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Exploring SU(6) based 5 dimensional Grand Unification Theory

Alejandro Pierola Cervantes

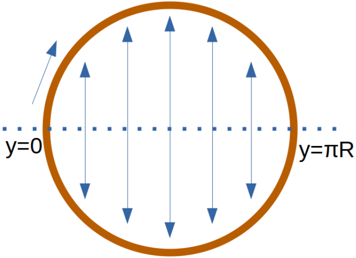
IN COLLABORATION WITH R.PASECHNIK, A.PREDA, G.CACCIAPAGLIA, W.ISNARD, A.DEANDREA...

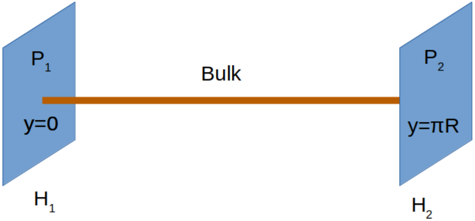
Why SU(6)?

- Previous work on SU(5) aGUT (G. Cacciapaglia et al., Phys. Rev. D **104**, 2021)
  - Yukawa couplings diverge
- SU(6) offers more freedom while staying minimal

Orbifold

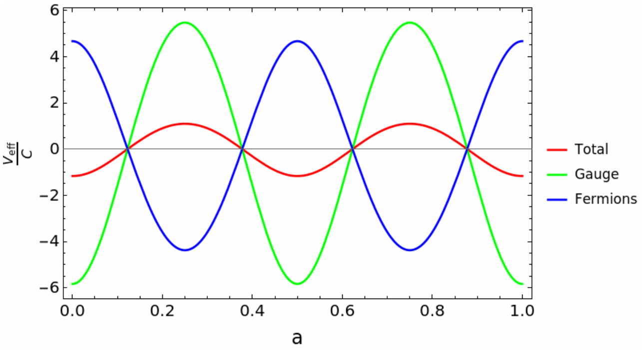
- Boundary conditions break SU(6) into H_1 and H_2
- 4D EFT $\subset H_1 \cap H_2$
- $m_{\mathrm{KK}} = 1/R$





Orbifold stability

- Gauge scalar: A_5
- One loop effective potential
- Stable orbifold if gauge contribution is minimum at $a = 0$



Field content

Why SU(6)?
Orbifold
Orbifold stability
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Phenomenology
Localised Yukawa interactions
Scalar sector
Anomalies, proton decay
Review
Extra slides

- SM fields embedded in SU(6) representations

- SU(6) representations can be decomposed:

$$\mathrm{F} = \mathbf{6} \to (\mathbf{3},\mathbf{1})_{-1/3,1/2} \oplus (\mathbf{1},\mathbf{1})_{0,-5/2} \oplus (\mathbf{1},\mathbf{2})_{1/2,1/2}$$

$$\mathrm{A} = \mathbf{15} \to (\bar{\mathbf{3}},\mathbf{1})_{-2/3,1} \oplus (\mathbf{1},\mathbf{1})_{1,1} \oplus (\mathbf{1},\mathbf{2})_{1/2,-2} \oplus (\mathbf{3},\mathbf{2})_{1/6,1} \oplus (\mathbf{3},\mathbf{1})_{-1/3,-2}$$

$$\overline{\mathrm{A}} = \overline{\mathbf{15}} \to (\mathbf{3},\mathbf{1})_{2/3,-1} \oplus (\mathbf{1},\mathbf{1})_{-1,-1} \oplus (\mathbf{1},\mathbf{2})_{-1/2,+2} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-1/6,-1} \oplus (\bar{\mathbf{3}},\mathbf{1})_{1/3,2}$$

- Minimal aGUT field content:

$$\psi_{15}^{(+,-)} \supset \mathrm{q}_L + \mathrm{l}_L^c \qquad \psi_{\overline{15}}^{(-,-)} \supset \mathrm{u}_R + \mathrm{e}_R + \mathrm{d}_R^c \qquad \phi_{15}^{(-,+)} \supset \varphi_H'$$

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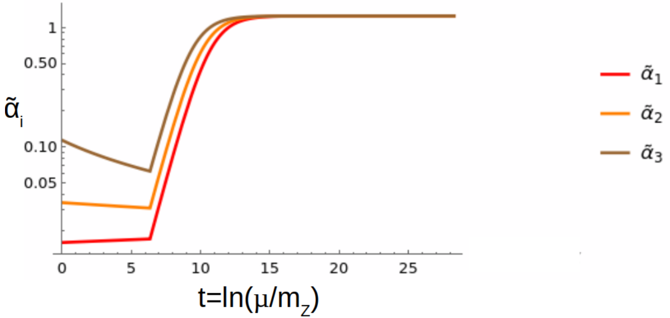
Phenomenology

