

Cosmology without sphaleron

Nodoka Yamanaka
(Nishina Center, Riken)

Based on

N. Yamanaka, arXiv:2212.10994 [hep-th]

N. Yamanaka, arXiv:2212.11820 [hep-ph]

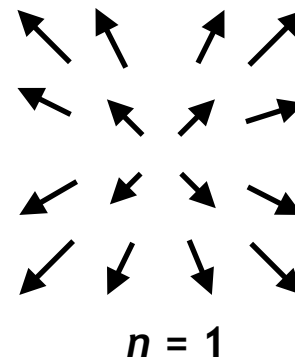
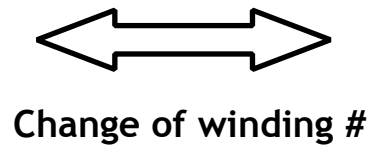
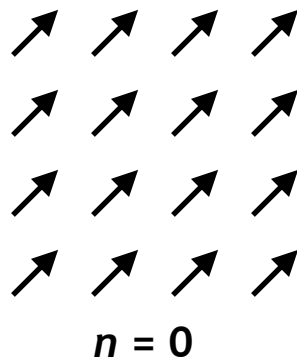
T. Tanii and N. Yamanaka, <https://zenodo.org/records/20805793>

See also <https://www2.yukawa.kyoto-u.ac.jp/~nodoka.yamanaka/topologicalcharge/>

2026/08/06
U. Tokyo

Topological charge

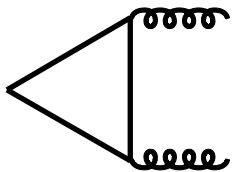
In QCD, there are non-trivial topological gauge configurations



(3-dimension
in reality)

Can be probed with the **topological charge**

$$\frac{\alpha_s}{8\pi} \int d^4x F_{\mu\nu,a} \tilde{F}_a^{\mu\nu} = \int d^4x \partial_\mu K^\mu$$



$$= \frac{ig_s\alpha_s}{24\pi} \int d^3\vec{x} f_{abc} \epsilon_{ijk} A_{ia}(\vec{x}) A_{jb}(\vec{x}) A_{kc}(\vec{x}) \Bigg|_{t=-\infty}^{t=+\infty}$$

$$= \Delta n \quad \text{Integer!}$$

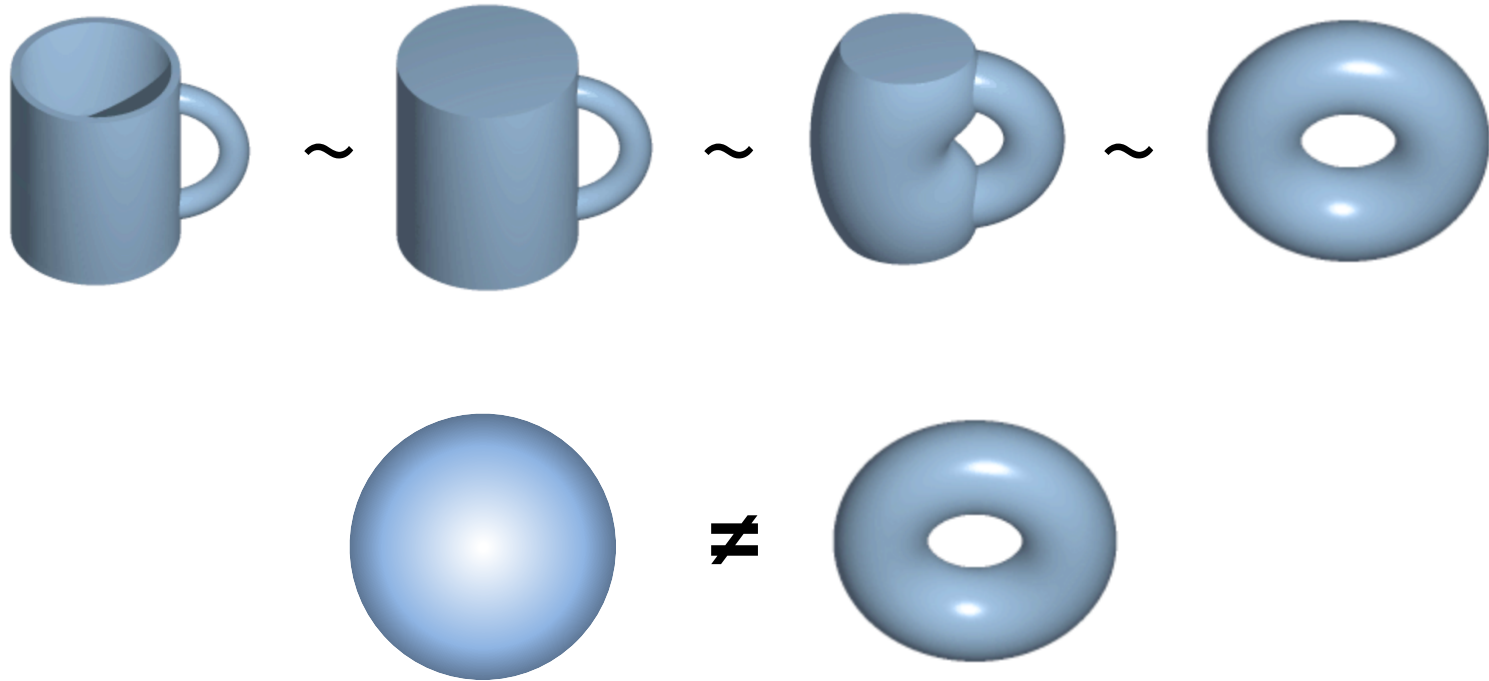
$$\begin{aligned} \tilde{F}_a^{\mu\nu} &\equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^a \\ K_\mu &\equiv \frac{\alpha_s}{8\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu F_a^{\rho\sigma} - \frac{g_s}{3} f_{abc} A_a^\nu A_b^\rho A_c^\sigma \right] \end{aligned}$$

