Radiative Breaking of Two-Zero Neutrino Mass Minors

2607.06102 and on going work

Workshop on Muon LFV and Beyond 2026 @UTokyo

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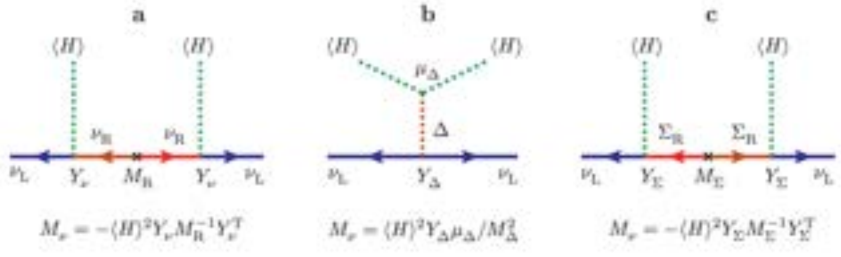
Introduction

The origin of Neutrino mass is open question

$$m_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}$$

Seesaw, Radiative model, and so on …



Tommy Ohlsson, and Shun Zhou
1311.3846

Introduction

# of Observables < # of Model Parameters

$$\theta_{12}, \quad \theta_{23}, \quad \theta_{13}, \quad \delta_{\rm CP}, \quad \Delta m^2_{21}, \quad \Delta m^2_{31}$$

$$m_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}$$

Enhance model predictive power using textures

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Of course there are other types, which omitted here

Introduction

Flavor symmetry naturally implements these textures

(UV perspective)

$$
\begin{aligned}
\mathcal{L} = &\, L_i^\dagger i\bar{\sigma}^\mu D_\mu L_i + \bar{N}_\alpha^\dagger i\bar{\sigma}^\mu \partial_\mu \bar{N}_\alpha + D^\mu H^\dagger D_\mu H - \lambda_H |H|^4 \\
&- \frac{1}{2} M_{\alpha\beta} \bar{N}_\alpha \bar{N}_\beta - \kappa_{i\alpha} \left(H \cdot L_i\right) \bar{N}_\alpha - y_{ij} H^\dagger \bar{E}_i L_j + \text{h.c.}
\end{aligned}
$$

Type-I Seesaw

	Field			$U(1)_{L_\mu - L_\tau}$
Leptons	L_e	$\bar{e}_e$	$\bar{N}_e$	0
	L_μ	$\bar{e}_\tau$	$\bar{N}_\tau$	+1
	L_τ	$\bar{e}_\mu$	$\bar{N}_\mu$	−1
Scalar		ϕ		+1

Flavor symmetry+SSB

$$
m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}
$$

Introduction

Textures are assumed at Low-energy

(IR perspective)

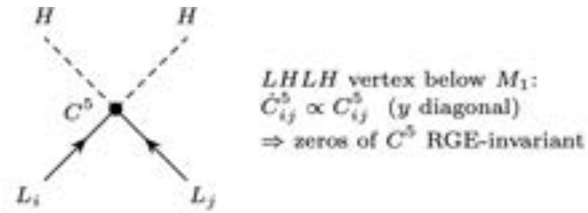
$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Recent cosmology and neutrino experiments give strong constraints for these textures

This indicate the corresponding UV theory is excluded?

How robust the exact texture at Low-energy?

Our Works



One-Loop RGE of the weinberg operator's flavor structure does not change

Evaluating radiative corrections of thresholds of Heavy particles

(UV perspective)

This indicate the corresponding UV theory is excluded?

How robust the exact texture at Low-energy?

