5D Unified Theories: from general principles to an SU(6) example

Anca Preda

in collaboration with G. Cacciapaglia, A. Deandrea, W. Isnard, R. Pasechnik and Z. Wang

LUND UNIVERSITY

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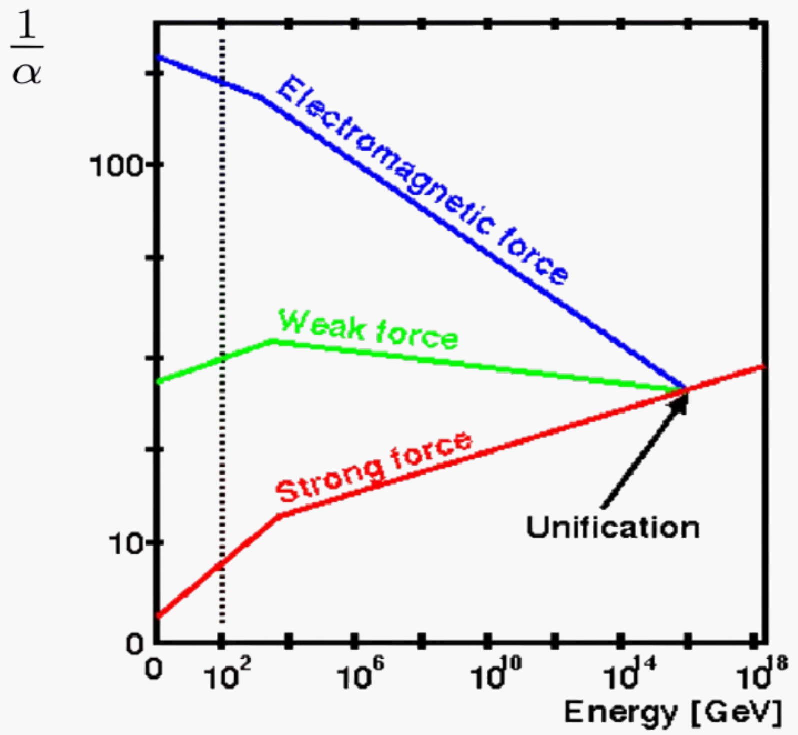
What happens in 5D?

Asymptotic Grand Unification Theories (GUTs)

4D theories

standard idea of gauge coupling unification

SU(5), SO(10) ...



Asymptotic Grand Unification Theories (aGUTs)

extra dimensional theories (5D)

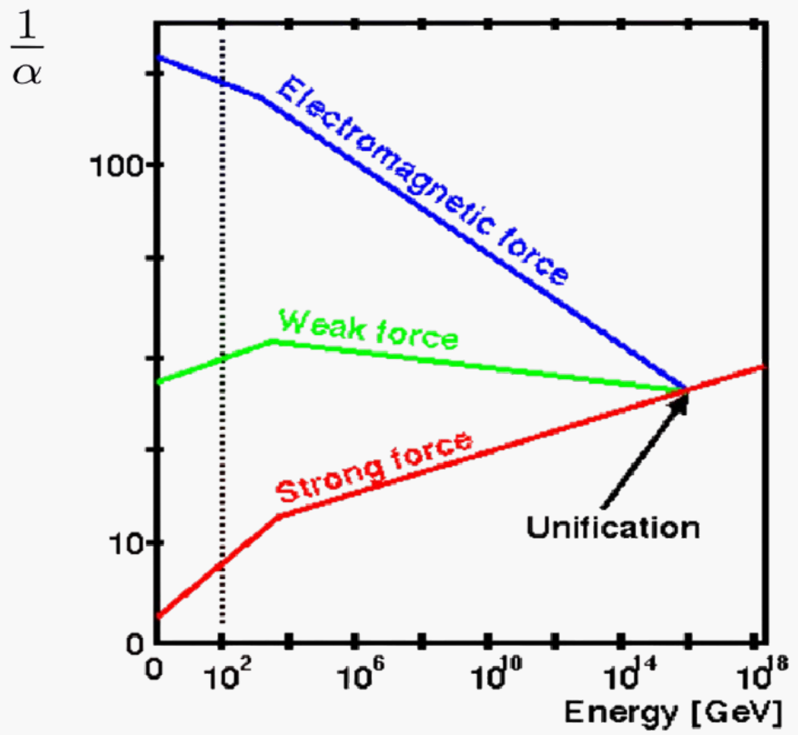
same models SU(5), SO(10) ...

different construction

4D theories

standard idea of gauge coupling unification

SU(5), SO(10) ...