$\Rightarrow \Delta n$ = change of **winding number** of gauge configurations

Topological charge

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Topology:

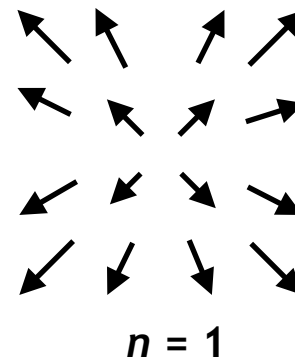
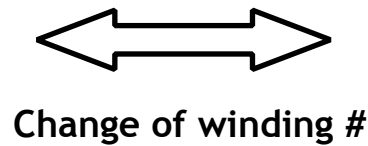
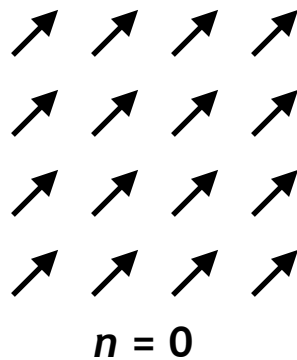


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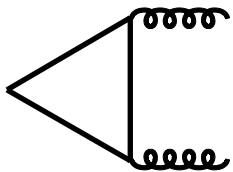
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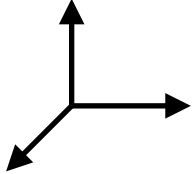
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Unphysical topological charge

Topological charge is a triple product of gauge fields

$$\frac{ig_s\alpha_s}{24\pi} \int d^3\vec{x} f_{abc} \epsilon_{ijk} A_{ia}(\vec{x}) A_{jb}(\vec{x}) A_{kc}(\vec{x}) \Big|_{t=-\infty}^{t=+\infty}$$

Time is frozen, restricted to 3-dimensional space

ϵ_{ijk} means  $\Rightarrow$ Covers all 3-dimensional directions!

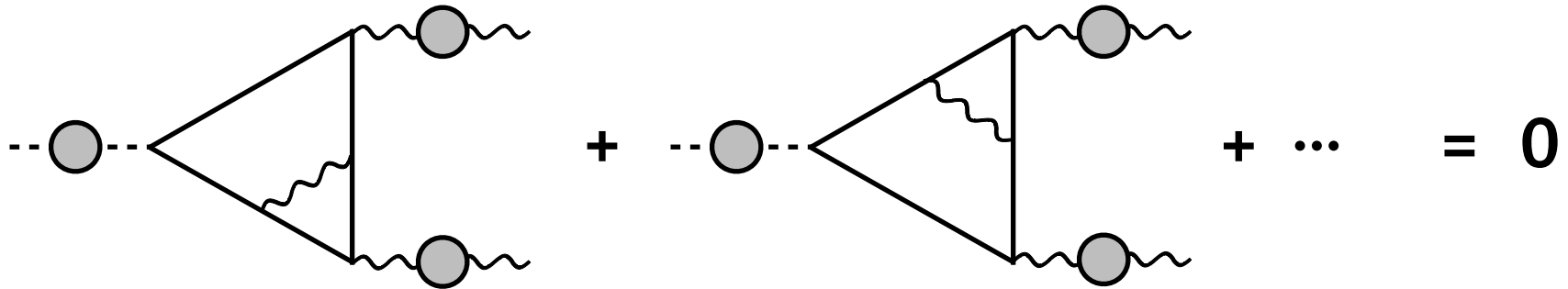
$\Rightarrow$ Also covers the **unphysical gauge freedom direction!**
 $(\vec{A}_a \rightarrow \vec{A}_a + \vec{\nabla} \chi_a)$

➡ Topological charge is unphysical !!

This only holds in perturbation theory ... Is it really OK ??

Adler-Bardeen theorem

Radiative corrections do not contribute to chiral anomaly
(up to renormalization of external fields)



S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969);
D. Anselmi, Phys. Rev. D **91**, 105016 (2015).

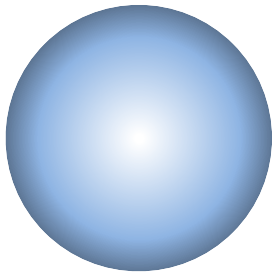
Chiral anomaly (= topological charge) does not change at any scale,
even if external fields are “dressed” by nonperturbative effects

⇒ Topological charge is strictly perturbation finite!

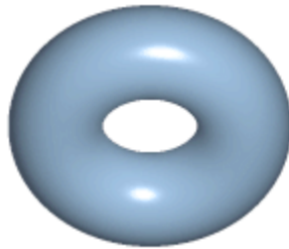
⇒ Unphysical longitudinal gauge component justified (BRST symmetry)

Why no radiative corrections?

