Cosmology without sphaleron

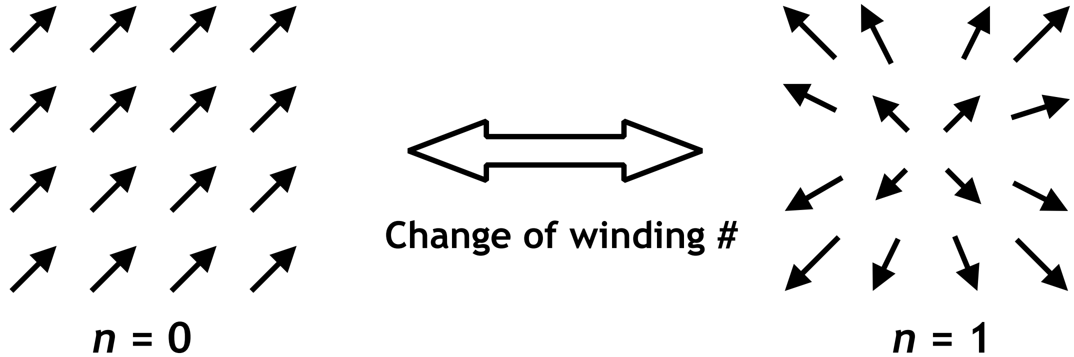
Nodoka Yamanaka
(Nishina Center, Riken)

Based on
N. Yamanaka, arXiv:2212.10994 [hep-th]
N. Yamanaka, arXiv:2212.11820 [hep-ph]
T. Tanii and N. Yamanaka, https://zenodo.org/records/20805793
See also https://www2.yukawa.kyoto-u.ac.jp/~nodoka.yamanaka/topologicalcharge/

2026/08/06
U. Tokyo

Topological charge

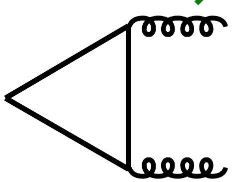
Non-abelian gauge theory has non-trivial topological gauge configurations



(3-dimension in reality)

Can be probed with the topological charge

$$\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}^{\mu\nu}_a = \int d^4x\, \partial_\mu K^\mu$$



$$= \frac{ig_s\alpha_s}{24\pi}\int d^3\vec{x}\, f_{abc}\epsilon_{ijk}A_{ia}(\vec{x})A_{jb}(\vec{x})A_{kc}(\vec{x})\Big|_{t=-\infty}^{t=+\infty}$$

$$= \Delta n$$ **Integer!**

$$\boxed{\begin{aligned}\tilde{F}^{\mu\nu}_a &\equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma}\\ K_\mu &\equiv \frac{\alpha_s}{8\pi}\epsilon_{\mu\nu\rho\sigma}\left[A^\nu_a F^{\rho\sigma}_a - \frac{g_s}{3}f_{abc}A^\nu_a A^\rho_b A^\sigma_c\right]\end{aligned}}$$

⇒ Δn = change of winding number of gauge configurations

Topological charge

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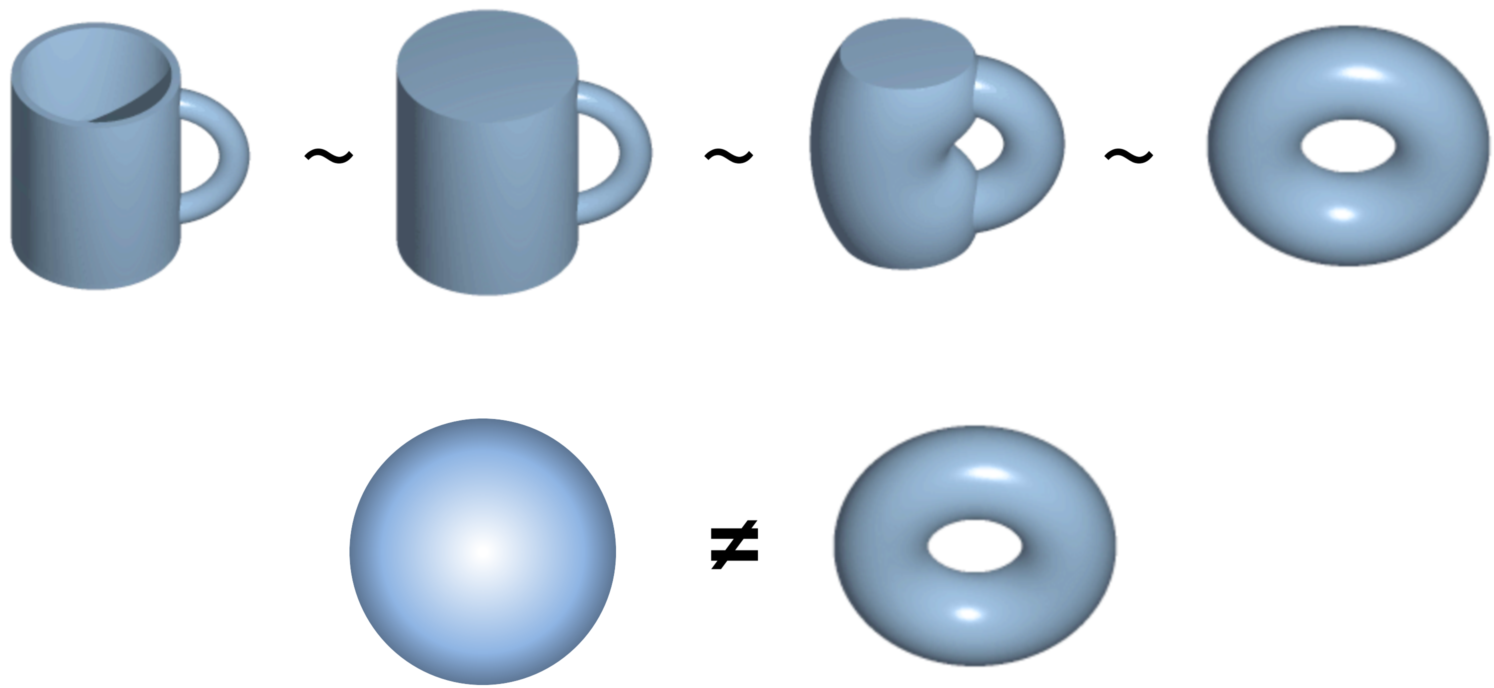
Topology:



Car

$$K_\mu \equiv \frac{\alpha_s}{8\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu F_a^{\rho\sigma} - \frac{g_s}{3} f_{abc} A_a^\nu A_b^\rho A_c^\sigma\right]$$

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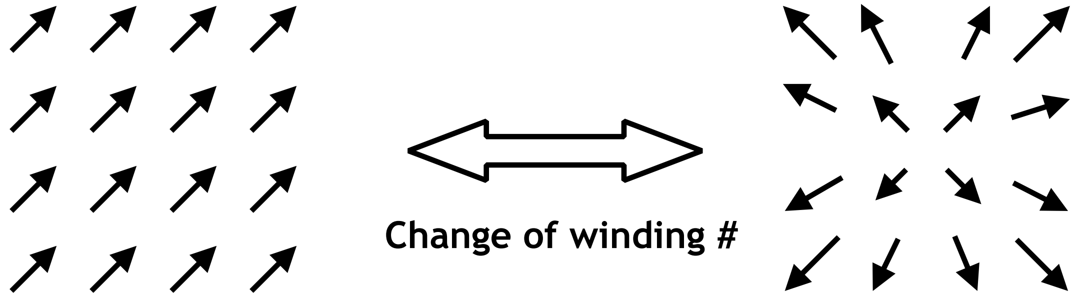
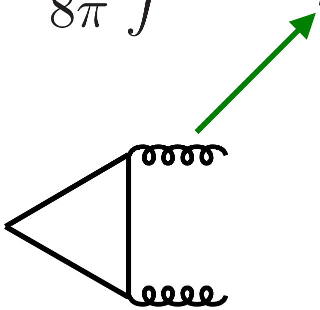


n = 0 n = 1

(3-dimension in reality)

Can be probed with the topological charge



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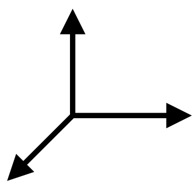
Unphysical topological charge

Topological charge is a triple product of gauge fields

$$\frac{i g_s \alpha_s}{24\pi} \int d^3\vec{x}\, f_{abc}\epsilon_{ijk} A_{ia}(\vec{x}) A_{jb}(\vec{x}) A_{kc}(\vec{x}) \Big|_{t=-\infty}^{t=+\infty}$$

Time is frozen, restricted to 3-dimensional space

ε_{ijk} means  ⇒ Covers all 3-dimensional directions!

⇒ Also covers the unphysical gauge freedom direction!

$$(\vec{A}_a \to \vec{A}_a + \vec{\nabla}\chi_a)$$

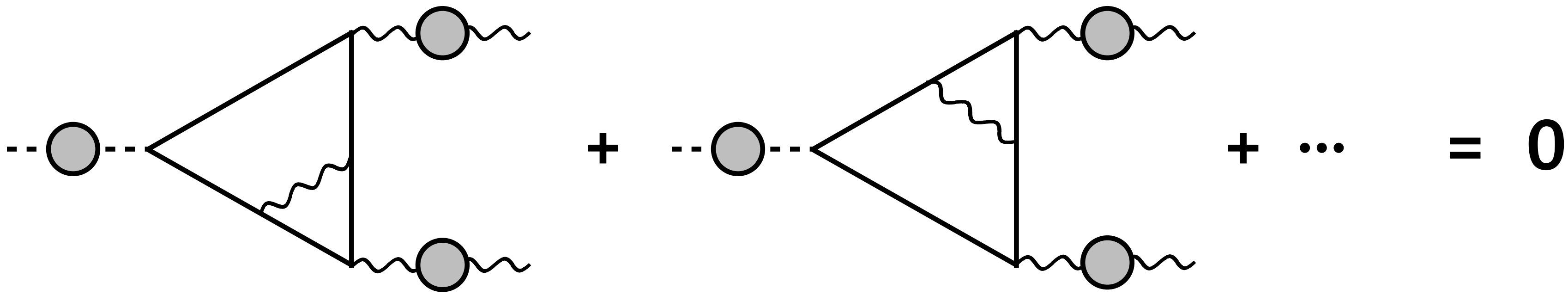
Topological charge is unphysical !!

This only holds in perturbation theory … Is it really OK ??

NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

Adler-Bardeen theorem

Radiative corrections do not contribute to chiral anomaly
(up to renormalization of external fields)



S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969);
D. Anselmi, Phys. Rev. D **91**, 105016 (2015).

Chiral anomaly (= topological charge) does not change at any scale, even if external fields are “dressed” by nonperturbative effects

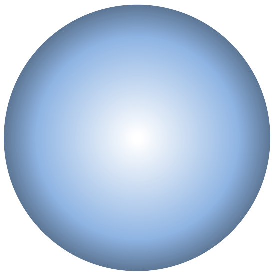
⇒ Topological charge is strictly perturbation finite!

⇒ Unphysical longitudinal gauge component justified (BRST symmetry)

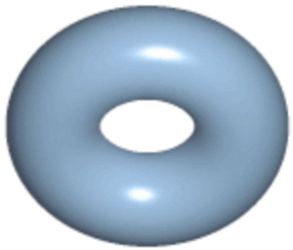
For detail, see NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph]

Why no radiative corrections?

