Machine Learning for Economic Demography: Bayesian Approach to Analysing Ageing and Productivity

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Make it (Net)Work! - MLA2S Networking Seminar #10
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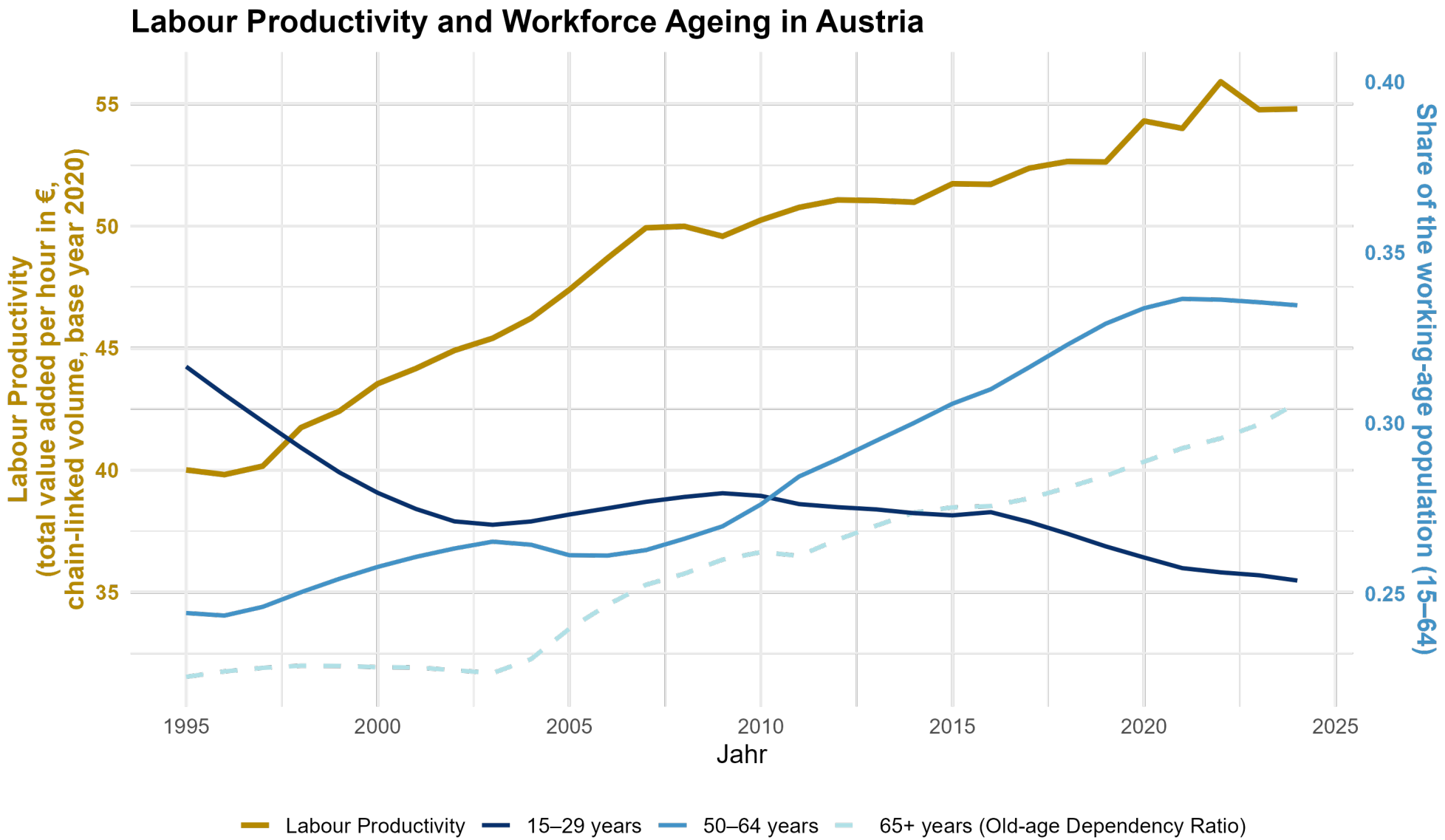


Source: OECD Data Explorer.

Data – Linked Employer-Employee Data Analysis (LEDA)

Firms

Structural Business Statistics Survey (LSE - RE)

firm size, investments, total value added, personnel costs, legal status, region, OENACE, year of foundation, some inventory, ...

Individuals

Register-based Labour Market Statistics (AEST REGZ)

age, gender, education level and field, nationality, part-time/full-time, occupation status (self-employed/employed/parental leave/etc.), number of children, ...

(1) Individuals are **linked** to firms & (2) staff information is **aggregated to firm level**

Result: firm-level data (2013-2022)
~250.000 firms with ~2 million actively working individuals per year

Full list of firm characteristics, workforce characteristics and NACE sectors

Research Questions

- What is the **relationship between productivity and the age** composition of the workforce at the firm level?

- How does this relationship **vary with firm heterogeneity**, such as firm size or sector?

- What role does the **educational attainment of the workforce** play?

Model & Methodology I

production, where the most important input factors are capital and labour:

$$Y_f = A_f K_f^{\alpha} L_f^{1-\alpha}$$

here we assume that labour is heterogeneous across age groups:

$$L_f = \theta_y L_{fy} + \theta_p L_{fp} + \theta_o L_{fo}$$

By dividing by L_f and assuming $\ln(1+x) \approx x$, we obtain:

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \beta_y s_{fy} + \beta_p s_{fp} + \beta_o s_{fo} + \ln A_f + \varepsilon_f$$

When further decomposing A_f into observed (firm- & workforce-level) controls and time shocks and drop the share of prime-aged workers as the reference group, we are left with the following for the regression:

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \boldsymbol{\beta_y} s_{fy} + \boldsymbol{\beta_o} s_{fo} + Z_f^F \gamma + Z_f^W \delta + \omega_t + \varepsilon_f$$

ÖAW VID

Model & Methodology II

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{L_f}\beta_i + Z_f^F\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$

Kuhn, M., Prskawetz, A., Wrzaczek, S., & Feichtinger, G. (2007). *Health, Survival and Consumption over the Life Cycle: An Optimal Control Approach*. Working paper.

$\frac{Y_f}{L_f}$ *Labour Productivity*

def. as *total value added per worker* (in an extension *per fte*)

$\frac{L_{fi}}{L_f}$ *Age Groups*

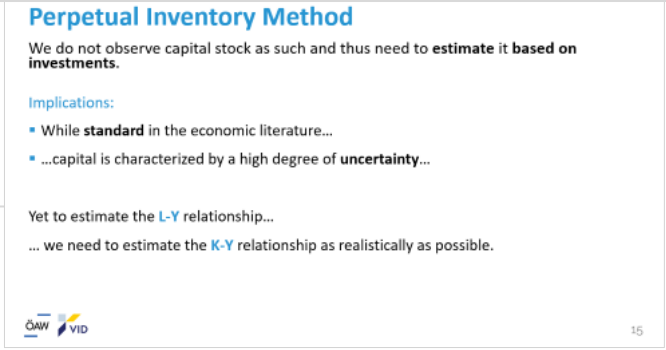
(1) Broad age groups (15-29, 30-49, 50+)
(2) 5-year age groups
(3) Joint shares of age & education

$\frac{K_f}{L_f}$ *Capital per worker*

estimated based on investments and the *perpetual inventory method*

Bayesian time-FE model for each NACE sector to understand between-firm variation in labour productivity

Annex: priors



Shortly about Bayesian Econometrics

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{L_f}\beta_i + Z_f^F\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$

Central idea of Bayesian updating:

$$p(\beta|y) \propto p(y|\beta)p(\beta)$$

Posterior distribution — Likelihood — Prior distribution

Selection of priors

- Weakly informative priors for the variables of interest (i.e., age)
- Priors for firm characteristics based on economic theory & similar studies