Topological charge is an integer #



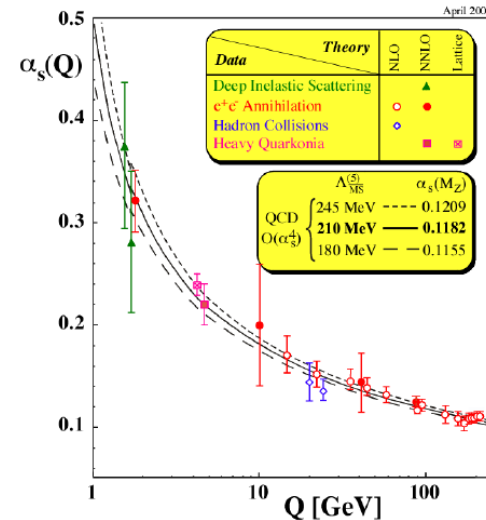
$n=0$



$n=1$

of holes is an integer !

Radiative corrections are real #



Running is continuous



Topological charge/chiral anomaly
do not have radiative corrections !

Definition of “unphysical”

“Unphysical” = “**Probability is zero**”

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0|F\tilde{F}(x)|\Omega\rangle \langle \Omega|F\tilde{F}(y)|0\rangle = 0$$

while matrix element (amplitude) $\langle 0|F\tilde{F}(x)|\Omega\rangle \neq 0$

Sum of absolute values cancels due to indefinite metric (= BRST symmetry)

Can be derived from Ward-Takashi identity of BRST symmetry

For arbitrary Green's functions, topological charge can be factorized

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0|\phi(x')|0\rangle \langle 0|F\tilde{F}(x)|\Omega\rangle \langle \Omega|F\tilde{F}(y)|0\rangle \langle 0|\phi(y')|0\rangle = 0$$

ϕ : arbitrary scalar BRST singlet function

Composite operator of $F\tilde{F}$ does not include $F\tilde{F}$ (Adler-Bardeen)

➡ Topological charge cannot appear in observables !

Consequences of unphysical topological charge

- Resolution of Strong CP problem

Axions not needed

- U(1) symmetries (axial, B+L) are not broken by anomaly

Sphaleron does not happen

NY, arXiv:2212.10994 [hep-th], 2212.11820 [hep-ph].

η' is NG boson, but no issues of U(1) problem

NY, arXiv:2411.02792 [hep-ph].

- Magnetic monopoles are unphysical

GUT transition after inflation

T. Tanii and NY (2024), <https://zenodo.org/records/20805793>

- SUSY gauge theory is conformal

No SUSY GUT

NY (2026), <https://zenodo.org/records/20914734>

Chiral Ward-Takahashi identity

$$\partial_\mu J_5^\mu \equiv \partial_\mu \sum_{\psi}^{N_f} \bar{\psi} \gamma^\mu \gamma_5 \psi = -2 \sum_{\psi}^{N_f} \left[m_\psi \bar{\psi} i \gamma_5 \psi \right] - \underbrace{\frac{N_f \alpha_s}{8\pi} F_{\mu\nu, a} \tilde{F}_a^{\mu\nu}}$$

Integrate : LHS $\rightarrow$ axial/B+L U(1) charge, RHS $\rightarrow$ unphysical topological charge

$\Rightarrow$ Conservation of U(1)_{A,B+L} !

(up to current fermion mass m_ψ)

What is the unphysical part of LHS (fermion) ?

Atiyah-Singer's theorem:

Number of chiral Dirac zero-modes = topological charge

$$\text{ind}(\not{D}) = -\frac{\alpha_s}{8\pi} \int d^4x F_{\mu\nu, a} \tilde{F}_a^{\mu\nu}$$

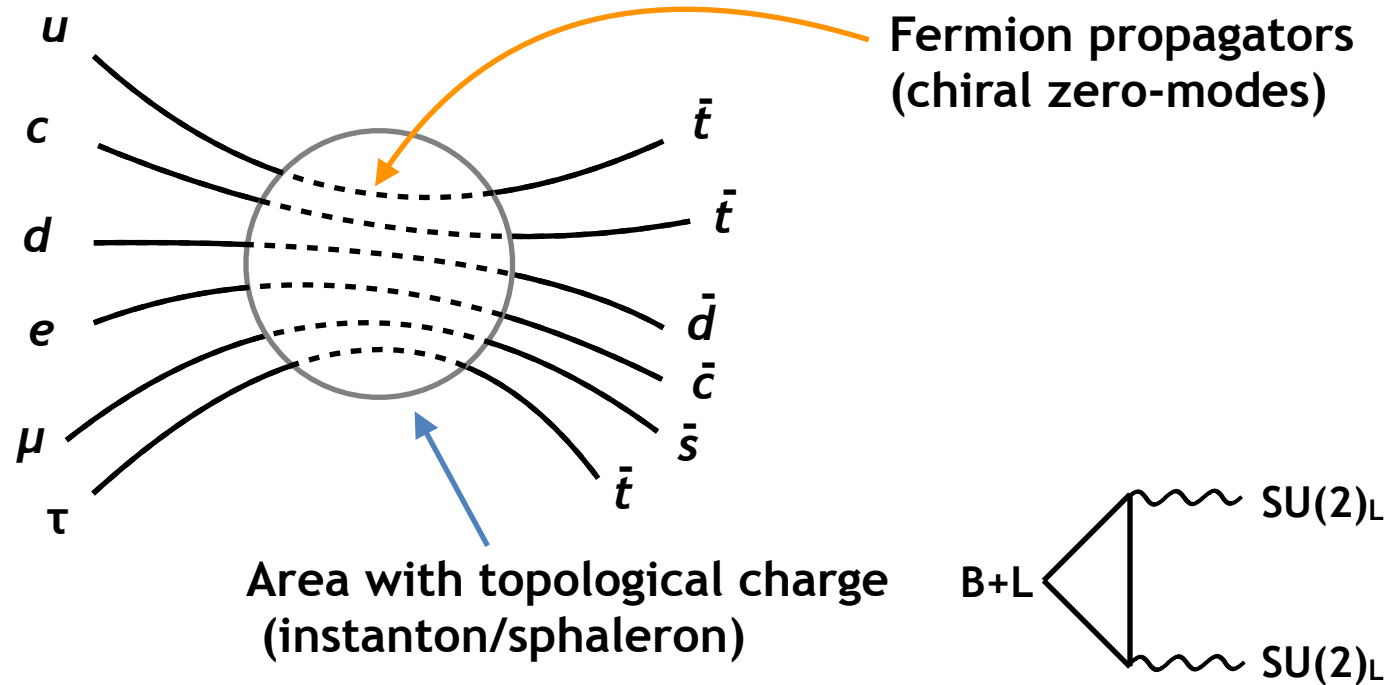
Dirac zero-mode :
solution of $\not{D}\psi = \lambda\psi$