2607.06102: Type-I seesaw + Lmu-Ltau

	Field			$\mathrm{U}(1)_{L_\mu - L_\tau}$
Leptons	L_e	$\bar{E}_e$	$\bar{N}_e$	0
	L_μ	$\bar{E}_\tau$	$\bar{N}_\tau$	+1
	L_τ	$\bar{E}_\mu$	$\bar{N}_\mu$	−1
Scalar		ϕ		+1

$$\mathcal{L} = L_i^\dagger i\bar{\sigma}^\mu D_\mu L_i + \bar{N}_\alpha^\dagger i\bar{\sigma}^\mu \partial_\mu \bar{N}_\alpha + D^\mu H^\dagger D_\mu H - \lambda_H |H|^4 - \frac{1}{2} M_{\alpha\beta} \bar{N}_\alpha \bar{N}_\beta - \kappa_{i\alpha} (H \cdot L_i) \bar{N}_\alpha - y_{ij} H^\dagger \bar{E}_i L_j + \text{h.c.} \,,$$

$$y = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad \kappa = \begin{pmatrix} \kappa_e & 0 & 0 \\ 0 & \kappa_\mu & 0 \\ 0 & 0 & \kappa_\tau \end{pmatrix}$$

$$M = \begin{pmatrix} M_{ee} & \lambda_{e\mu} \langle \phi \rangle & \lambda_{e\tau} \langle \phi^* \rangle \\ \lambda_{e\mu} \langle \phi \rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau} \langle \phi^* \rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

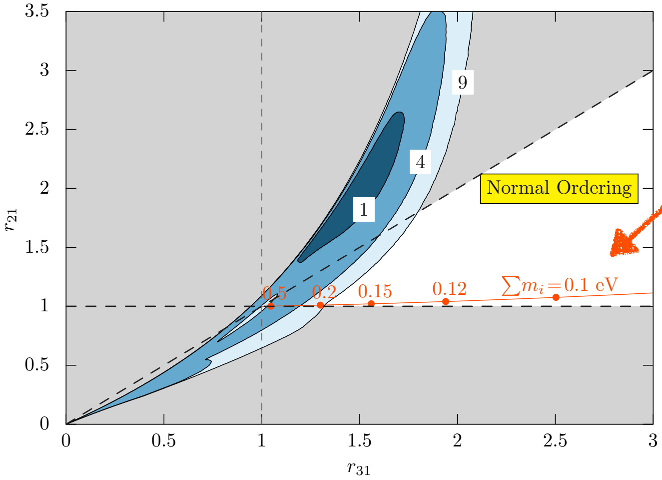
$$M_\nu = U^* \hat{m}_\nu U^\dagger \,, \quad \hat{m}_\nu = \mathrm{diag}(m_1, m_2, m_3) \,,$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Tree Level

$$r_{21} := \frac{m_2}{m_1} = \frac{c_{12}\left[-c_{12}\cos 2\theta_{23} + e^{i\delta_{CP}} s_{12}s_{13}\sin 2\theta_{23}\right]}{s_{12}\left[\cos 2\theta_{23}s_{12} + e^{i\delta_{CP}} c_{12}s_{13}\sin 2\theta_{23}\right]} \times e^{-2i(\eta_1-\eta_2)},$$

$$r_{31} := \frac{m_3}{m_1} = \frac{c_{12}c_{13}^2\left(c_{12}\cos 2\theta_{23} - 2e^{i\delta_{CP}} c_{23}s_{12}s_{13}s_{23}\right)}{e^{i\delta_{CP}} s_{13}\left[-e^{i\delta_{CP}}\cos 2\theta_{12}\cos 2\theta_{23}s_{13} + 2c_{12}c_{23}s_{12}\left(1+e^{2i\delta_{CP}} s_{13}^2\right)s_{23}\right]} \times e^{-2i\eta_1}$$



When $\sum m_i$ is fixed, m1,m2,m3 are given

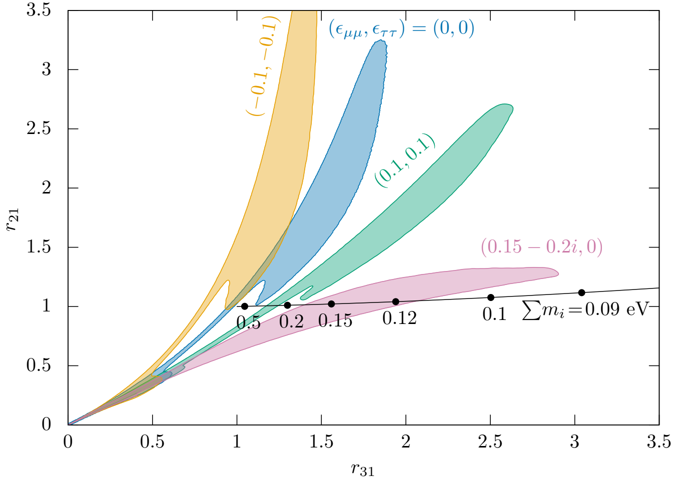
$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Two-Zero minor leads the above equations

