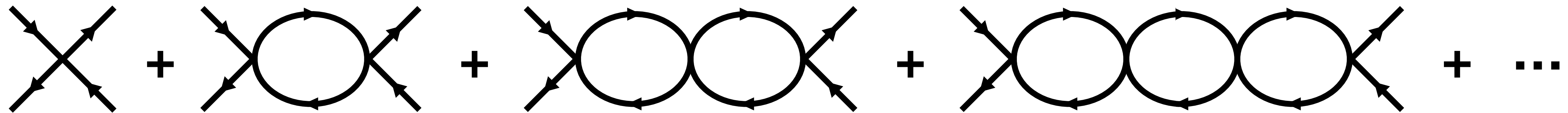


# Renormalisable four-fermion interactions in four dimensions



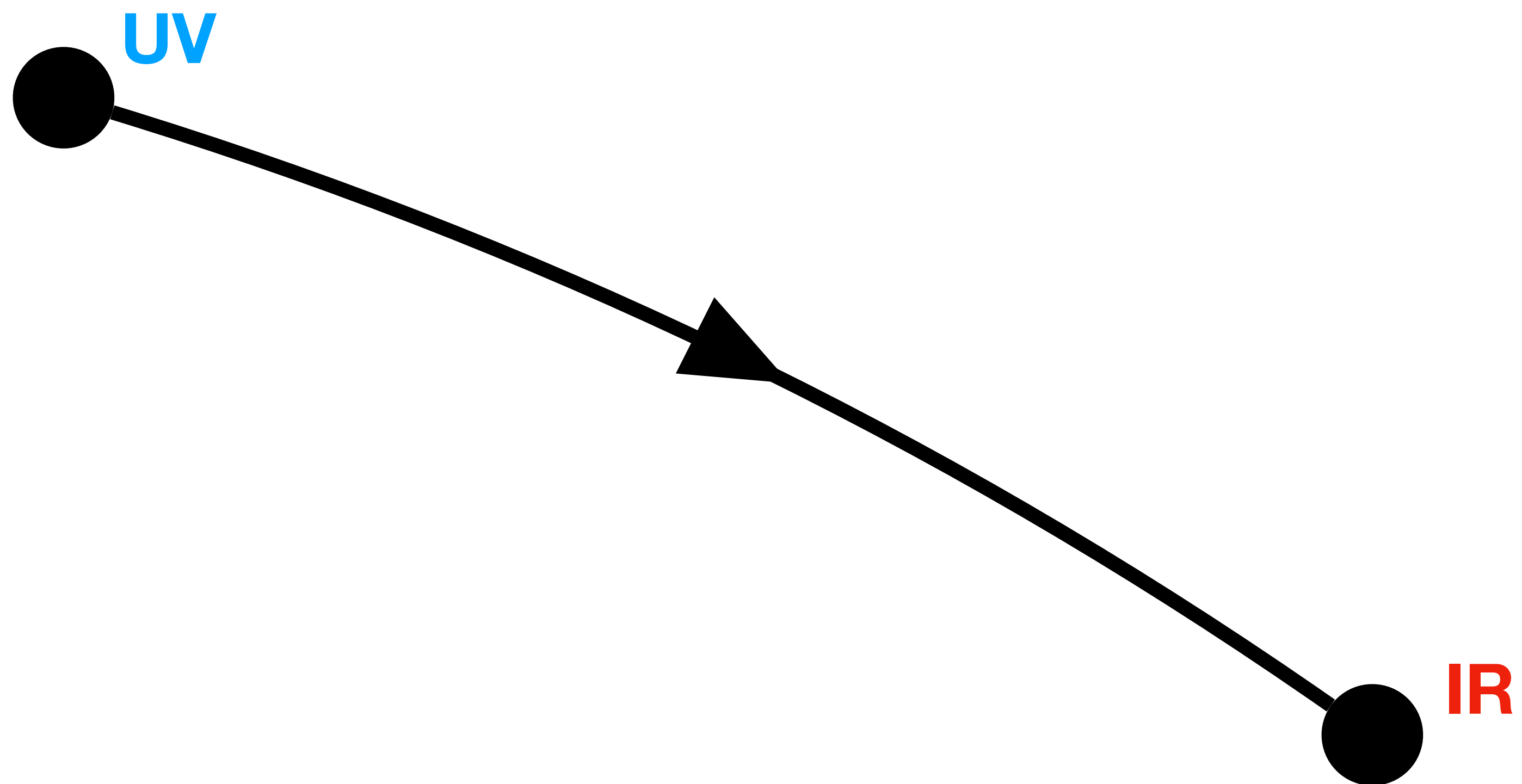
**Charlie Cresswell-Hogg**  
*Chalmers University*