Since the topological charge is unphysical,

fermion chiral Dirac zero-modes are also unphysical !

Sphaleron induced baryogenesis

't Hooft vertex : multi-fermion vertex generated by chiral Dirac zero-modes



G. 't Hooft, PRL 37, 8 (1976), PRD 14, 3432 (1976).

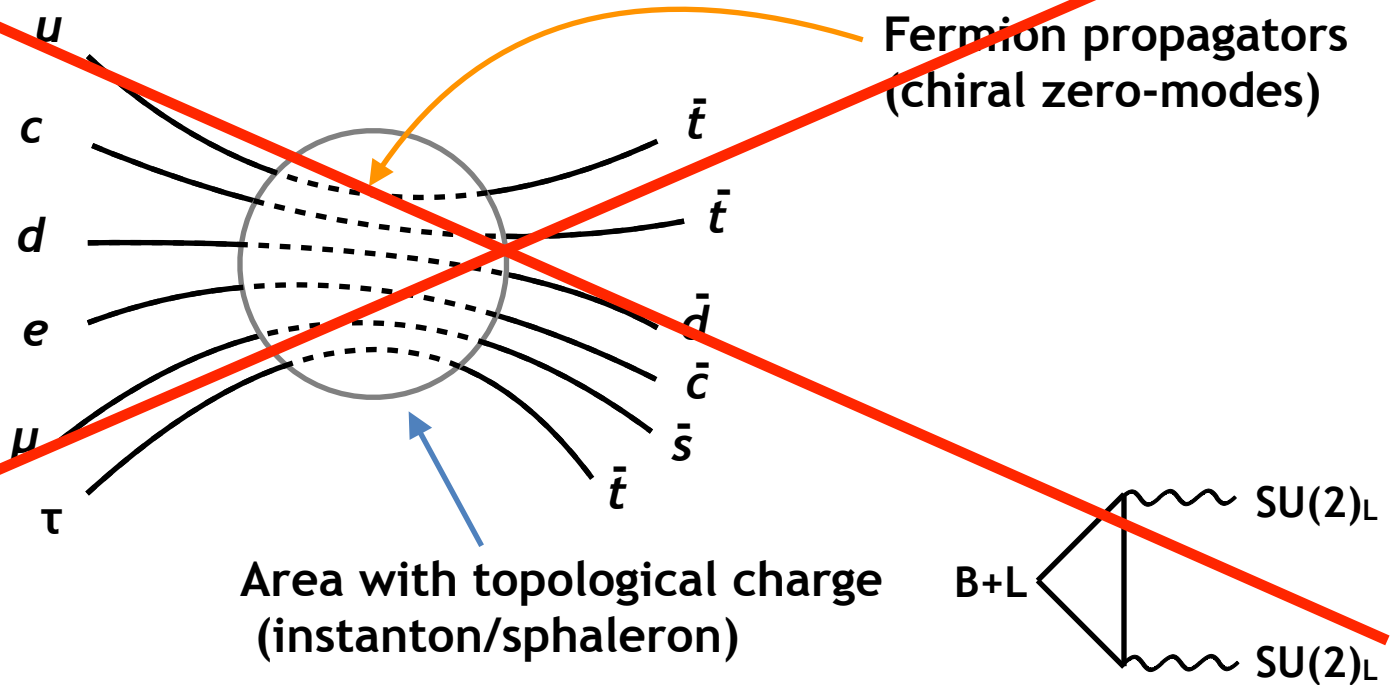
(Nonlocal) B+L violating 12-fermion interaction

⇒ Baryon and lepton numbers may be generated with weak interaction!

V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985), 36;
M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