Gauge RG flow

- Presents unified UV fixed point



- Running changes at $m_{\text{KK}} = 1/R$
- Extra loop factor: $\xi(d) = \frac{\Omega(d)}{(2\pi)^d} 4\pi\tilde{\alpha} \approx 0.05$

Yukawa RG flow

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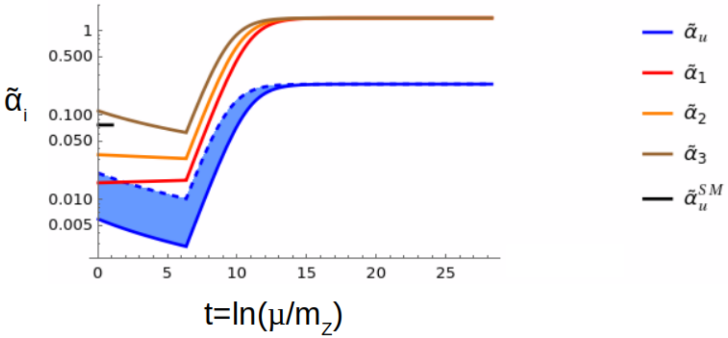
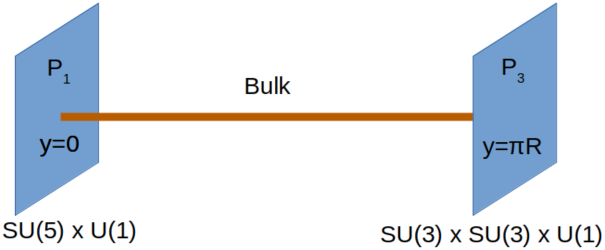


- Solution: Localised Yukawa interactions

Localised Yukawa interactions

- Couplings localised on a boundary
- $y = \pi R$ is the better choice



Scalar sector

- Localised Yukawa interactions require two scalar fields:

$$\Phi_1 \to H_1 + S_1^- \qquad \Phi_2 \to H_2 + S_2^0 + T$$

- Scalar potential:

$$V = m_1^2 \Phi_1^\dagger \Phi_1 + \frac{m_2^2}{2}\Phi_2^2 + \mu \Phi_1^\dagger \Phi_2 \Phi_1 + \eta\, \mathrm{tr}\Phi_2^3 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 \mathrm{tr}(\Phi_2^2)^2 + \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)\mathrm{tr}(\Phi_2^2) + \lambda_4 \Phi_1^\dagger \Phi_2^2 \Phi_1$$

Work in progress...

Generate full 2HDM with RG flow

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Anomalies, proton decay

- Green-Schwarz mechanism cancels anomalies
- Bulk fields:

$$\psi_{15}^{(+,-)} \supset \mathrm{q}_L + \mathrm{l}_L^c \qquad \psi_{\overline{15}}^{(-,-)} \supset \mathrm{u}_R + \mathrm{e}_R + \mathrm{d}_R^c$$

- Quarks and leptons have different chirality, no proton decay
- Can bring compactification scale down to $m_{\mathrm{KK}} \sim 30 - 40$ TeV

Review

- Minimal, stable SU(6) aGUT
- Can embed SM fields in SU(6) reps.
- Gauge couplings flow towards an UV fixed point
- Yukawa interactions are localised on the boundary
- Scalar sector WIP
- Anomalies cancel and no proton decay

Extra slides

One loop effective potential

- Gauge-scalar $\longleftrightarrow$ scalar potential

$$V_{\text{eff}}/C = -3\nu_{\text{R}_\text{G}} + 4\sum_\text{f} \nu_{\text{R}_\text{f}} - \sum_\text{s} x_\text{s} \nu_{\text{R}_\text{s}} \,, \quad \nu_\text{R} = \sum_i \mathscr{F}^+(a_i) + \sum_j \mathscr{F}^-(a_j) \,,\ a \propto \text{VEV}$$

$$\mathscr{F}^+(a) = \frac{3}{2}\sum_{n=1}^{\infty} \frac{\cos(2n\pi a)}{n^5} \,, \qquad \mathscr{F}^-(a) = \frac{3}{2}\sum_{n=1}^{\infty} (-1)^n \frac{\cos(2n\pi a)}{n^5}$$

- VEV not at 0 breaks the orbifold!