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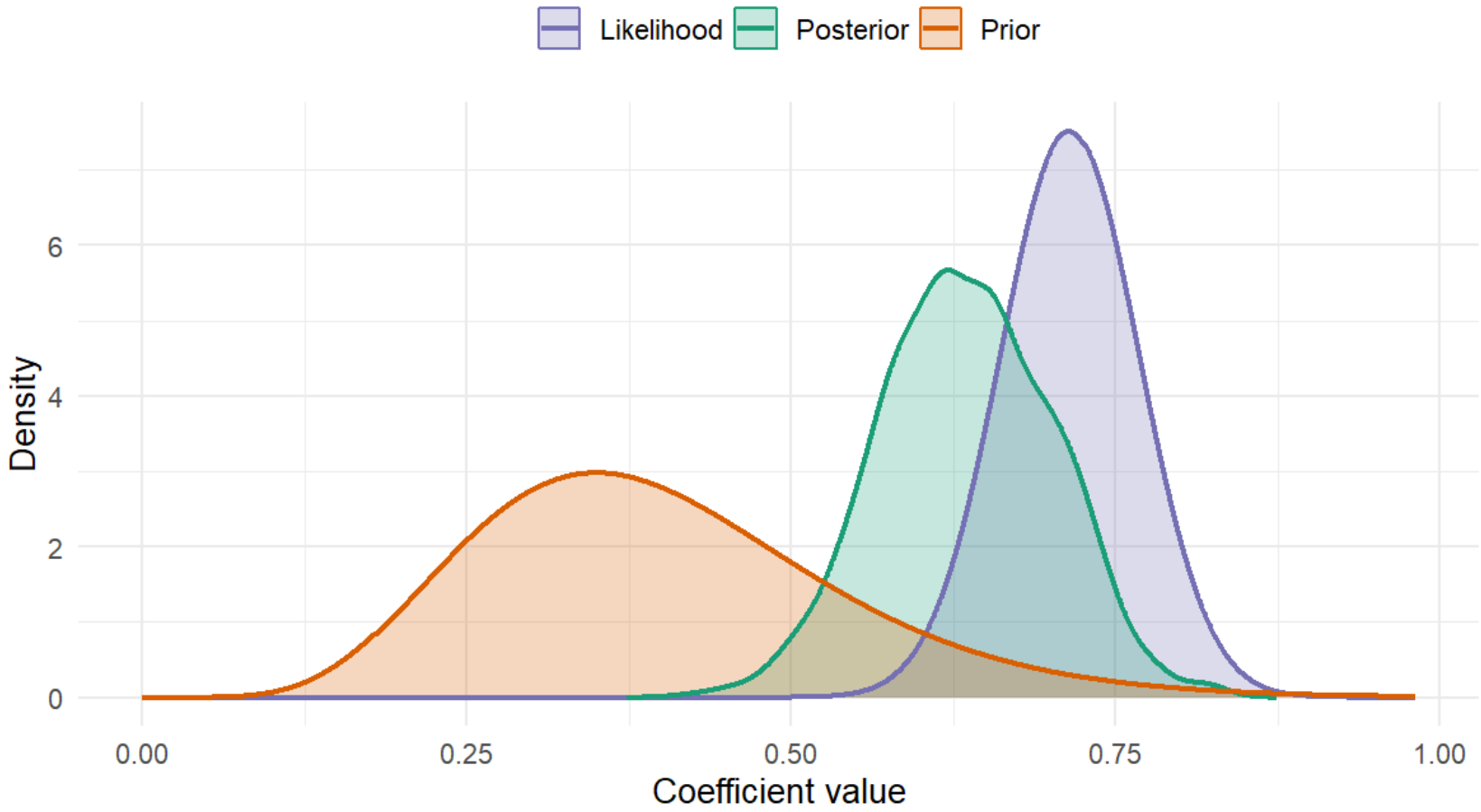
Prior on Capital

$$\ln\left(\frac{Y_f}{L_f}\right) = \boldsymbol{\alpha}\, \boldsymbol{ln}\left(\frac{\boldsymbol{K_f}}{\boldsymbol{L_f}}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{L_f}\beta_i + Z_f^F\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$



Results

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{L_f}\beta_i + Z_f^F\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$

- **hump-shaped** relationship between **age shares** and **productivity**…
- …partly driven by **extremes of the age** distributions…
- …where **low education may strengthen** this negative relationship.

→ These results differ by economic activity

Getting back to the bigger picture:

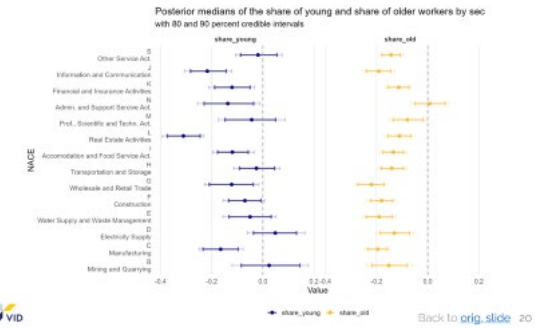
$$\ln\left(\frac{Y_f}{\boldsymbol{L_f}}\right) = \alpha \ln\left(\frac{\boldsymbol{K_f}}{\boldsymbol{L_f}}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{\boldsymbol{L_f}}\beta_i + \boldsymbol{Z_f^F}\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$

- Firm characteristics – strongly related to productivity
- Labour intensity – results differ when using *per head* or *per fte* measure

Results

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha \ln\left(\frac{K_f}{L_f}\right) + \sum_{i=1}^{N-1} \frac{\boldsymbol{L_{fi}}}{\boldsymbol{L_f}} \beta_i + Z_f^F \gamma + Z_f^W \delta + \omega_t + \varepsilon_f$$

Spec. 1 – Broad age groups



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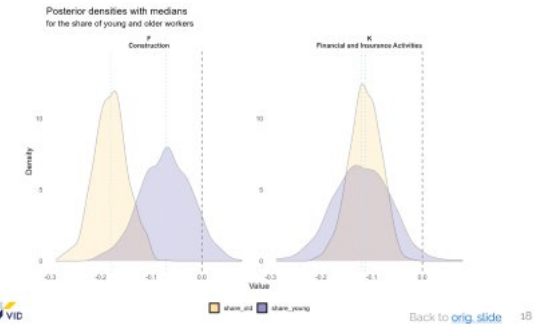

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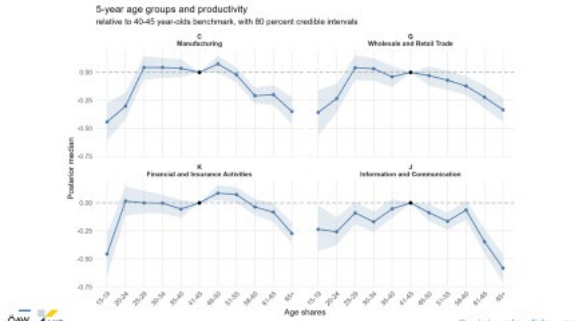
Spec. 2 – Narrow age groups



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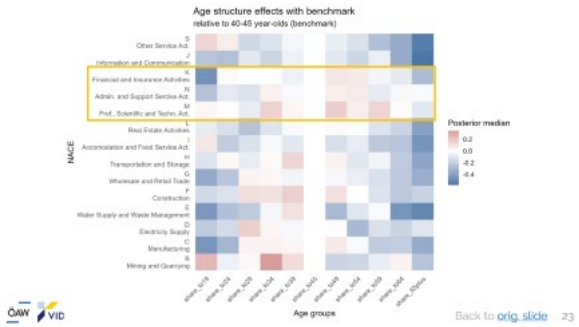

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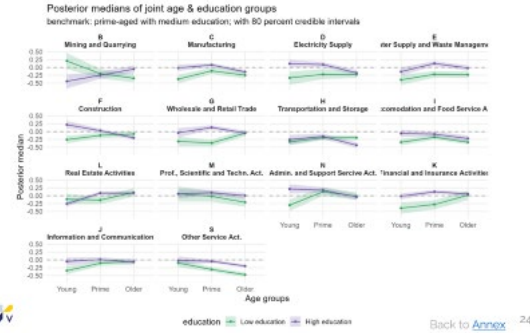
Spec. 3 – Age & education



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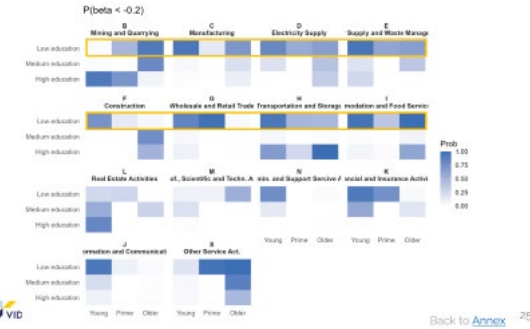

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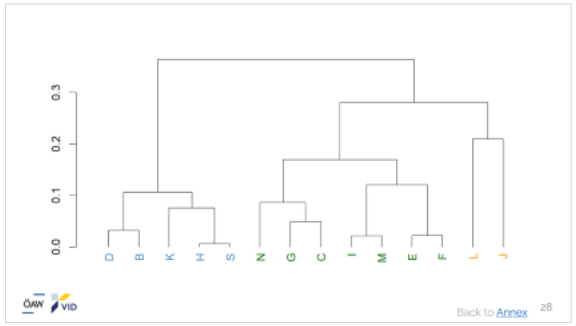
Further possible ML applications

$$\ln\left(\frac{Y_f}{L_f}\right) = \alpha\, ln\left(\frac{K_f}{L_f}\right) + \sum_{i=1}^{N-1} \frac{L_{fi}}{L_f}\beta_i + Z_f^F\gamma + Z_f^W\delta + \omega_t + \varepsilon_f$$

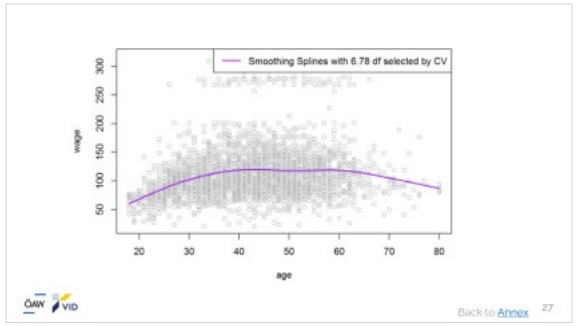
Clustering

- based on selected characteristics such as age
- supports interpretation regarding NACE-sector similarities