Sphaleron induced baryogenesis

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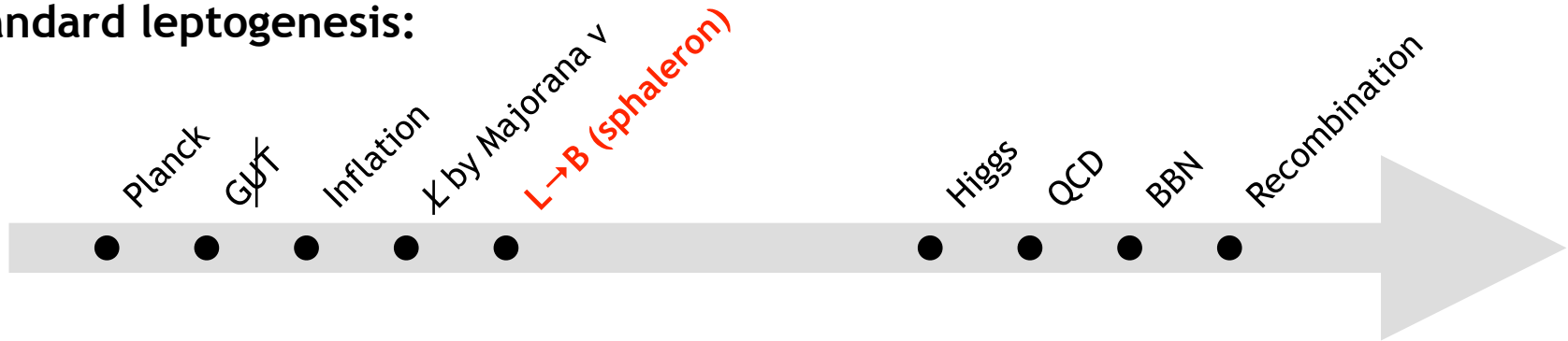
Atiyah-Singer theorem : topological charge = # of chiral Dirac zero-modes

$\Rightarrow$ 't Hooft vertex is unphysical !

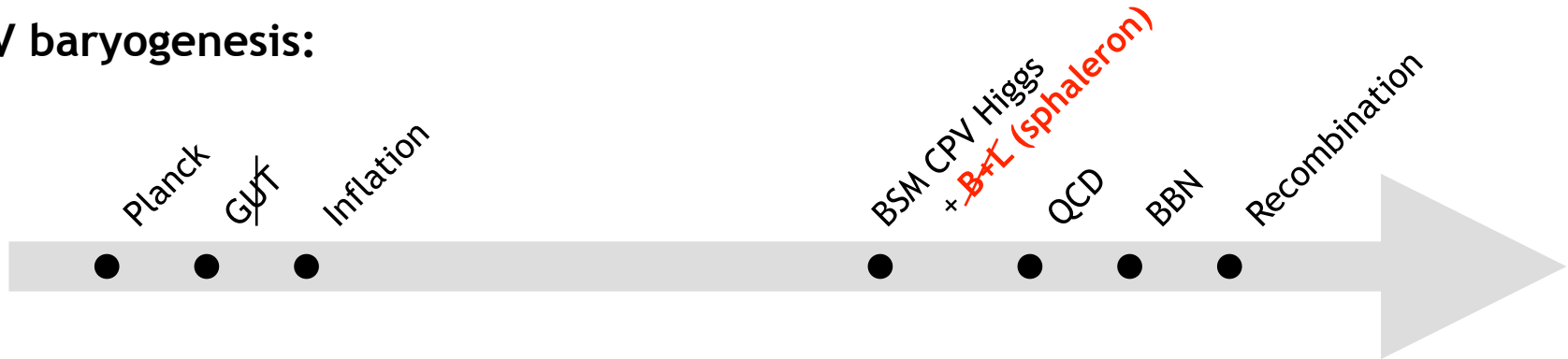
since 't Hooft vertex is generated by the propagation of zero-modes

Summary of baryogenesis scenarios

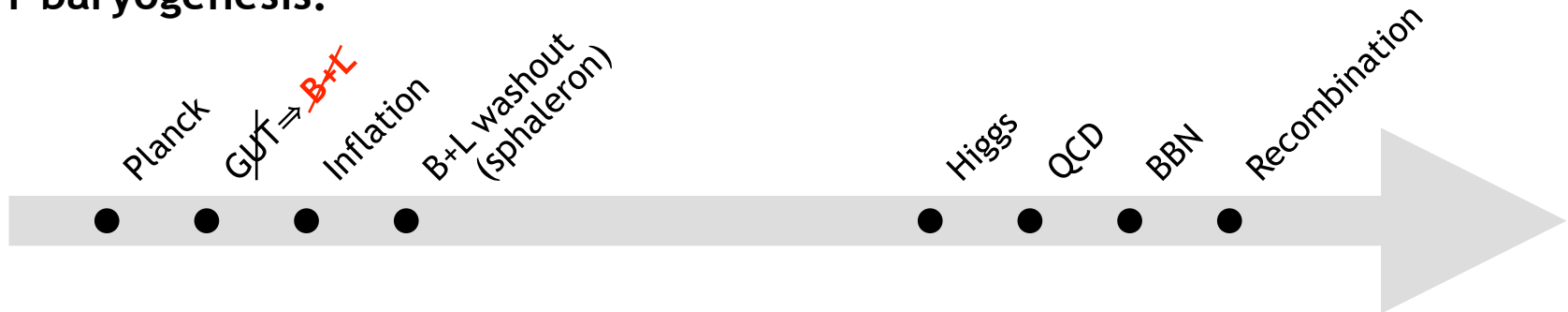
Standard leptogenesis:



TeV baryogenesis:



GUT baryogenesis:



Summary of baryogenesis scenarios

~~Standard leptogenesis:~~

Since sphaleron process does not occur, **these scenarios are impossible**

TeV baryogenesis:

GUT baryogenesis:

Summary of baryogenesis scenarios

Standard leptogenesis:

Planck GUT Inflation ~~Majorana v~~ ~~B (sphaleron)~~ ~~ags~~ ~~CD~~ ~~BN~~ ~~Recombination~~

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TeV baryogenesis:

Planck GUT Inflation BSM CPV Higgs + ~~B+L (sphaleron)~~ QCD BBN Recombination

GUT baryogenesis:

Planck GUT $\Rightarrow$ ~~B+L~~ Inflation BBN Recombination

Only allowed scenario:

$\Rightarrow$ B+L asymmetry by GUT SSB

... Diluted by inflation ??