Quasi Two-Zero Minor

$$r_{21}^{(\epsilon)} = \frac{c_{12}\left[-c_{12}\cos 2\theta_{23} + e^{i\delta_{CP}} s_{12}s_{13}\sin 2\theta_{23}\right]}{s_{12}\left[\cos 2\theta_{23}s_{12} + e^{i\delta_{CP}} c_{12}s_{13}\sin 2\theta_{23}\right] - c_{23}^2\,\epsilon_{\mu\mu} + s_{23}^2\,\epsilon_{\tau\tau}} \times e^{-2i(\eta_1-\eta_2)}\,,$$

$$r_{31}^{(\epsilon)} = \frac{c_{12}c_{13}^2\left(c_{12}\cos 2\theta_{23} - 2e^{i\delta_{CP}} c_{23}s_{12}s_{13}s_{23}\right)}{e^{i\delta_{CP}} s_{13}\left[-e^{i\delta_{CP}}\cos 2\theta_{12}\cos 2\theta_{23}s_{13} + 2c_{12}c_{23}s_{12}\left(1+e^{2i\delta_{CP}}s_{13}^2\right)s_{23}\right] - K_{31}} \times e^{-2i\eta_1}\,,$$



$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & \epsilon_{\mu\mu}e^{2i\eta_1}/m_1 & * \\ * & * & \epsilon_{\tau\tau}e^{2i\eta_1}/m_1 \end{pmatrix}$$

Quasi Two-Zero minor change the prediction

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	Field			$U(1)_{L_\mu - L_\tau}$
	L_e	$\bar{E}_e$	$\bar{N}_e$	0
Leptons	L_μ	$\bar{E}_\tau$	$\bar{N}_\tau$	+1
	L_τ	$\bar{E}_\mu$	$\bar{N}_\mu$	−1
Scalar		ϕ		+1

$$\mathcal{L} = L_i^\dagger i\bar{\sigma}^\mu D_\mu L_i + \bar{N}_\alpha^\dagger i\bar{\sigma}^\mu \partial_\mu \bar{N}_\alpha + D^\mu H^\dagger D_\mu H - \lambda_H |H|^4$$
$$- \frac{1}{2} M_{\alpha\beta} \bar{N}_\alpha \bar{N}_\beta - \kappa_{i\alpha} \left(H \cdot L_i\right) \bar{N}_\alpha - y_{ij} H^\dagger \bar{E}_i L_j + \text{h.c.} \ ,$$

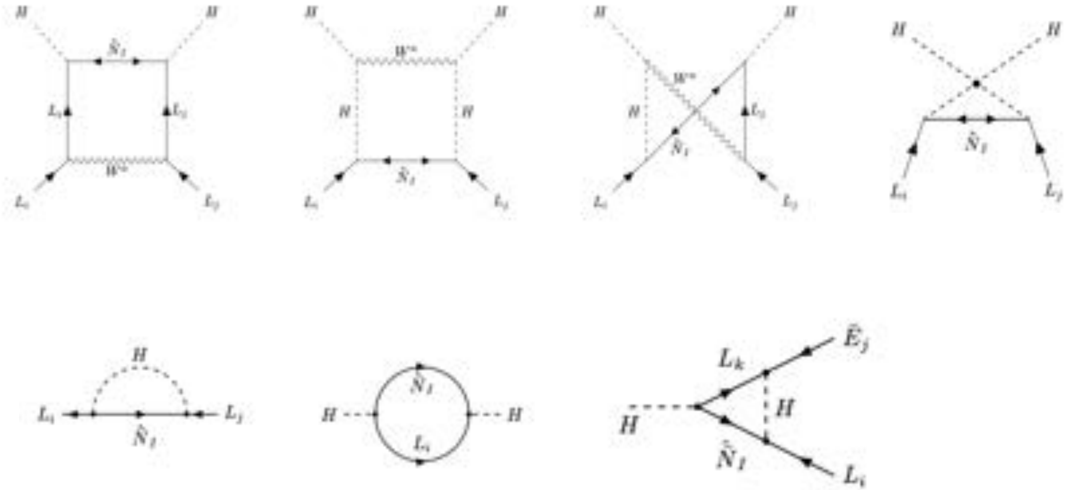
$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & \epsilon_{\mu\mu} e^{2i\eta_1}/m_1 & * \\ * & * & \epsilon_{\tau\tau} e^{2i\eta_1}/m_1 \end{pmatrix}$$