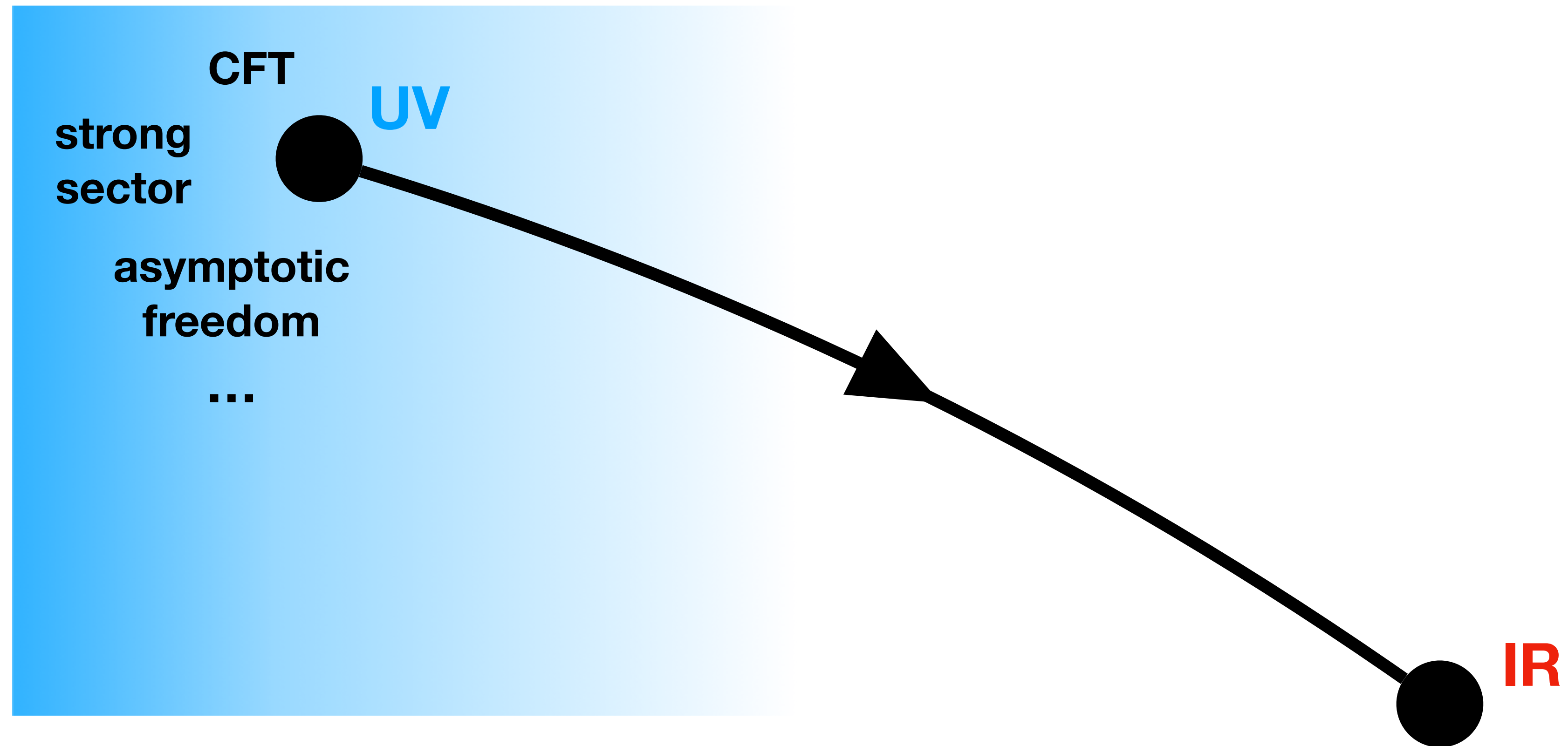
*based on 2603.19357 with Daniel Litim @ Sussex U.*