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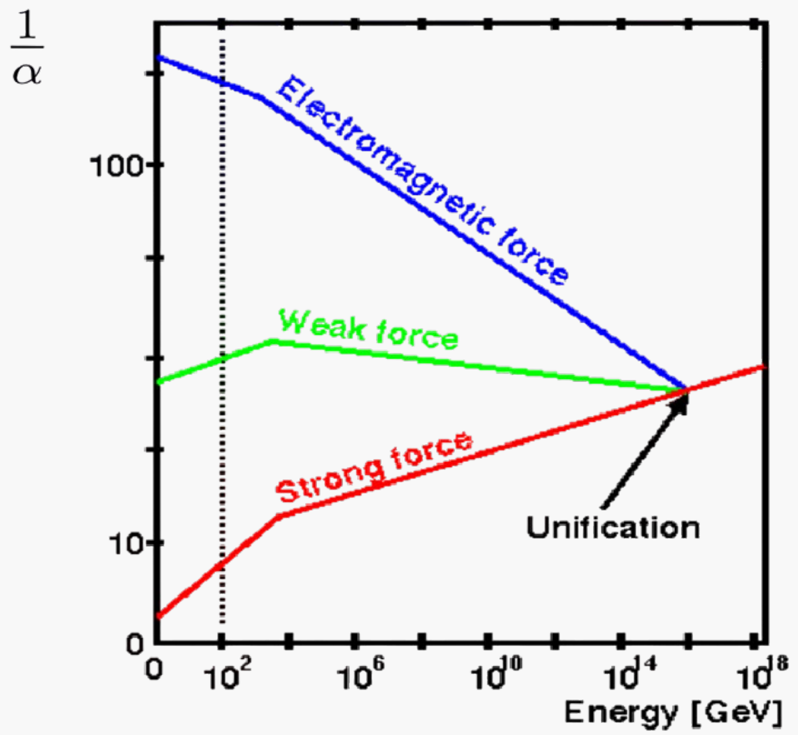
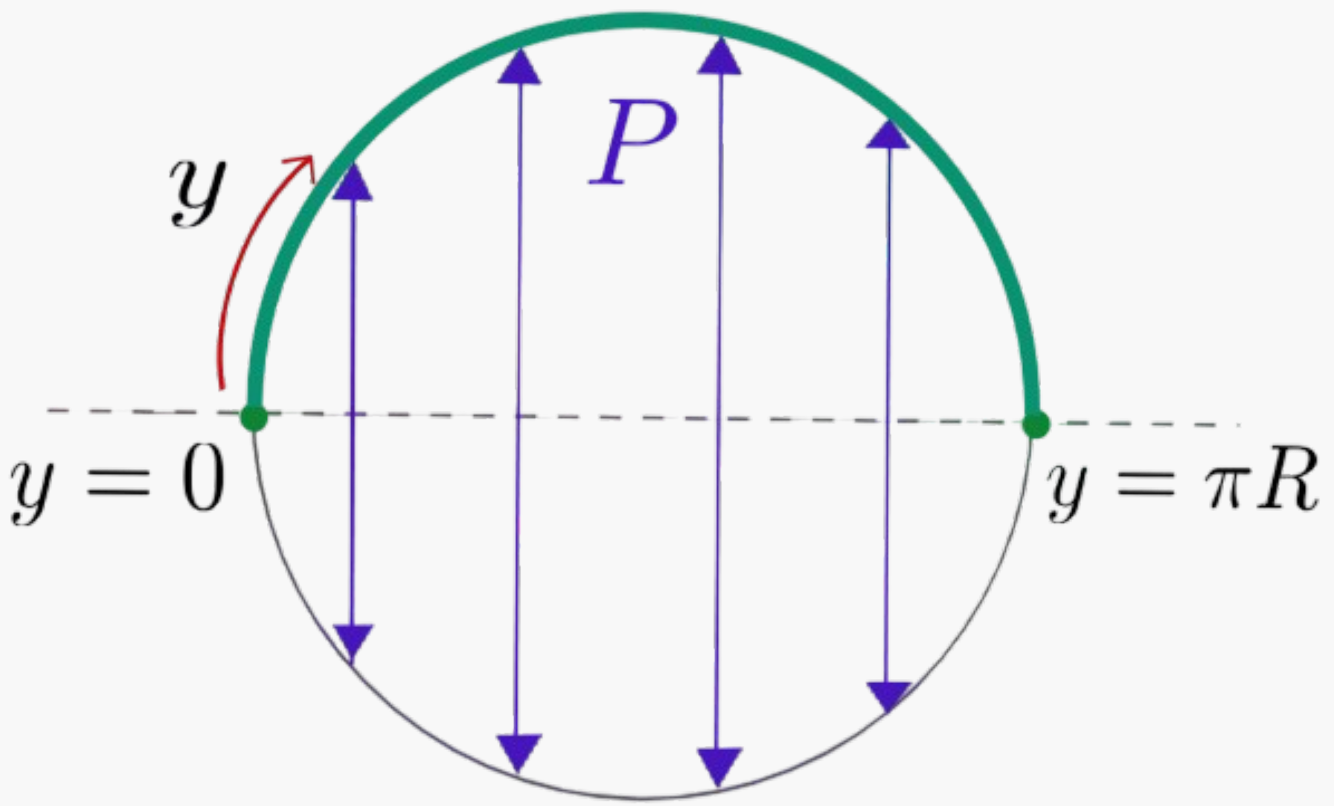
Why?

- lower unification scale
- less parameters
- hierarchy problem (?)