Without a specific UV model, how large ϵ are allowed?

In Bottom-Up way, it may put ϵ as free parameter,

But nobody knows it comes from some Flavor symmetry

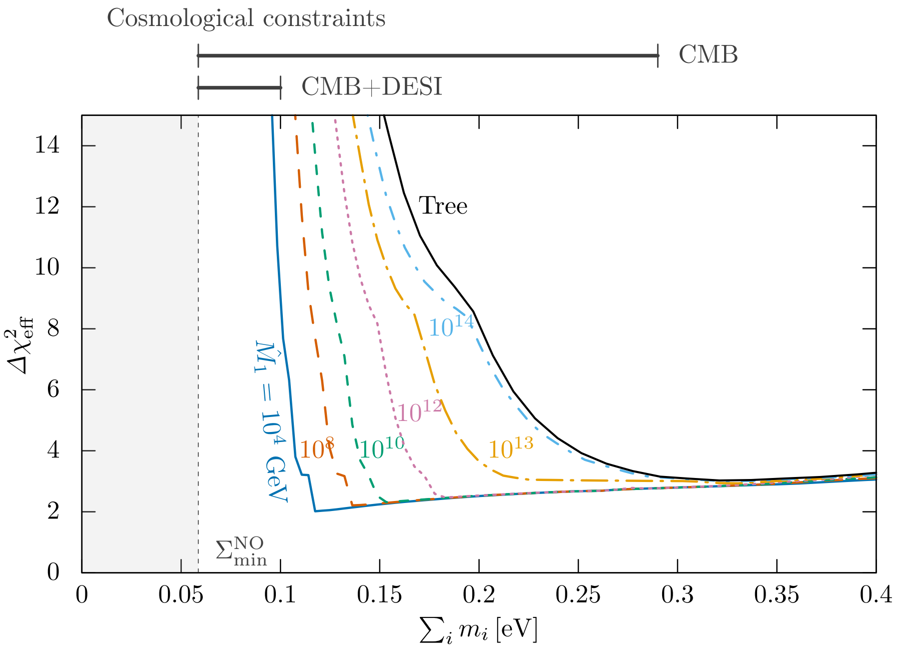
Relevant Diagrams for One Loop Matching



In this work, we have neglected hidden sector loop

Run and match at each thresholds may give more precise predictions

Results



$\hat{M}$ are mass Eigen values of RHNs

$\hat{M}_1$ is the Lightest RHN

The y axis indicates how worse the flavor model is comparing generic model

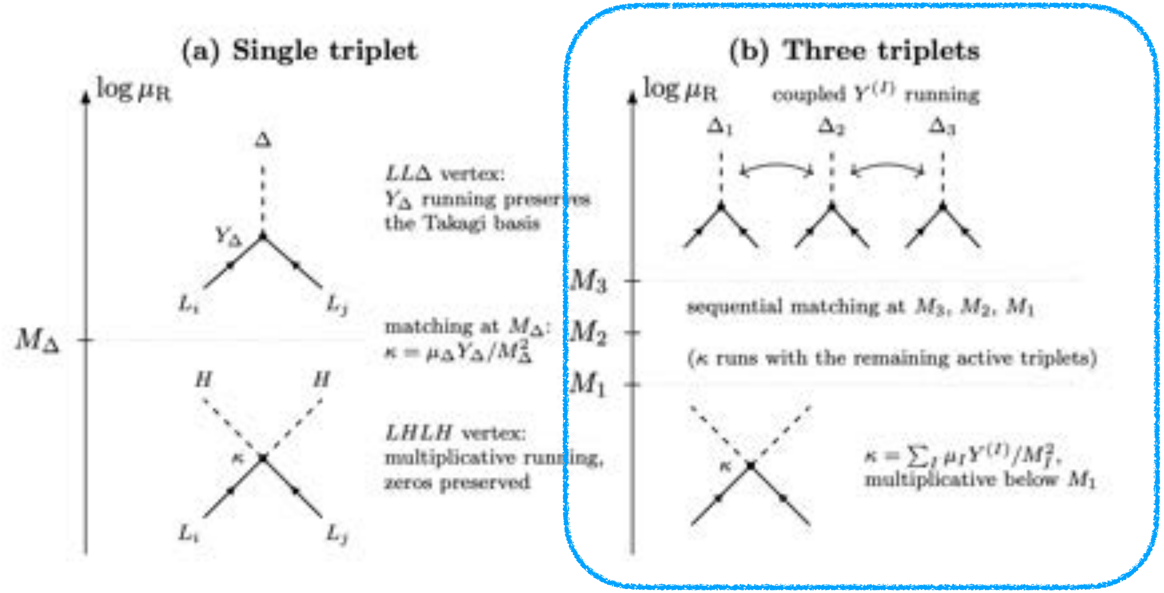
On going works (Type-II seesaw)

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_{L_\mu - L_\tau}$
L_e	$\mathbf{2}$	$-1/2$	0
L_μ	$\mathbf{2}$	$-1/2$	$+1$
