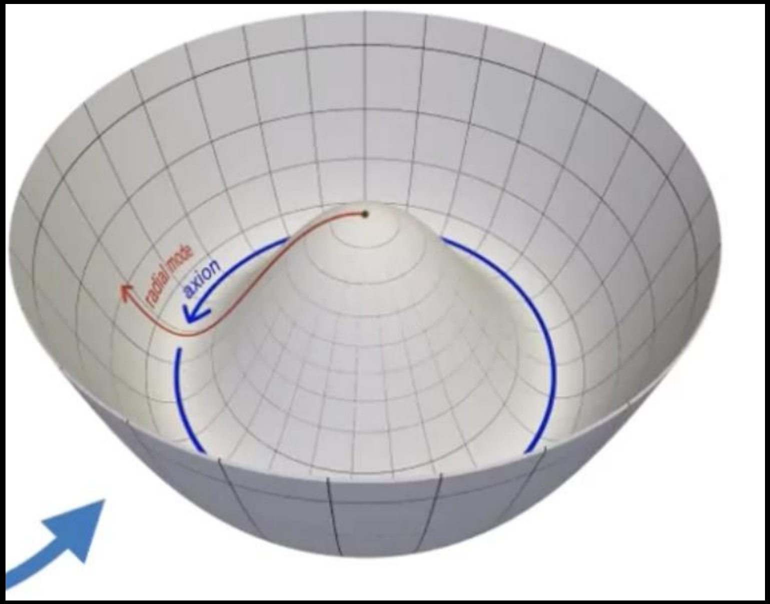
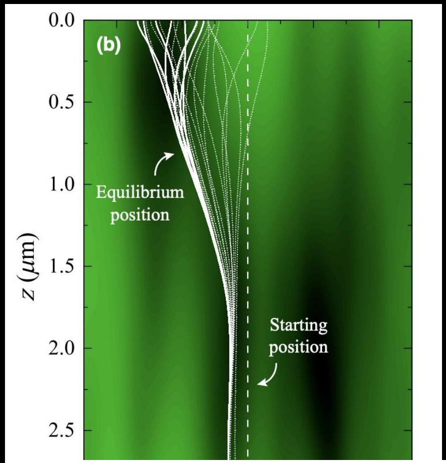
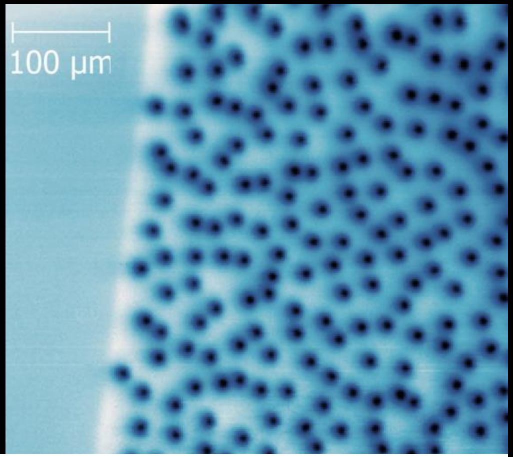
Theory Behind Axion Searches

Jamie McDonald | University of Manchester

data analysis workshop Sheffield | 19 - 21 May 2026







$$\eta_0 \dot{u}(t,z) = \varepsilon u''(t,z) - \kappa_{\mathrm{p}} u(t,z) + \gamma \cos(\omega t) e^{-z/\lambda}$$

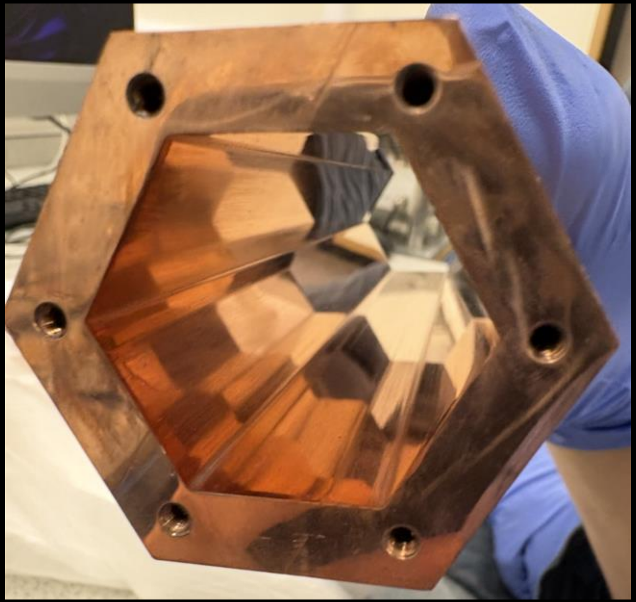
$$\nabla \cdot \mathbf{D} = -g_{\mathrm{a}\gamma\gamma}\mathbf{B}$$

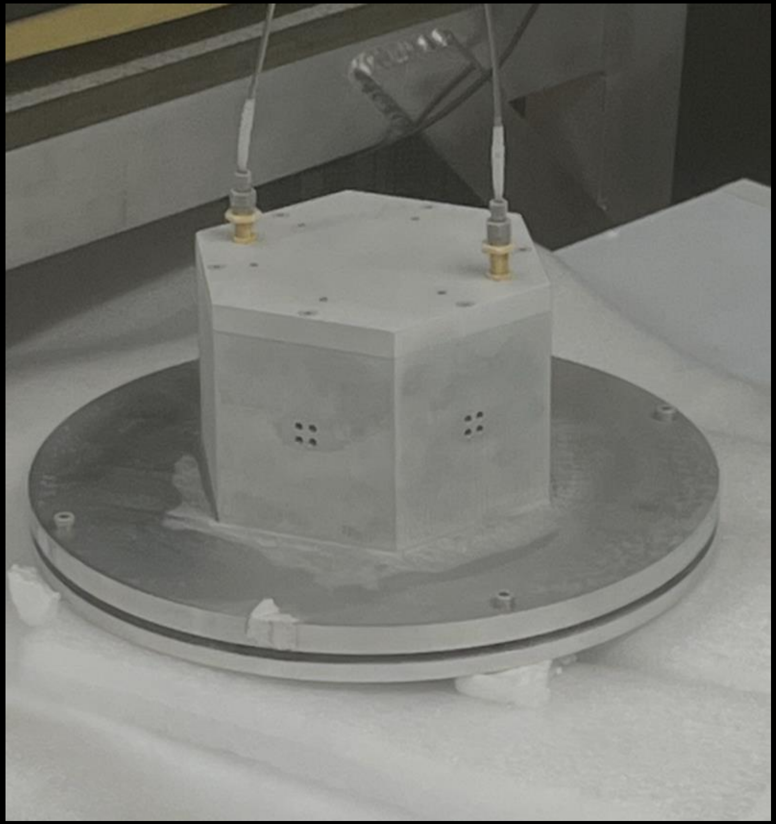
$$\nabla \times \mathbf{B} - \dot{\mathbf{D}} = g_{\mathrm{a}}$$

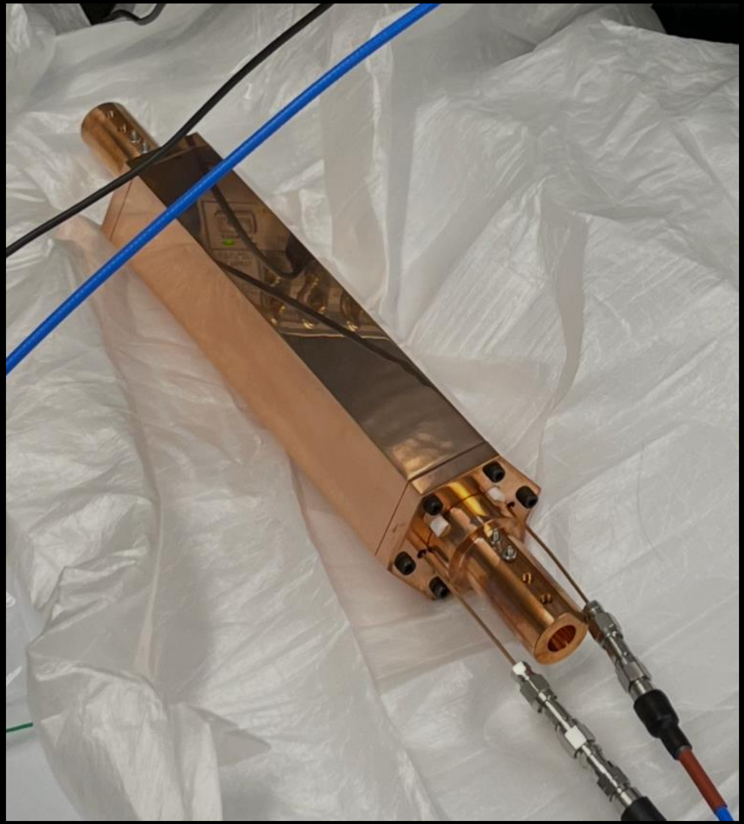
$$\nabla \cdot \mathbf{B} = 0$$

$$\dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

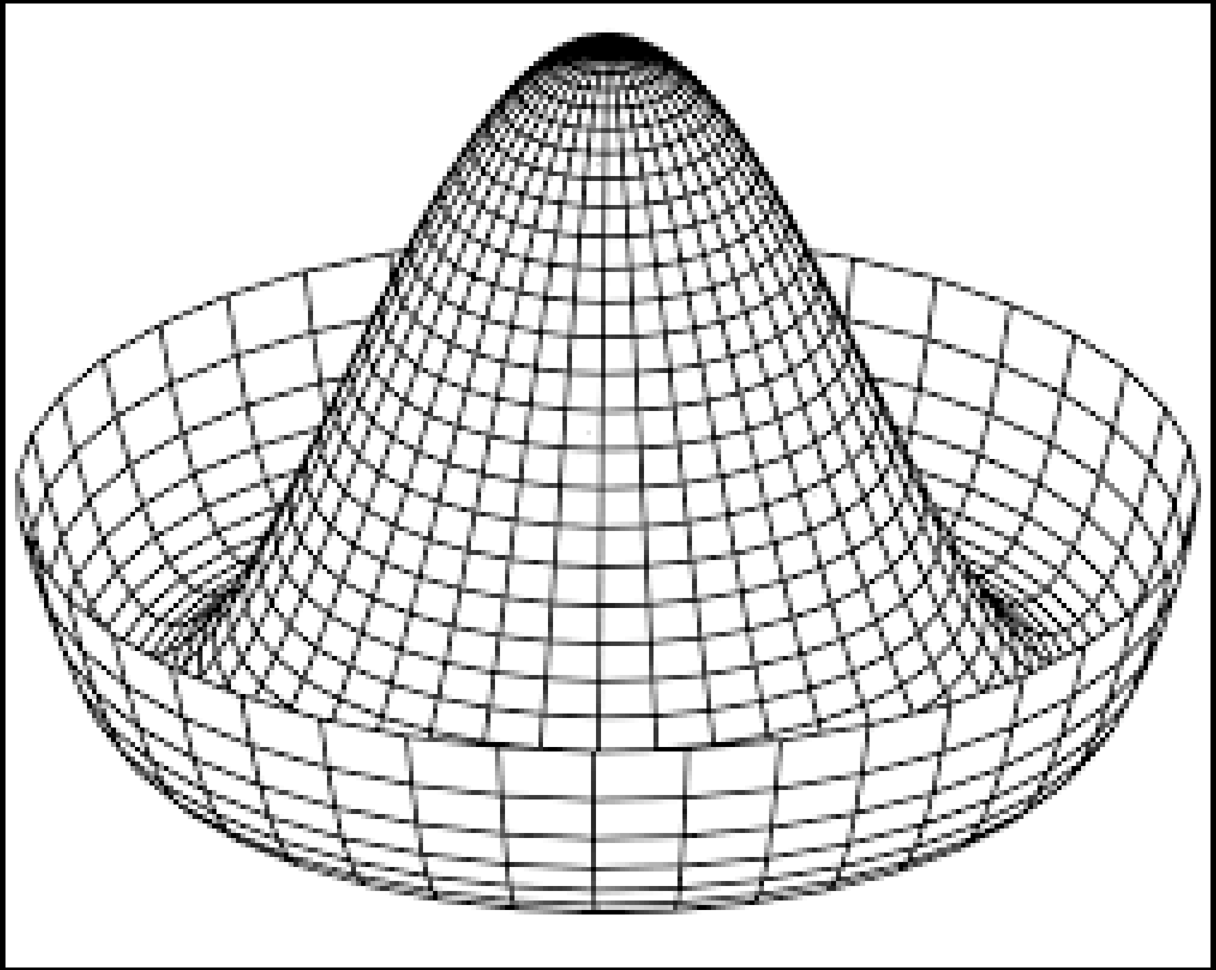
What I normally do these days…

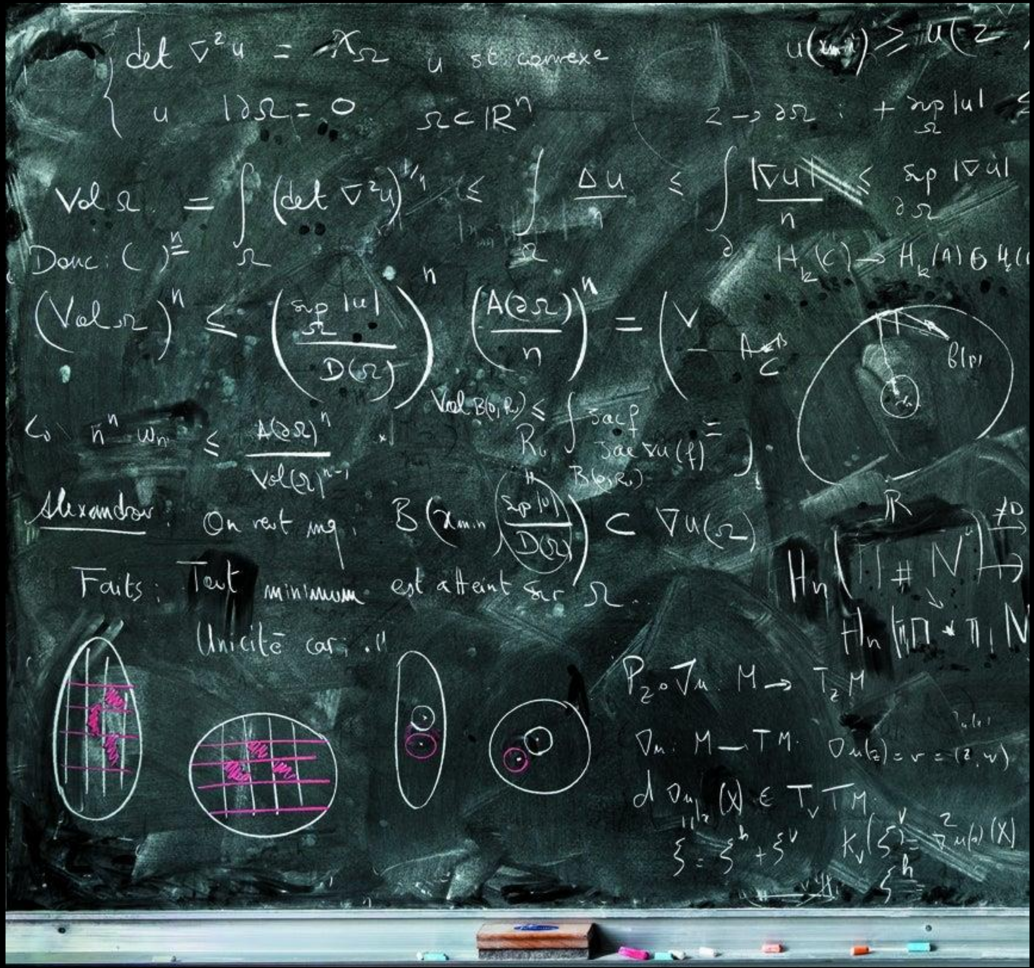






What I think you might want me to do in the next 30 mins…





Theory Behind Axion Searches

Axions in the Early Universe

Sensitivity/Axion Electrodynamics

Superconductors

Stochastic processes/signal synthesis

Condensed matter/materials

Open quantum systems/cavity QED

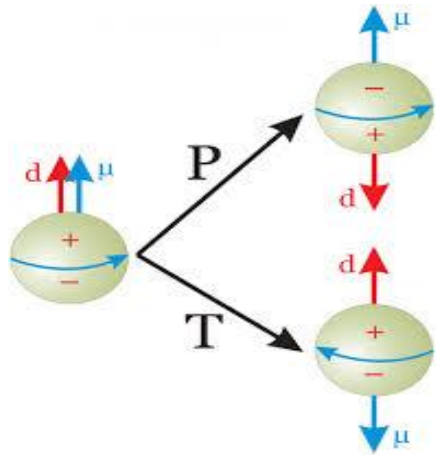
(101)

Fundamental Theory

(102)

Practical Theory

Axions



Solve strong CP problem in QCD

(why neutron EDM so small?)