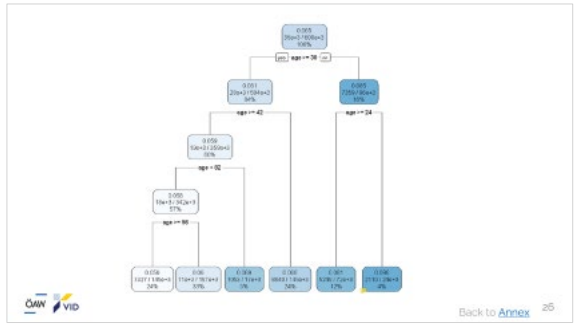
Smoothing splines

- Depart from linearity & one coefficient for each variable
- Relates to idea of benefits of age diversity ↔ disadvantage of age concentration



Random Forests

- Partitioning of feature space to obtain homogeneous subsets for regression
- Coefficient on age shares may depend on other firm characteristics



Conclusion on the Ageing-Productivity Nexus

- **Inverse U-relationship** between Ageing and Productivity
- More decisive: **capital, technology, firm size, age of the firm**

On a more general note

- **Socio-demographic composition** of age groups (such as education or part-time rates) relevant
- Definition of **labour intensity** matters (*per fte* instead of *per head*)
- **Register data:** allows for detailed country-specific analysis and shows **strong heterogeneity in firms** (computational limits, however)

Thank you!

LEDA project

Period: June 2024 – November 2026

Funding Body: Data:Research:Austria funding programme of the Austrian Academy of Sciences (Call 2023).

Contacts: alexia.fuernkranz-prskawetz@oeaw.ac.at & isabel.gerstner@oeaw.ac.at

Website: https://www.oeaw.ac.at/vid/research/research-projects/leda
or scan the QR code:

Annex

Perpetual Inventory Method

We do not observe capital stock as such and thus need to **estimate** it **based on investments**.

Implications:

- While **standard** in the economic literature…
- …capital is characterized by a high degree of **uncertainty**…

Yet to estimate the **L-Y** relationship…

… we need to estimate the **K-Y** relationship as realistically as possible.

Model & Methodology II

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$\frac{Y_f}{L_f}$ *Labour Productivity*

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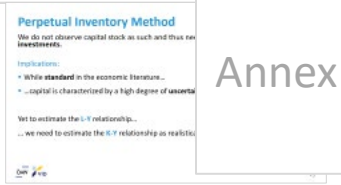

Bayesian time-FE model for each NACE sector to understand between-firm variation in labour productivity

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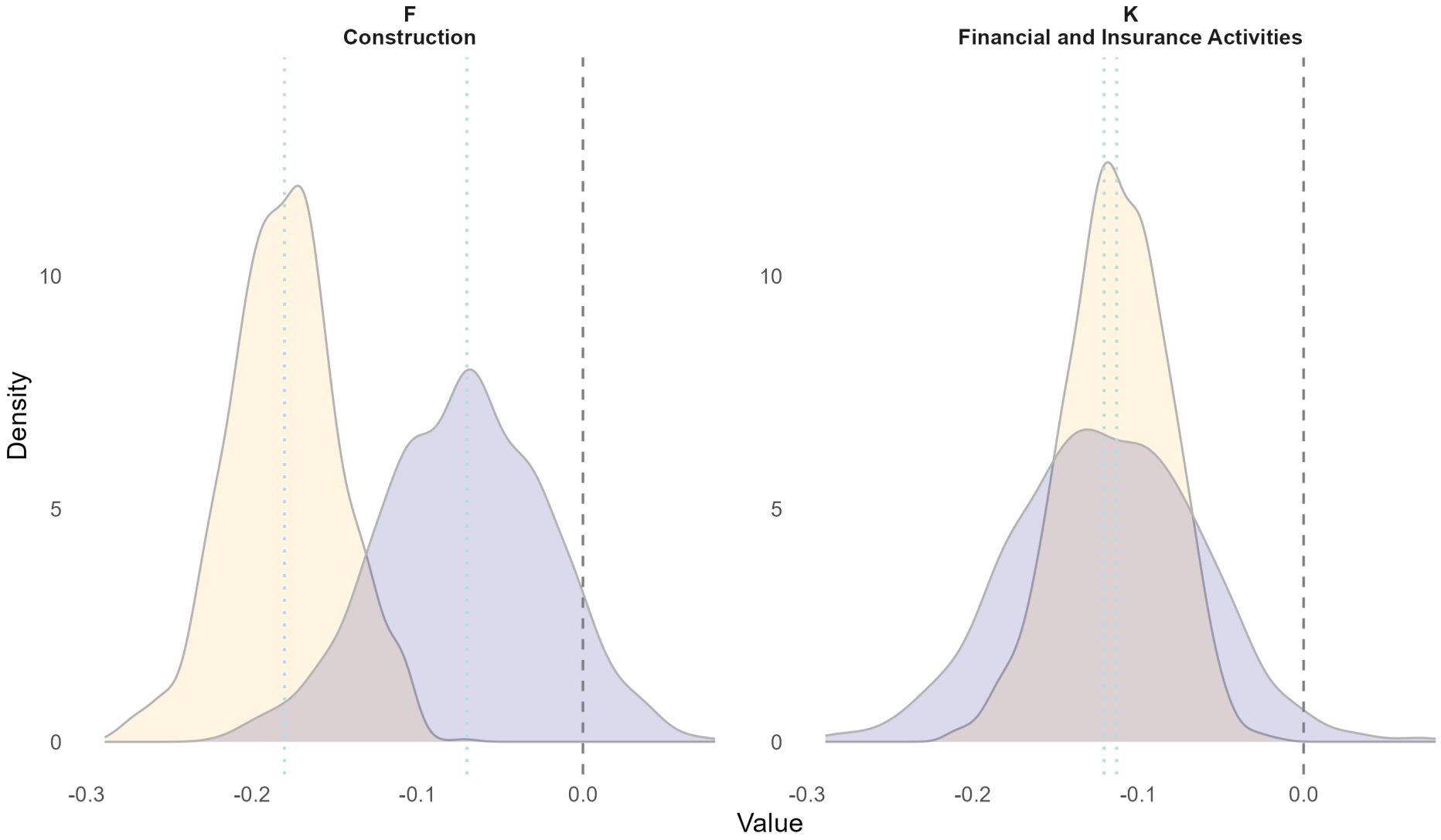
Posterior densities with medians