Topological charge is an integer #



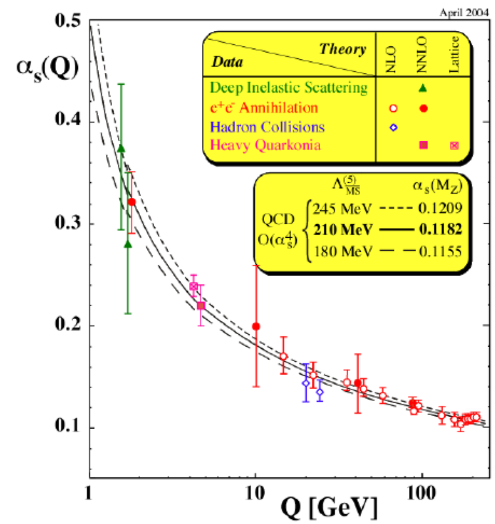
n=0



n=1

# of holes is an integer !

Radiative corrections are real #



Running is continuous

Topological charge/chiral anomaly do not have radiative corrections !

Definition of "unphysical"

"Unphysical" = "Probability is zero"

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0|F\tilde{F}(x)|\Omega\rangle\langle\Omega|F\tilde{F}(y)|0\rangle = 0$$

while matrix element (amplitude) $\langle 0|F\tilde{F}(x)|\Omega\rangle \neq 0$

Sum of absolute values cancels due to indefinite metric (= BRST symmetry)

Can be derived from Ward-Takashi identity of BRST symmetry

For arbitrary Green's functions, topological charge can be factorized

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0|\phi(x')|0\rangle\langle 0|F\tilde{F}(x)|\Omega\rangle\langle\Omega|F\tilde{F}(y)|0\rangle\langle 0|\phi(y')|0\rangle = 0$$

ϕ : arbitrary scalar BRST singlet function

Composite operator of $F\tilde{F}$ does not include $F\tilde{F}$ (Adler-Bardeen)

➡ Topological charge cannot appear in observables !

For detail, see NY, arXiv:2212.11820 [hep-ph]

Consequences of unphysical topological charge

- Resolution of Strong CP problem

  Axions not needed

- U(1) symmetries (axial, B+L) are not broken by anomaly

  Sphaleron does not happen NY, arXiv:2212.10994 [hep-th], 2212.11820 [hep-ph].

  η' is NG boson, but no issues of U(1) problem NY, arXiv:2411.02792 [hep-ph].

- Magnetic monopoles are unphysical

  GUT transition after inflation T. Tanii and NY (2024), https://zenodo.org/records/20805793

- SUSY gauge theory is conformal

  No SUSY GUT NY (2026), https://zenodo.org/records/20914734

Chiral Ward-Takahashi identity

$$\partial_\mu J_5^\mu \equiv \partial_\mu \sum_\psi^{N_f} \bar{\psi}\gamma^\mu\gamma_5\psi = -2\sum_\psi^{N_f}\left[m_\psi \bar{\psi} i\gamma_5\psi\right] - \underline{\frac{N_f\alpha_s}{8\pi}F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}}$$

Integrate : LHS → axial/B+L U(1) charge, RHS → unphysical topological charge

⇒ Conservation of $U(1)_{A,B+L}$!

(up to current fermion mass m_ψ)

What is the unphysical part of LHS (fermion) ?

Atiyah-Singer's theorem:

Number of chiral Dirac zero-modes = topological charge

$$\mathrm{ind}(\not{D}) = -\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}$$

Dirac zero-mode : solution of $\not{D}\psi = \lambda\psi$

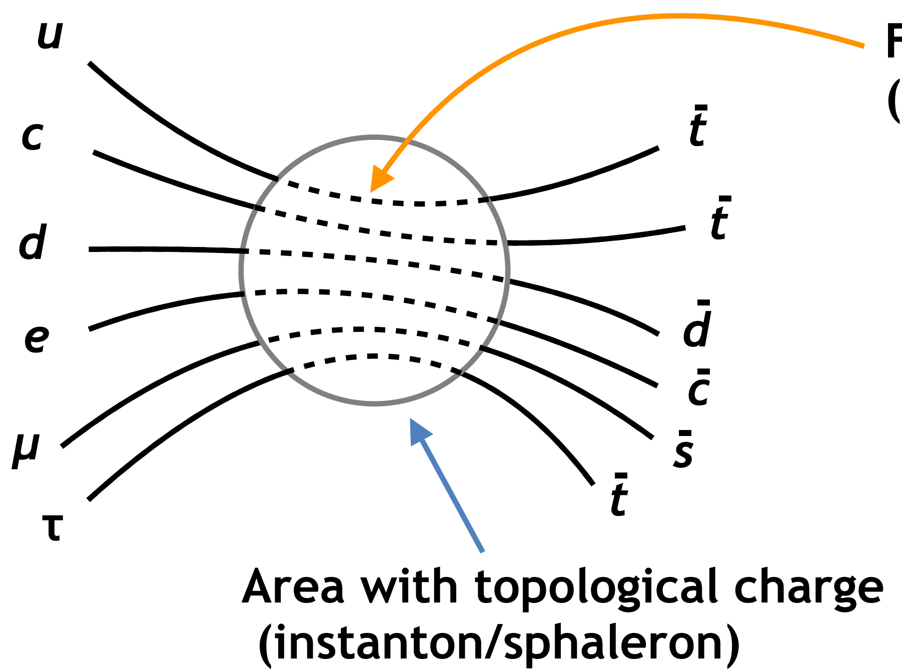
Since the topological charge is unphysical,

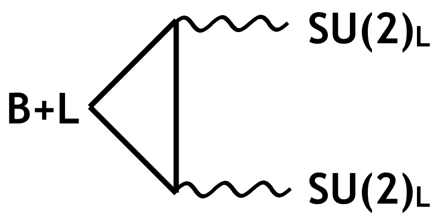
fermion chiral Dirac zero-modes are also unphysical !

NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph]

Sphaleron induced baryogenesis

’t Hooft vertex : multi-fermion vertex generated by chiral Dirac zero-modes





G. ’t Hooft, PRL **37**, 8 (1976), PRD **14**, 3432 (1976).

(Nonlocal) B+L violating 12-fermion interaction

⇒ Baryon and lepton numbers may be generated with weak interaction!

