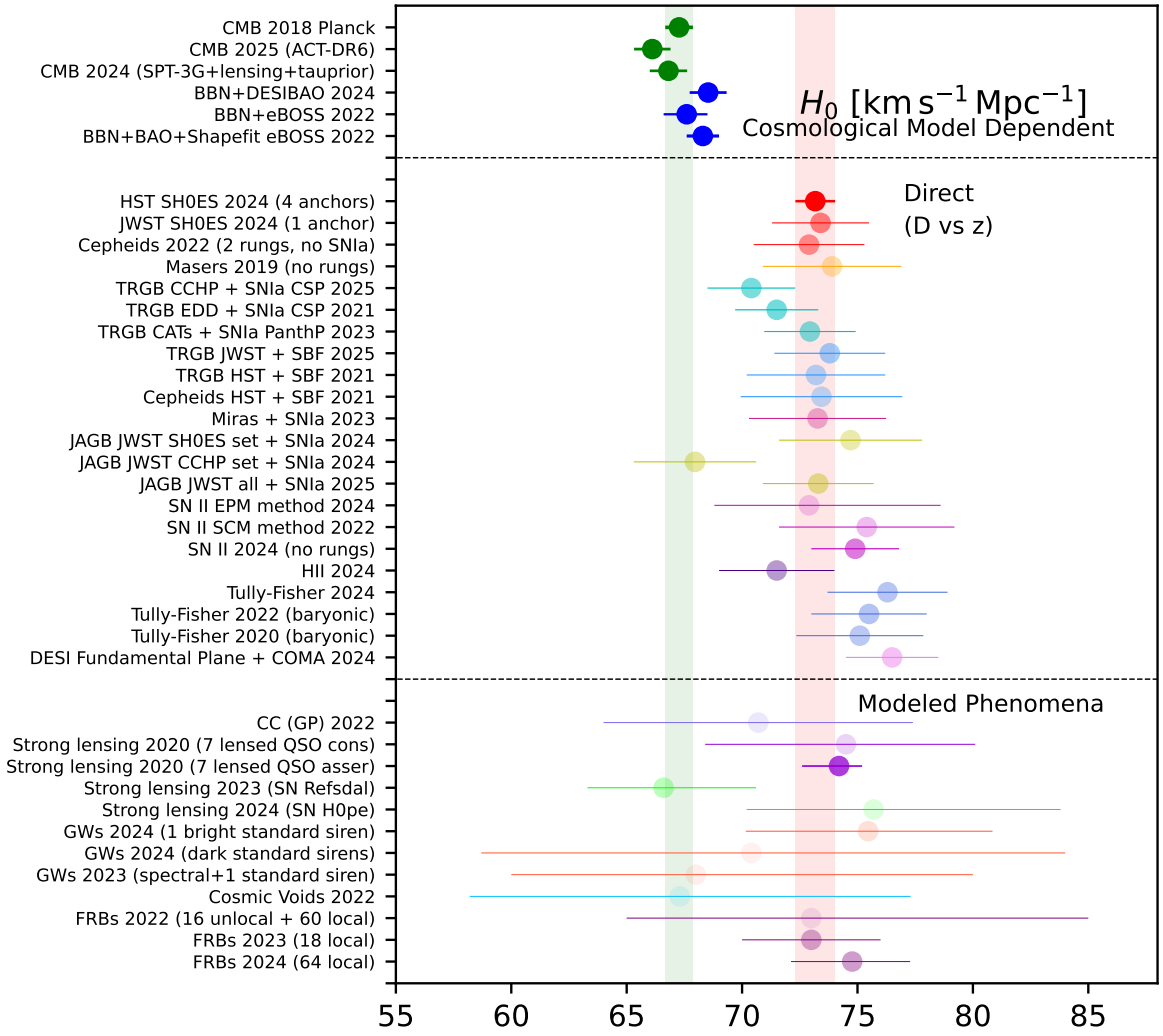
- The TRGB method uses the theoretically well-understood and observationally well-defined discontinuity in the luminosity function of stars evolving up the red giant branch in old, metal-poor stellar populations.



Tip of the Red Giant Branch (TRGB)



The Hubble tension



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Relevant papers for this lecture

- Testing General Relativity in Cosmology M. Ishak Living Reviews in Relativity 22, 1 (2019)
- S. Perlmutter. Supernovae, dark energy, and the accelerating universe. Physics Today, 56, 53 (2003).
- Wendy L. Freedman and Barry F. Madore, The Hubble Constant, Annual Review of Astronomy and Astrophysics 48 (2010) 673-710, arXiv:1004.1856 [astro-ph.CO].
- Riess et al., A redetermination of the Hubble constant with the Hubble Space Telescope from a differential distance ladder Astrophysical Journal, 699, 539 (2009)
- Casertano et al., The Local Distance Network: a community consensus report on the measurement of the Hubble constant at 1% precision A&A 708, A166 (2026) arXiv:2510.23823

Relevant papers: SNIa compilations

- Scolnic et al., The Pantheon+ Analysis: The Full Data Set and Light-curve Release, Astrophysical Journal 938, 113 (2022), arXiv:2112.03863
- Brout et al., The Pantheon+ Analysis: Cosmological Constraints, Astrophysical Journal 938, 110 (2022), arXiv:2202.04077
- Rubin et al., Union Through UNITY: Cosmology with 2,000 SNe Using a Unified Bayesian Framework, arXiv:2311.12098
- DES Collaboration, The Dark Energy Survey: Cosmology Results with ∼1500 New High-redshift Type Ia Supernovae Using the Full 5-year Data Set, Astrophysical Journal Letters 973, L14 (2024), arXiv:2401.02929
- Popovic et al., DES-SN5YR reanalysis with the Dovekie calibration (2025)
- Efstathiou, Evolving dark energy or supernovae systematics?, MNRAS 538, 875 (2025), arXiv:2408.07175
- DESI Collaboration, DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints, arXiv:2503.14738