Peccei & Quinn (1977)

$$\mathcal{L} = \frac{\theta}{16\,\pi^2} G_{\mu\nu} G^{\mu\nu}$$

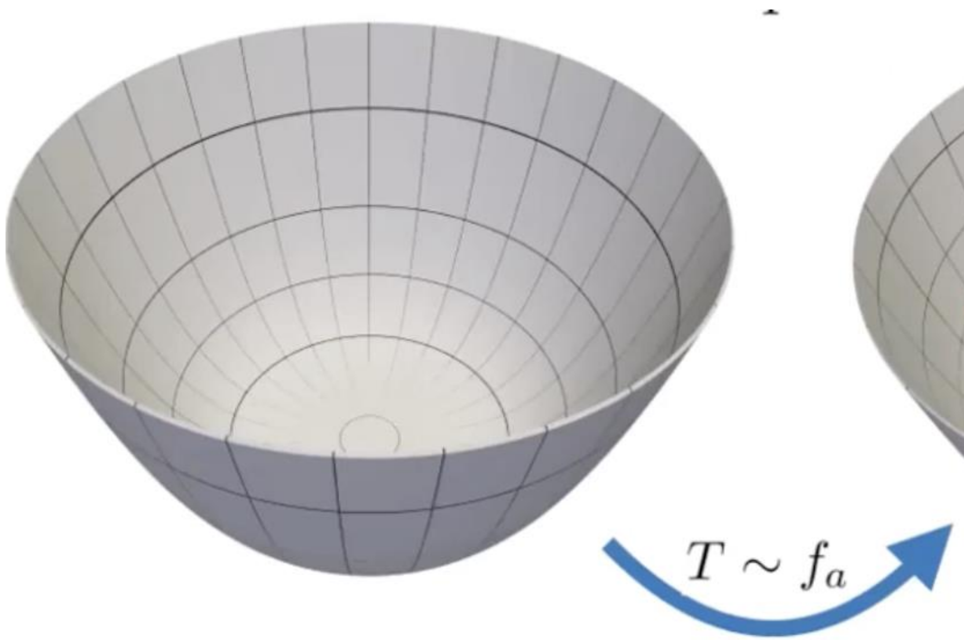
$$d_n \simeq \frac{e\, g_{\pi NN}\, \bar{g}_{\pi NN}}{4\pi^2 m_N} \ln\left(\frac{m_N}{m_\pi}\right) \sim \mathcal{O}(10^{-16})\, \bar{\theta}\, e\,\mathrm{cm}$$

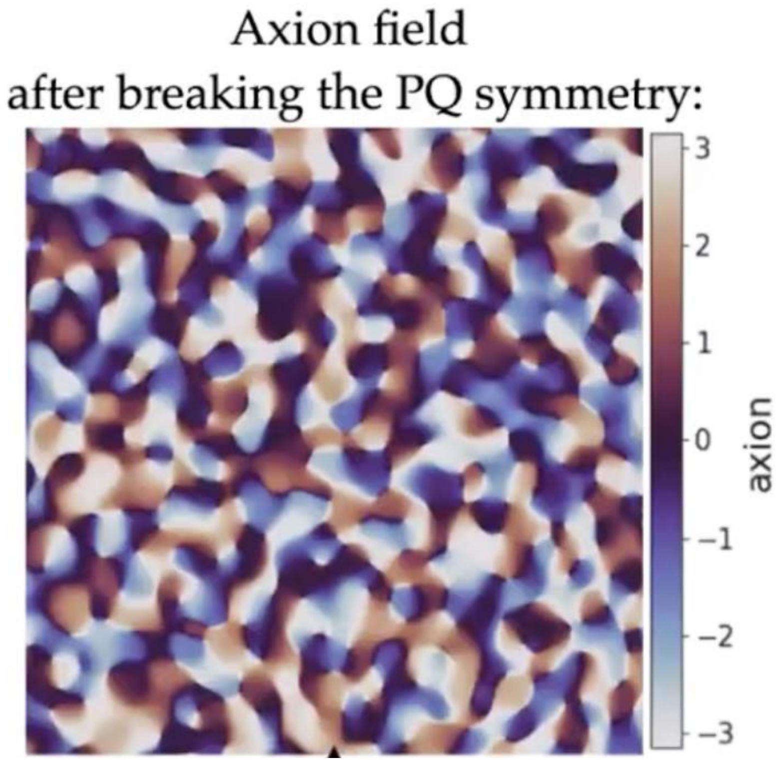
Leading candidates to explain dark matter

($m_{DM} \sim 10^{-6}\mathrm{eV}$ << GeV)

Dine & Fischler (1983), Abbott & Sikivie (1983), Preskill, Wise & Wilczek (1983).

Axion Symmetry Breaking





$$\Phi = |\Phi| \, e^{i\, a\,/f_a}$$

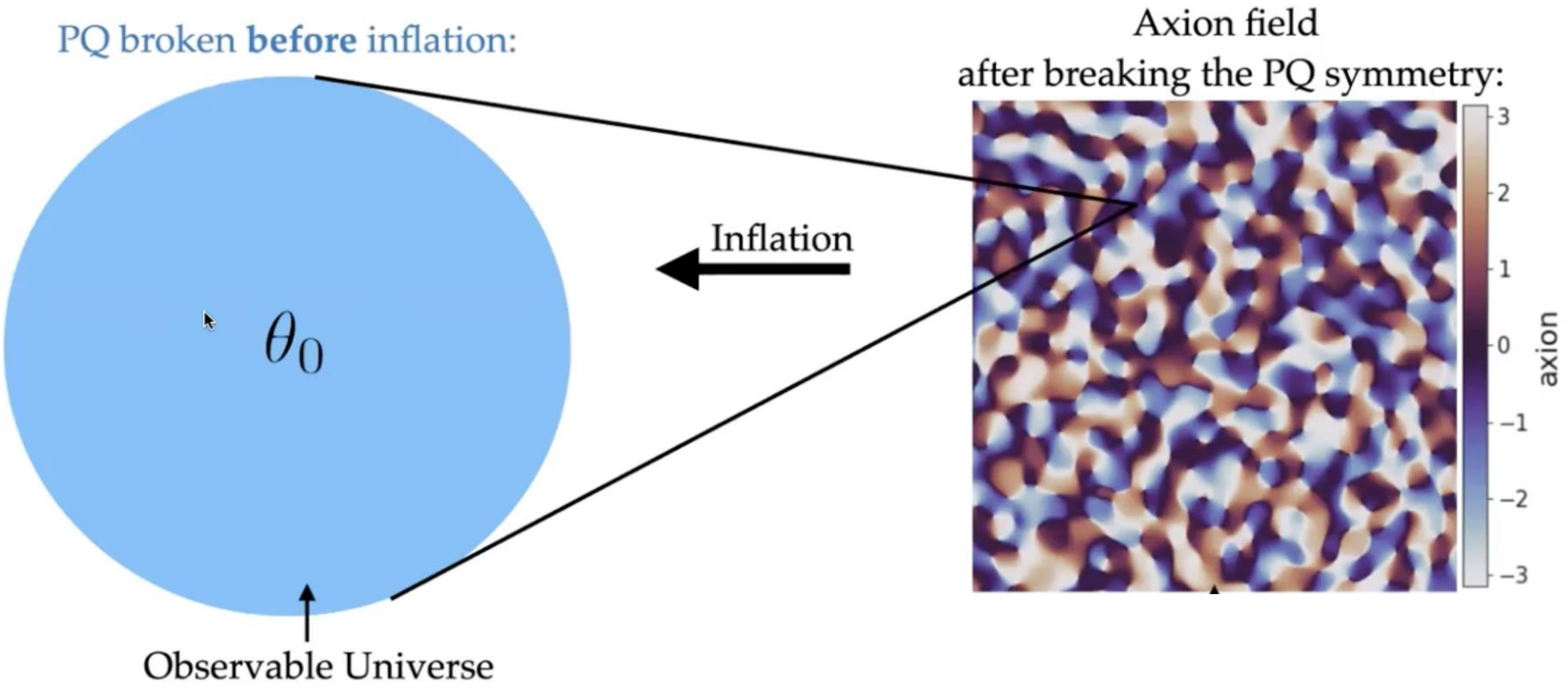
$$\theta = a\,/f_a$$

(Slide courtesy of Malte Buschmann)

(field takes values between 0 and π)

Symmetry Breaking (Before) Inflation

$\boldsymbol{\theta(x) = a(x)/f_a}$



Dark matter abundance

Two free parameters: f_a, θ_0

$$\Omega_a \sim \theta_0^2 / f_a^2$$

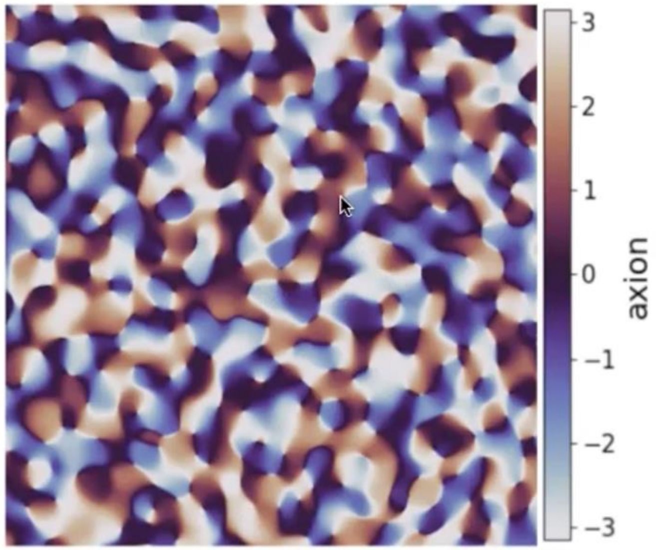
No fine-tuning of θ

$\rightarrow m_a \sim \mu eV$

(Slide courtesy of Malte Buschmann)

Symmetry Breaking (after) Inflation

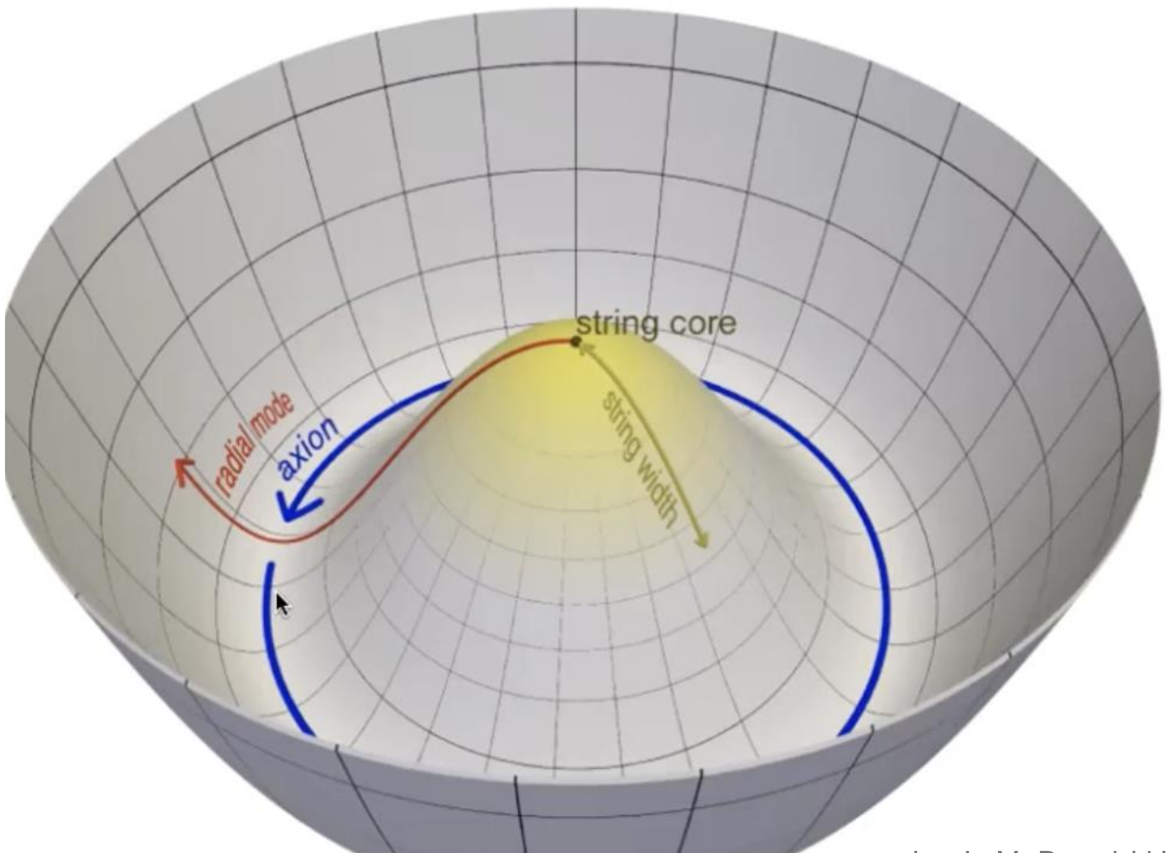
PQ broken **after** inflation:



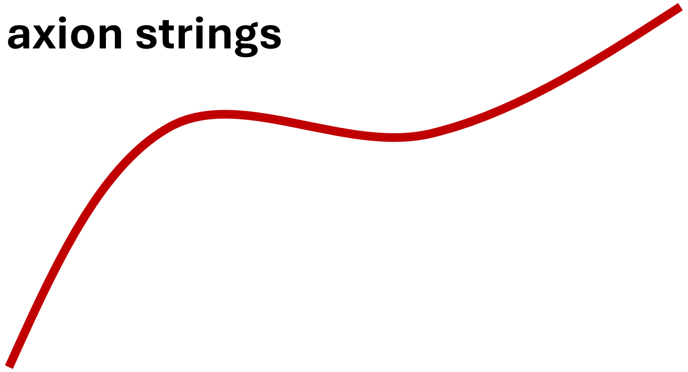
Requires us to understand the cosmological evolution of the axion field ← We can predict the axion mass! →