V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985), 36;
M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

Sphaleron induced baryogenesis

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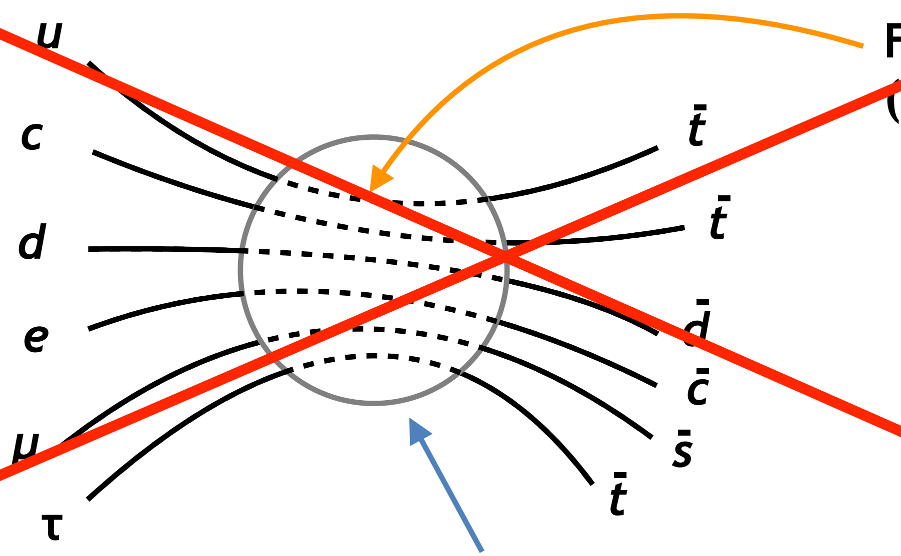
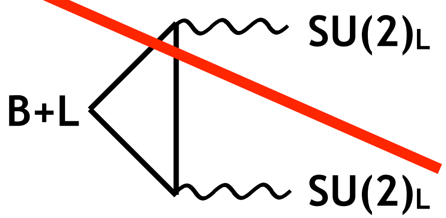




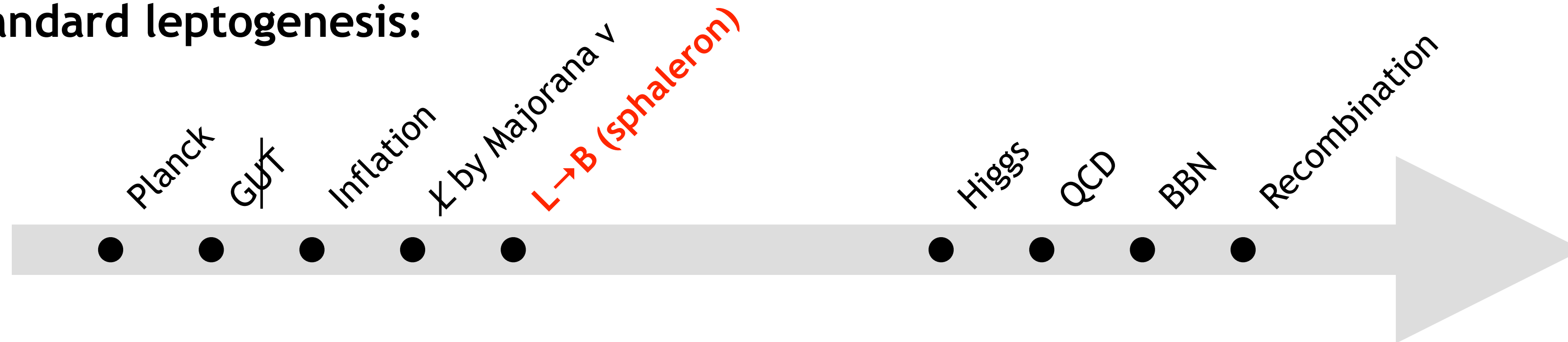
Atiyah-Singer theorem : topological charge = # of chiral Dirac zero-modes

⇒ 't Hooft vertex is unphysical !

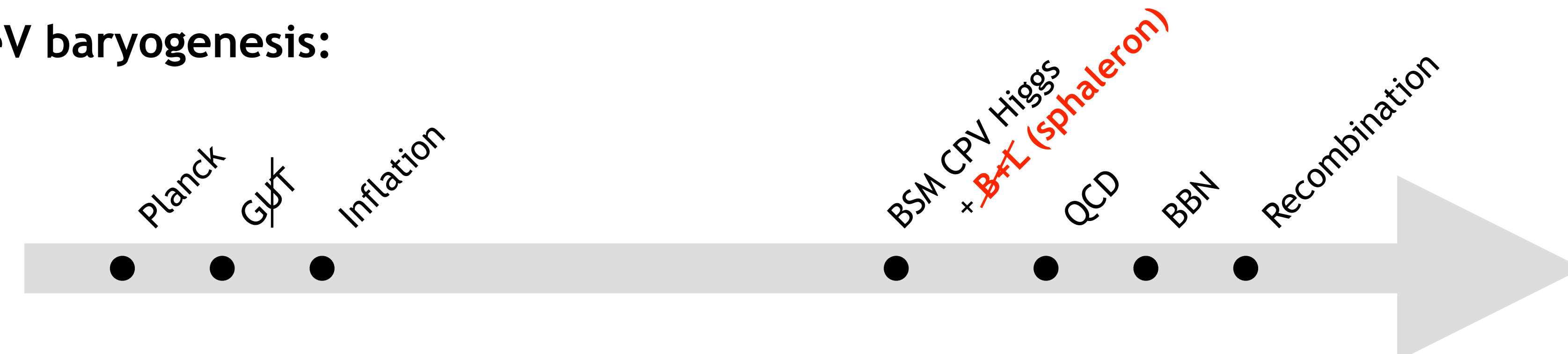
since 't Hooft vertex is generated by the propagation of zero-modes