L_τ	$\mathbf{2}$	$-1/2$	-1
H	$\mathbf{2}$	$+1/2$	0
$\bar{E}_e$	$\mathbf{1}$	$+1$	0
$\bar{E}_\mu$	$\mathbf{1}$	$+1$	-1
$\bar{E}_\tau$	$\mathbf{1}$	$+1$	$+1$
Δ_e	$\mathbf{3}$	$+1$	0
Δ_μ	$\mathbf{3}$	$+1$	$+1$
Δ_τ	$\mathbf{3}$	$+1$	-1
ϕ	$\mathbf{1}$	0	-1

$$\mathcal{L} \supset -Y_{ee} L_e^T i\sigma_2 \Delta_e L_e - 2Y_{\mu\tau} L_\mu^T i\sigma_2 \Delta_e L_\tau - 2Y_{e\mu} L_e^T i\sigma_2 \Delta_\tau L_\mu - 2Y_{e\tau} L_e^T i\sigma_2 \Delta_\mu L_\tau \\ - \mu_e H^T i\sigma_2 \Delta_e^\dagger H - \kappa_\tau \phi H^T i\sigma_2 \Delta_\tau^\dagger H - \kappa_\mu \phi^\dagger H^T i\sigma_2 \Delta_\mu^\dagger H + \text{h.c.}$$

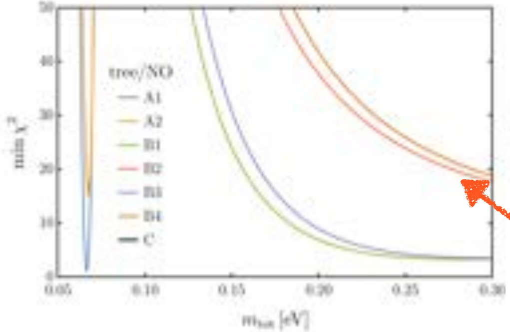
Here we ignore the scalar sector

On going works (Type-II seesaw)