Stability criterion [Cacciapaglia et al., PRD, 2025]

Absolute minimum has to be at $a = 0$

Parity selection

- SU(6) gauge group:

$$P_1 = \mathrm{diag}(+,+,+,+,+,-) \qquad \mathrm{SU}(6) \to \mathrm{SU}(5) \times \mathrm{U}(1)$$
$$P_2 = \mathrm{diag}(+,+,+,-,-,+) \qquad \mathrm{SU}(6) \to \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$
$$P_3 = \mathrm{diag}(+,+,+,-,-,-) \qquad \mathrm{SU}(6) \to \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)$$

- Different combinations:

$P_1 \times P_2$ $\qquad$ $P_1 \times P_3$ $\qquad$ $P_2 \times P_3$

$$P_1 \times P_3 \downarrow \text{Minimal} + \text{stable}$$

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Gauge RGE

$$2\pi \frac{d\alpha_i}{dt} = b_i^{\mathrm{SM}} \alpha_i^2 + (S(t)-1) b_5 \alpha_i^2 \qquad S(t) = \begin{cases} 1 & : \mu < 1/R \\ \mu R = m_z R e^t & : \mu > 1/R \end{cases}$$

$$b_5 = -\left(\frac{11}{3} - \frac{1}{6}\right) C_2(\mathrm{Adj}) + \frac{4}{3} \sum_f T(R_f) + \frac{1}{3} \sum_s T(R_s)$$

- Effective 't Hooft coupling:

$$\tilde{\alpha}_i = \begin{cases} \alpha_i & : \mu < \frac{1}{R} \\ \alpha_i \mu R & : \mu > \frac{1}{R} \end{cases}$$

- Full equation:

$$2\pi \frac{d\tilde{\alpha}_i}{dt} = \begin{cases} b_i^{\mathrm{SM}} \tilde{\alpha}_i^2 & : \mu < 1/R \\ 2\pi \tilde{\alpha}_i + b_5 \tilde{\alpha}_i^2 + \underbrace{\frac{1}{\mu R}\left(b_i^{\mathrm{SM}} \tilde{\alpha}_i^2 - b_5 \tilde{\alpha}_i^2\right)}_{O\left(\frac{1}{\mu R}\right)} & : \mu > 1/R \end{cases}$$

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Perturbative gauge fixed point?

- Fixed point at $\tilde{\alpha}_i^* = -\frac{2\pi}{b_5} = -\frac{6\pi}{16n_g - 61} \underset{n_g=3}{\approx} 1.45 > 1$
- We are working with a perturbative theory, but the gauge fixed point $\tilde{\alpha}_i^* > 1$. Is this catastrophic?
- Have to add an additional loop factor due to the extra dimension:

$$\xi(d) = \frac{\Omega(d)}{(2\pi)^d} 4\pi\tilde{\alpha}$$

- Using $\Omega(d) = \frac{2\pi^{d/2}}{\Gamma(d/2)}$ with $d = 5$ yields:

$$\xi_i^*(5) = \frac{2}{13\pi} \approx 0.049 < 1$$

Yukawa RGE

$$2\pi\frac{d\alpha_y}{dt} = \underbrace{\left(\sum_{y'} a_{yy'}\alpha_{y'}\right)\alpha_y}_{\text{SM part}} + (S(t)-1)\left[\left(\sum\nolimits_{y'} c_{yy'}\alpha_{y'}\right) - d_y\alpha_5\right]\alpha_y$$

$$\mathrm{S(t)}=\begin{cases} 1 & : \mu < 1/R \\ \mu R = m_z R e^t & : \mu > 1/R \end{cases}, \qquad \tilde{\alpha}_i = \begin{cases} \alpha_i & : \mu < \frac{1}{R} \\ \alpha_i \mu R & : \mu > \frac{1}{R} \end{cases}$$

- Full equation:

$$2\pi\frac{d\tilde{\alpha}_y}{dt} = \begin{cases} \left(\sum\nolimits_{yy'} a_{yy'}\tilde{\alpha}_{y'}\right)\tilde{\alpha}_y & : \mu < 1/R \\ \left(2\pi + \sum\nolimits_{y'} c_{yy'}\tilde{\alpha}_{y'} - d_y\tilde{\alpha}_5\right)\tilde{\alpha}_y + \\ \frac{1}{\mu R}\left[\underbrace{\left(\sum_{y'} a_{yy'}\tilde{\alpha}_{y'}\right)\tilde{\alpha}_y}_{\text{SM part}} - \left(\sum\nolimits_{y'} c_{yy'}\tilde{\alpha}_{y'} - d_y\tilde{\alpha}_5\right)\tilde{\alpha}_y\right] & : \mu > 1/R \end{cases}$$

Kaluza-Klein modes

[Dienes et al., PRL, 2003]

- 5D fields can be decomposed as follows:

$$\phi = \phi_+ + \phi_-$$

where

$$\phi_+(x^\mu, y) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right)$$

$$\phi_-(x^\mu, y) = \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right)$$

- $\phi_\pm^{(n)}$ are 4D fields with mass $\frac{n}{R}$
- $\phi_+^{(0)}$ is massless: “zero mode"