One free parameter: f_a

$$\Omega_a \sim \langle \theta^2 \rangle / f_a^2$$



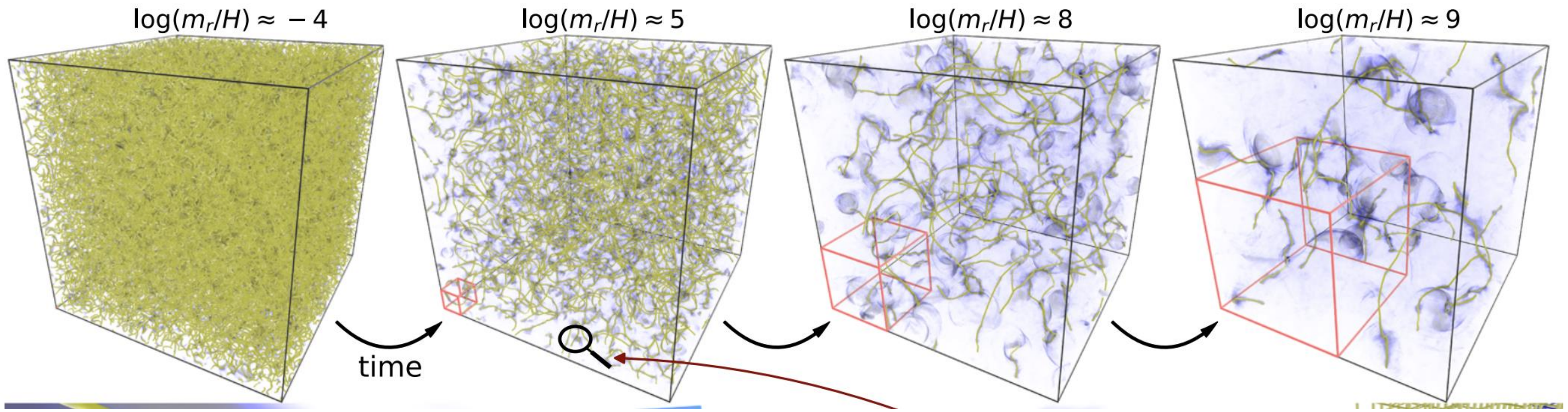
axion strings



(Slide courtesy of Malte Buschmann)

supercomputer simulations of axion strings

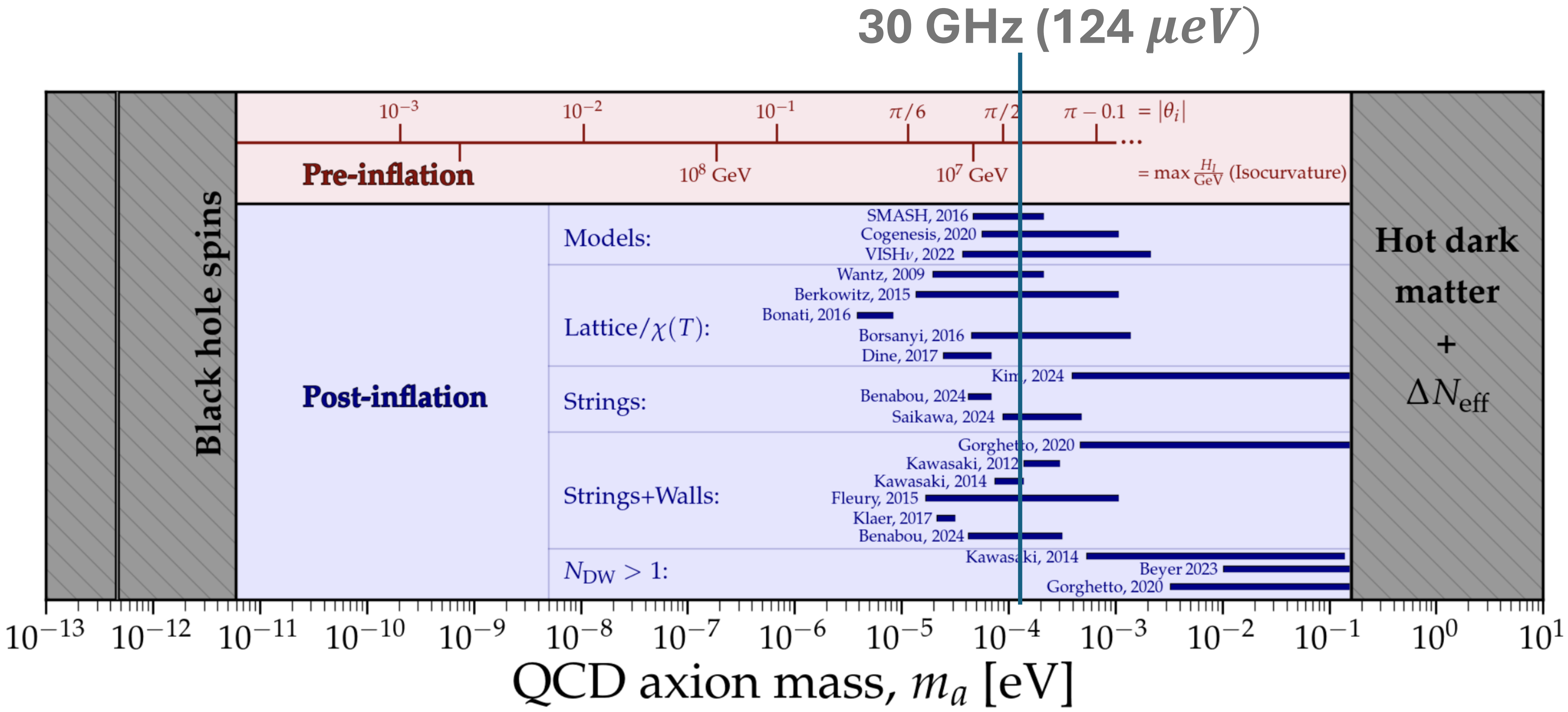
$$\mathcal{L}_{PQ} = |\partial\Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 - \frac{\lambda T^2}{3}|\Phi|^2$$



Buschmann et al Nature Communications
volume 13, Article number: 1049 (2022)

(Slide courtesy of Malte Buschmann)

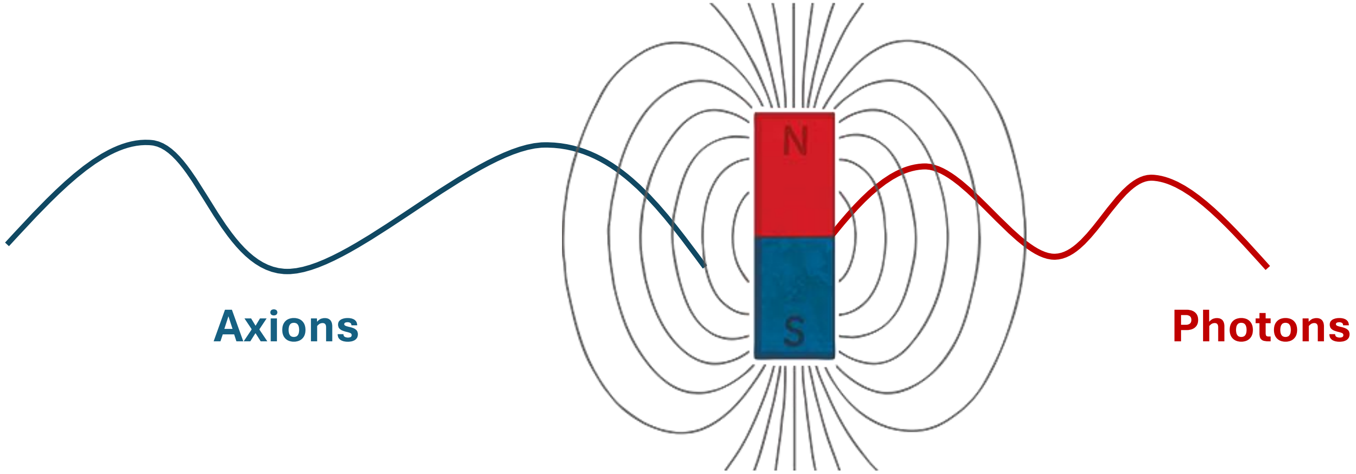
Theory predictions for axion mass



Plot Courtesy: Ciaran O'Hare

Axion Electrodynamics

$$\mathcal{L} = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \, \boldsymbol{E}.\boldsymbol{B}$$



Axion Electrodynamics

$$\nabla \cdot \mathbf{D} = -g_{\mathrm{a}\gamma\gamma}\mathbf{B} \cdot \nabla a$$

$$\nabla \times \mathbf{B} - \dot{\mathbf{D}} = g_{\mathrm{a}\gamma\gamma}\dot{a}\mathbf{B} - g_{\mathrm{a}\gamma\gamma}\mathbf{E} \times \nabla a$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

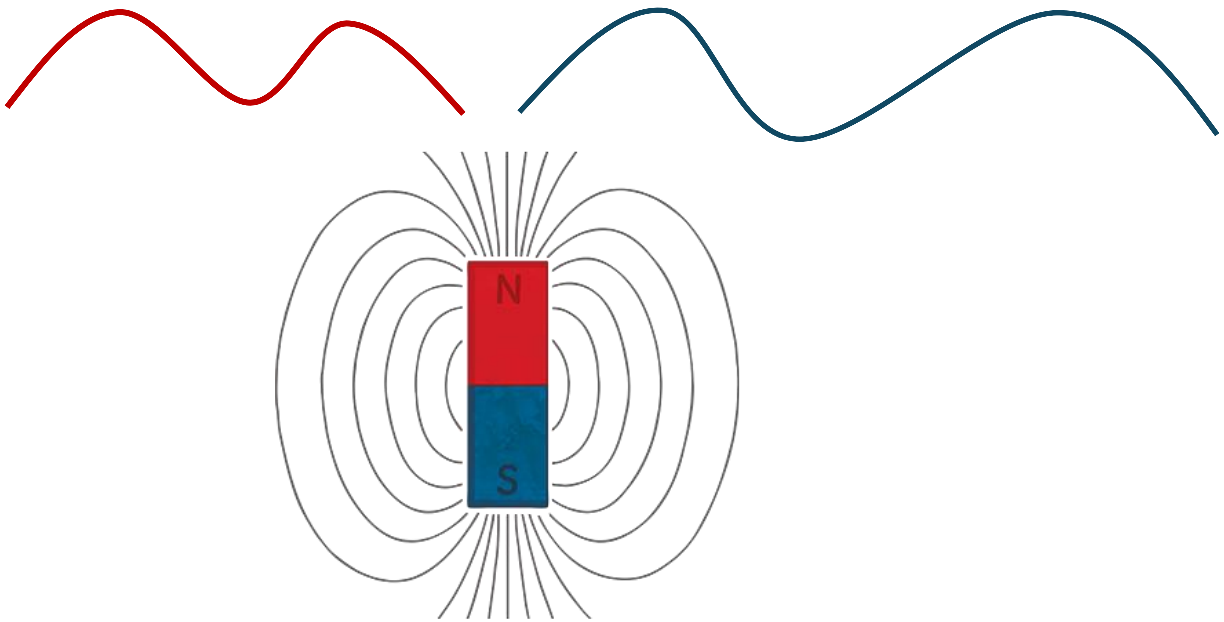
Maxwell's Equations + axions

Axion Electrodynamics

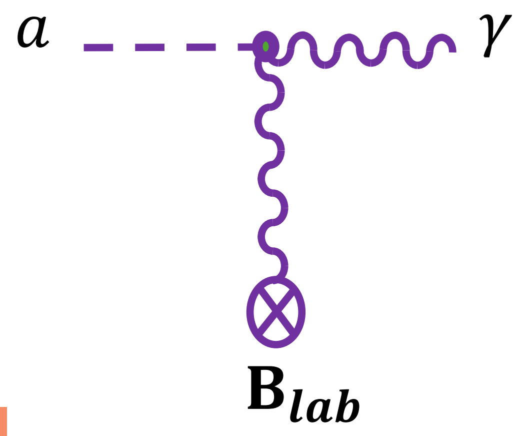
$$-\boldsymbol{\nabla}^2\,\mathbf{E} + \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot \boldsymbol{E}) - \boldsymbol{\omega}^2\varepsilon.\,\boldsymbol{E} = \boldsymbol{g}_{a\,\gamma\gamma}\boldsymbol{\omega}^2\boldsymbol{a}\,\boldsymbol{B}$$

Light

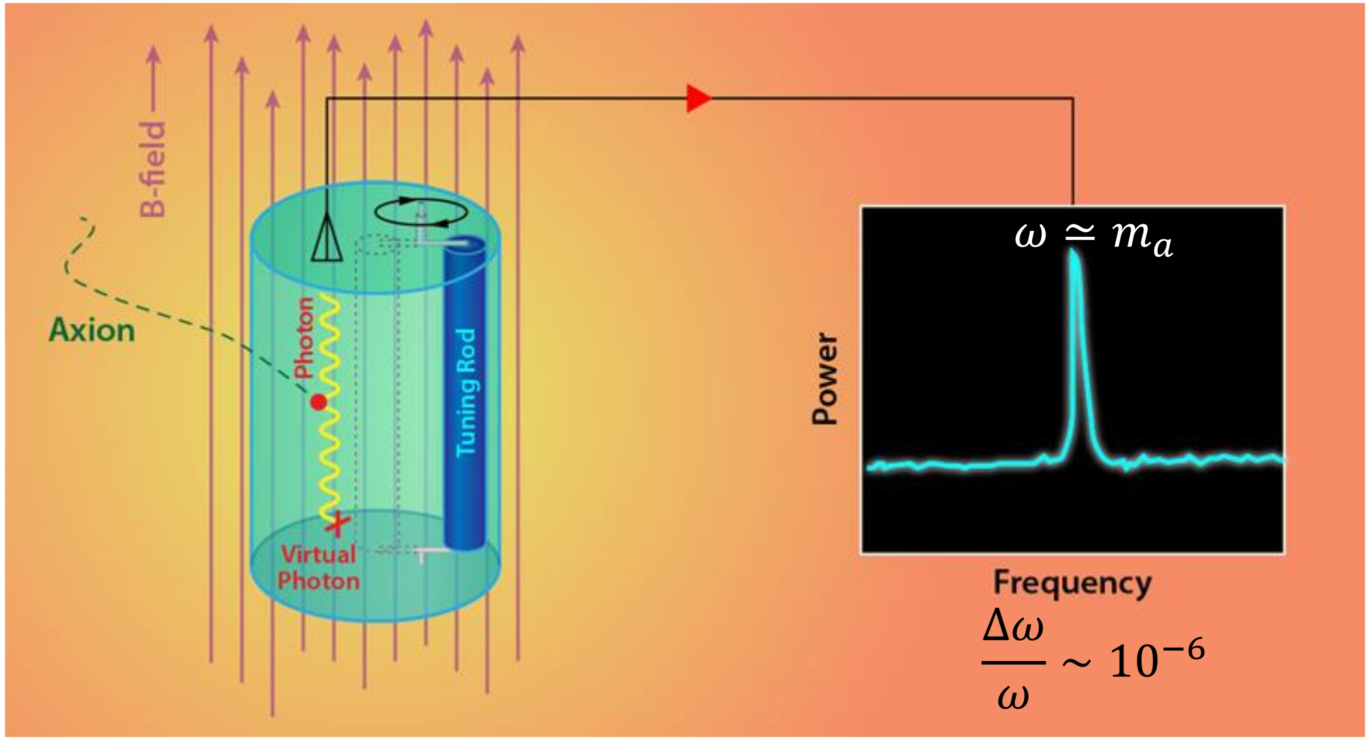
Axion field



Axion Dark Matter Detection



$a \propto e^{i\omega t}$ $\omega \simeq m_a + \frac{1}{2} m_a v_{DM}^2$

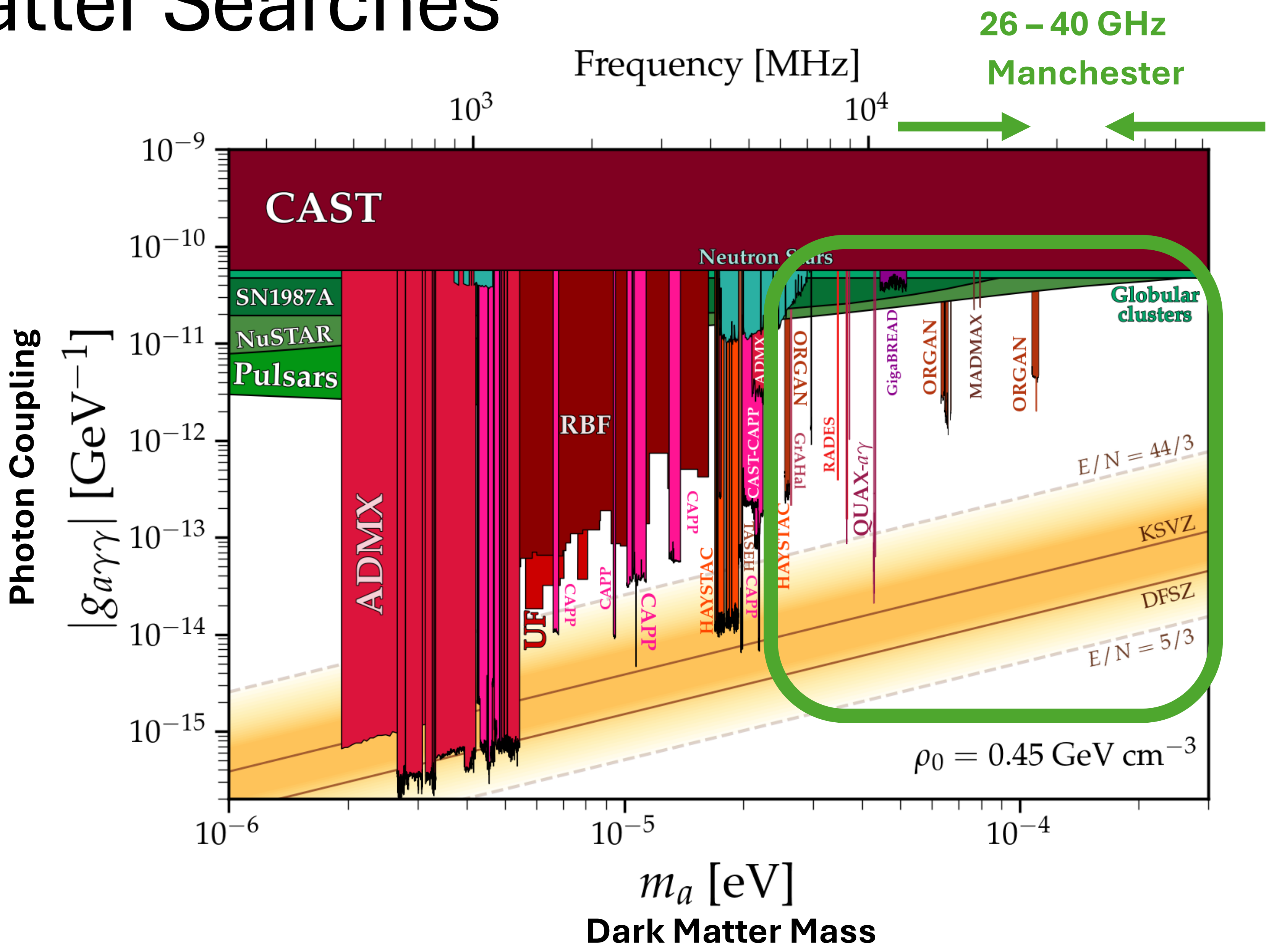


$V_{DM} \sim 200\ km/sec$

$\rho = 0.45\ GeV\ /cm^3$

Axion Dark Matter Searches

$$\mathcal{L} = -\frac{g_{a\gamma\gamma}}{4} a\, F_{\mu\nu}\, \tilde{F}^{\mu\nu}$$