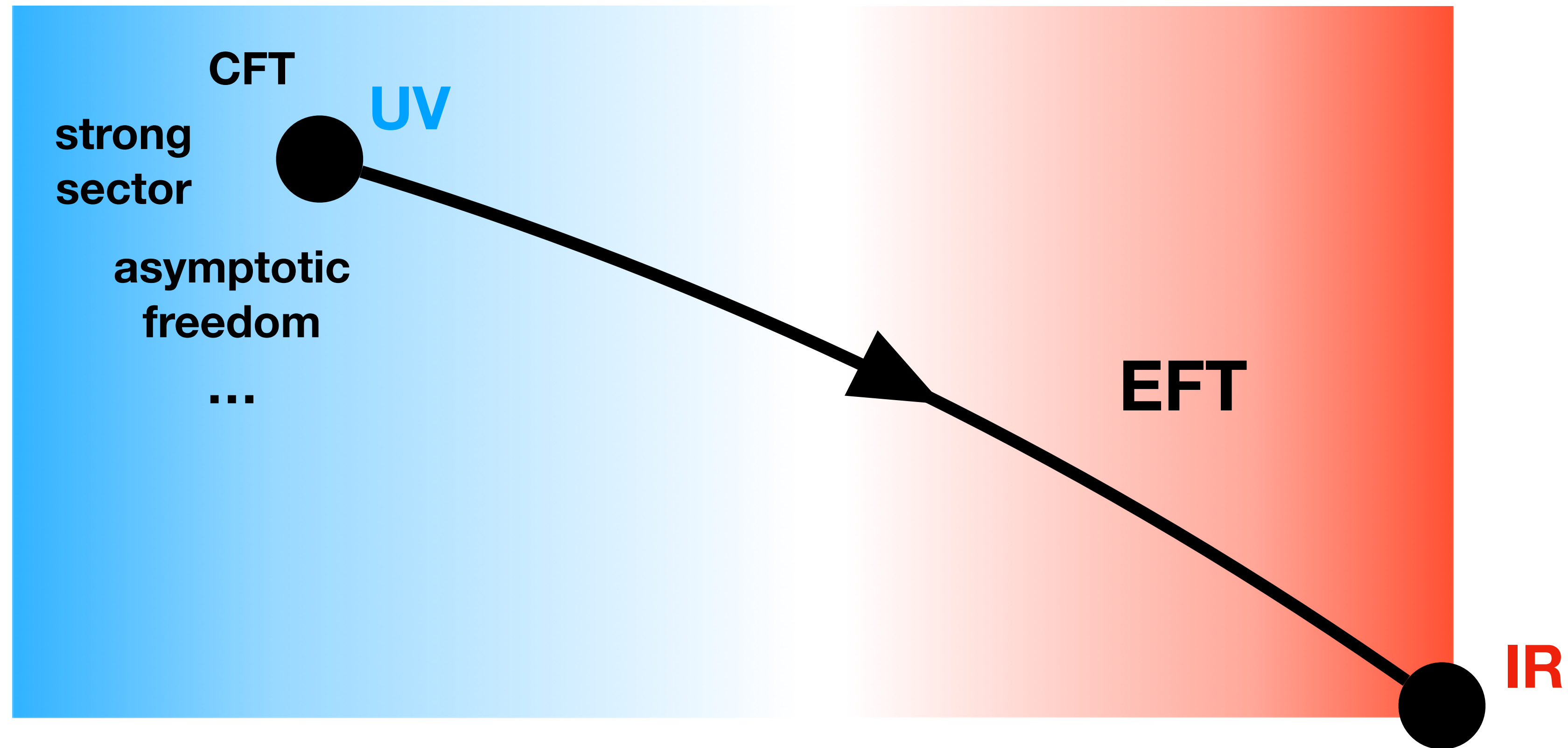
**Partikeldagarna 2026 @ Lund**

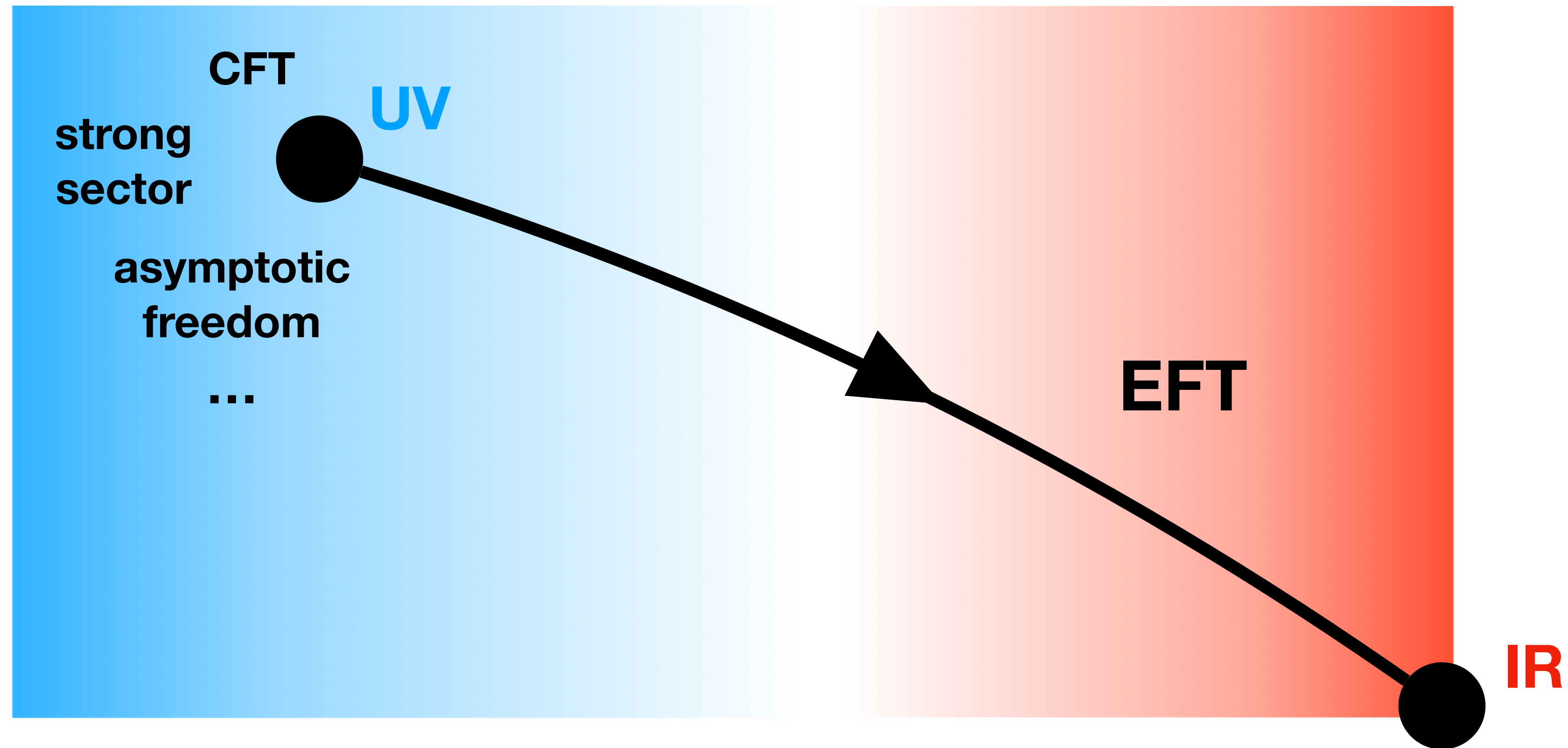