Magnetic monopole and behavior at long distance

SSB of gauged scalar field theory where U(1) remains
(e.g. adjoint Higgs, $SU(2) \rightarrow U(1)$)

$$\mathcal{L} = D_\mu \phi_a D^\mu \phi_a^\dagger - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{\lambda}{4} (|\phi_a|^2 - v^2)^2$$

$\Rightarrow$ VEV is like $\langle \phi \rangle \propto \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (= \tau_3) \Rightarrow$ Commute with A_3^μ
 $\Rightarrow$ Massless U(1) gauge remains

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Energy of (static) magnetic monopoles :

$$E = \int d^3x \left[|\dot{\phi}_a|^2 + \frac{1}{2} |\dot{A}_a|^2 + D_i \phi_a D_i \phi_a^\dagger + \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{\lambda}{4} (|\phi|^2 - v^2)^2 \right]$$

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ϕ may have topological defects, but energy diverges?

Angular mode: $\vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\varphi \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \Rightarrow$ No E density damp at $r \rightarrow \infty$

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$\Rightarrow$ Cancellation between derivative and gauge in $D_i \phi_a = (\nabla_i + ig A_{bi} T_b) \phi_a$

$\Rightarrow \nabla_i \phi \sim g A_i \phi$ at large r !

Magnetic monopoles are unphysical

Typical GUT SSB : $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

$U(1)$ appears $\Rightarrow$ magnetic monopoles at GUT transition

$\Rightarrow$ Too many monopoles at GUT, Universe would collapse : “monopole problem”

If inflation happens, dilute monopole density

$\Rightarrow$ Conventional resolution of monopole problem

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Our resolution :

$$\begin{aligned}\text{Magnetic monopole} &= -\frac{1}{2} \int_V \epsilon_{ijk} \partial_i F_{jk} dV \\ &= -\frac{1}{2gv^3} \int_V \epsilon^{ijk} f^{abc} (\partial_i \phi^a) (\partial_j \phi^b) (\partial_k \phi^c) dV \quad \ni \underbrace{\epsilon_{ijk} A_i^d A_j^e A_k^f}_{r \rightarrow \infty}\end{aligned}$$

$\Rightarrow$ Triple product again !

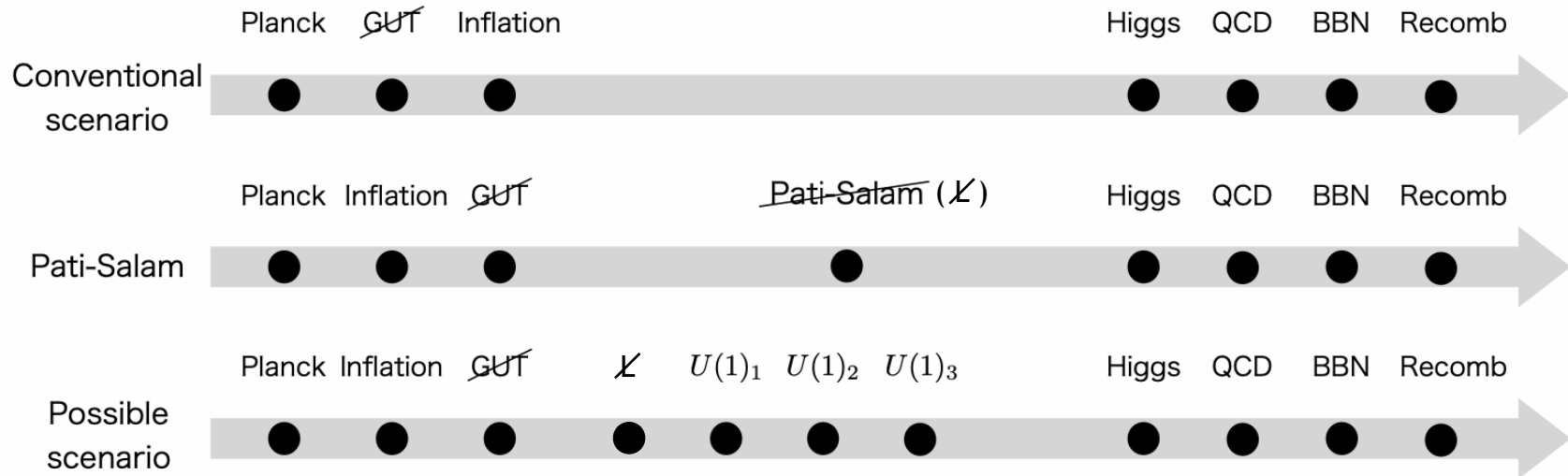
(Due to scalar-gauge
cancellation of $|D\phi|$)

$\Rightarrow$ Unphysical ! No monopole problem !