Plot Courtesy: Ciaran O'Hare

Theory Behind Sensitivity of Axion Experiments

$$\frac{df}{dt} \propto \frac{B^4\, V^2 \mathrm{C}^2}{T_{sys}} \min(Q_c, Q_a)$$

(a story through a formula)

Sensitivity of Axion Experiments

$$\frac{df}{dt} \propto \frac{B^4\, V^2 C^2}{T_{sys}} \min(Q_c, Q_a)$$

B 

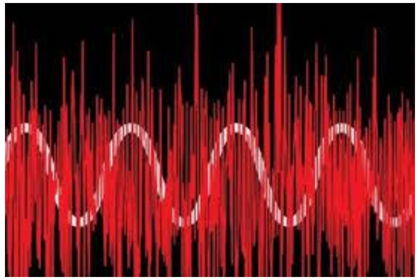
$\mathbf{C} = \frac{\left(\int dV\, E.B_{lab}\right)^2}{V B_{lab}^2 \int dV\, |E|^2}$

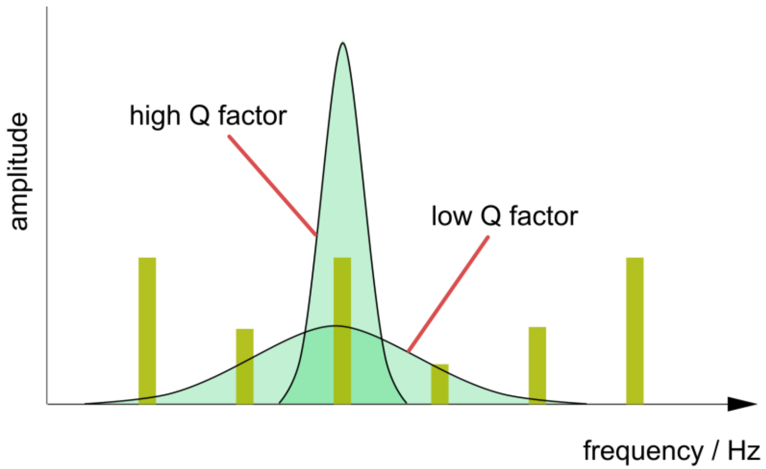
$Q_a \sim 10^6$

$$Q_c = \frac{\int_V dV\, |H|^2}{R \int dS |n \times H|^2 + \tan\delta \int dV |H|^2}$$

Surface Resistance

Losses from bulk materials

T_{sys} 



Sensitivity of Axion Experiments

$$\frac{df}{dt} \propto \frac{\boxed{B^4} V^2 C^2}{T_{sys}} \min(Q_c, Q_a)$$

Manchester

~ few cm bore
~ 3 Tesla field

Sheffield

Initial Magnet at Sheffield

20 cm bore
8 T field

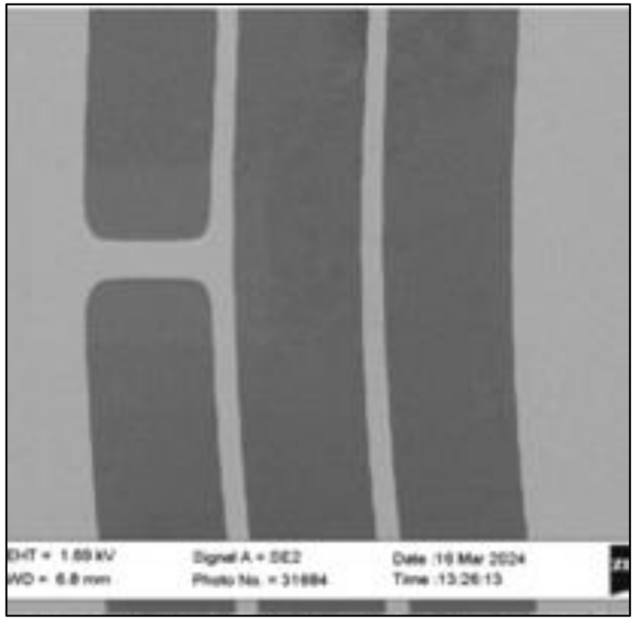
~ 10^2 gain in scan rate

*

Sensitivity of Axion Experiments

$$\frac{df}{dt} \propto \frac{B^4\, V^2 C^2}{\boxed{T_{sys}}} \min(Q_c, Q_a)$$

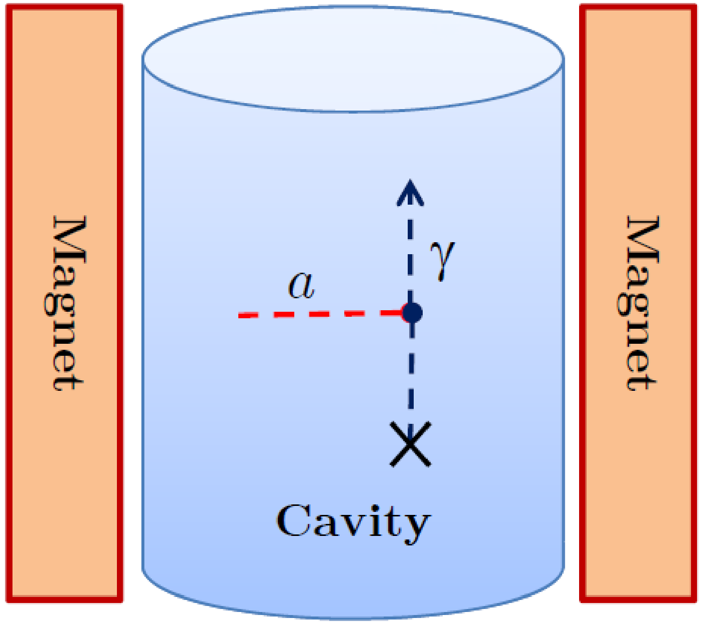
Quantum Noise Limited amplifiers



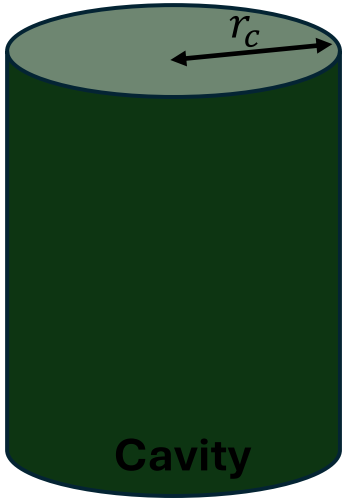
Sensitivity of Axion Experiments

$$\frac{df}{dt} \propto \frac{B^4 \boxed{V^2}\boxed{C^2}}{T_{sys}} \min(\boxed{Q_c}, Q_a)$$

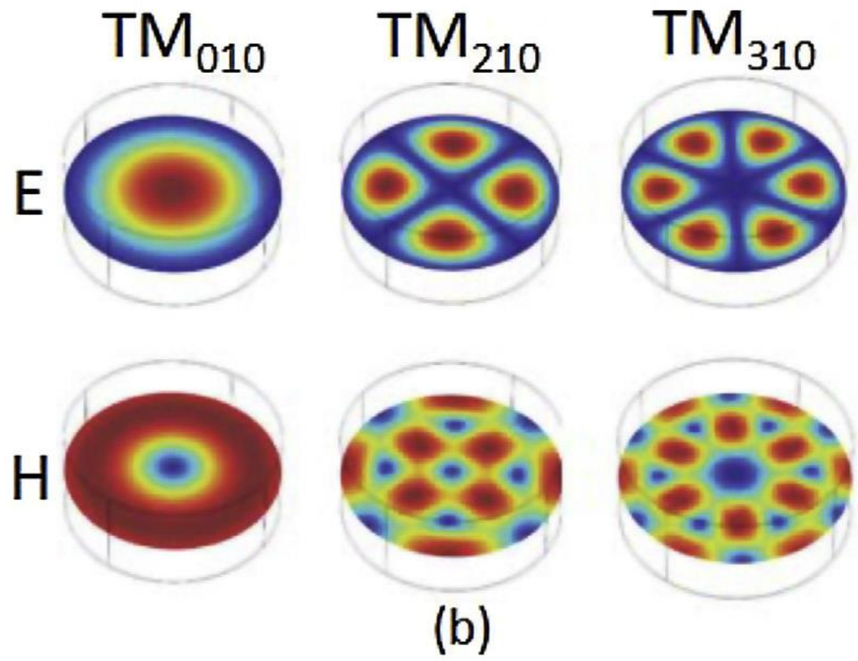
Cavities



Cavity Searches for Axion Dark Matter



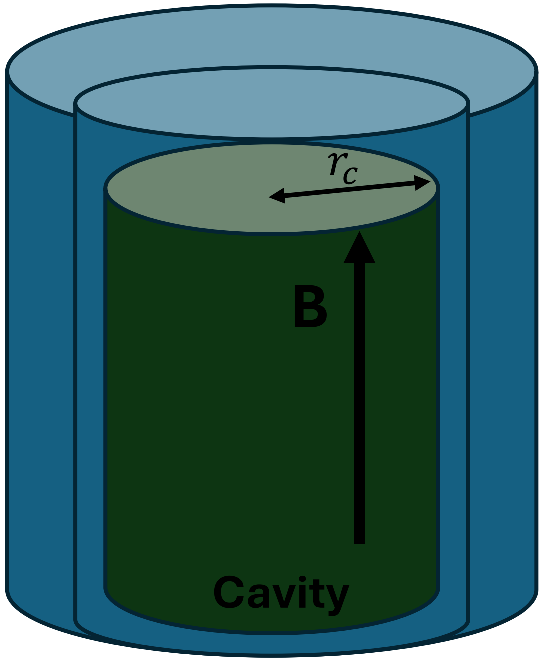
Cavity Modes

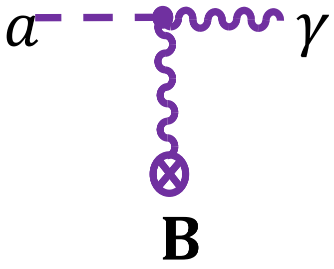


$$f_{lmn}$$

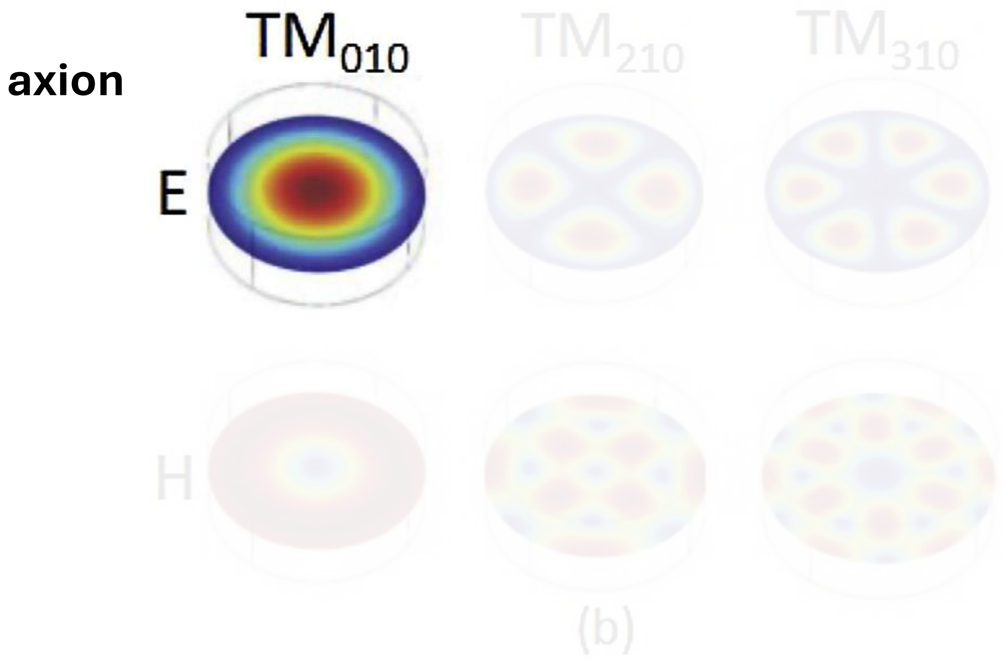
(eigenfrequencies)