Summary of baryogenesis scenarios

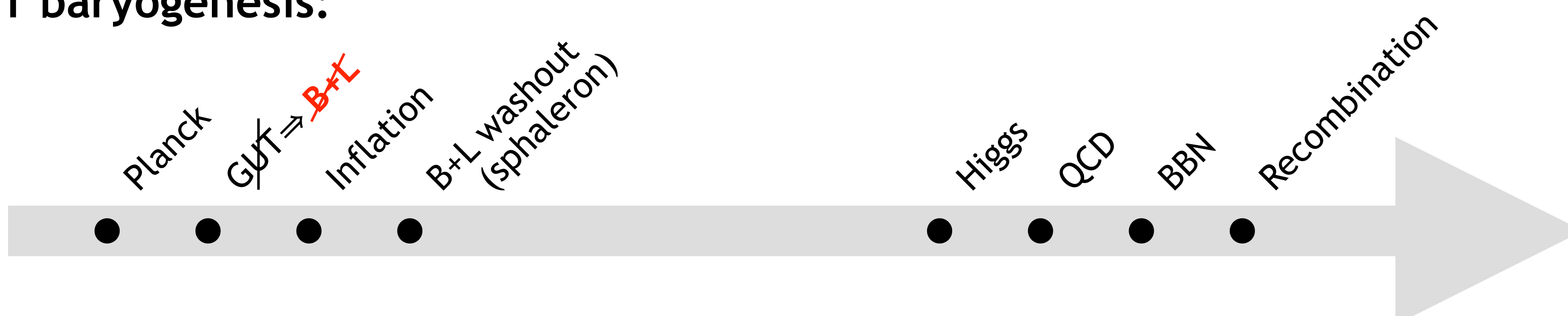
Standard leptogenesis:



TeV baryogenesis:



GUT baryogenesis:

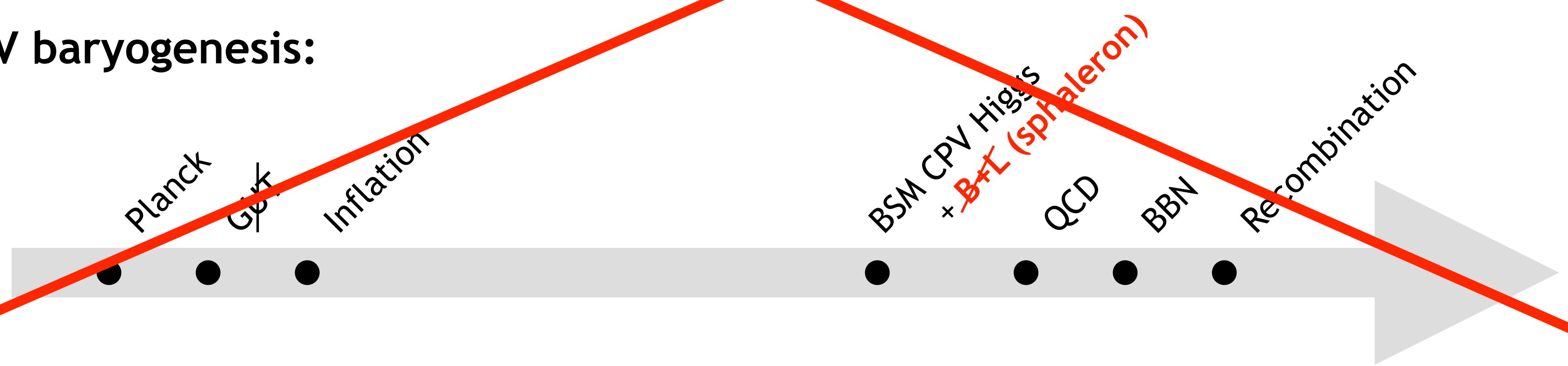


Summary of baryogenesis scenarios

Standard leptogenesis:

Since sphaleron process does not occur, these scenarios are impossible

TeV baryogenesis:



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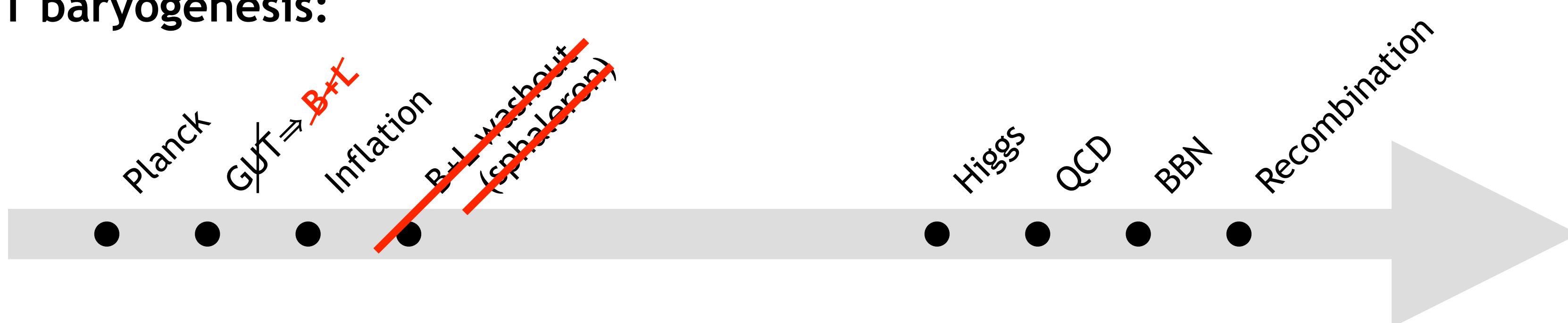