Asymptotic GUTs

- **5D models**: one extra dimension [1] compactified on $K = S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$



[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

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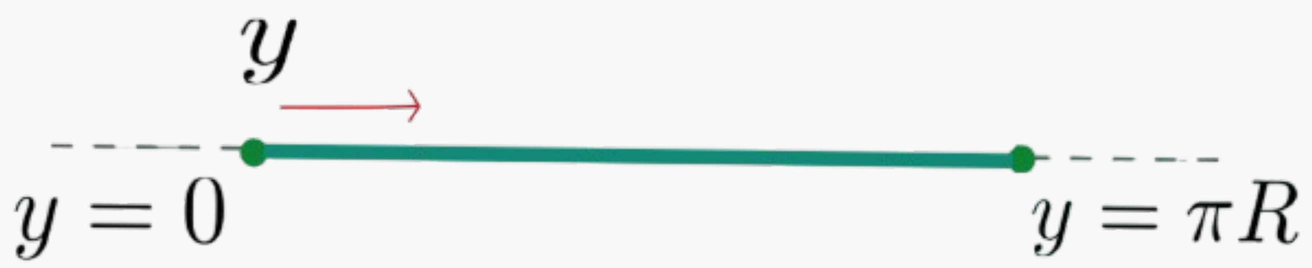

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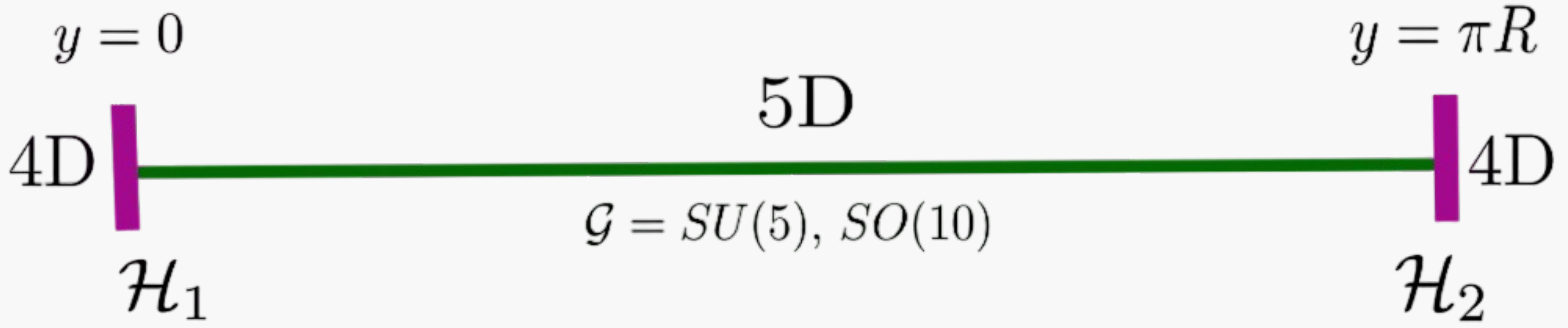

- Symmetry is broken via orbifolding ($\mathcal{G} \to \mathcal{H}_i$ on each boundary [2])

$$\mathcal{G}_{\text{4D}} \equiv \mathcal{H}_i \cap \mathcal{H}_j \supset \mathcal{G}_{\text{SM}}$$

remnant 4D theory

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

General principles:

1. Understanding the vacuum of the theory

Asymptotic GUTs – Orbifold stability

5D field

$\underbrace{A_\mu (\mu = 1 \dots 4)}_{\text{4D}}$ and $\underbrace{A_5}_{\text{extra dimension}}$

A_5 behaves as a scalar field in 4D

$\equiv$ **Higgs field**

→ Study the vacuum!

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Study the vacuum!

set of criteria: orbifold stability [3]

$$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2 \quad \text{only if} \quad p \geq N/2$$

[3] G. Cacciapaglia, A. Deandrea, A. Cornell, W. Isnard, R. Pasechnik, **AP** and Z. Wang, Phys. Rev. D 111, (2025) & EPJC 85, (2025)

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examples:

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$
$$SU(6) \to SU(3) \times SU(2) \times U(1)^2$$

[3] G. Cacciapaglia, A. Deandrea, A. Cornell, W. Isnard, R. Pasechnik, **AP** and Z. Wang, Phys. Rev. D 111, (2025) & EPJC 85, (2025)

General principles:

2. Are the theories finite (renormalizable)?

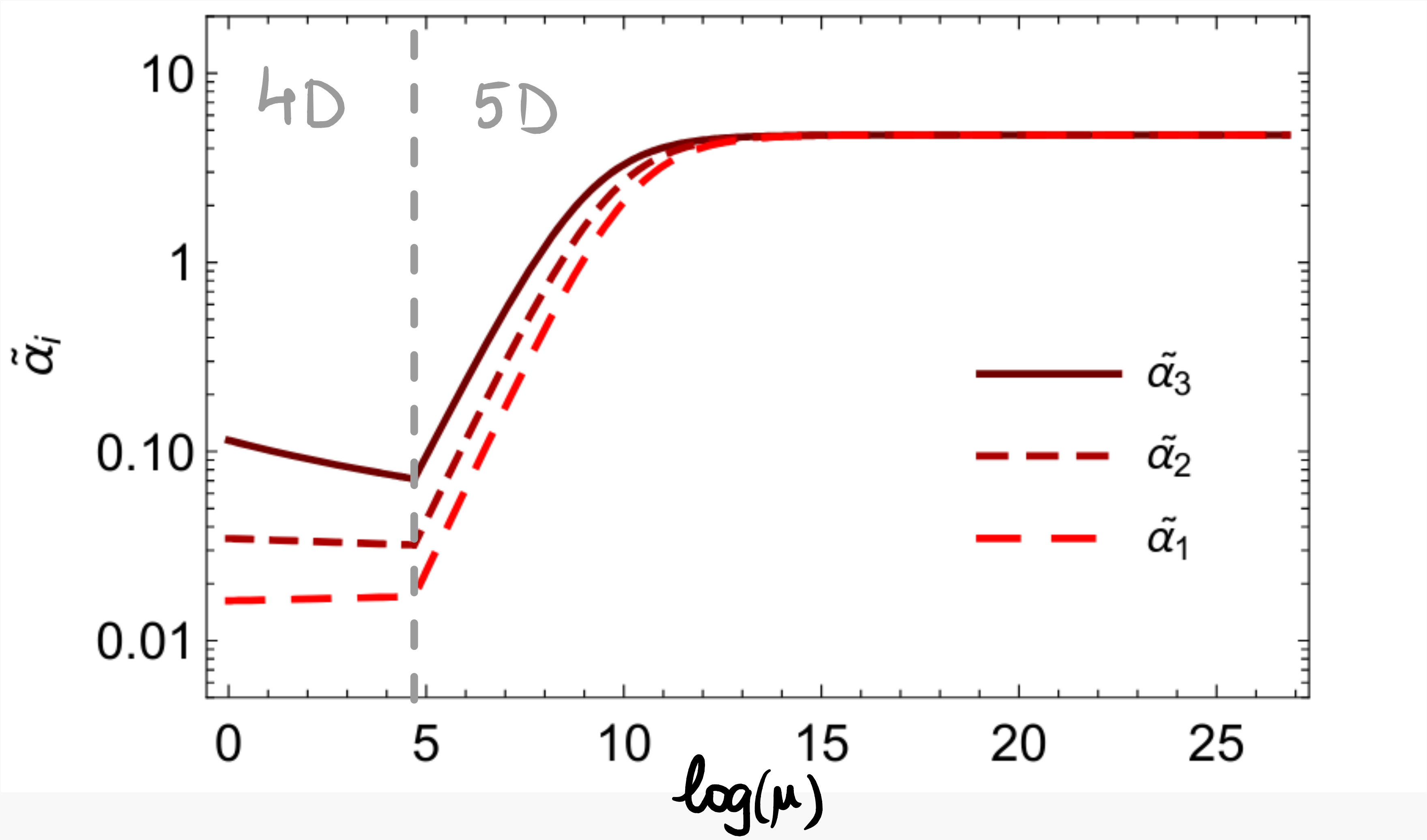
Evolution of couplings: from 4D to 5D

- 4D: logarithmic running

- 5D: power-law running [2]

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t)-1) b_5^i g_i^3$$

where $S(t) = \begin{cases} \mu\ \mathrm{R}{=}M_Z\ \mathrm{R}\ e^t, & \text{for } \mu \geq 1/R \\ 1, & \text{otherwise} \end{cases}$



G. Cacciapaglia ,et.al., 104, PRD 2021

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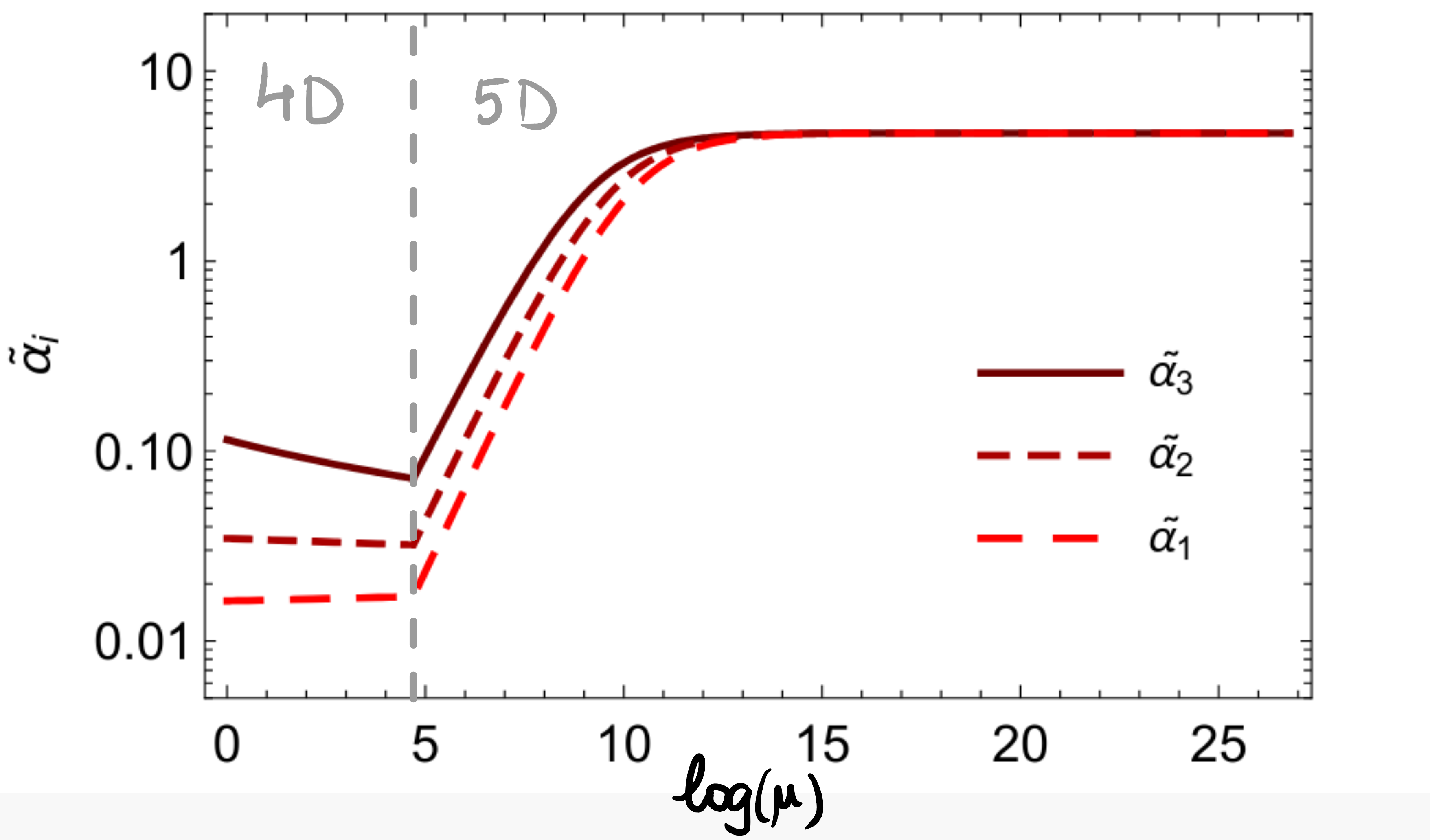