Possible cosmological scenarios without monopoles

Irrelevance of magnetic monopoles opens up new scenarios

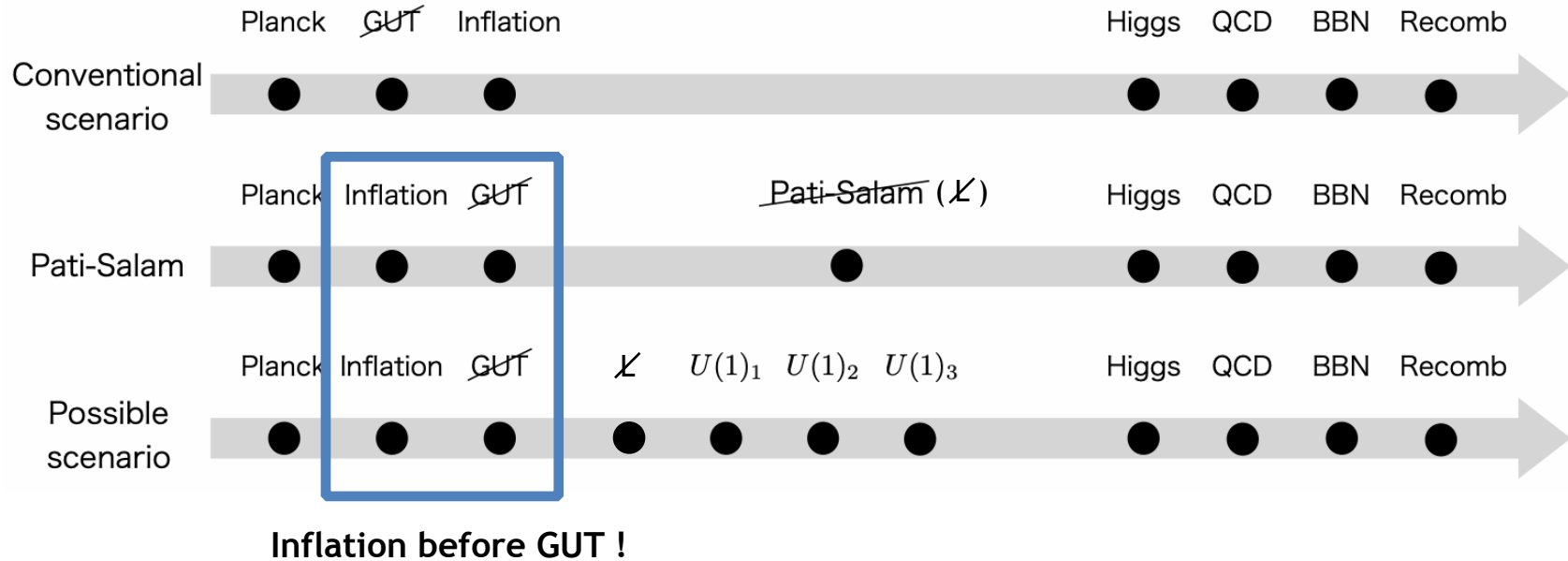
Transitions generating U(1) are allowed (e.g. Pati-Salam)



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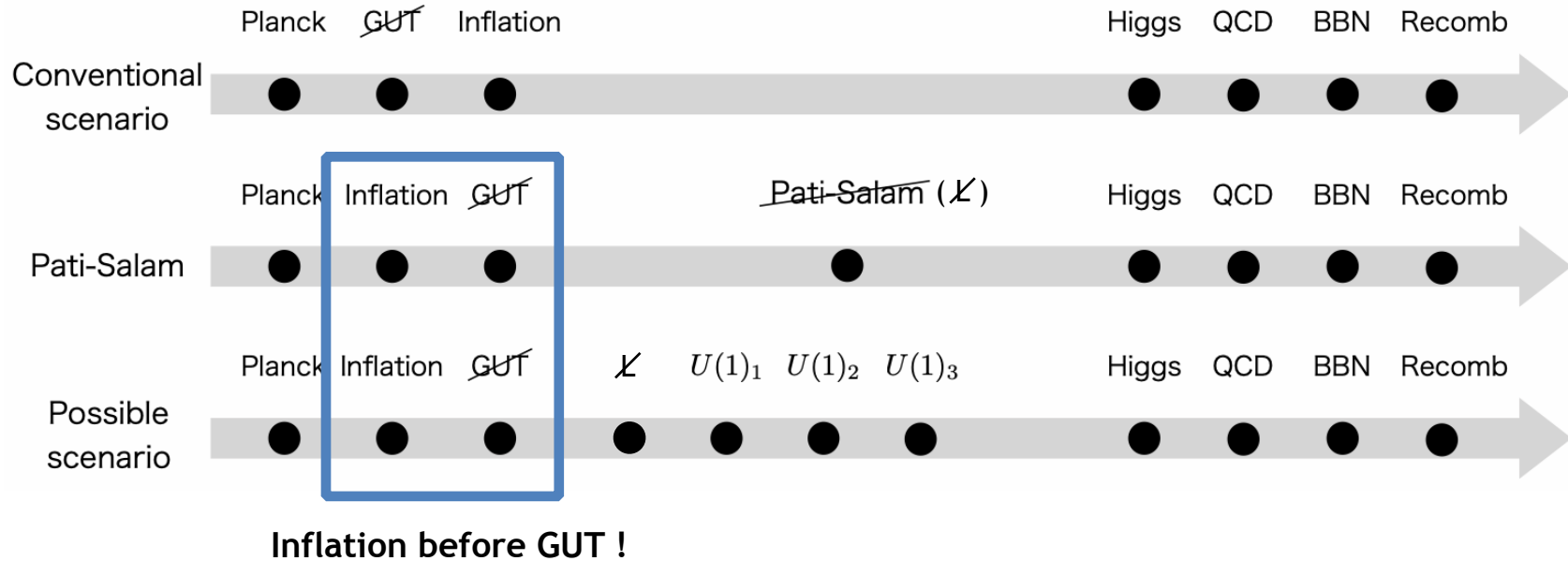
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GUT baryogenesis OK, multi-step unification,... and even no-inflation?