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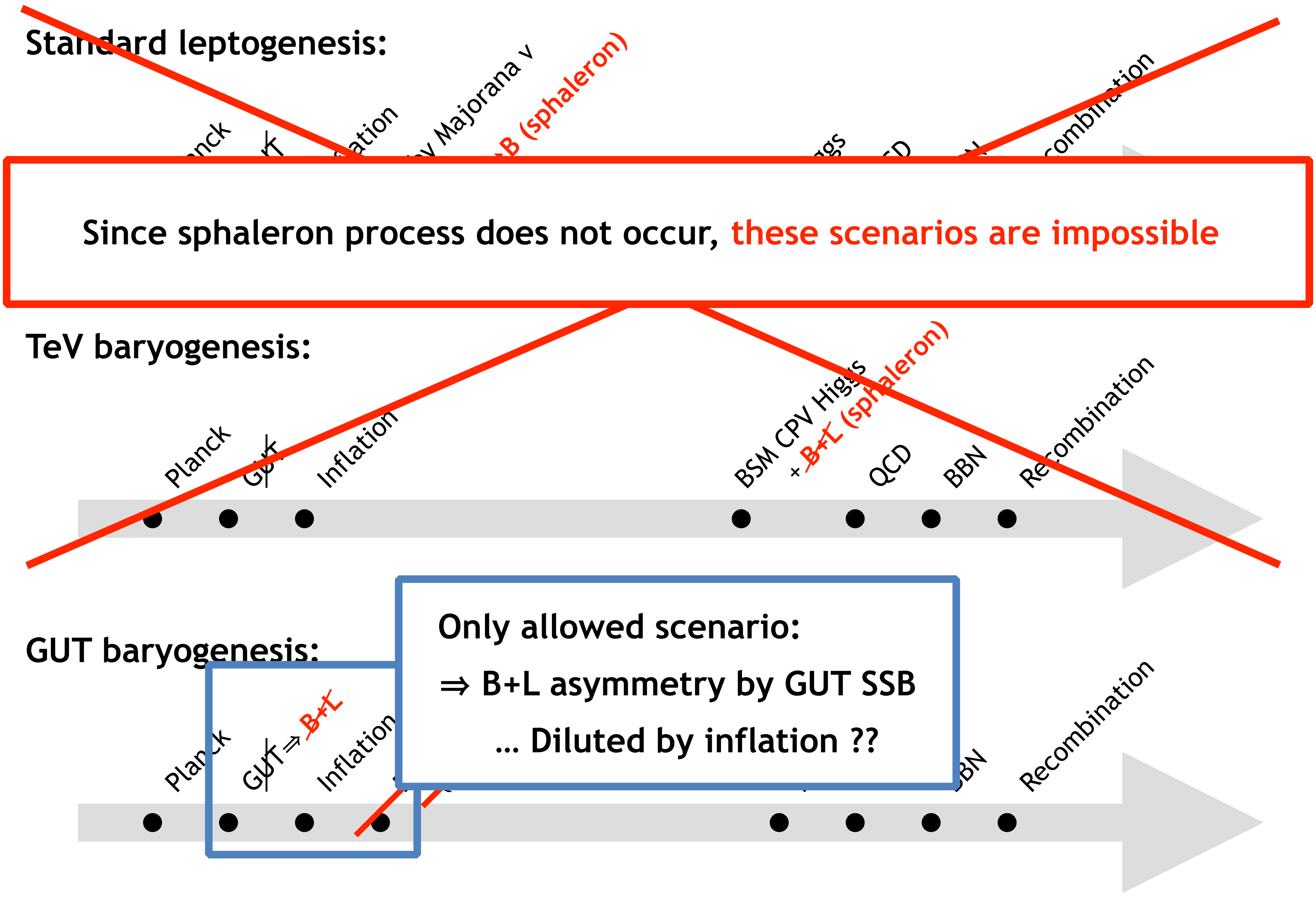

Magnetic monopole and behavior at long distance

SSB of gauged scalar field theory where U(1) remains (e.g. adjoint Higgs, SU(2) → U(1))

$$\mathcal{L} = D_\mu \phi_a D^\mu \phi_a^\dagger - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{\lambda}{4}\left(|\phi_a|^2 - v^2\right)^2$$

⇒ **VEV is like** $\langle\phi\rangle \propto \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; (= \tau_3)$ ⇒ **Commute with** A_3^μ

⇒ **Massless U(1) gauge remains**

G. 't Hooft, Nucl. Phys. B **79**, 276 (1974);
A. M. Polyakov, JETP Lett. **20**, 194 (1974).

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Energy of (static) magnetic monopoles :

$$E = \int d^3x \left[|\dot{\phi}_a|^2 + \frac{1}{2}|\dot{A}_a|^2 + D_i \phi_a D_i \phi_a^\dagger + \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 \right]$$

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ϕ **may have topological defects, but energy diverges?**

Angular mode: $\vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{\underline{r}} \frac{\partial}{\partial \theta} + \vec{e}_\varphi \frac{1}{\underline{r} \sin\theta} \frac{\partial}{\partial \varphi}$ ⇒ **No E density damp at** $r \to \infty$

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⇒ Cancellation between derivative and gauge in $D_i \phi_a = (\nabla_i + igA_{bi}T_b)\phi_a$

⇒ $\nabla_i \varphi \sim gA_i \varphi$ at large r !

G. 't Hooft, Nucl. Phys. B **79**, 276 (1974);
A. M. Polyakov, JETP Lett. **20**, 194 (1974).

Magnetic monopoles are unphysical

Typical GUT SSB : SU(5) → SU(3) x SU(2) x U(1)

U(1) appears ⇒ magnetic monopoles at GUT transition

⇒ Too many monopoles at GUT, Universe would collapse : “monopole problem”

If inflation happens, dilute monopole density

⇒ Conventional resolution of monopole problem

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Our resolution :

Magnetic monopole = $-\frac{1}{2}\int_V \epsilon_{ijk}\partial_i F_{jk} dV$

$$= -\frac{1}{2gv^3}\int_V \epsilon^{ijk} f^{abc}(\partial_i\phi^a)(\partial_j\phi^b)(\partial_k\phi^c)dV \quad \ni \underline{\epsilon_{ijk}A_i^d A_j^e A_k^f}\Big|_{r\to\infty}$$

(Due to scalar-gauge cancellation of |Dφ|)

⇒ Triple product again !