for the share of young and older workers



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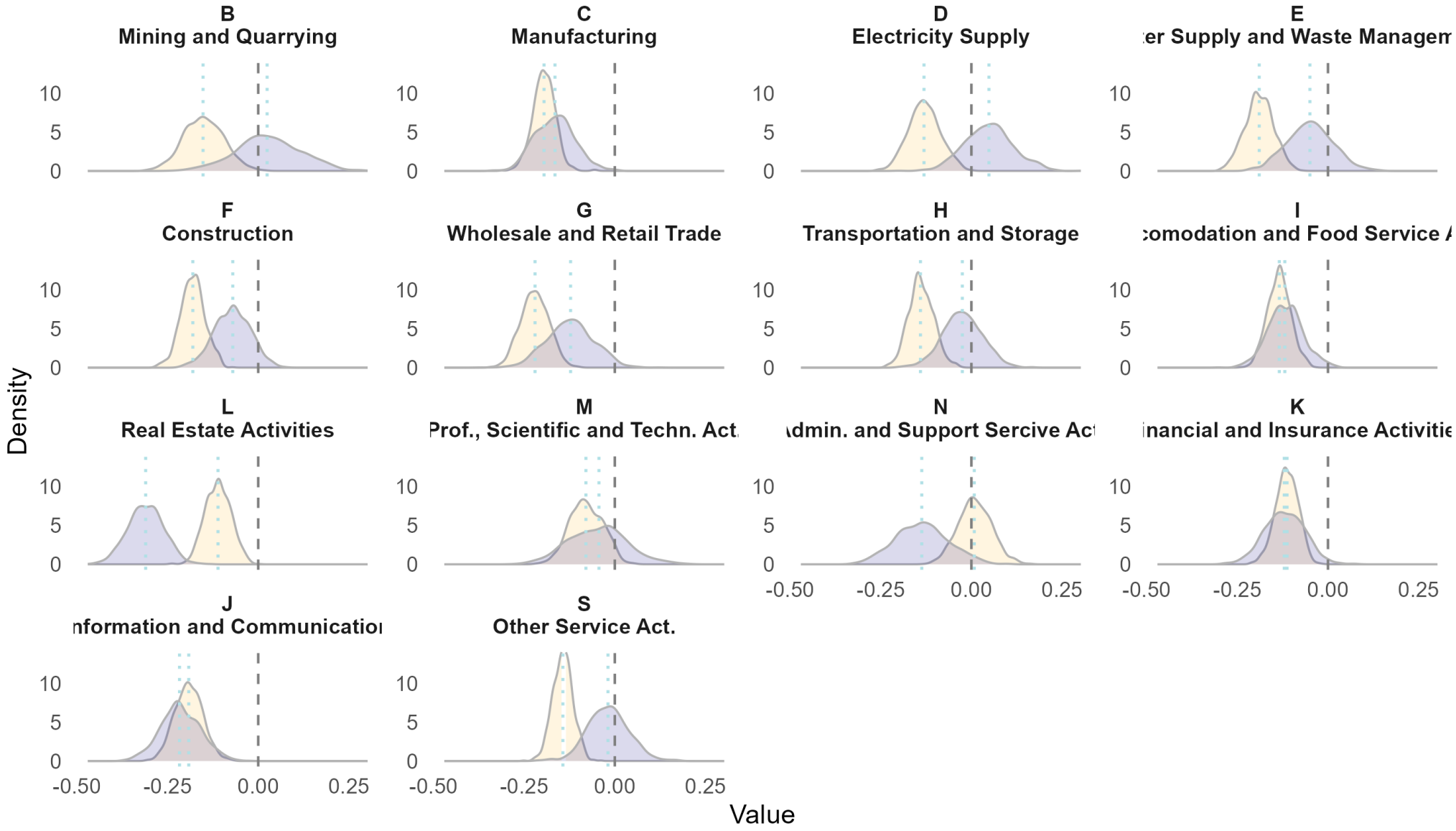
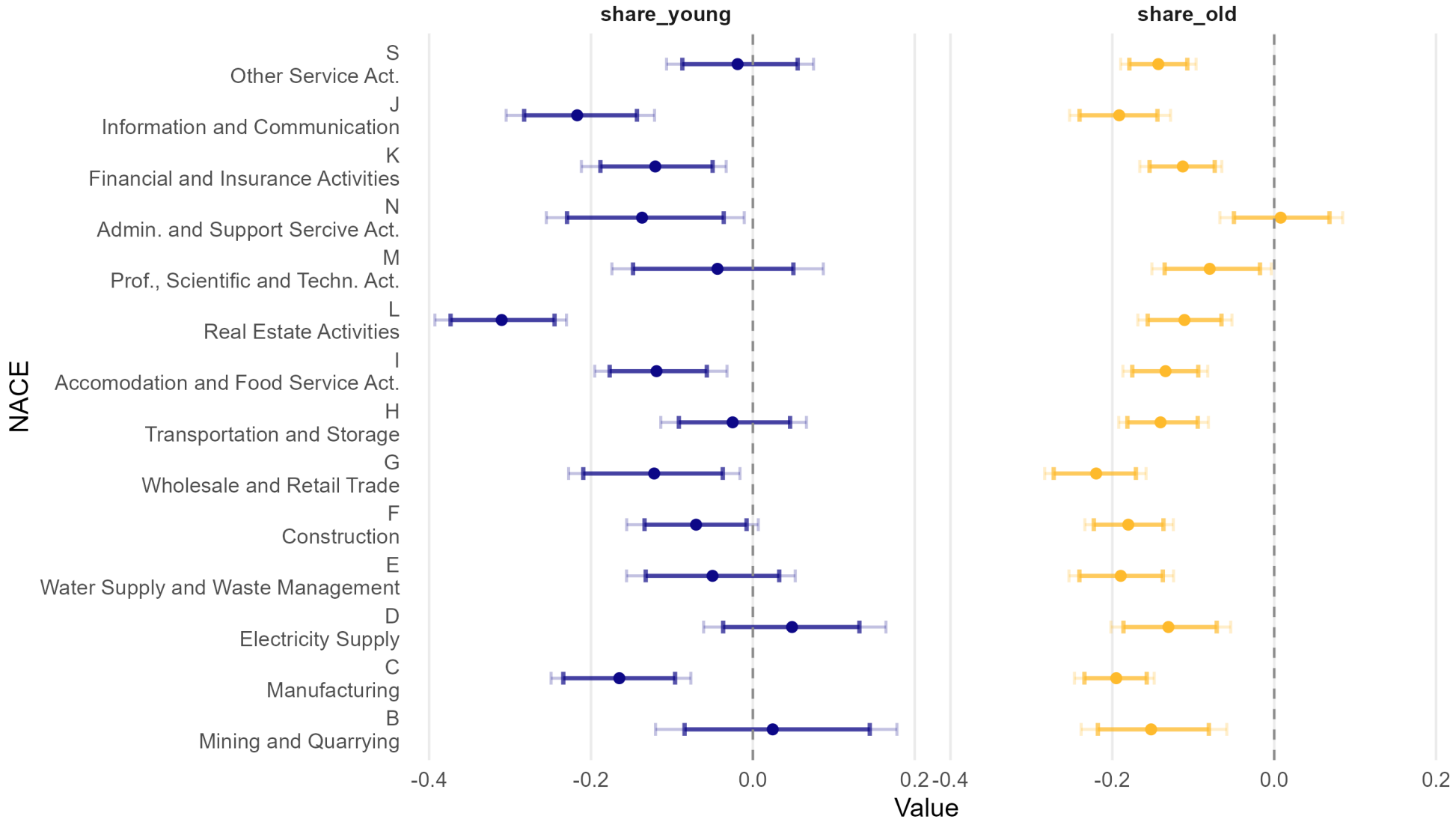


Posterior medians of the share of young and share of older workers by sec

with 80 and 90 percent credible intervals



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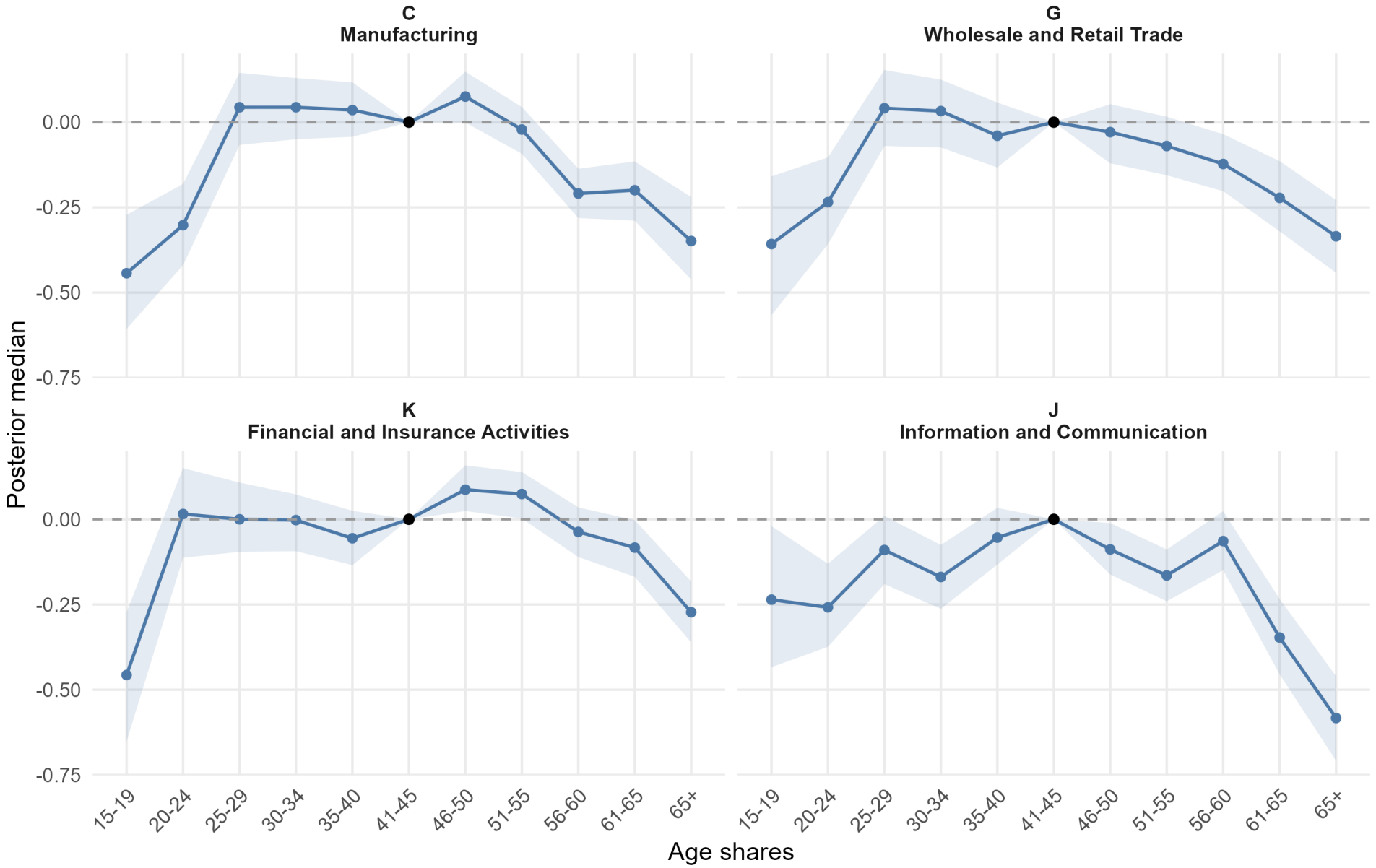
5-year age groups and productivity