G. Cacciapaglia ,et.al., 104, PRD 2021

5D couplings flow asymptotically towards a UV fixed point

It´s existence ⟺ good behavior in the UV

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

What does finite mean?

Couplings have perturbative **fixed points**

+

Finite number of counterterms to absorb the power law divergencies

Renormalization

Loop diagrams may be divergent $(+\infty)$

Introduce counterterms to cancel divergencies

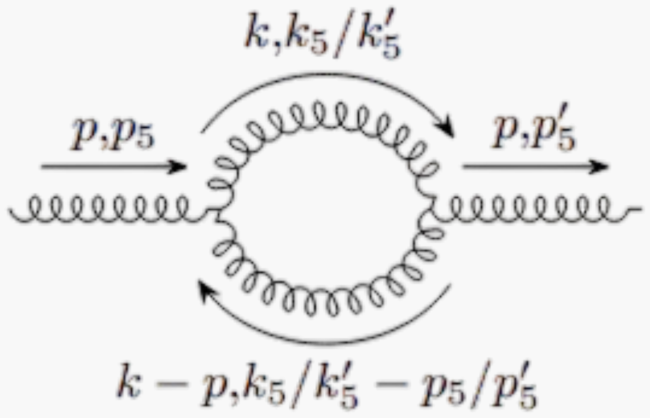
$(-\infty)$

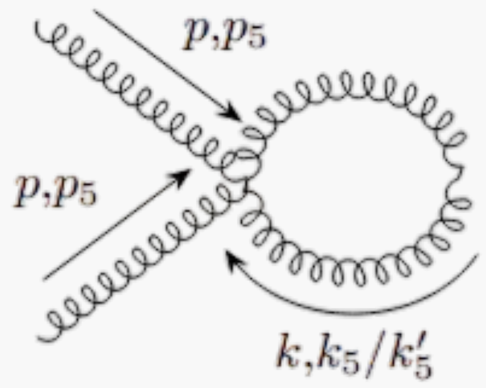
If the number of counterterms is finite: theory is **renormalizable**

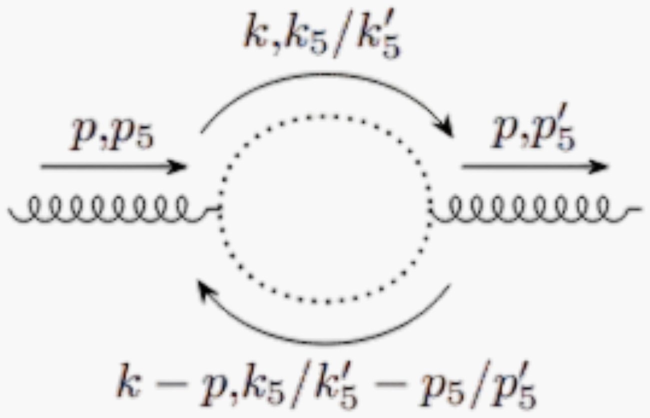
(finite)

Renormalization of 5D gauge theories

Pure Yang Mills theory







! Theory is renormalizable both in all regions of spacetime (at one loop) [4]

(finite number of operators to cancel divergencies)