Majorana neutrino and large $\mathbb{X}$ originate from Pati-Salam?

Generate U(1) symmetries associated to flavor?

Lepton number asymmetry as a probe of our statement??

We saw that sphaleron induced B+L violation is unphysical

⇒ No wash-out

Baryon number of Universe is known : $B / s = 10^{-10}$

What about lepton number ?

 $L / s \sim 10^{-5}$ (from recent analysis of metal poor galaxies)

A. Matsumoto et al., Astrophys. J. 941 (2022) 167

$L \gg B$??

Wash-out would yield $B \sim L$, very unlikely to have had sphaleron process

⇒ Support our statement!

(There are other scenarios using “L-balls” to avoid wash-out,
admitting sphaleron process)

Kawasaki and Murai, JCAP 08 (2022) 041

Summary

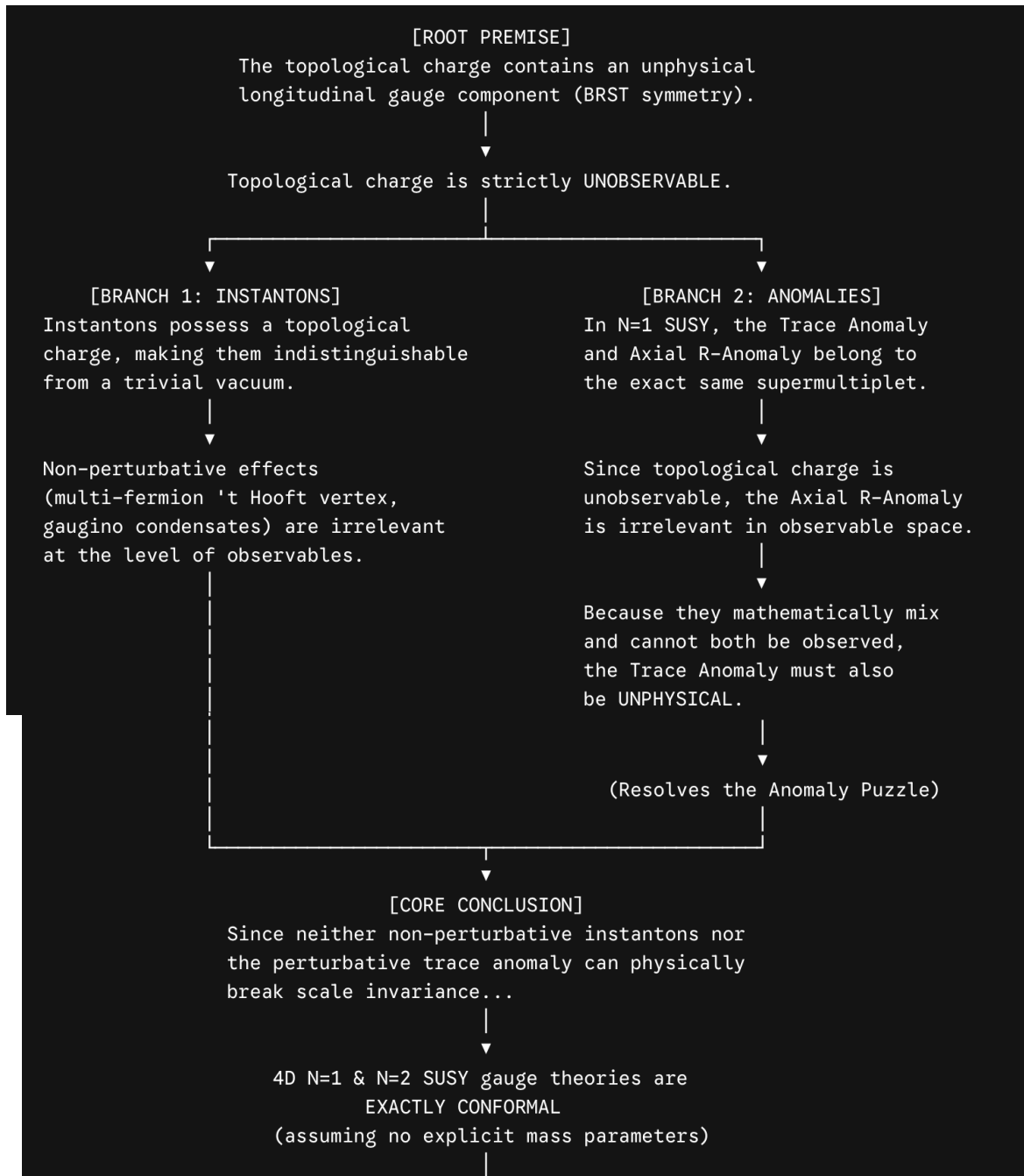
- My claim : **topological charge is unphysical, chiral anomaly is unphysical, sphaleron does not occur.**
- Sphaleron induced baryogenesis is excluded: need GUT to generate baryon number asymmetry.
- For not diluting B+L asymmetry generated by GUT, inflation must occur before GUT transition.
- **Magnetic monopoles are also unobservable** : no problem with inflation before GUT transition.
- Many scenarios are possible, including Pati-Salam, BSM U(1) associated to flavor,...

Homeworks:

- Confirm large lepton number asymmetry.
- Road to incorporating gauged flavors ?

Backup slides

Why is SUSY conformal ?



NY (2026),
<https://zenodo.org/records/20914734>

(Summarized by Gemini)