relative to 40-45 year-olds benchmark, with 80 percent credible intervals



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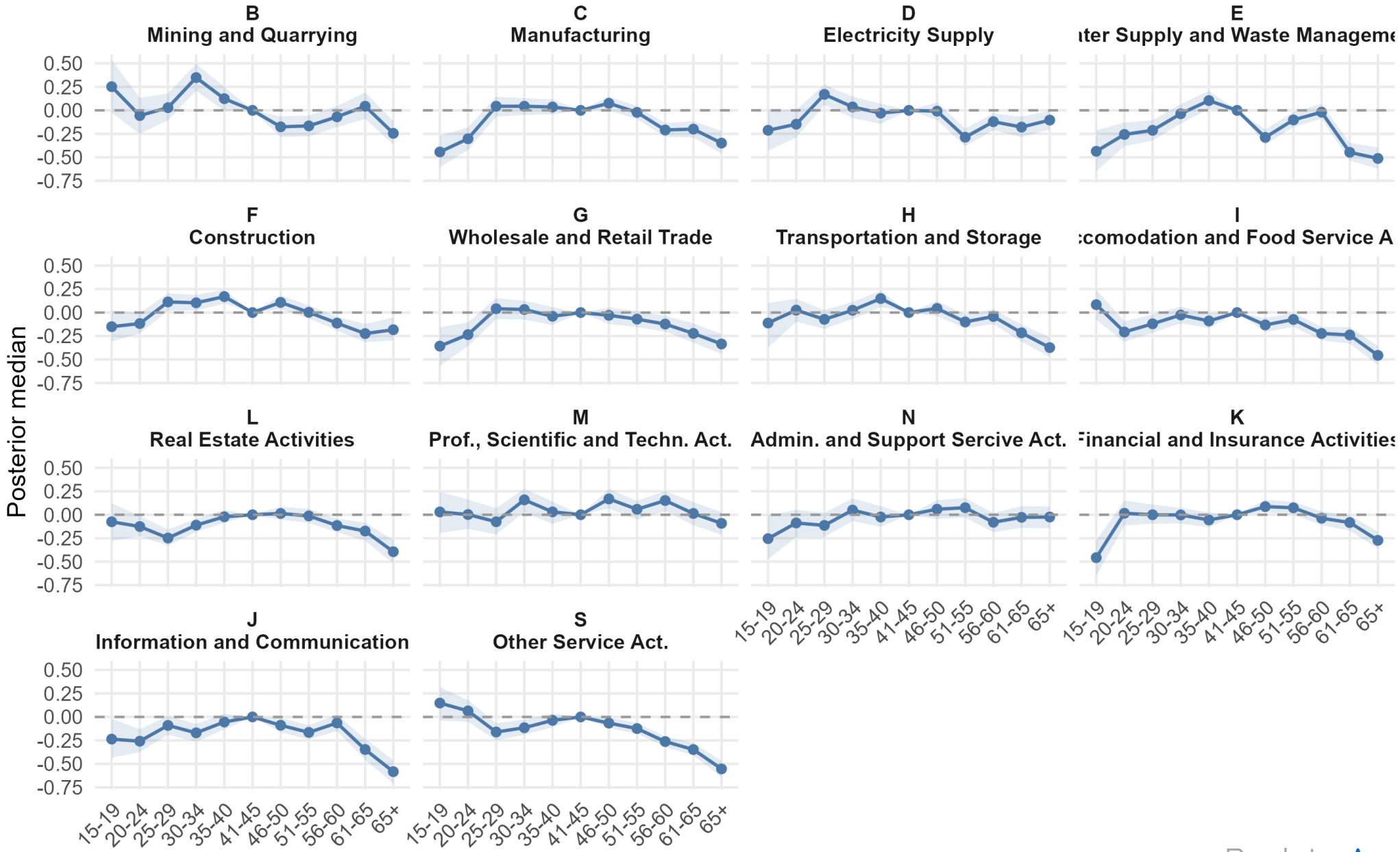
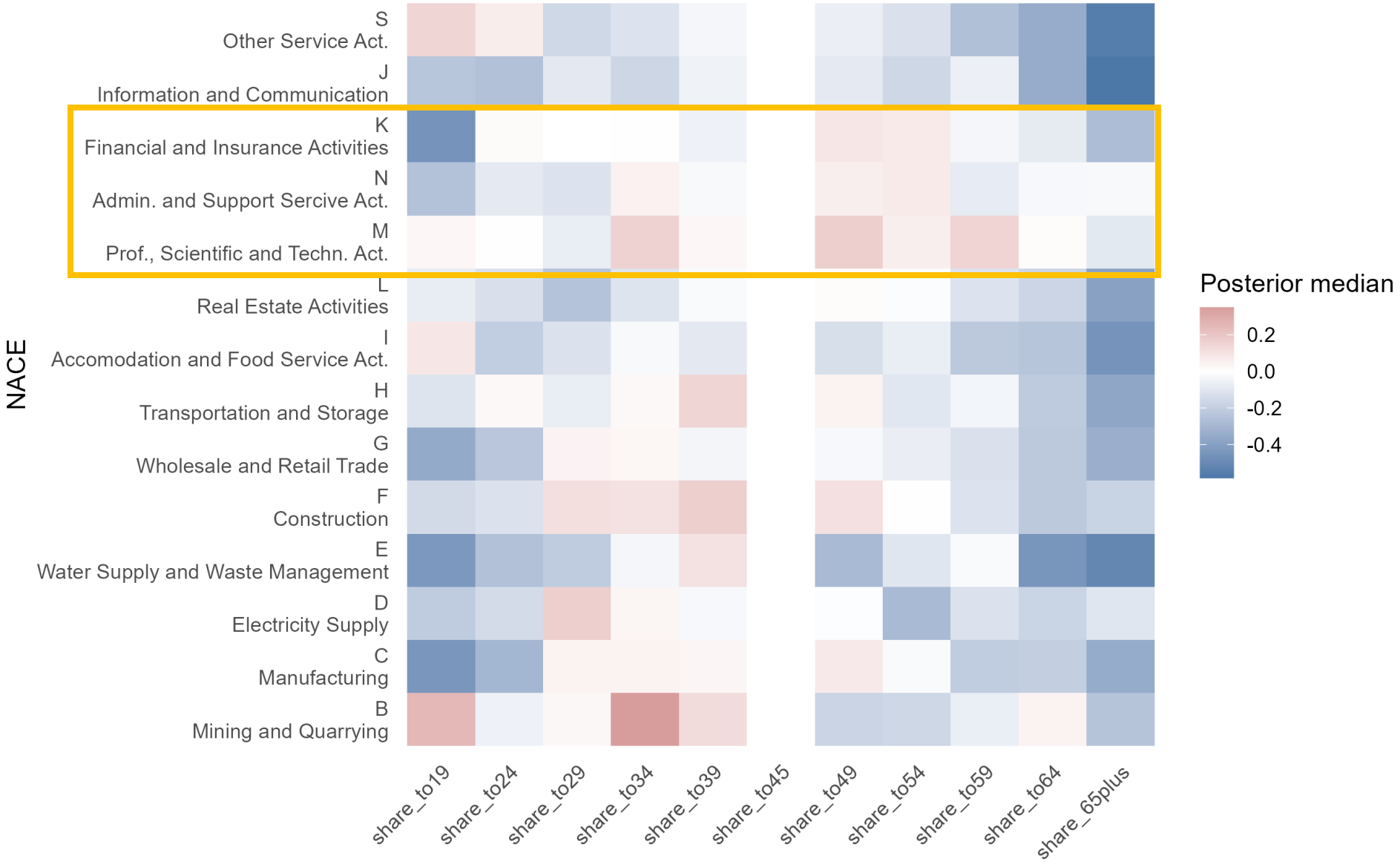


Age structure effects with benchmark

relative to 40-45 year-olds (benchmark)