**CHALMERS**  
TEKNISKA HÖGSKOLA

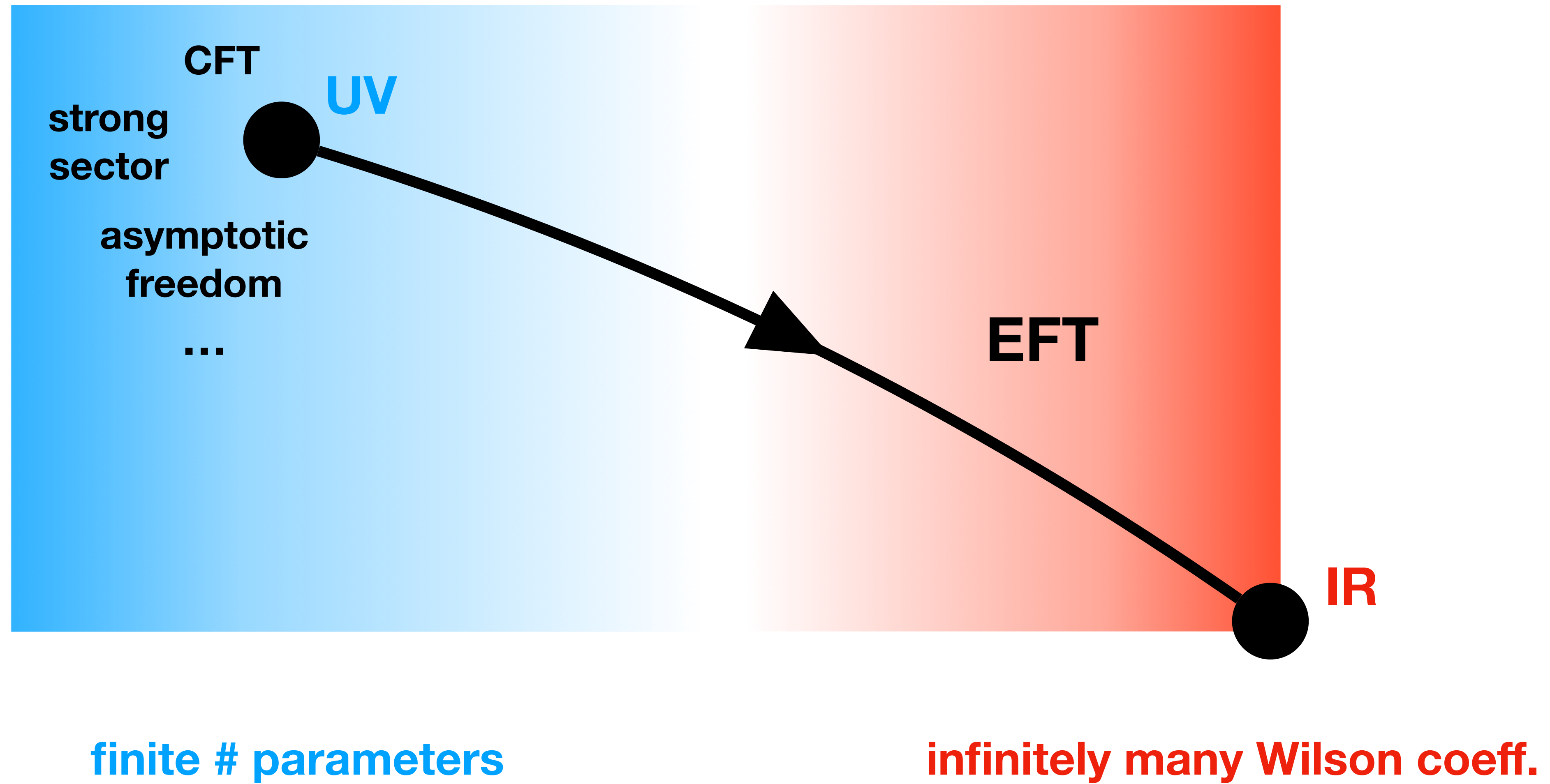


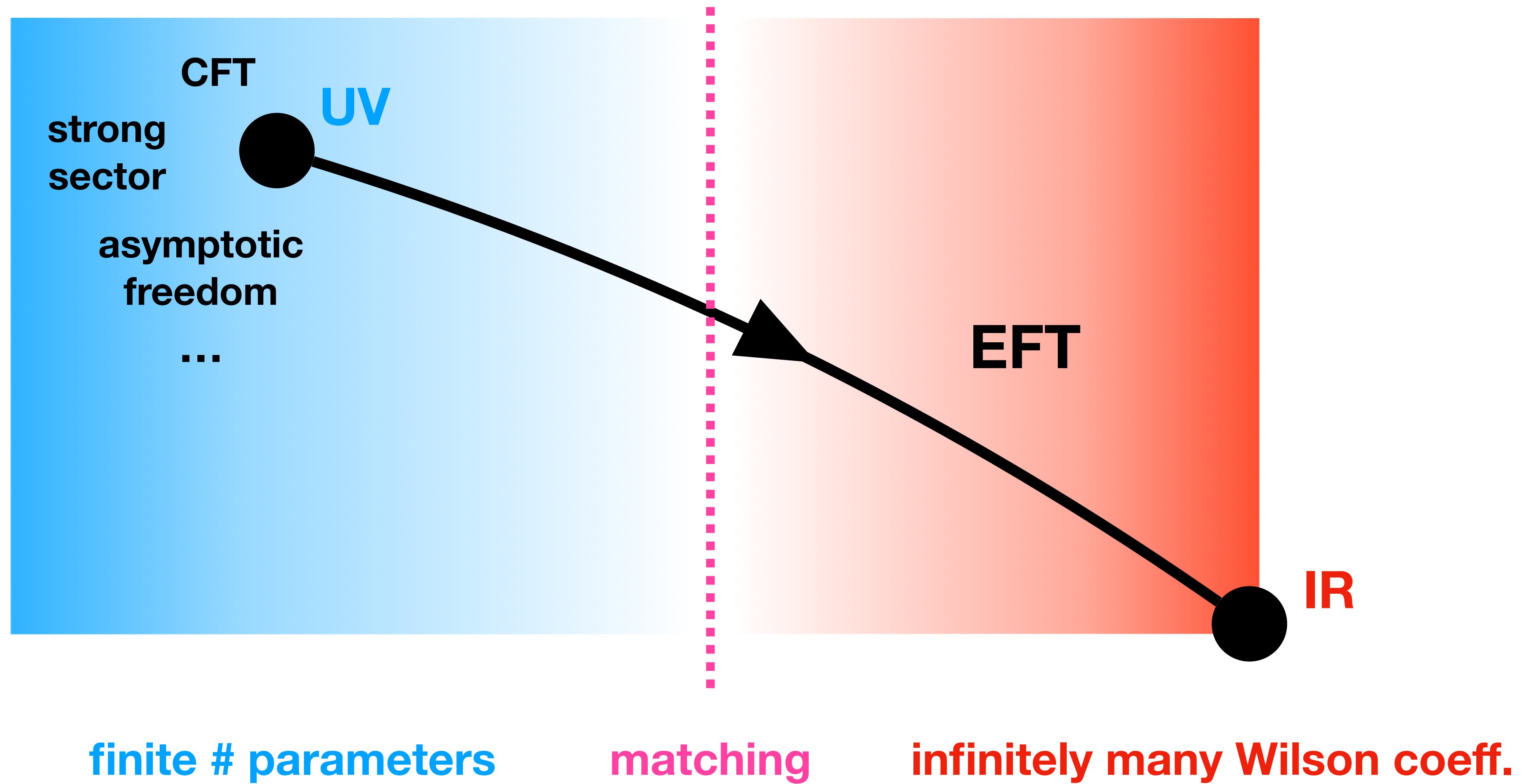