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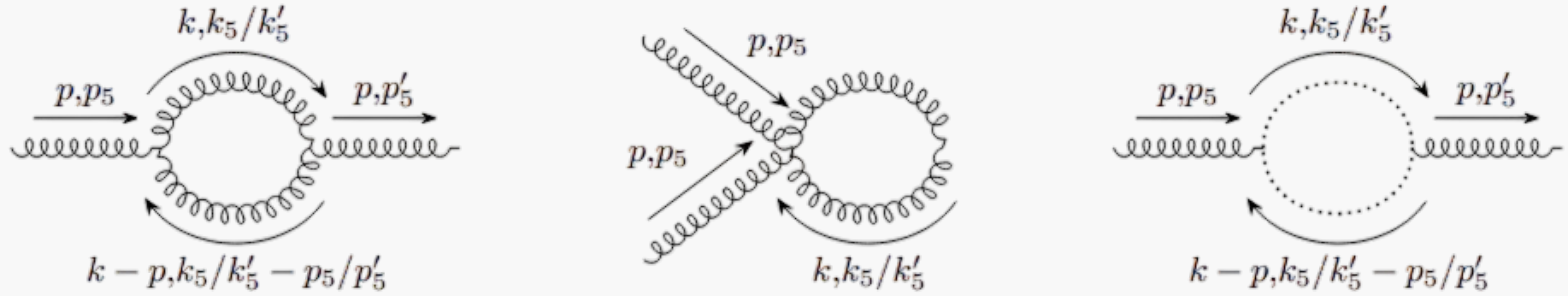

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Going beyond Yang Mills?

➡ theory stays finite when **fermions** and **scalars** are included

[4] G. Cacciapaglia, W. Isnard, R. Pasechnik, **AP** [arXiv:2601.00453]

A case example: $SU(6)$ unification

Is the model well defined and physical?

- The orbifold is **stable**: $SU(6) \to SU(3) \times SU(2) \times U^2(1)$ $\boxed{3 \geq 6/2 \quad \text{orbifold stability}}$

- Gauge couplings exhibit a **fixed point**: good UV behaviour

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- Gauge couplings exhibit a **fixed point**: good UV behaviour

+

- The theory can be made anomaly free
- Account for fermion masses
- Proton is stable: unification scale can be as low as TeV

Summary and conclusions

- 5D theories: interesting theoretical playground for new physics
- Vacuum constrains the allowed breaking patterns $\longrightarrow$ orbifold stability
- More formal side: theories are finite at one loop

First candidate: SU(6) asymptotic GUT

$\longrightarrow$ passes the minimal physical criteria (more on that in Alejandro's talk)

Backup slides

Kaluza Klein decomposition

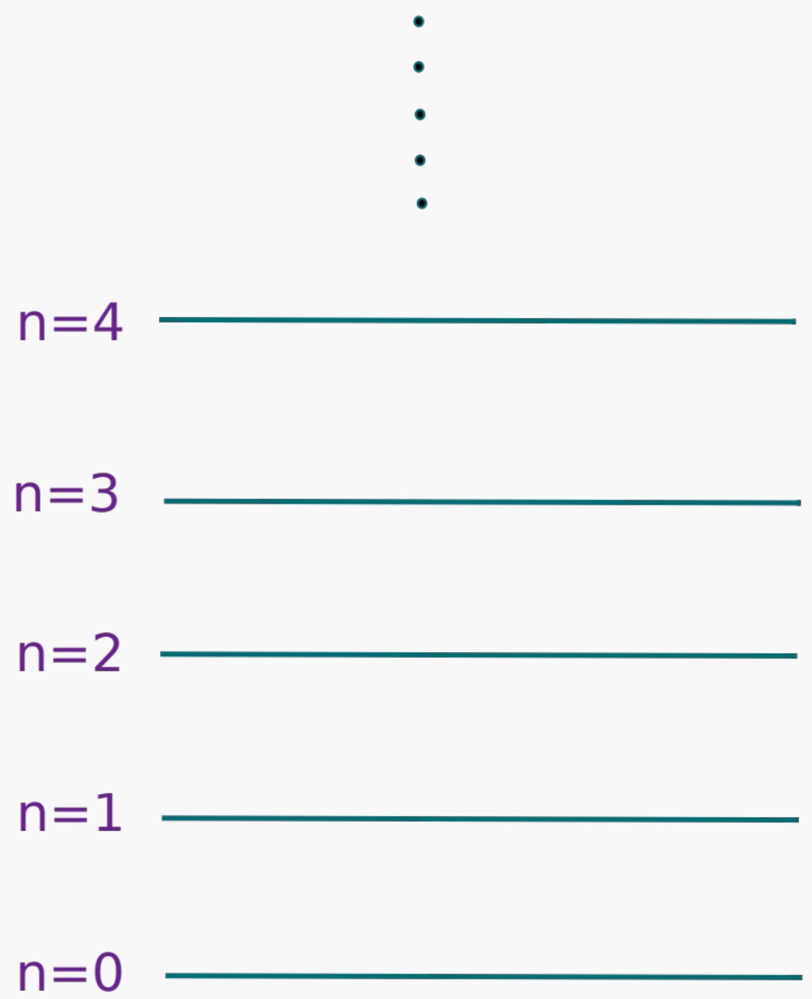
$$\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\cos(ny/R) + \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu)\sin(ny/R)$$