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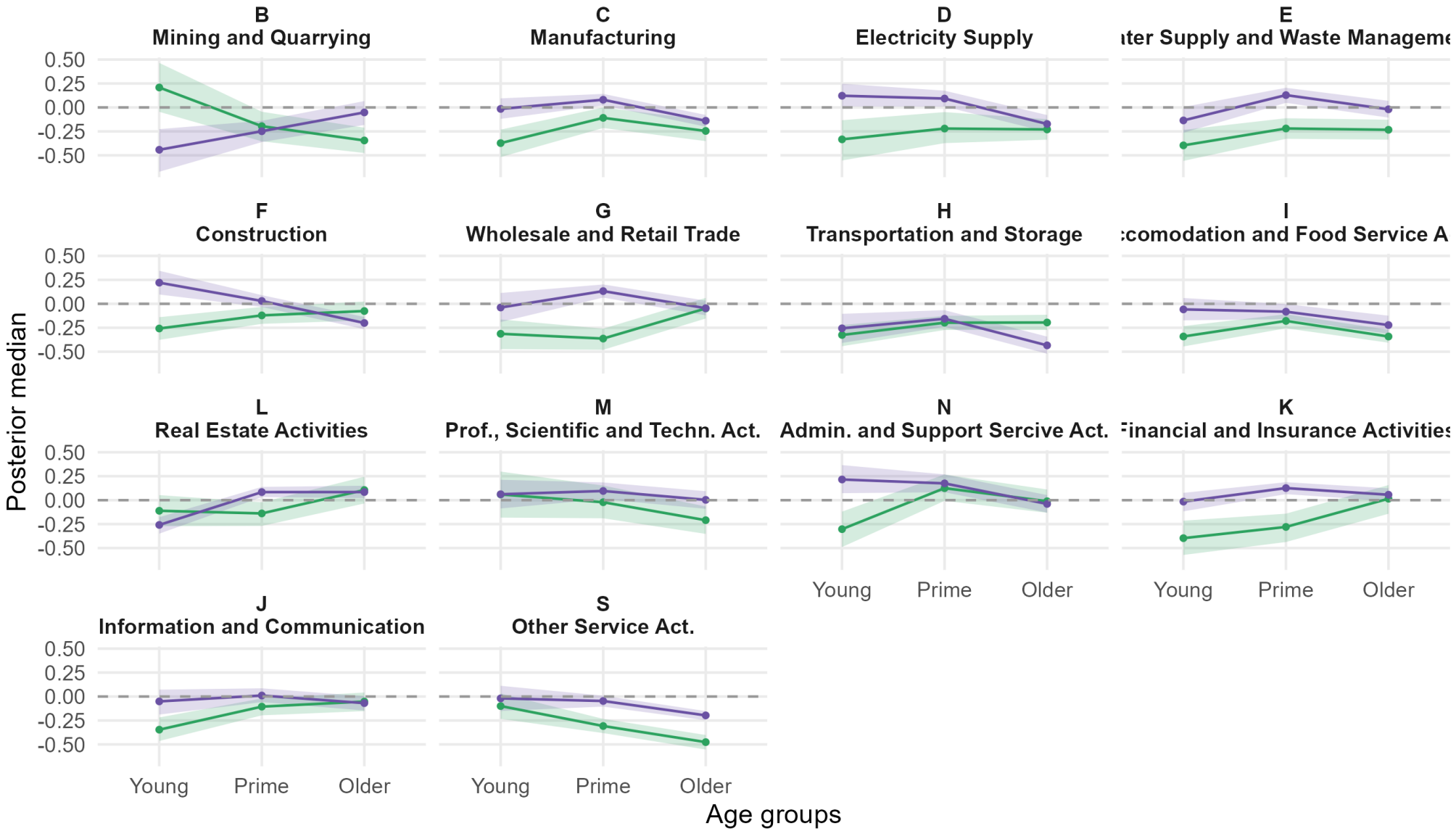
Posterior medians of joint age & education groups

benchmark: prime-aged with medium education; with 80 percent credible intervals



education Low education High education

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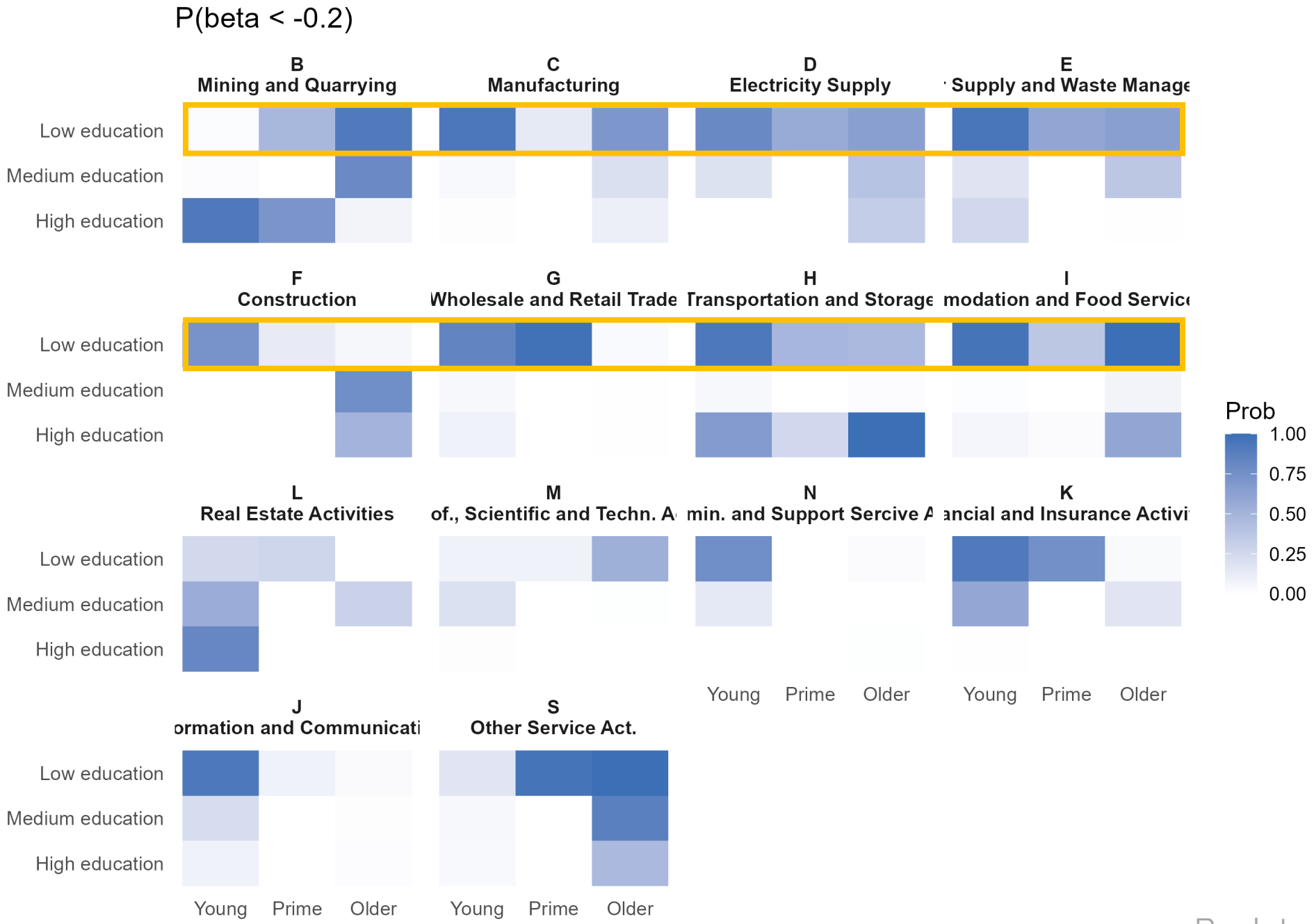

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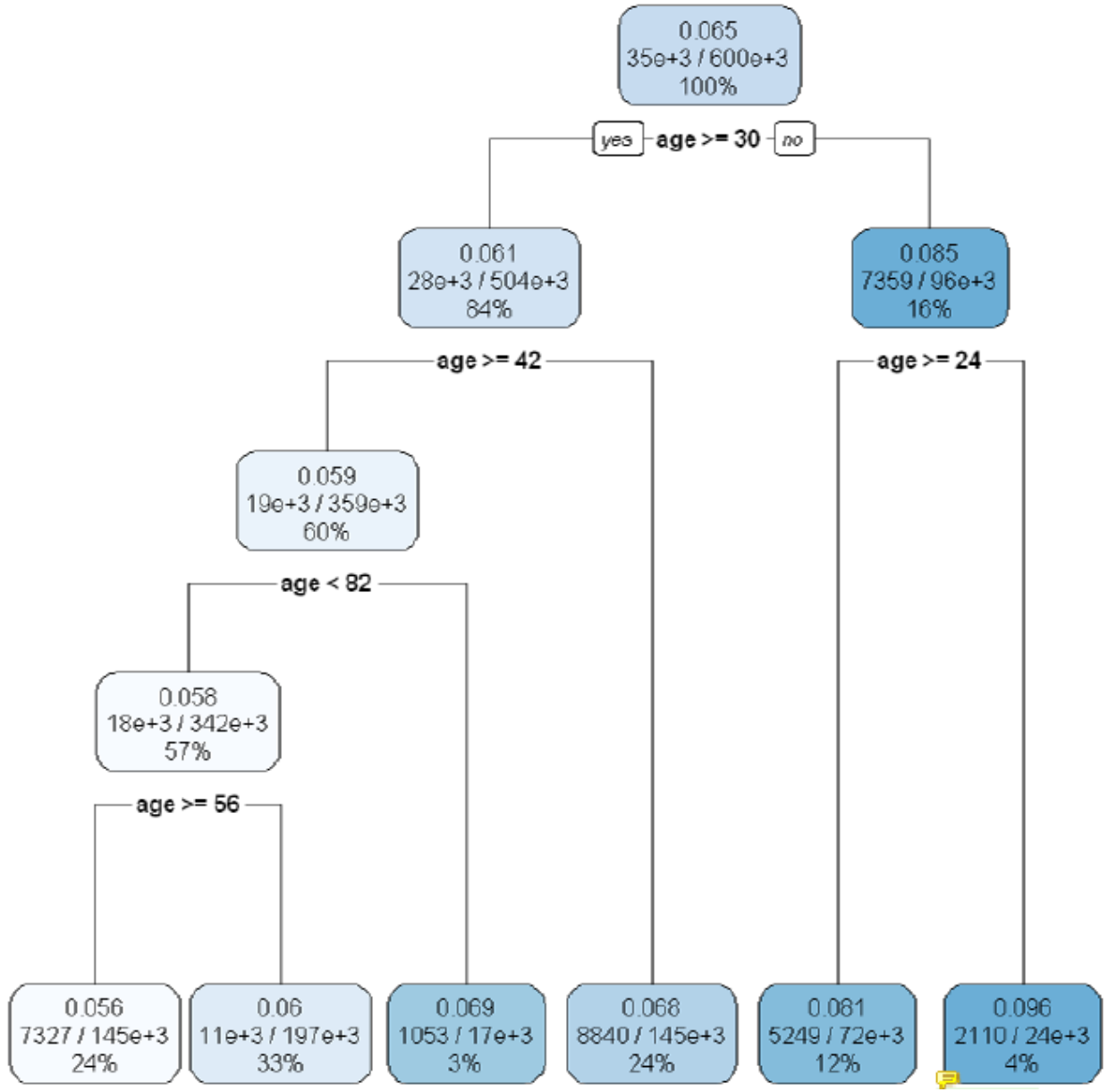