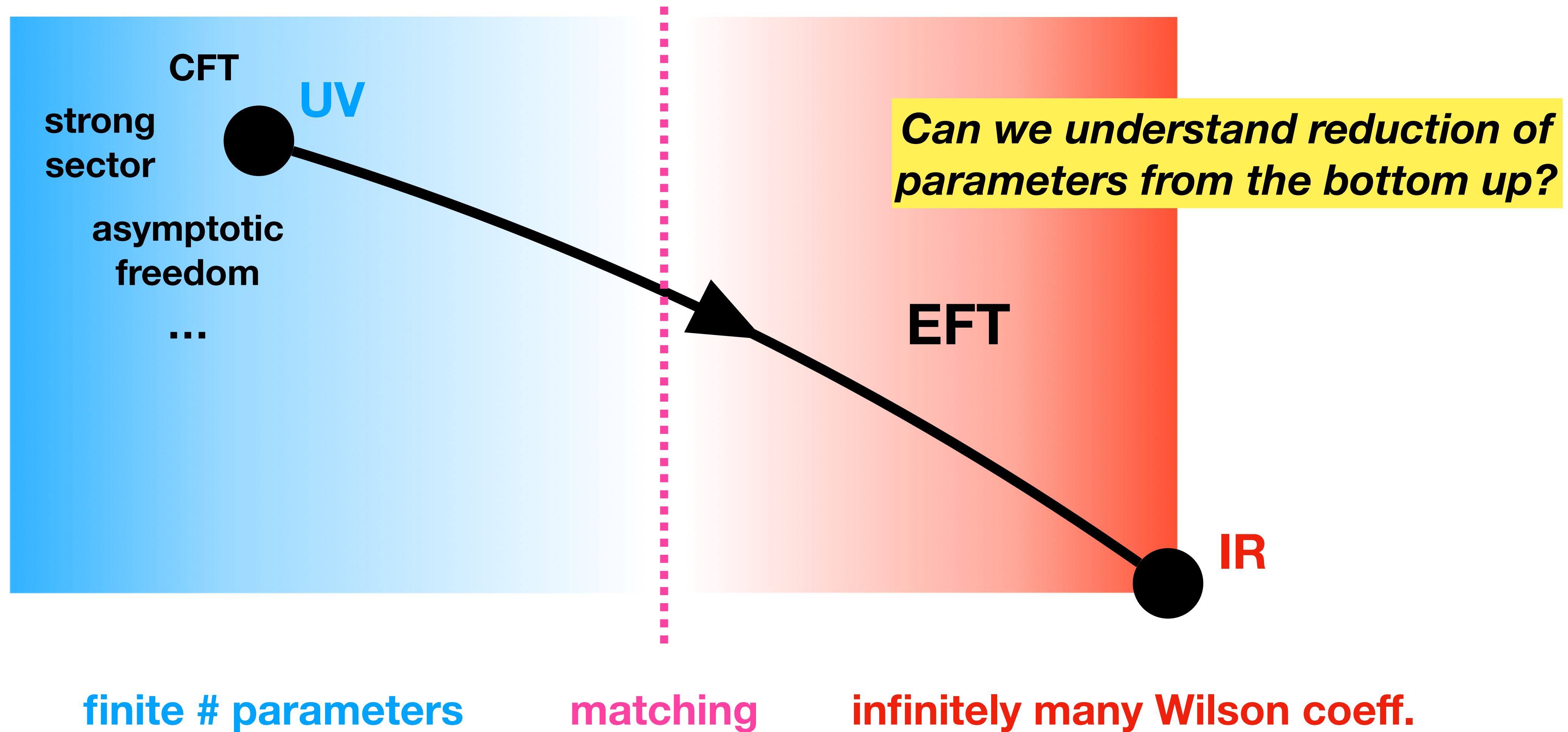




finite # parameters







# A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \dots$$

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**2+1 dimensions:**  $1/N$  renormalisable & conformal at high energy