Cavity Searches for Axion Dark Matter





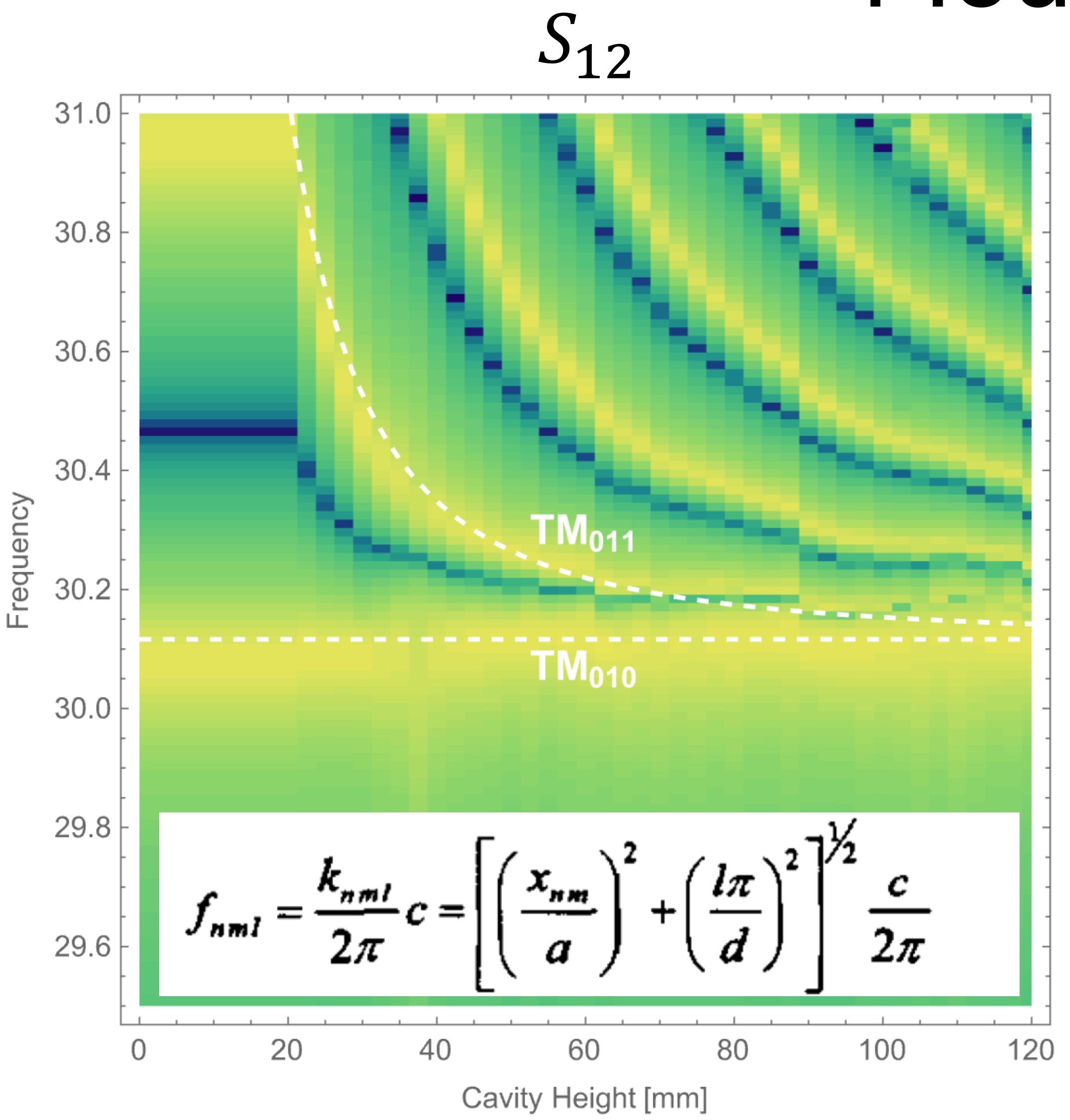
Cavity Modes



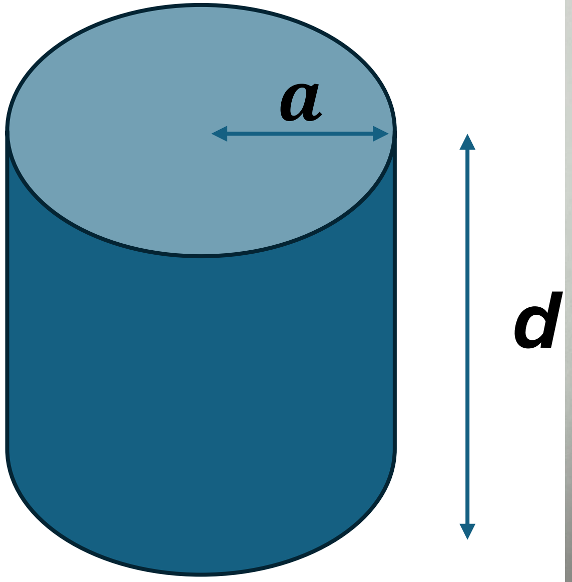
$$f_{010} \sim 1/r_c \sim \nu_{axion} \sim m_a$$

(eigenfrequencies)

Mode Crowding

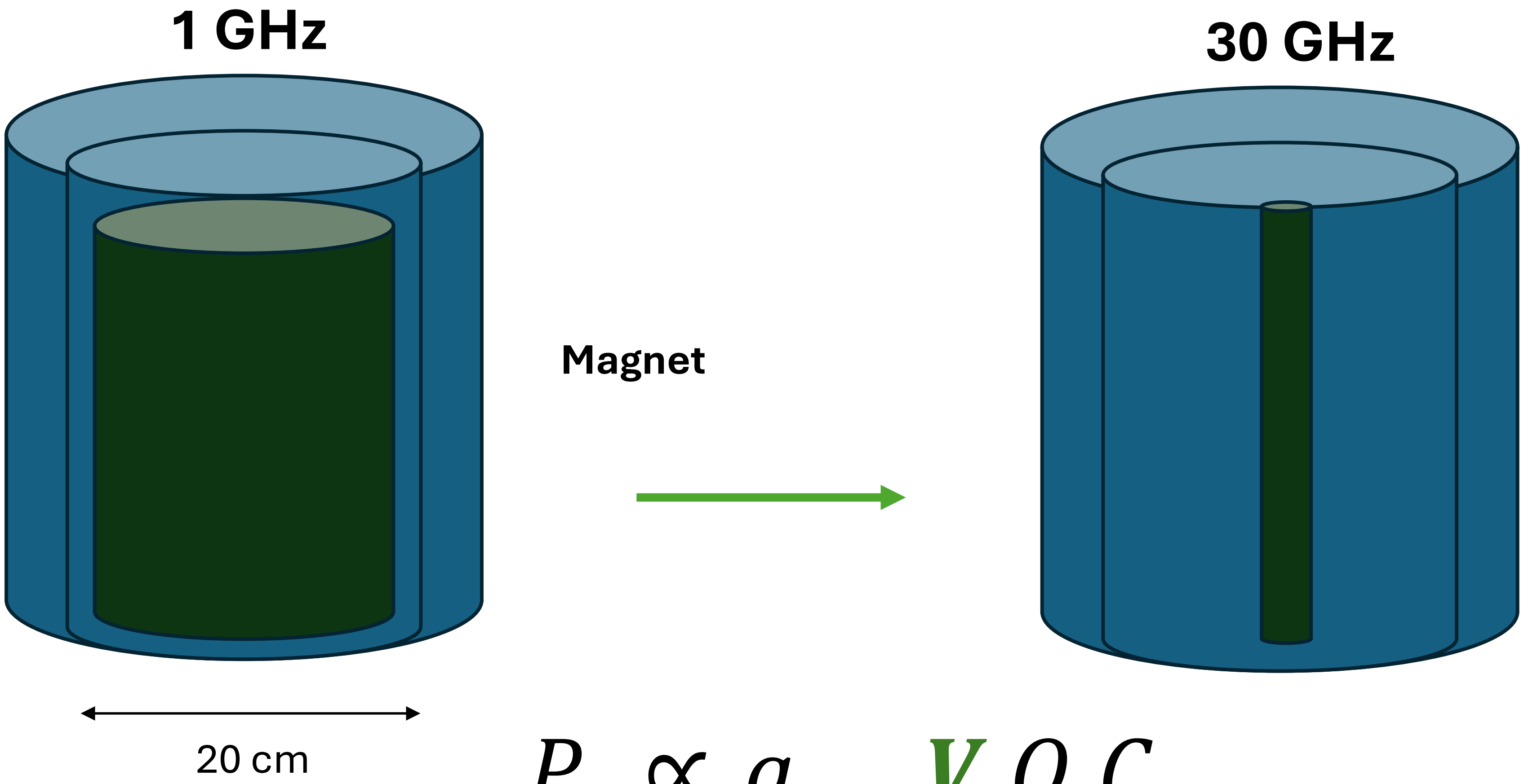


$$f_{nml} = \frac{k_{nml}}{2\pi} c = \left[\left(\frac{x_{nm}}{a} \right)^2 + \left(\frac{l\pi}{d} \right)^2 \right]^{1/2} \frac{c}{2\pi}$$



Volume Challenge

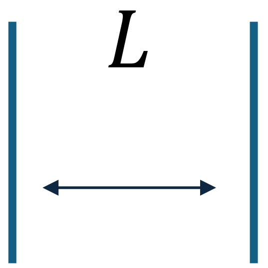
$$L \sim 1/m_a \sim 1/\nu$$



$$P_a \propto g_{a\gamma\gamma} \boldsymbol{V}\, Q\, C$$

Volume Challenge

$$L \sim 1/m_a \sim 1/\nu$$



Repeated Structures

Recover Larger Volume

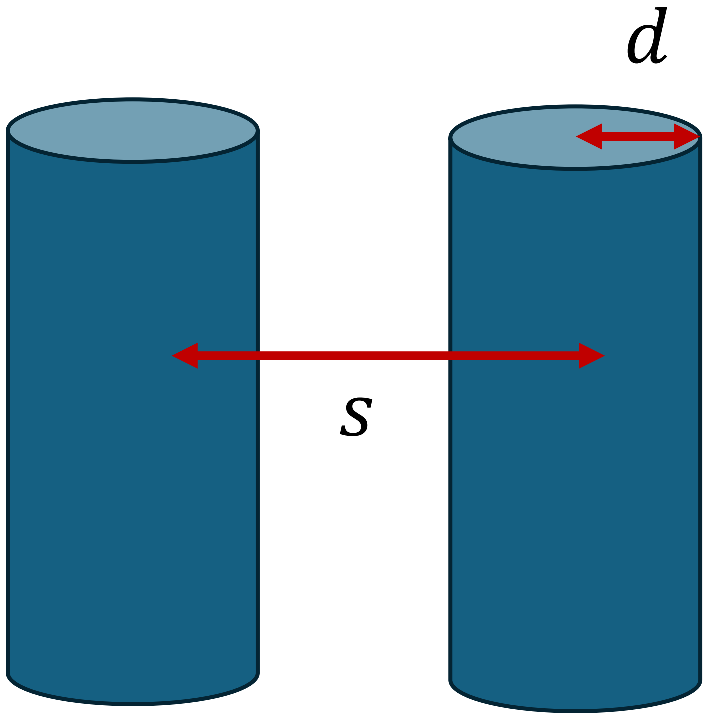
Volume Challenge

Repeated Structures

$$\omega_p^2 = \frac{2\pi}{s^2 \log(s/d)}$$

Resonant axion conversion

$$\omega_p^2 = m_a^2$$

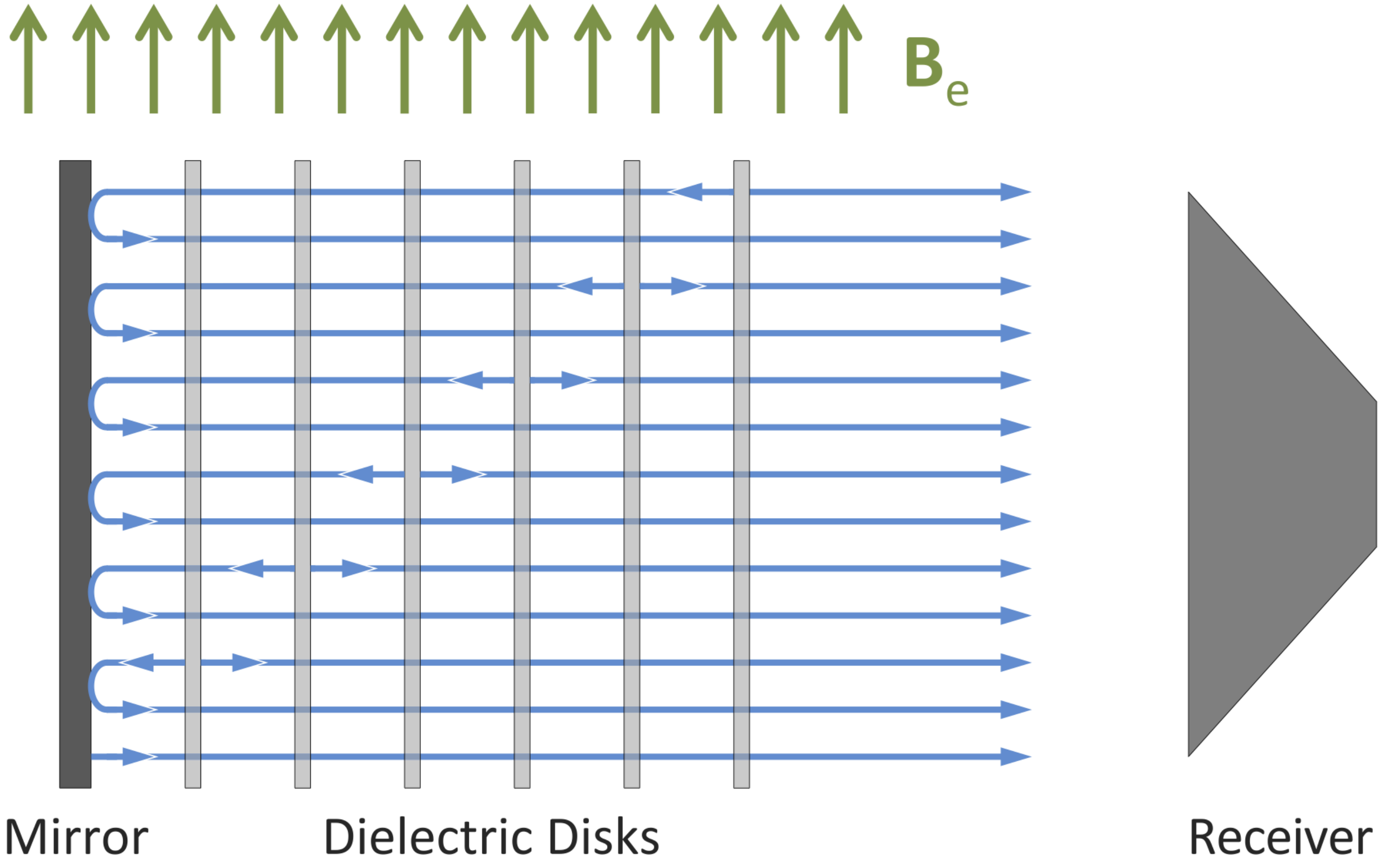


Wires

Lawson et al PRL 123 (2019) 14, 141802

Volume Challenge

Repeated Structures

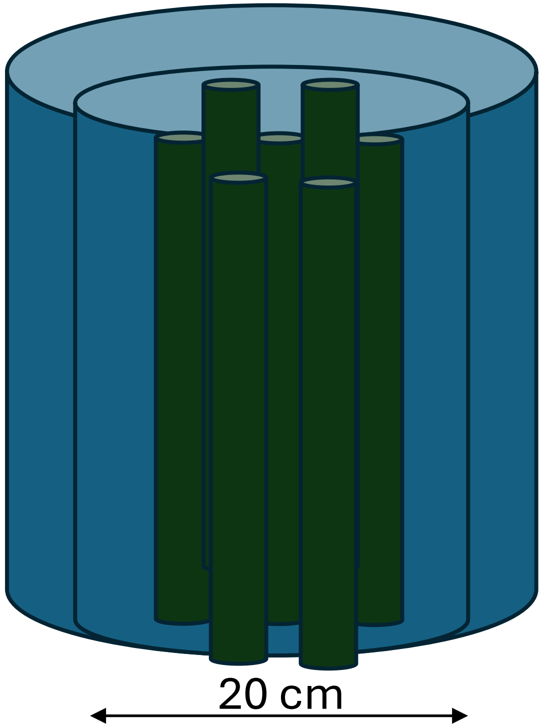


MADMAX – concept (Caldwell et al PRL 118, 091801 (2017))

Volume Challenge

$$L \sim 1/m_a \sim 1/\nu$$

Multi-Cavity Arrays

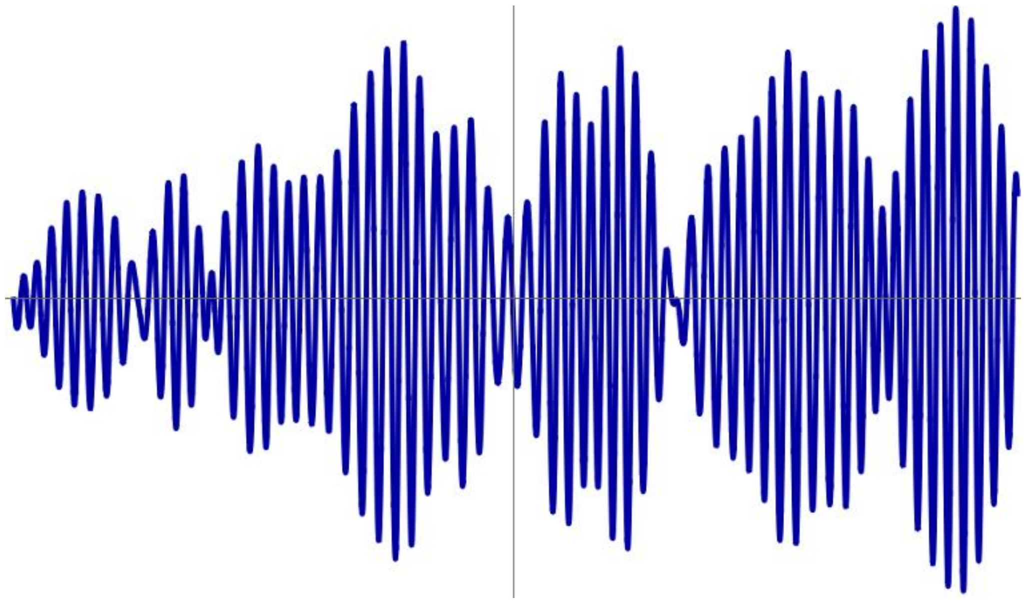


Magnet

See also: ORGAN, ADMX-ERF, CAPP

$$\frac{df}{dt} \propto \frac{B^4\, V^2 \mathrm{C}^2}{T_{sys}} \min(Q_c, \boxed{Q_a})$$

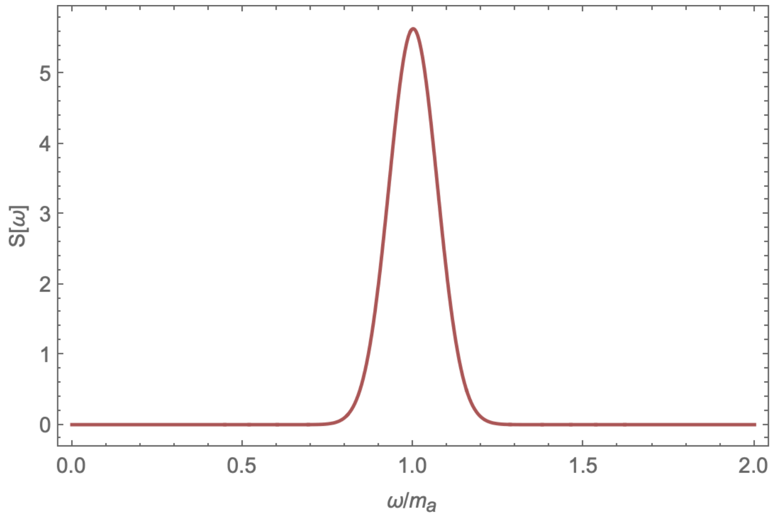
Signal Synthesis



1

Question: how do I model a given power spectrum as a stochastic process ?