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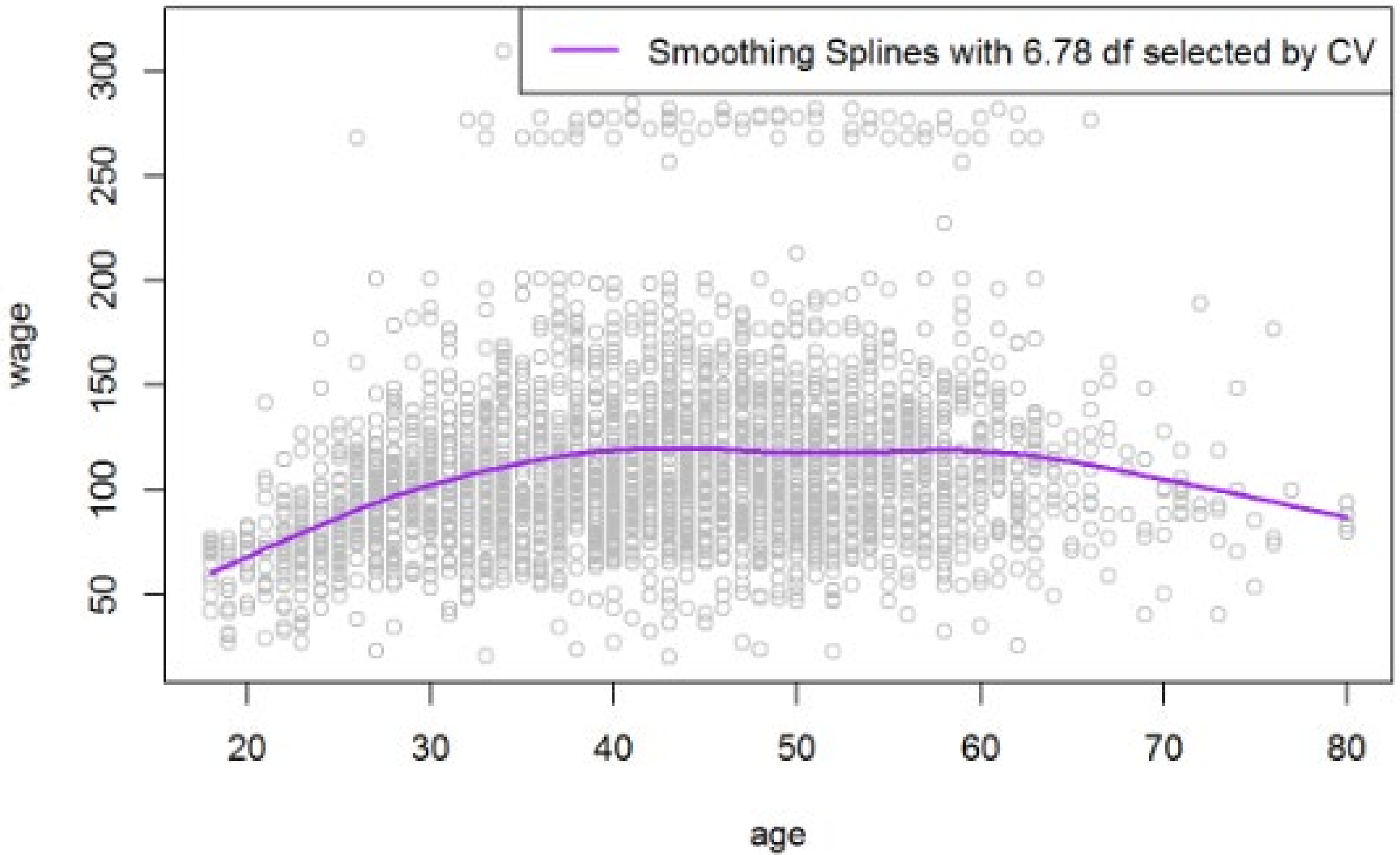
Smoothing Splines with 6.78 df selected by CV
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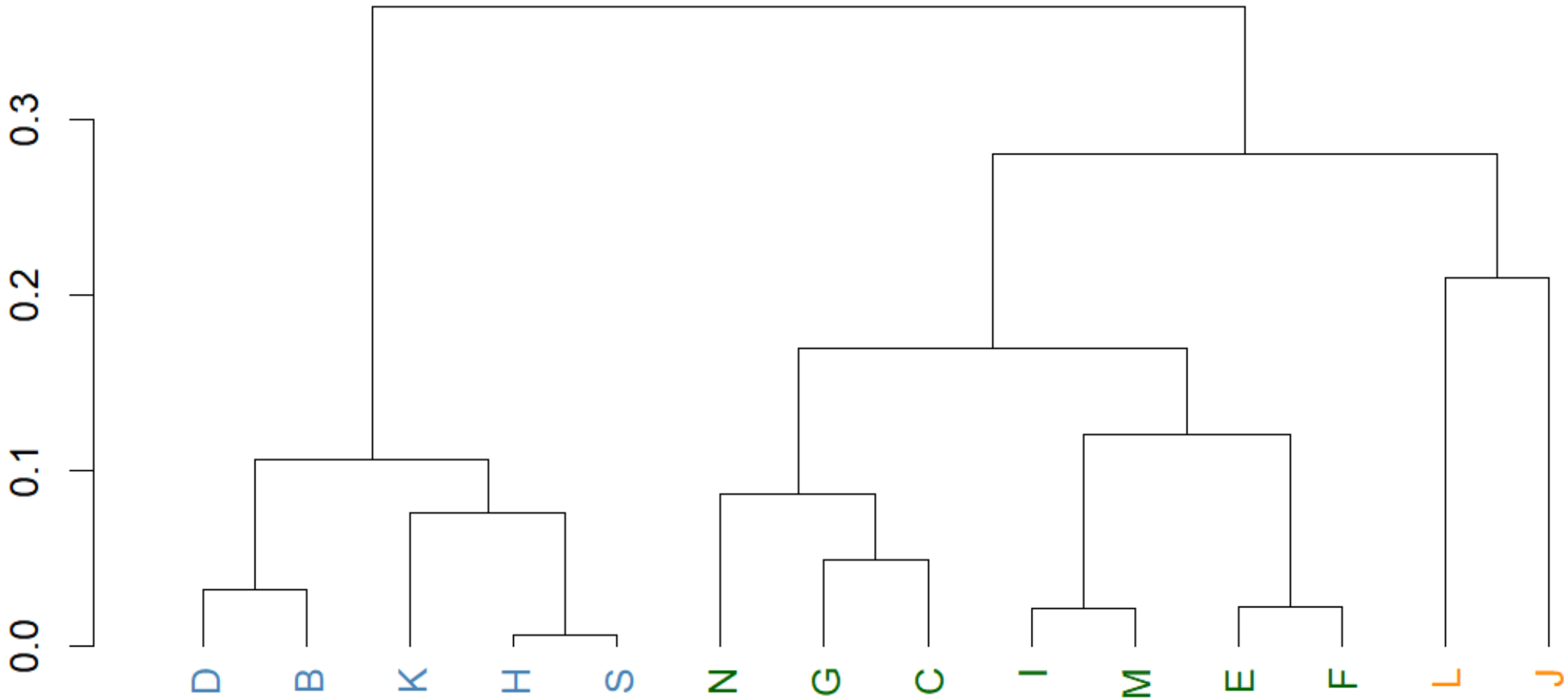
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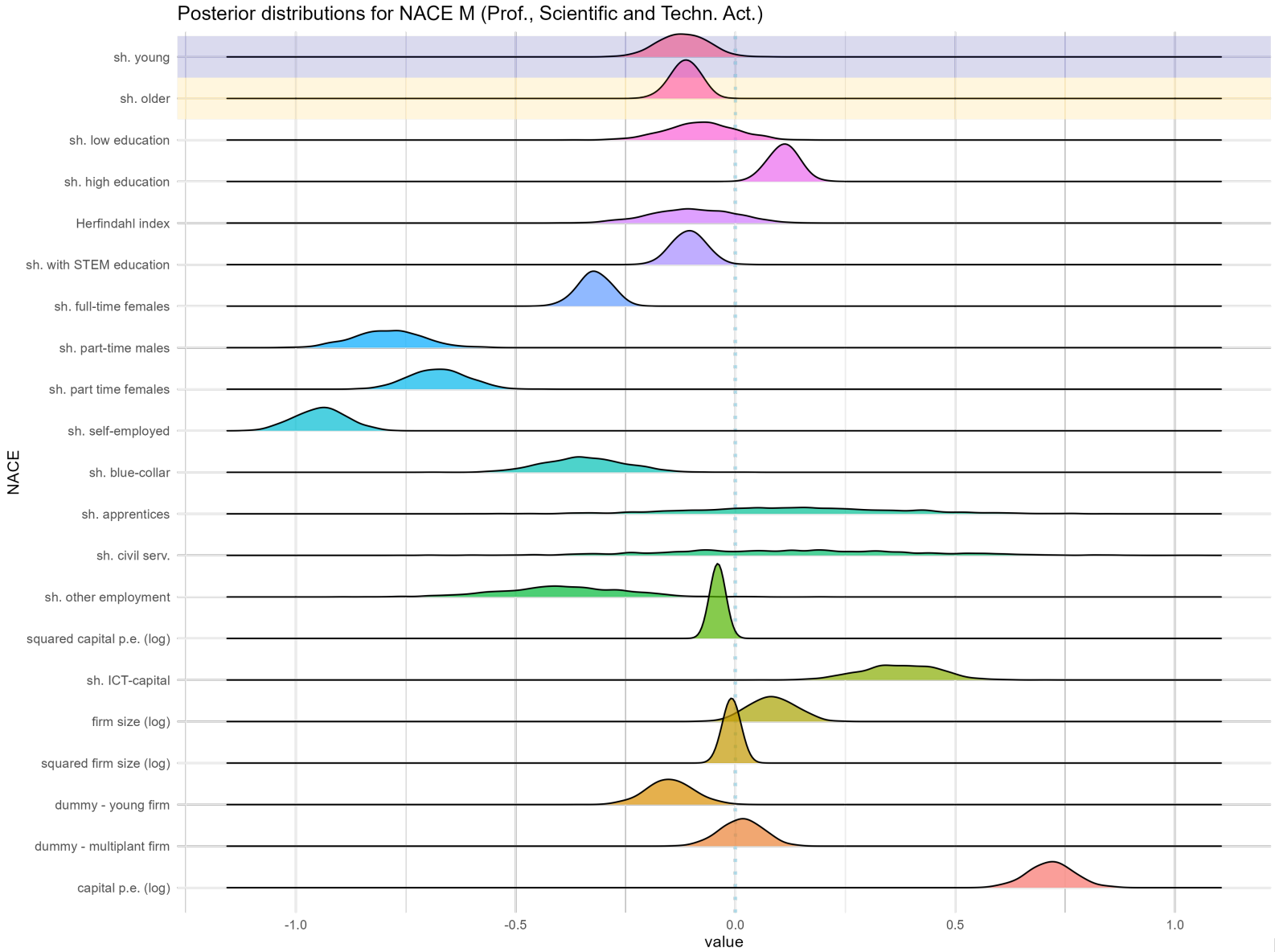
Results for NACE M