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Wilson '73

Gross, Neveu '74

Parisi '75

Gawędzki, Kupiainen '85

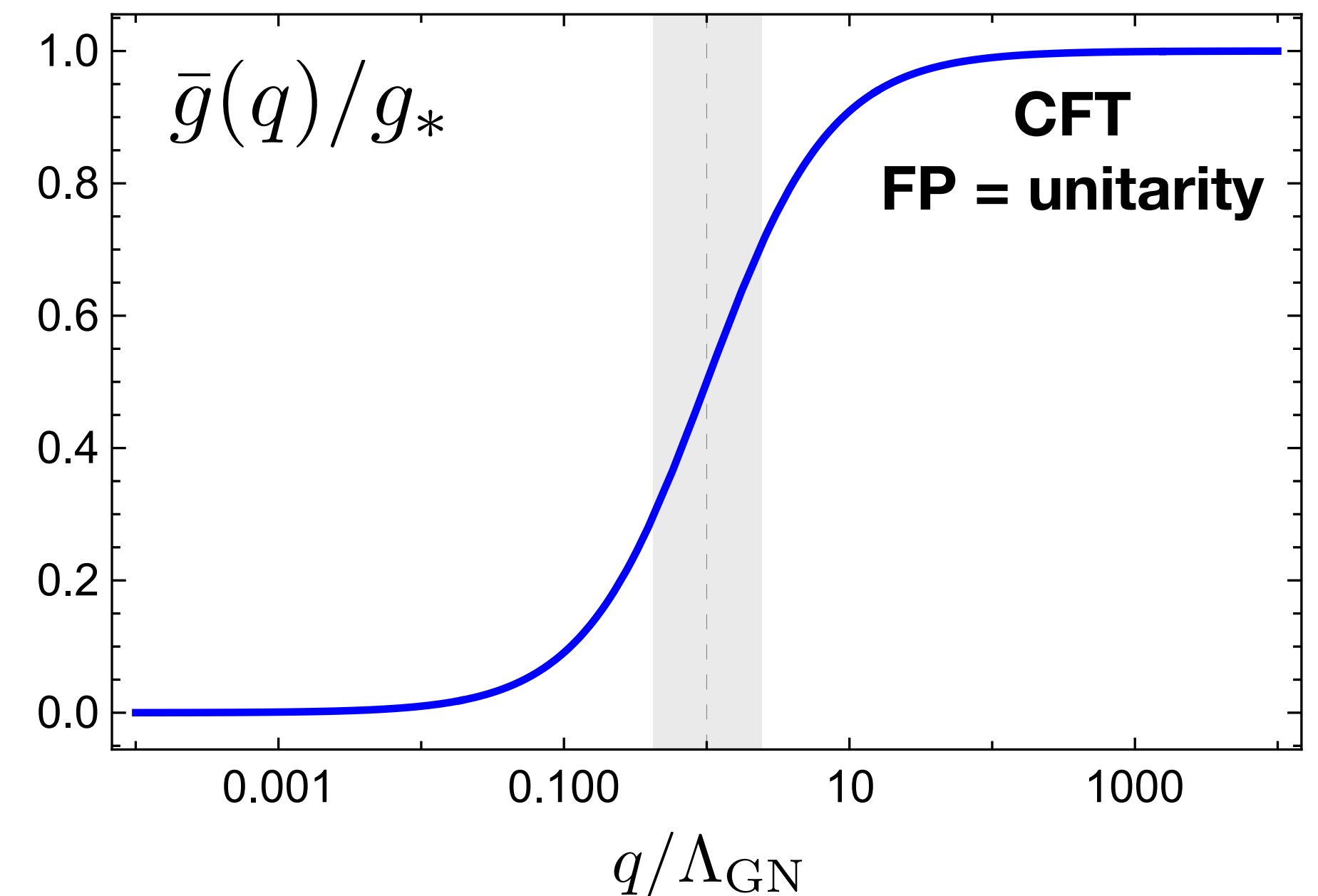
**TIP! Rosenstein, Warr, Park '88 (PRL)**

**Rosenstein, Warr, Park '91 (Phys.Rept.)**

Gat, Kovner, Rosenstein, Warr '90

de Calan, Faria Da Veiga, Magnen, Seneor '91

+ many more



# A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \dots$$

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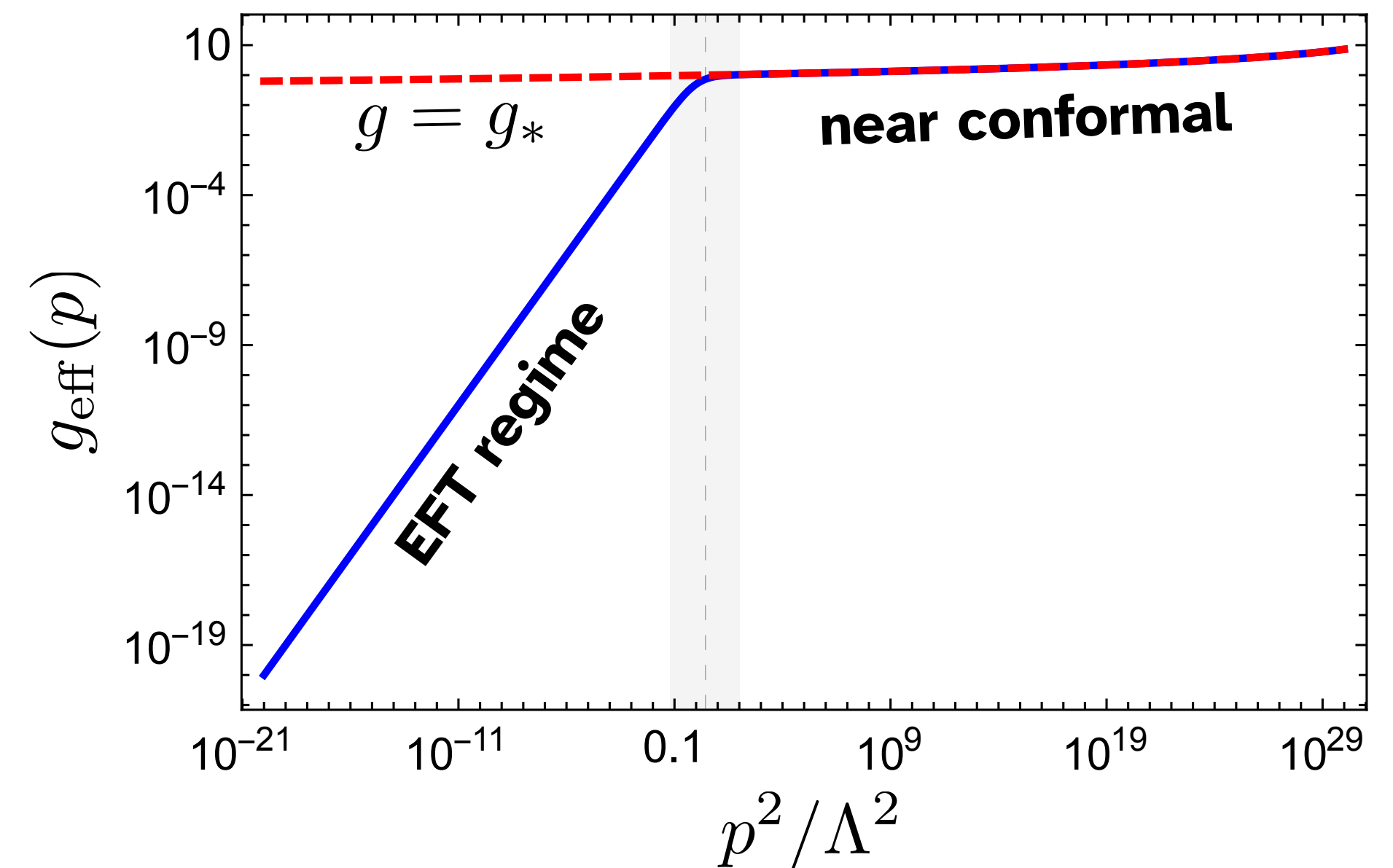
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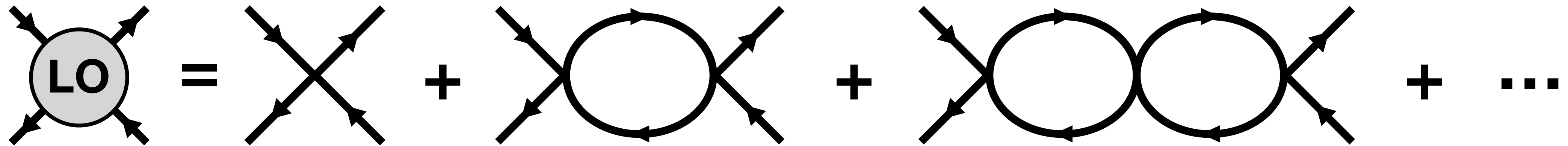
**Formal renormalisability in  $1/N$  dictates:**