Any 5D field

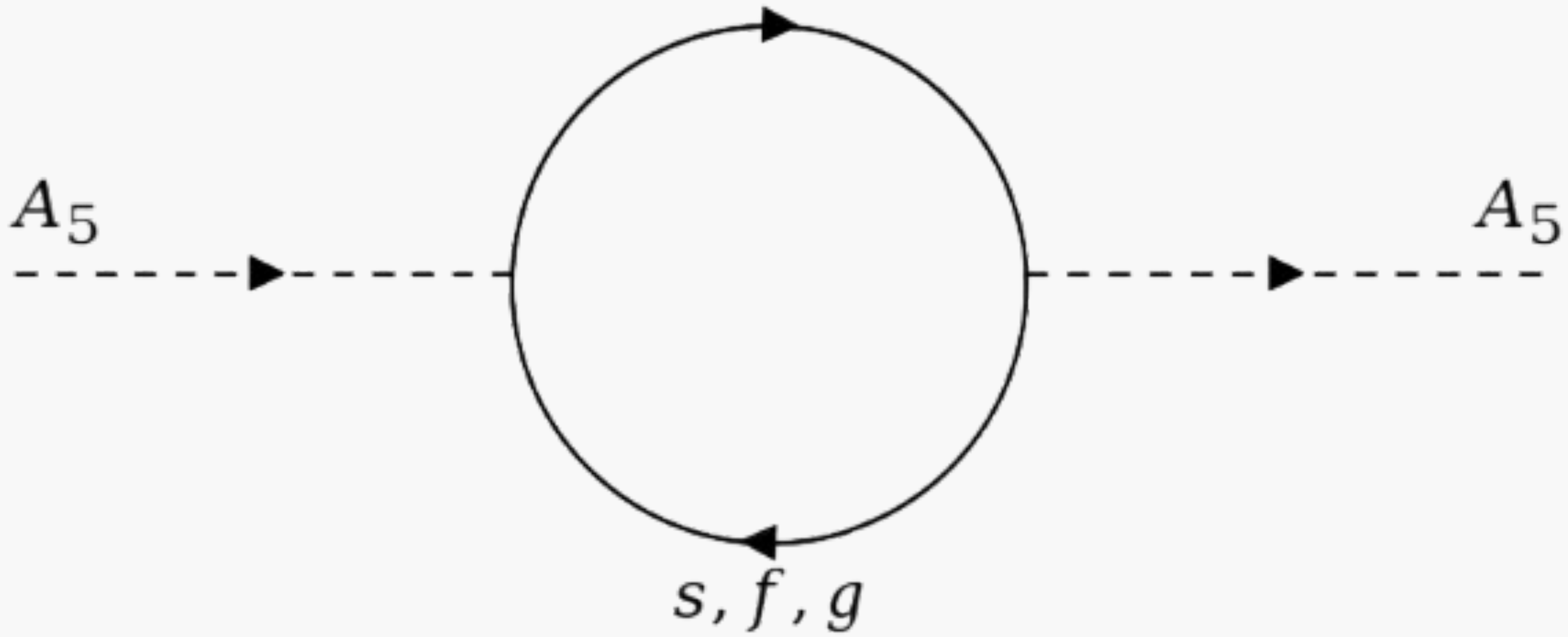
$$\Phi_{5\mathrm{D}}$$

(scalar, fermion, gauge boson)

Infinite tower of 4D fields



One loop effective potential



Total potential given by:

$$V_{\text{eff}}^{\text{total}}(\alpha) = V_{\text{eff}}^{\text{gauge}}(\alpha) + V_{\text{eff}}^{\text{scalar}}(\alpha) + V_{\text{eff}}^{\text{fermionic}}(\alpha)$$

Minimum of $V_{\text{eff}}^{\text{gauge}}(\alpha)$ must be at $\alpha = 0$.

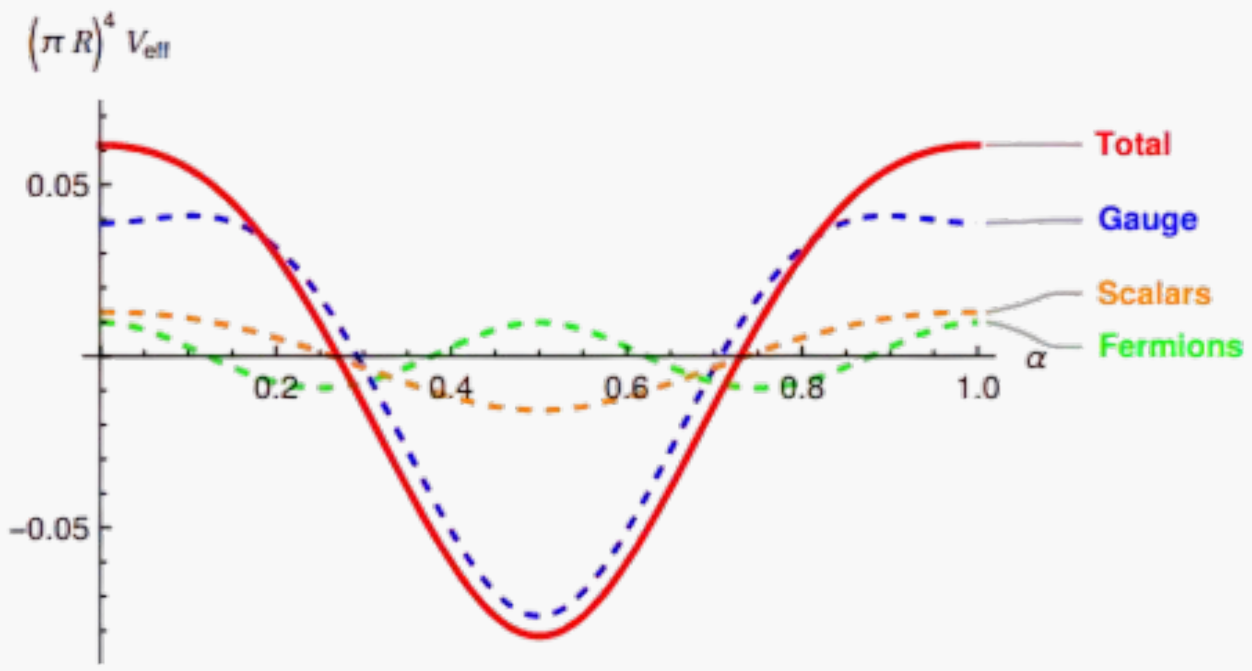


Figure: Potential of an $SU(6)$ model.