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Variables – Employee Characteristics

Category	Variable Name	Description / Notes
Dependent Variable	ln_val_add_e_d_AEST	Log of value added per employee
	L.ln_val_add_e_d_AEST	Lagged value added
Age Structure	share_young	Share of young (15–29 years)
	share_middle	Share of middle aged
	share_old	Share of old (50+ years)
	herf_age	Herfindahl index of age concentration
Birth Cohorts	sharecohort_generation_z	Share of Gen Z cohort (1997–2010)
	sharecohort_generation_y	Share of Gen Y cohort (1981–1996)
	sharecohort_generation_x	Share of Gen X cohort (1965–1980)
	sharecohort_baby_boomer	Share of Baby Boomer cohort (≤1964)
	share_kid15	Share of employees with children under 15
Education	share_loweducated	Share of low-educated employees (primary/lower secondary)
	share_mediumeducated	Share of medium-educated (upper secondary)
	share_higheducated	Share of highly educated employees (mainly tertiary)
Employment Composition	female_share_AEST	Female employee share
	part_time_share_AEST	Part-time employee share
	part_female_share_AEST	Part-time female employee share
	blue_collar_share_AEST	Blue-collar employee share
	white_collar_share_AEST	White-collar employee share
	apprentices_share_AEST	Apprentice share
	other_employed_share_AEST	Other employed share

Variables – Firm Characteristics

Category	Variable Name	Description / Notes
Firm Characteristics	ln_size_firm_AEST	Log of firm size
	ln_capital_total_e_d_AEST	Log of total capital per employee
	multiplant_dummy	Dummy for multi-plant firms
	young_firm_dummy	Dummy for young firms
	ICT_capital_share	Share of ICT capital
Legal Form Dummies	lf_AG_dummy	Corporation dummy
	lf_GEN_dummy	General partnership dummy
	lf_GES_dummy	Limited partnership dummy
	lf_partnership_dummy	Other partnership dummy
Industry Dummies (NACE)	nace_B_dummy	Mining and quarrying
	nace_C_dummy	Manufacturing
	nace_D_dummy	Electricity, gas, steam
	nace_E_dummy	Water supply, sewerage
	nace_F_dummy	Construction
	nace_H_dummy	Transportation and storage
	nace_I_dummy	Accommodation and food service
	nace_J_dummy	Information and communication
	nace_K_dummy	Financial and insurance activities
	nace_L_dummy	Real estate activities
	nace_M_dummy	Professional, scientific and technical services
	nace_N_dummy	Administrative and support services
	nace_S_dummy	Other service activities
Region Dummies	9 federal state dummies	Burgenland to Vorarlberg (NUTS-2)

Priors – Firm characteristics

Priors

- Log(capital/head) ~Gamma(8,20) (right skewed)
- Log(capital/head)^2 ~normal(-0.05, 0.1) (decr. returns to scale)
- ICT_capital_share ~normal(0.1, 0.1)
- Log(firmsize) ~normal(0.1, 0.2)
- Log(firmsize)^2 ~normal(-0.1, 0.2) (decr. returns to scale)
- Young_firm_dummy ~normal(-0.1, 0.2)
- Multiplant_dummy ~normal(0.05, 0.1)

Priors – Employee characteristics

Priors

- Share_young, share_old ~normal(0, 0.3)
- Herf_age ~normal(0, 0.3)
- Share_loweducated, share_higheducated ~normal(0, 0.3)
- Share_STEM_education ~normal(0, 0.3)
- Share_fullt_female ~normal(0, 0.2)
- Share_partt_male, share_partt_female ~normal(-0.1, 0.3)
- Self_employed_share, blue_collar_share, apprentices_share, other_empl_share ~normal(0, 0.3)

NACE sectors

- ÖNACE - Klassifikation der Wirtschaftstätigkeiten – WKO

nace	sector	nace_name	nace_name_english
B	production	Bergbau	Mining and Quarrying
C	production	Herstellung von Waren	Manufacturing
D	production	Energieversorgung	Electricity Supply
E	production	Wasserversorgung und Abfallentsorgung	Water Supply and Waste Management
F	production	Bautaetigkeiten	Construction
G	service	Gross- und Einzelhandel	Wholesale and Retail Trade
H	service	Verkehr und Lagerei	Transportation and Storage
I	service	Beherbergung und Gastronomie	Accomodation and Food Service Act.
J	ICT	Information und Kommunikation	Information and Communication
K	financial	Finanz- und Verischerungsdienstl.	Financial and Insurance Activities
L	service	Immobilien	Real Estate Activities
M	service	Freiberufliche/techn. Dienstl.	Prof., Scientific and Techn. Act.
N	service	Sonst. wirtschaftl. Dienstl.	Admin. and Support Sercive Act.
S	ICT	Sonst. Dienstl.	Other Service Act.

Results of Study by Firm Sizes

- inverse u-relationship between age and productivity…
  - … but only in micro (1-9 ind.) and small (10-49) firms
  - … and not in medium (50-249) or large (250+) firms

Limitations

- On the individual level: no information about
  - Experience
  - Skills
  - Tasks
  - Position within the firm
- On the firm level: no information about
  - capital stock observed
  - managerial practices
  - Technology usage