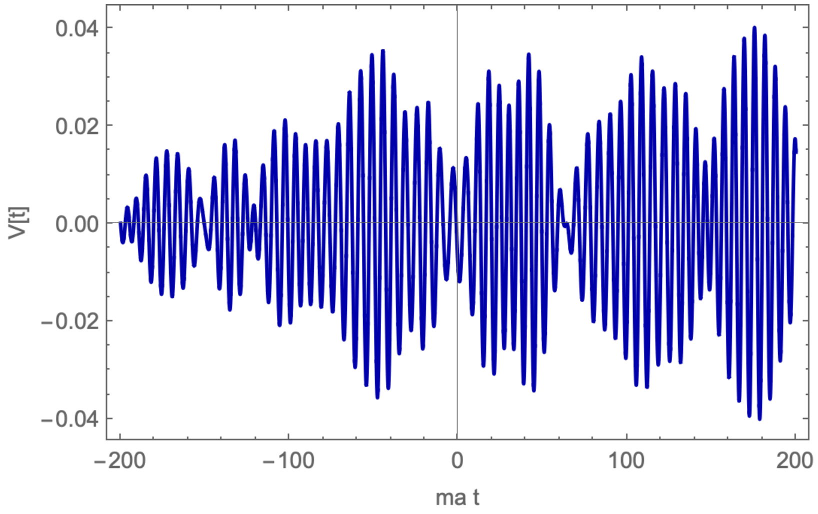
Power Spectrum (PSD)



Frequency

?

Stationary Time-Domain Process (Stochastic)



Time

2

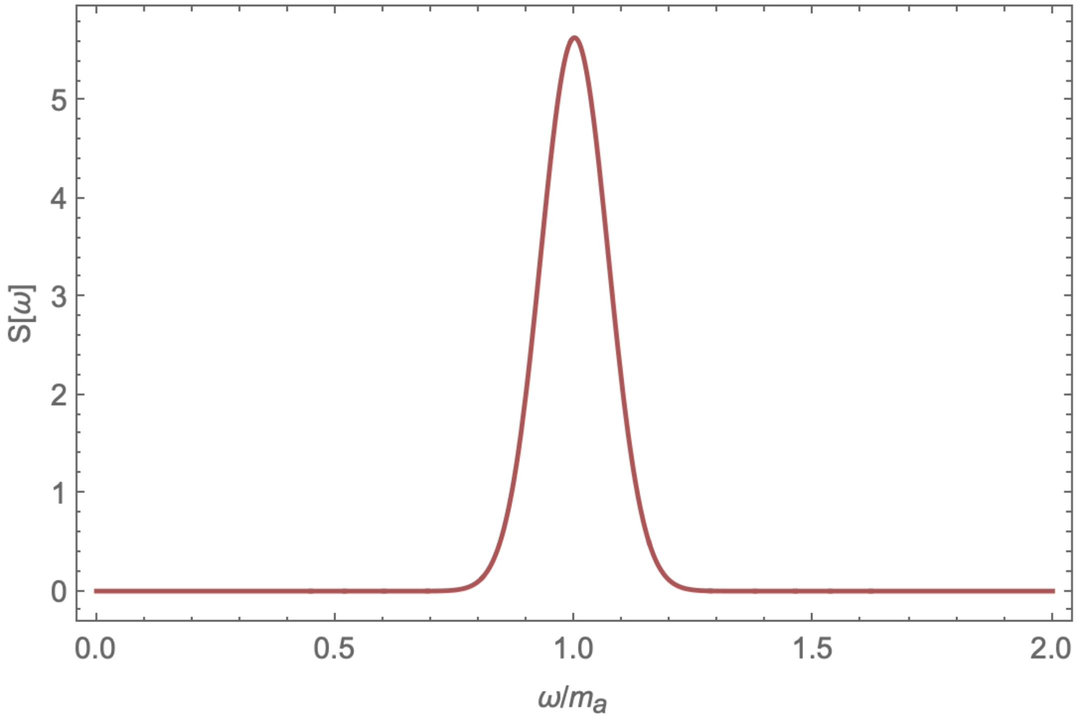
Let's consider a line signal

$$S(\omega) = \int d\tau\, R(\tau) e^{i\,\omega\,\tau}$$

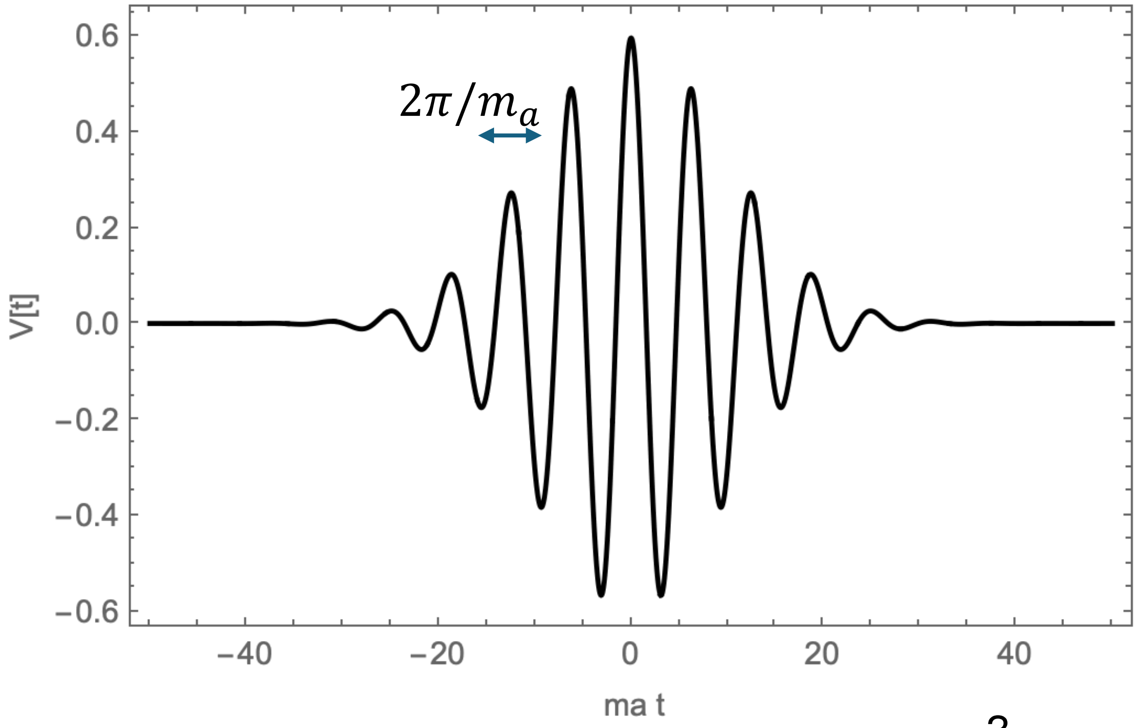
$$R(\tau) = < a(t)a(t+\tau) >$$

Characteristic Voltage:
$V_c(f) \equiv [S(f)]^{1/2}$

Power Spectral Density
$S(\omega)$



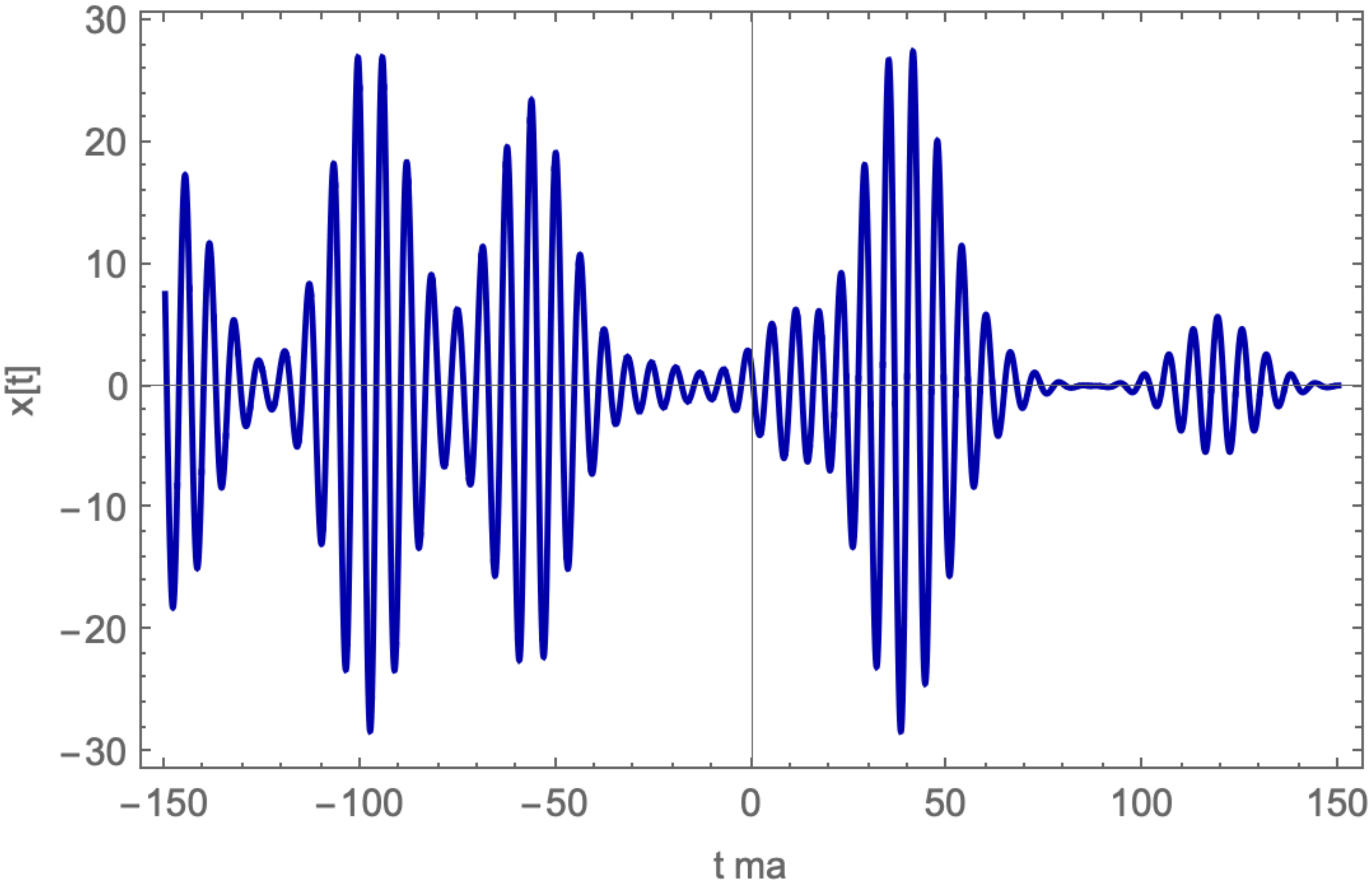
$V_c(t) \equiv \int d\omega\;\; e^{i\,\omega\,t} V_c(f)$



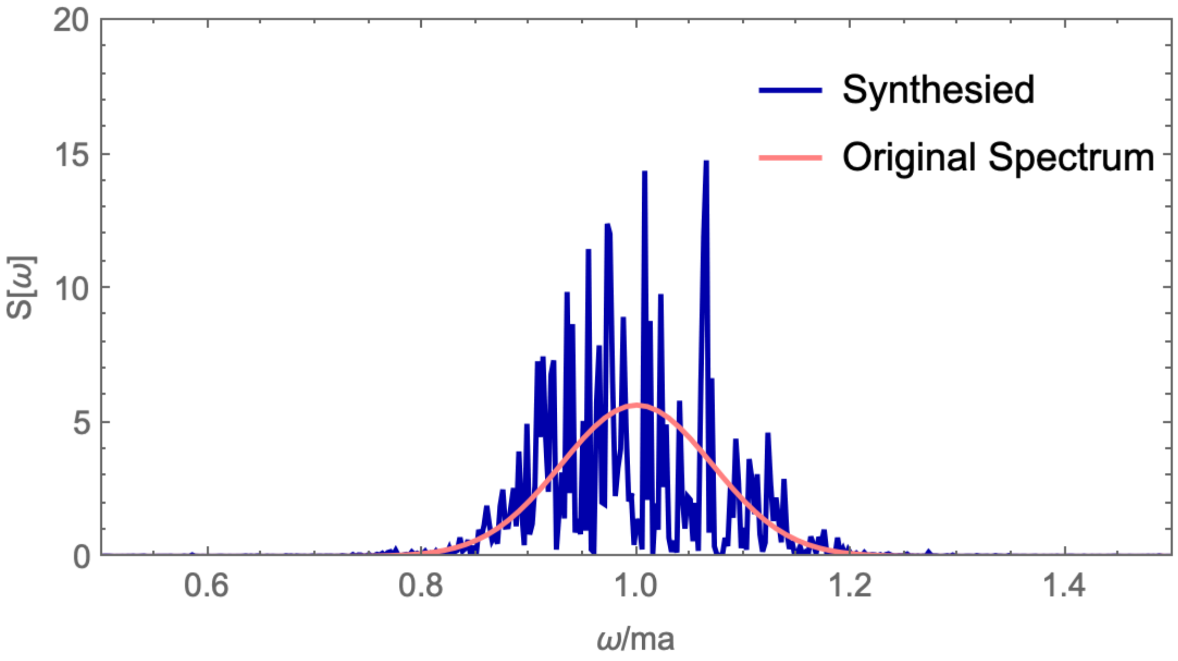
$t_{coh} \sim 1/(m_a v_0^2)$

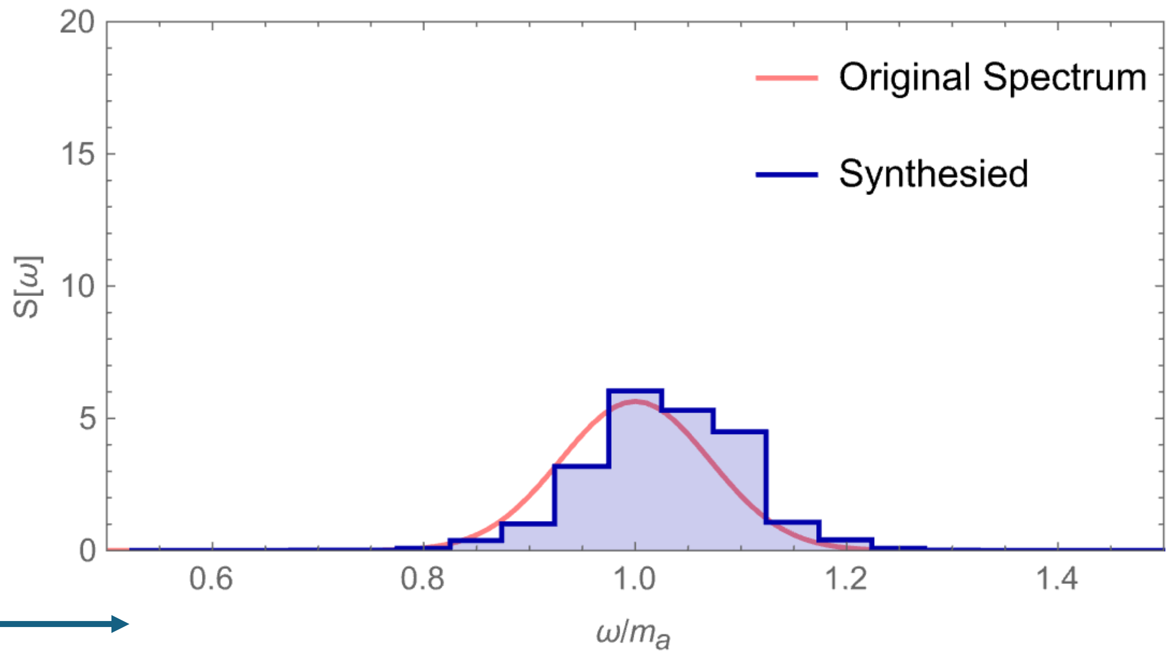
Synthesizing the Signal

$$V_{synth}(t) \equiv V_c(t) * W(t) = \int du\, V_c(t+u)W(u) = \sum_i V(t+u_i)W(u_i)$$ **(linear filter*)**



Double Check:
Do we Recover the PSD from this synthesized signal?