Gauge transformation to remove the VEV

⇩

change in the parity on one boundary

⇩

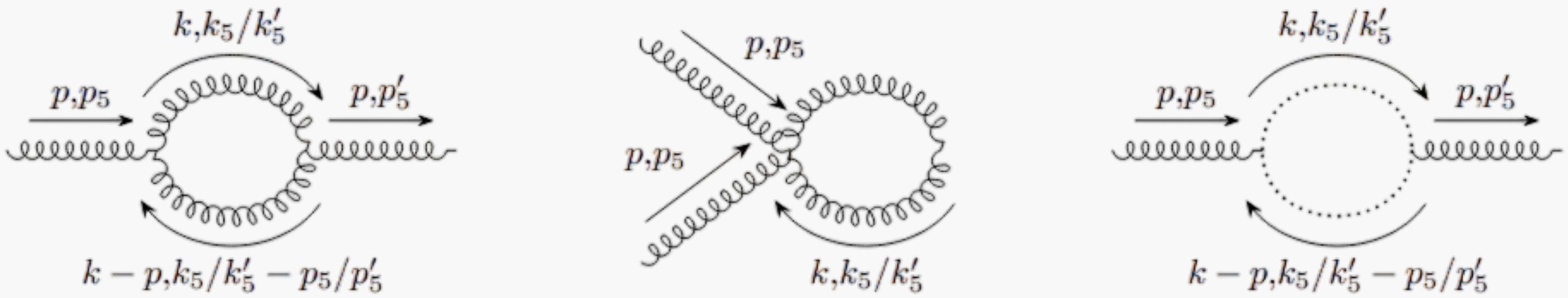
different breaking pattern

All possible working models for a $d = 5$ aGUT:

Model	**Breaking pattern**	**Stability criteria**
$\mathrm{SU}(N)$	$\mathrm{SU}(N) \to \mathrm{SU}(a) \times \mathrm{SU}(N-a) \times \mathrm{U}(1)$	stable $\forall$ a
	$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(s) \times \mathrm{U}(1)^2$	$p \geq N/2$
$\mathrm{Sp}(2N)$	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2a) \times \mathrm{Sp}(2(N-a))$	stable $\forall$ a
	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2p) \times \mathrm{Sp}(2q) \times \mathrm{Sp}(2s)$	$p \geq N/2$
	$\mathrm{Sp}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N)$	$\mathrm{SO}(2N) \to \mathrm{SO}(2a) \times \mathrm{SO}(2(N-a))$	stable $\forall$ a
	$\mathrm{SO}(2N) \to \mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$p \geq N/2$
	$\mathrm{SO}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N+1)$	$\mathrm{SO}(2N+1) \to \mathrm{SU}(2a+1) \times \mathrm{SU}(2(N-a))$	stable $\forall$ a
	$\mathrm{SO}(2N+1) \to \mathrm{SO}(2p+1) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$2p+1 \geq (2N+1)/2$

Renormalization of 5D gauge theories

Pure Yang Mills theory $$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{MN} F_{MN}$$



$$i\Sigma = \underbrace{[p_5 - \text{conserving terms}]}_{\text{bulk}} + \underbrace{[p_5 - \text{non-conserving terms}]}_{\text{boundary}}$$

Bulk: structure of divergencies same as in 4D (at one loop)

→ Scaling changes from logarithmic 4D ($\sim \log \Lambda$) to 5D linear ($\sim \Lambda$)

→ Same renormalization procedure as in 4D; **finite number of counterterms** are needed

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

→ less straightforward than bulk

Two-point function: [3,6]

$$i\Sigma = \frac{g^2}{64\pi^2}\frac{1}{\epsilon}\left[(g_{\mu\nu} - p_\mu p_\nu)\left(\frac{11}{3} - (\xi - 1)\right) C(G) + g_{\mu\nu}\frac{p_5^2 + {p'_5}^2}{2}(4 + (\xi - 1))C(G)\right]$$

To reconstruct the full counterterm on the boundary we need **higher vertex corrections** as well (3-pt and 4-pt functions)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\text{YM}} = -\frac{1}{4}F^{MN}F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

→ less straightforward than bulk

→ divergencies can be absorbed by a **finite number of counterterms**

→ **"magic gauge"** ($\xi = -3$): coefficients of the 2-,3- and 4-pt functions are the same

localized counterterm can be unified into a single term $-\frac{1}{4}K\delta\left(F_{\mu\nu}F^{\mu\nu}\right)$

! Theory is renormalizable both in the bulk and on the boundary (at one loop)

(finite number of operators to cancel divergencies)