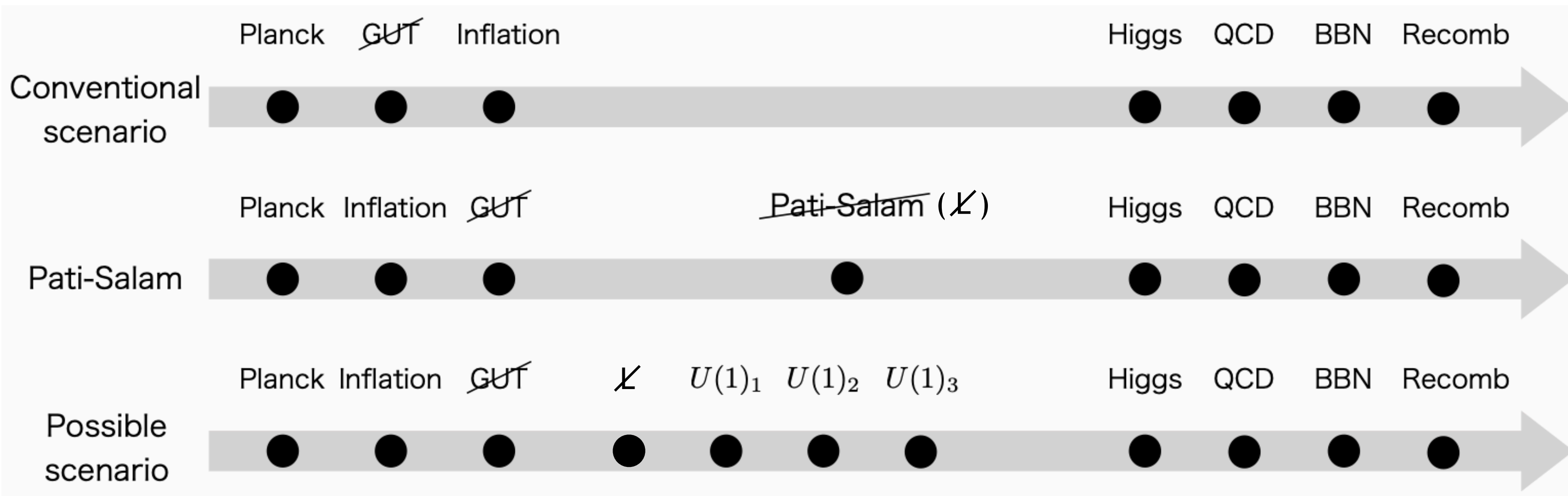
⇒ Unphysical ! No monopole problem !

T. Tanii and NY (2024), https://zenodo.org/records/20805793

Possible cosmological scenarios without monopoles

Irrelevance of magnetic monopoles opens up new scenarios

Transitions generating U(1) are allowed (e.g. Pati-Salam)



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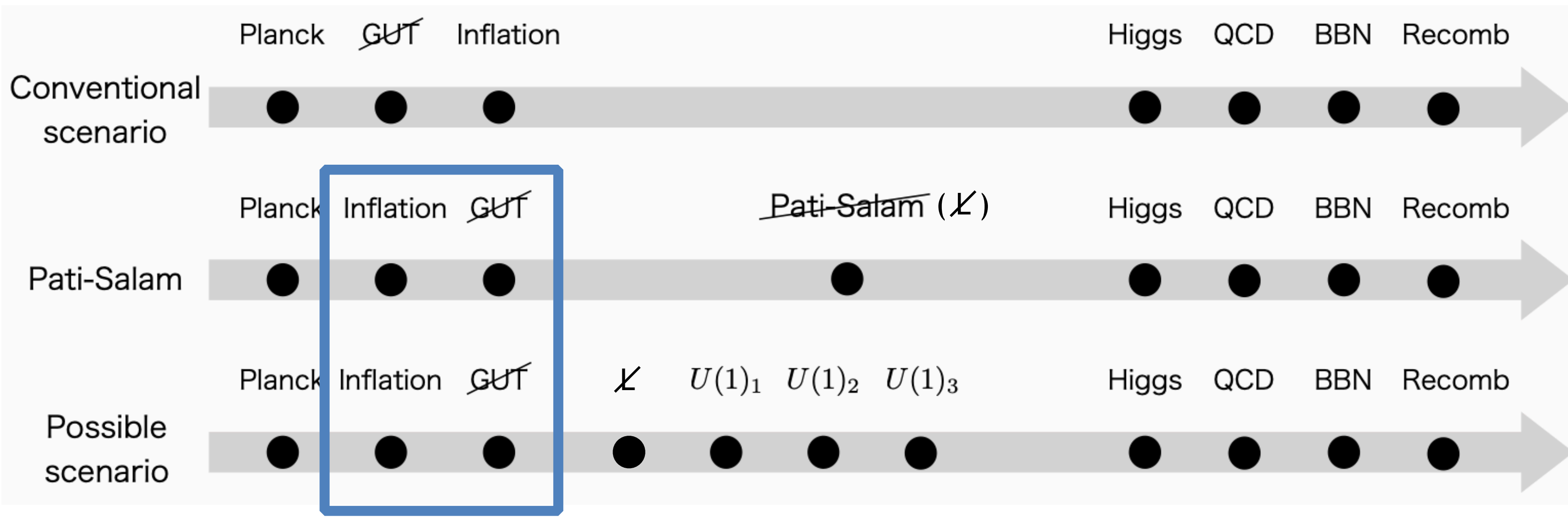

Inflation before GUT !