Re-bin

Back to sensitivity

$$\frac{df}{dt} \propto \frac{B^4\, V^2 \mathrm{C}^2}{T_{sys}} \min\left(\boxed{Q_c}, Q_a\right)$$

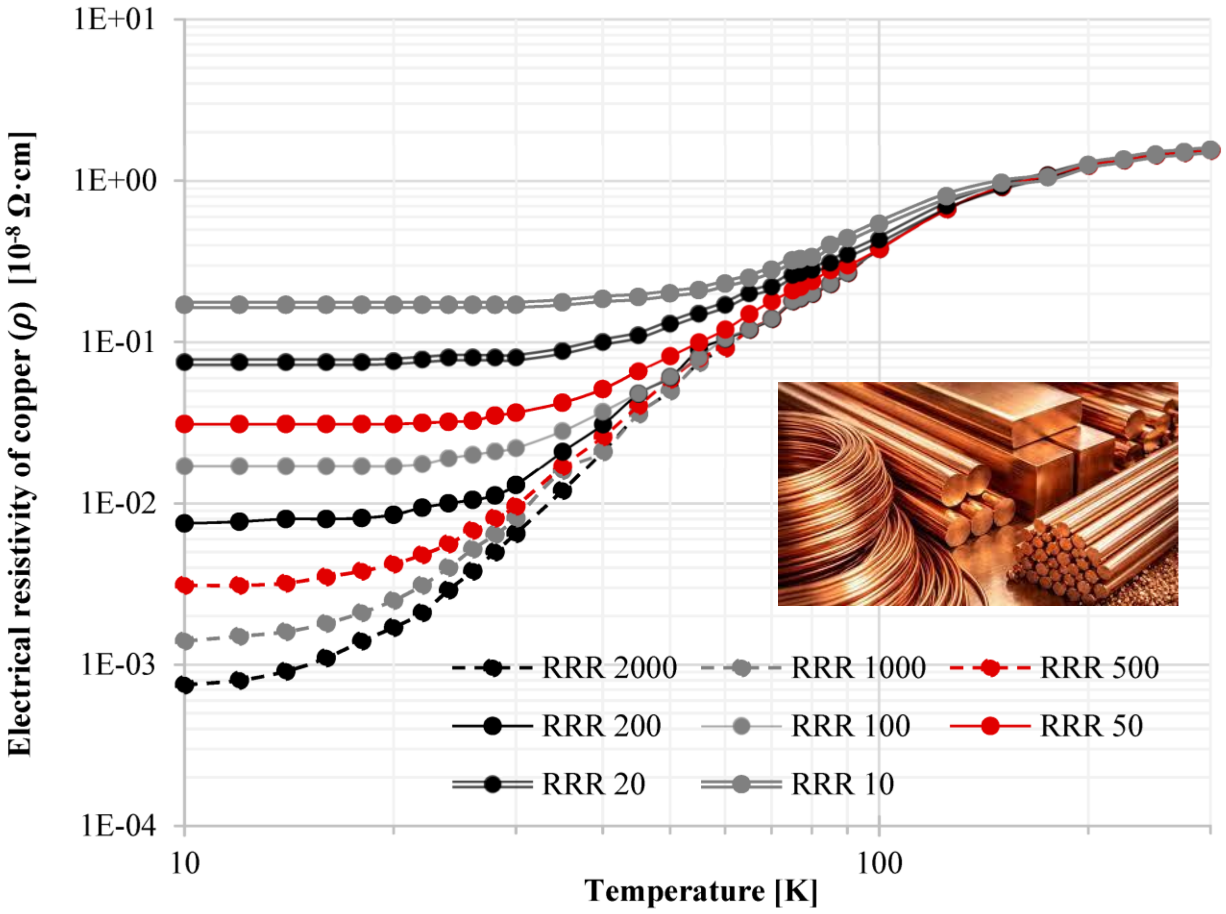
Theory of Losses (copper)

$$Q_c = \frac{2\omega \int_V \left(\frac{1}{2}\mu |H|^2 + \frac{1}{2}\epsilon |E|^2 \right) dV}{R_s \int_S |H_t|^2 \, dS}$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

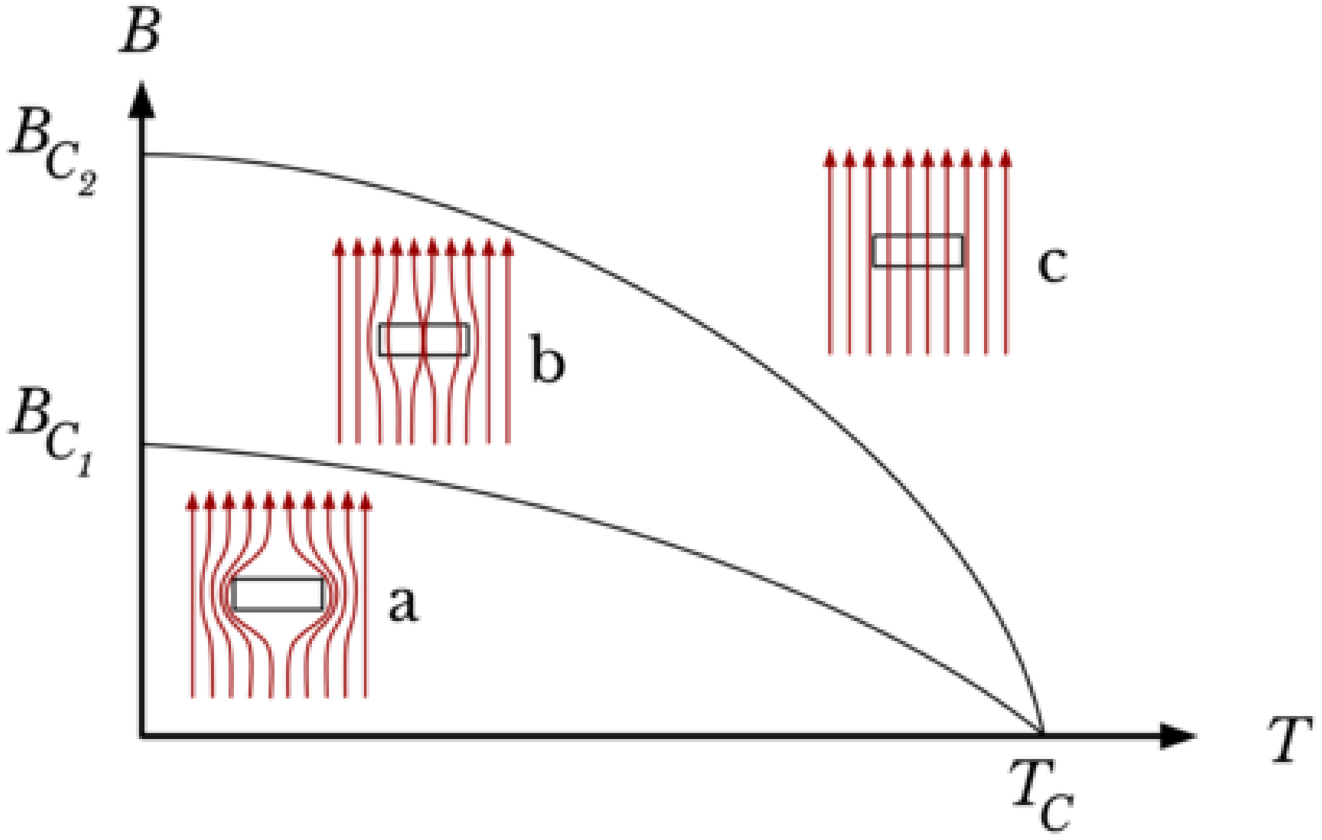
$$\text{RRR} = \frac{\sigma(T = 4K)}{\sigma(T = 300\ K)}$$

Copper



Theory of Losses (superconductors)

Type II Superconductors

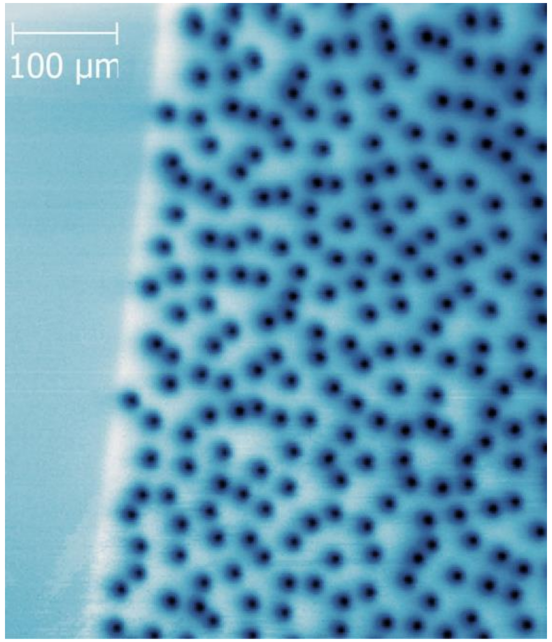


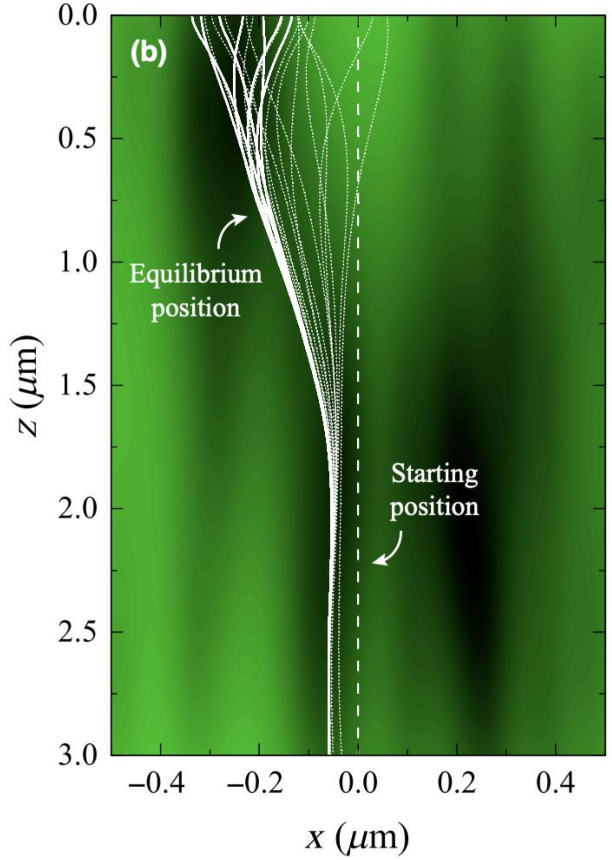
Theory of Losses (superconductors)

$$\eta_0 \dot{u}(t,z) = \varepsilon u''(t,z) - \kappa_{\rm p} u(t,z) + \gamma \cos(\omega t) e^{-z/\lambda}$$

$$R_{\rm fl} = 2\mu_0^2 \frac{N}{\Sigma} \frac{\langle P \rangle}{B_{\rm s}^2} = 2\mu_0^2 \frac{\langle P \rangle}{B_{\rm s}^2} \mathbf{B} \cdot \hat{\mathbf{n}}$$

$$P_{\rm c} = \frac{1}{2\mu_0^2} \int R_{\rm fl} B_{\rm s}^2 \, dA.$$





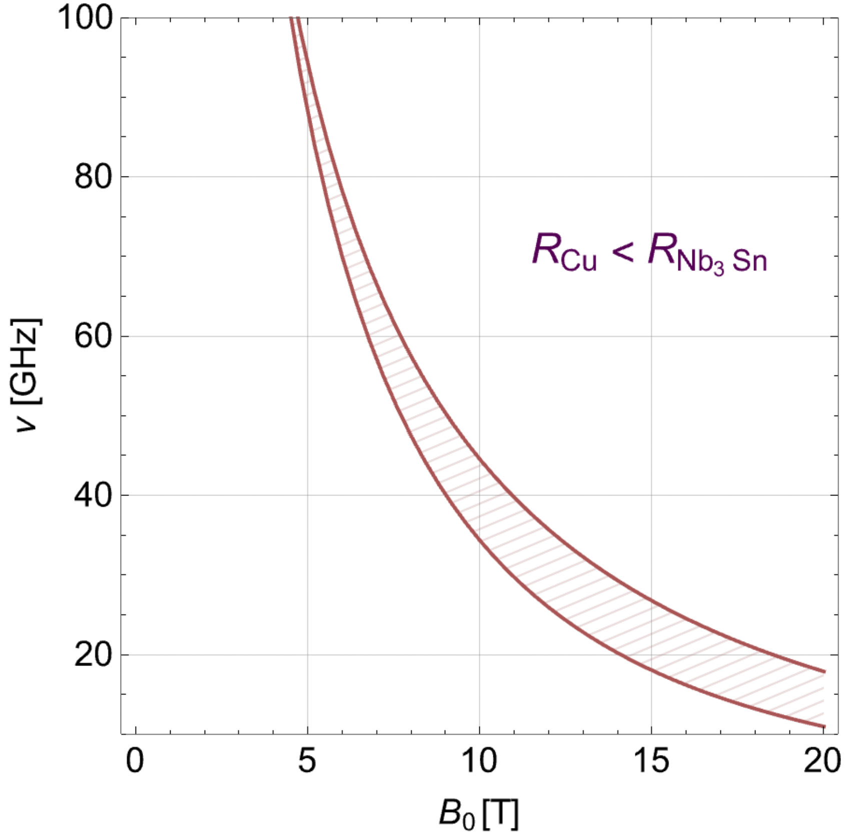
Mattia Checchin , A. Grassellino PRA 14, 044018 (2020)
Posen et al 2201.10733

Theory of Losses (superconductors)

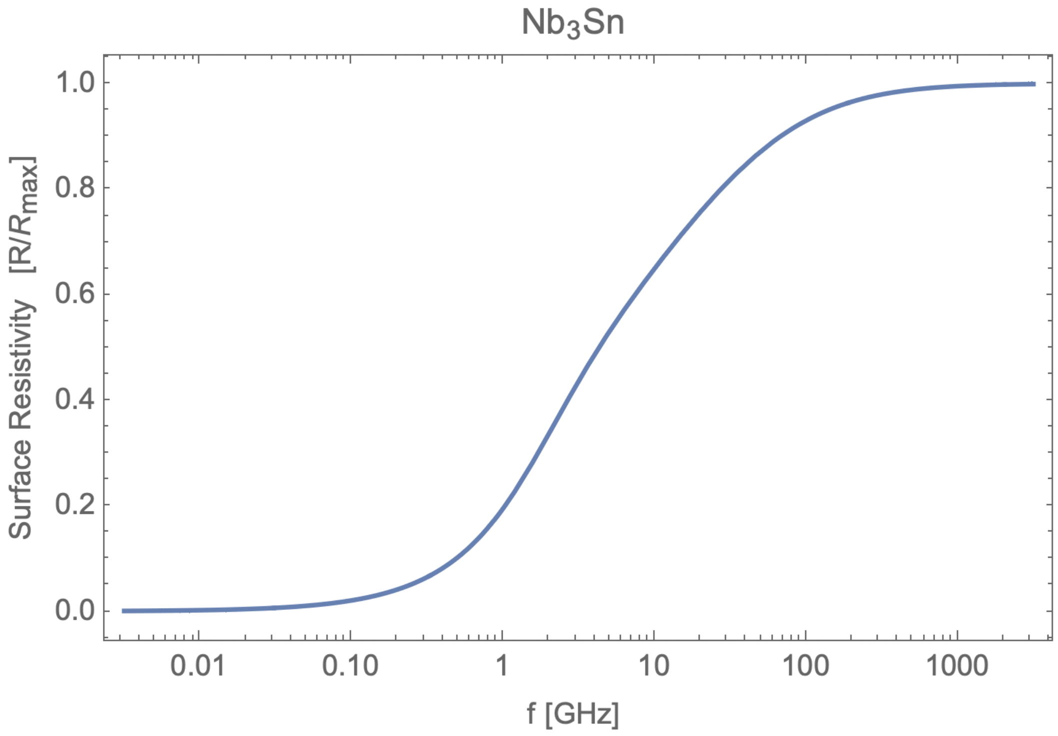
$$Q = \frac{\int dV \mid H|^2}{\int dS\, R\, |n\ \times H|^2}$$