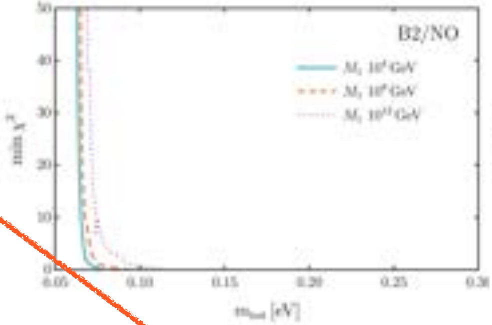


Type-II with flavor symmetry requires multi triplets

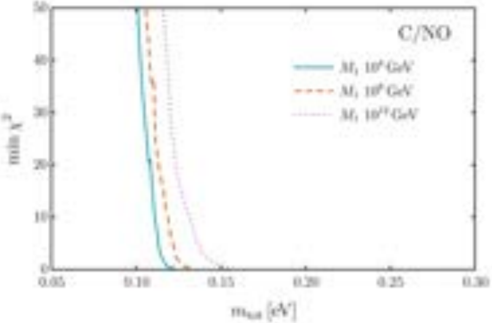
Preliminary Results



(a) Normal ordering.



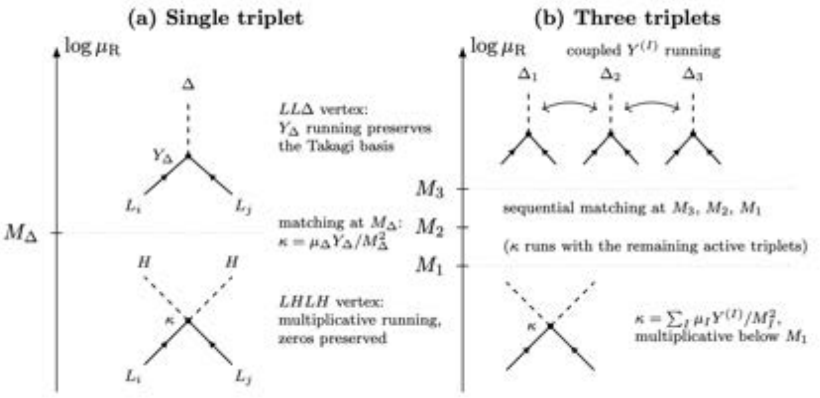
(d) B_2, normal ordering.



(e) C, normal ordering.

$$\chi^2 = \sum_{x \in \mathcal{O}} \left(\frac{x - x_{\rm bf}}{\sigma_x} \right)^2, \qquad \mathcal{O} = \{\theta_{12}, \theta_{23}, \theta_{13}, \Delta m^2_{21}, \Delta m^2_{3\ell}, \delta_{\rm CP}\}$$

Physical implications



The larger hierarchy leads to larger deviations from exact textures

The lightest triplet M_1 ~ 10TeV, and correlation with LFV is interesting

Conclusions/Future Prospects

We revisited (revisiting) Flavor Symmetric Seesaw Model.
The impact of radiative corrections should not be neglected.

Exact textures at Low-energy is not connected to flavor symmetry in general

Flavor symmetric Seesaw require 10 TeV the lightest BSM particle to meet the cosmological constraints (BAO+DESI) so that Parameter correlations with LFV is interesting

Bottom-Up way: The observed neutrino mass matrix can be connected with UV flavor symmetry, assuming some RGE may be interesting

Back Up

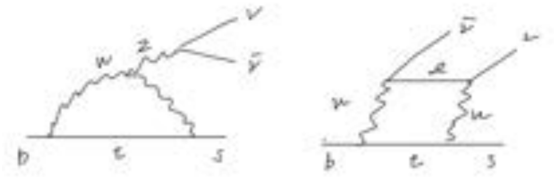
Leading Order = Loop Level

Tree level is not leading order for the zero entries
One loop level is leading order, and Two loop is NLO!!

$$(m_\nu^{-1})_{\mu\mu} = 0 + \epsilon_{\mu\mu} + \cdots$$

It must be checked perturbative expansion is valid,
not comparing tree with one loop, but LO with NLO

e.g. FCNC



$$i\mathscr{M}_{\text{SM}} = 0 + i\mathscr{M}^{\text{one-loop}}$$

In the SM, FCNC starts from one-loop