T. Tanii and NY (2024), https://zenodo.org/records/20805793

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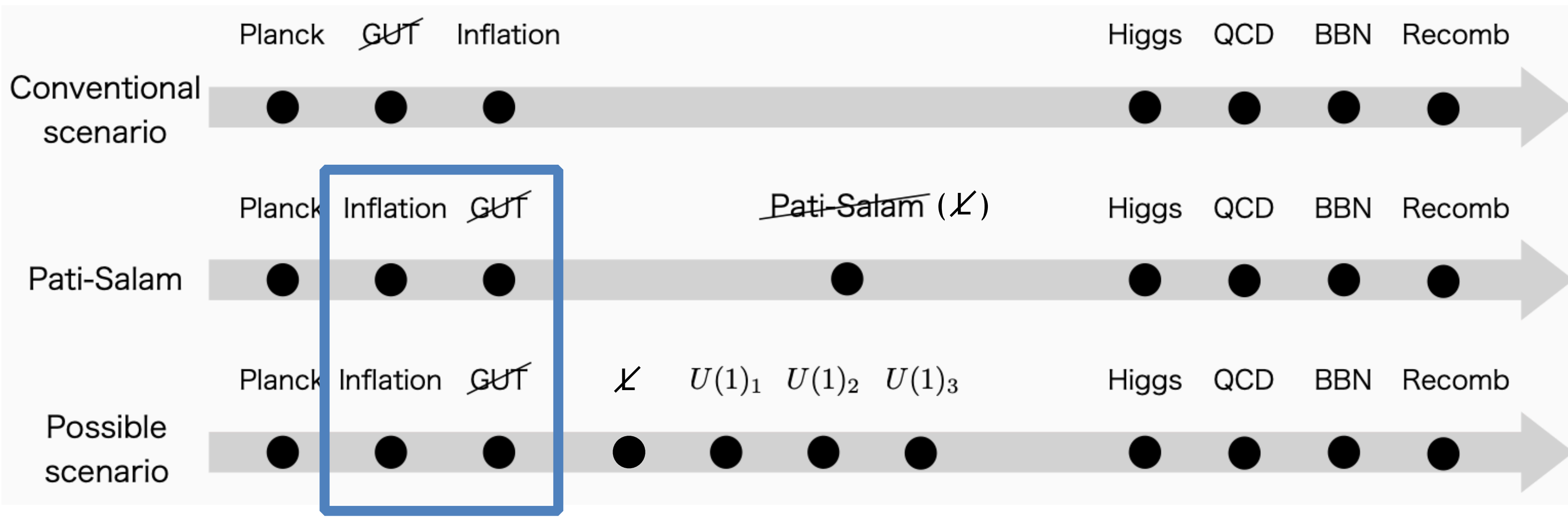

Inflation before GUT !

GUT baryogenesis OK, multi-step unification,… and even no-inflation?

Majorana neutrino and large $\not{L}$ originate from Pati-Salam?

Generate U(1) symmetries associated to flavor?

T. Tanii and NY (2024), https://zenodo.org/records/20805793

Lepton number asymmetry as a probe of our statement??

We saw that sphaleron induced B+L violation is unphysical

⇒ No wash-out

Baryon number of Universe is known : $B / s = 10^{-10}$

What about lepton number ?

➡ $L / s \sim 10^{-5}$ (from recent analysis of metal poor galaxies)

A. Matsumoto et al., Astrophys. J. **941** (2022) 167

$L >> B$??

Wash-out would yield $B \sim L$, very unlikely to have had sphaleron process

⇒ Support our statement!

(There are other scenarios using "L-balls" to avoid wash-out, admitting sphaleron process

Kawasaki and Murai, JCAP 08 (2022) 041)

Summary

- My claim : topological charge is unphysical, chiral anomaly is unphysical, sphaleron does not occur.
- Sphaleron induced baryogenesis is excluded: need GUT to generate baryon number asymmetry.
- For not diluting B+L asymmetry generated by GUT, inflation must occur before GUT transition.
- Magnetic monopoles are also unobservable : no problem with inflation before GUT transition.
- Many scenarios are possible, including Pati-Salam, BSM U(1) associated to flavor,…

Homeworks:

- Confirm large lepton number asymmetry.
- Road to incorporating gauged flavors ?

Backup slides

Why is SUSY conformal ?

```
                         [ROOT PREMISE]
               The topological charge contains an unphysical
               longitudinal gauge component (BRST symmetry).
                                 |
                                 ▼
              Topological charge is strictly UNOBSERVABLE.
                                 |
            ┌────────────────────┴────────────────────┐
            ▼                                         ▼
    [BRANCH 1: INSTANTONS]                     [BRANCH 2: ANOMALIES]
 Instantons possess a topological        In N=1 SUSY, the Trace Anomaly
 charge, making them indistinguishable   and Axial R-Anomaly belong to
 from a trivial vacuum.                  the exact same supermultiplet.
            |                                         |
            ▼                                         ▼
 Non-perturbative effects                Since topological charge is
 (multi-fermion 't Hooft vertex,         unobservable, the Axial R-Anomaly
 gaugino condensates) are irrelevant     is irrelevant in observable space.
 at the level of observables.                         |
            |                                         ▼
            |                            Because they mathematically mix
            |                            and cannot both be observed,
            |                            the Trace Anomaly must also
            |                            be UNPHYSICAL.
            |                                         |
            |                                         ▼
            |                              (Resolves the Anomaly Puzzle)
            |                                         |
            └────────────────────┬────────────────────┘
                                 ▼
                         [CORE CONCLUSION]
             Since neither non-perturbative instantons nor
             the perturbative trace anomaly can physically
             break scale invariance...
                                 |
                                 ▼
                4D N=1 & N=2 SUSY gauge theories are
                        EXACTLY CONFORMAL
               (assuming no explicit mass parameters)
                                 |
```

NY (2026),
https://zenodo.org/records/20914734

(Summarized by Gemini)