Surface Resistivity: $R \propto B.n$

Type II Superconductors don't like B fields



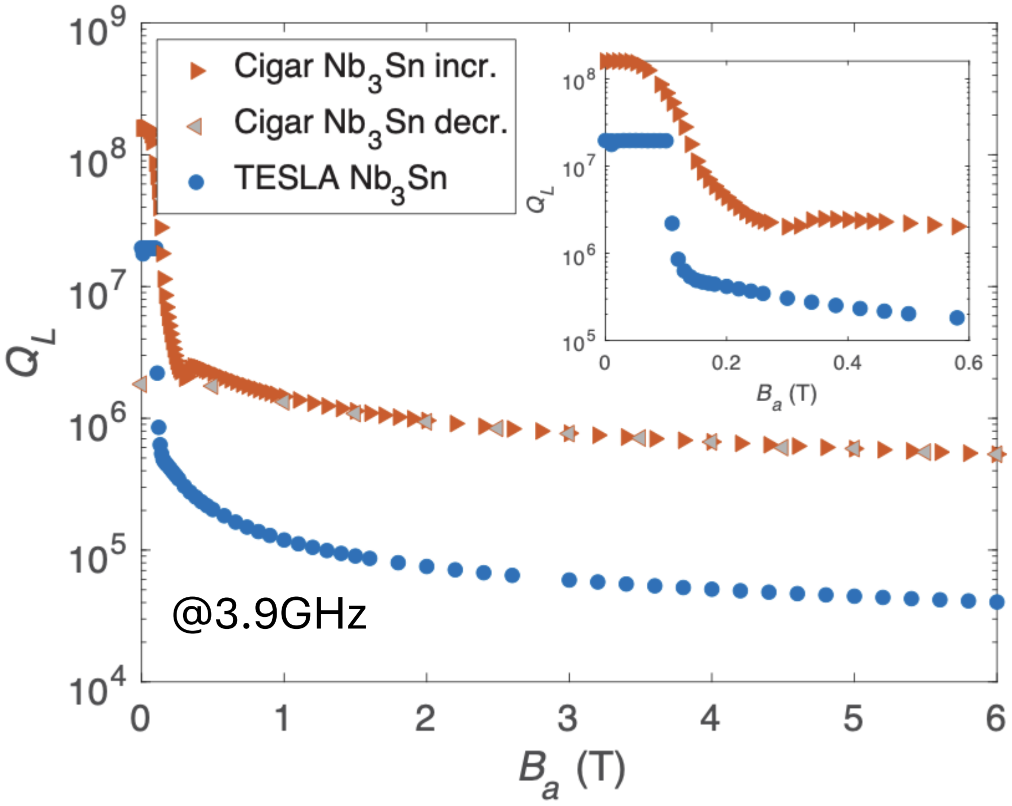
Surface Resistivity Gets Worse at High Frequency



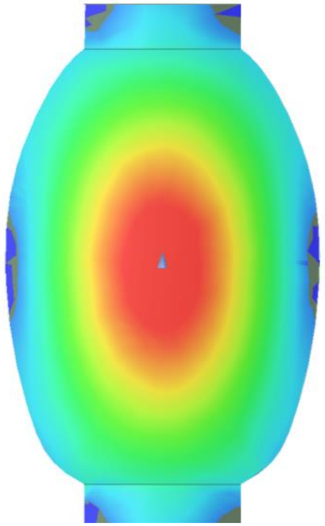
See https://arxiv.org/pdf/2201.10733

Theory of Losses (superconductors)

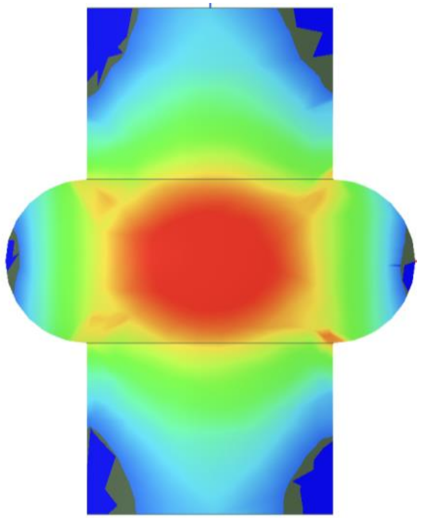
Surface Resistivity: $R \propto B.n$



Cigar



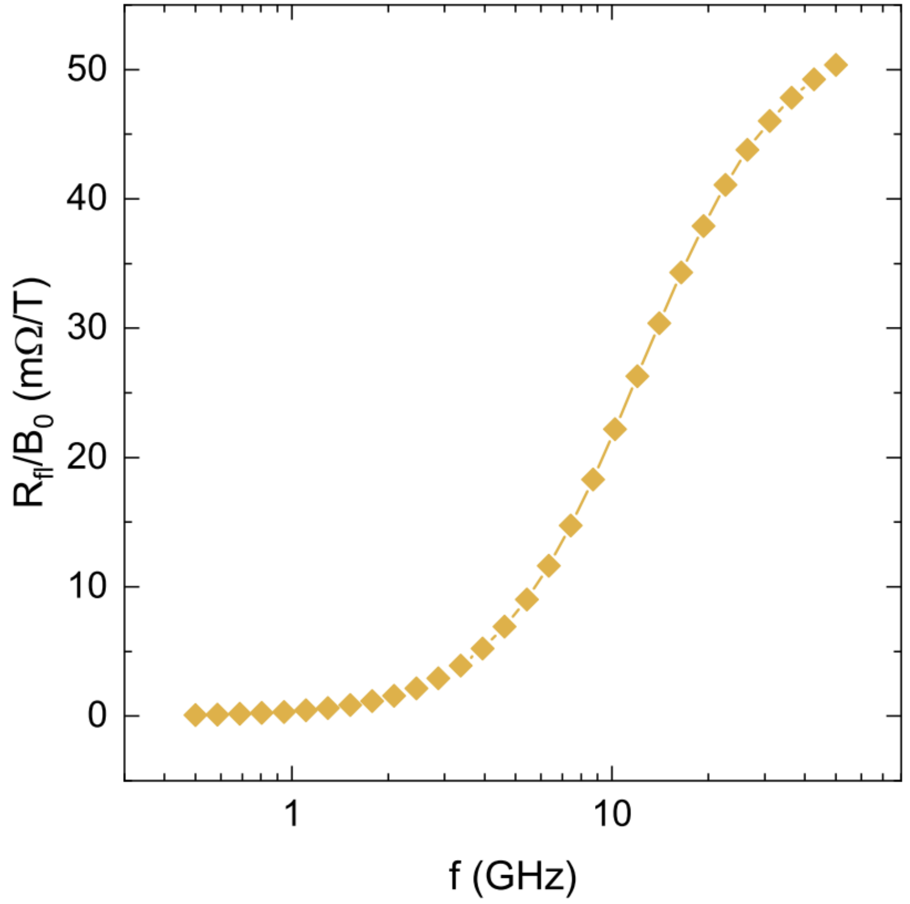
Tesla

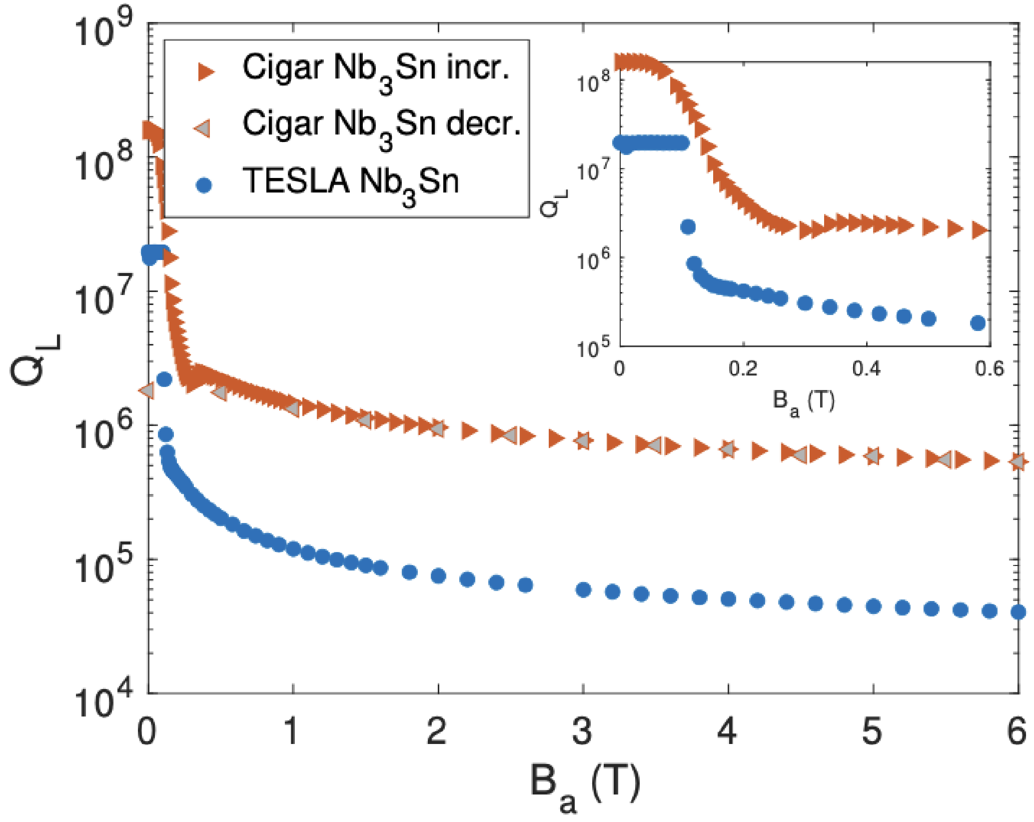


(not clear how to tune)

Posen et al (Fermilab group)

Theory of Losses (superconductors)





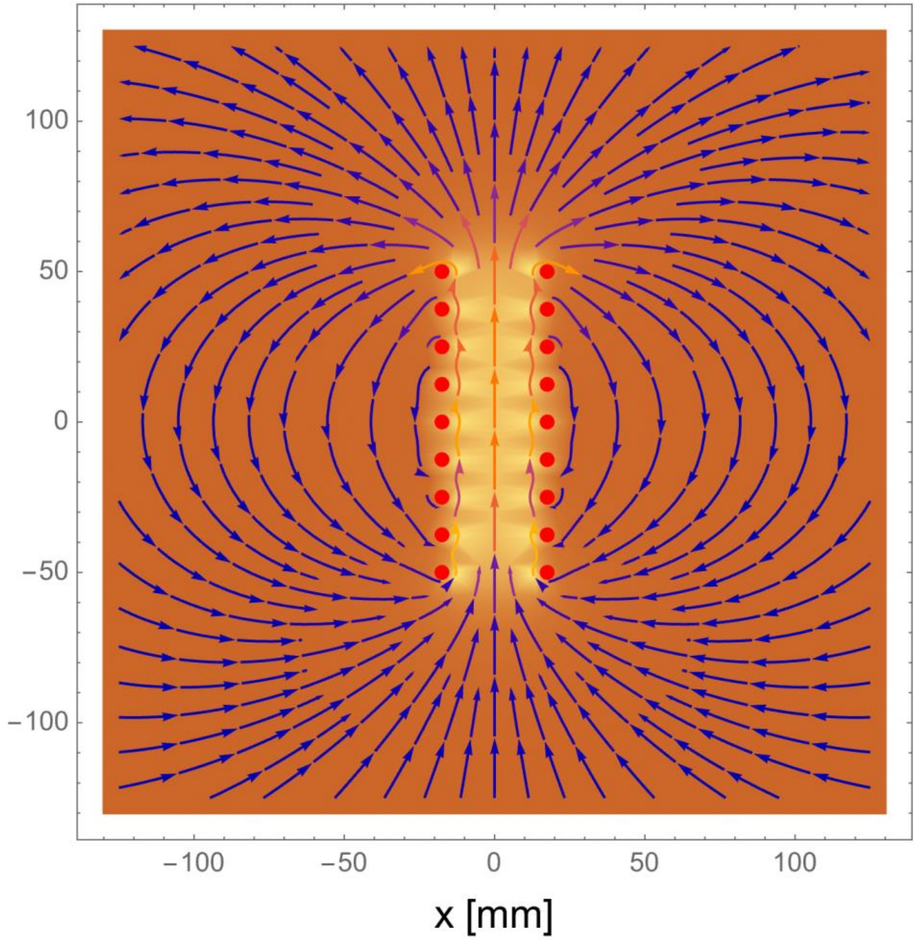
Posen et al 2201.10733

$$\eta_0 \dot{u}(t,z) = \varepsilon u''(t,z) - \kappa_{\mathrm{p}} u(t,z) + \gamma \cos(\omega t) e^{-z/\lambda}$$

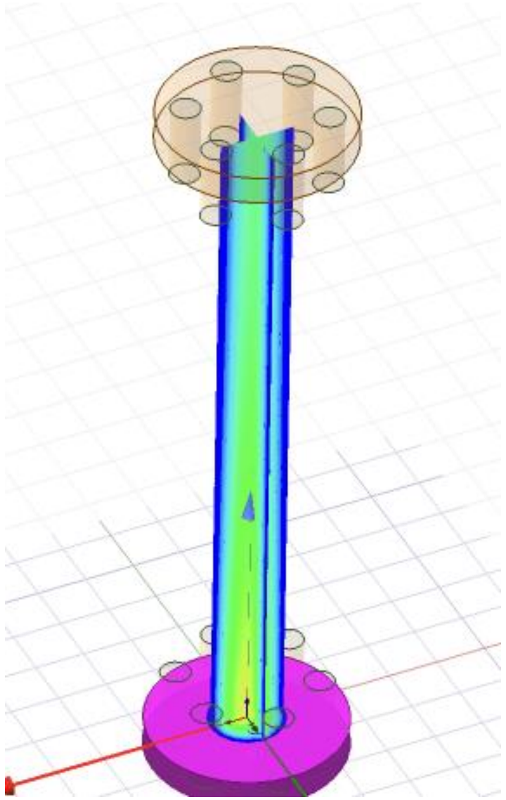
Back to sensitivity

$$\frac{df}{dt} \propto \frac{B^4\, V^2 \boxed{\mathrm{C}^2}}{T_{sys}} \min(Q_c, Q_a)$$

Modelling the C Factor



Magnetic Solenoid



Electric Field

$$C = \frac{\left| \int_V \mathbf{E}_c(\mathbf{x}) \cdot \mathbf{B}_0(\mathbf{x}) \, dV \right|^2}{B_0^2 V \int_V \epsilon(\mathbf{x}) |\mathbf{E}_c(\mathbf{x})|^2 \, dV}$$

Conclusions

- Axion theory predicts a broad range of masses should look everywhere.

- Post-inflationary scenario (e.g., strings) motivates higher axion mass searches.

- Lots of maths underlying the understanding of axion experiments and how to increase their sensitvity

Backup: Axion mechanism

$$\mathscr{L}_{\text{axion}} = \frac{1}{2}\,\partial_\mu a \partial^\mu a + \left(\theta + \frac{a(x)}{f_a}\right) \frac{g_s^2}{32\pi^2}\, F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}$$