- tower of higher-derivative operators
- eight-fermion interactions (but no higher)
- only three fundamental parameters



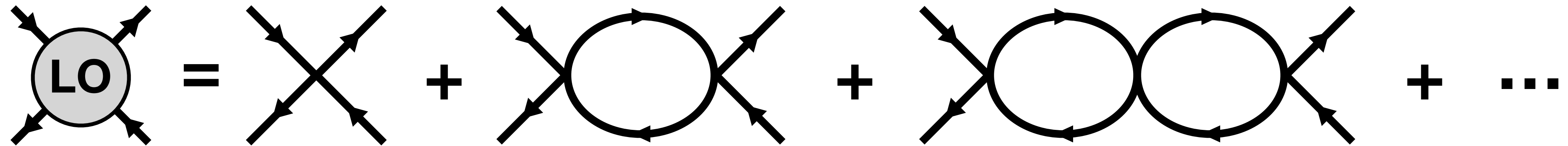
# Large- $N$ diagrammatics

Four-point function:

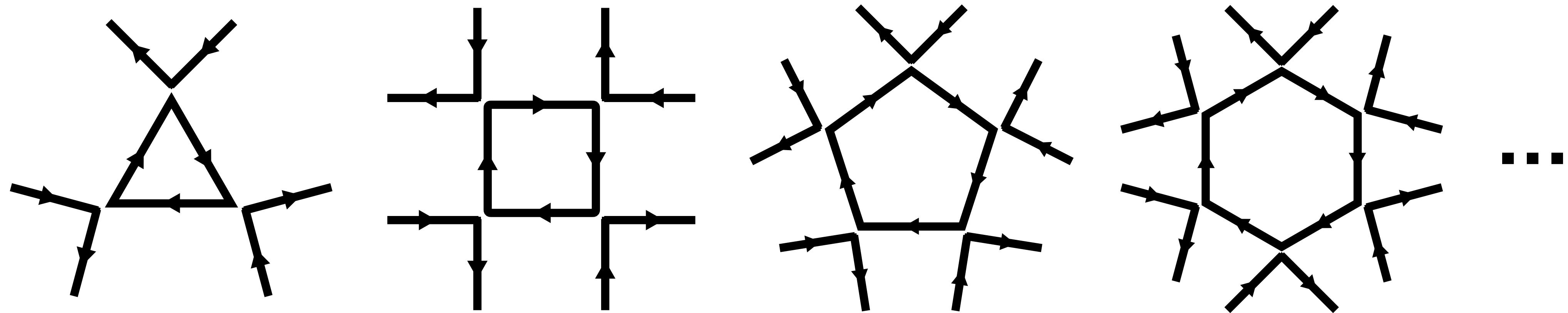


# Large- $N$ diagrammatics

Four-point function:

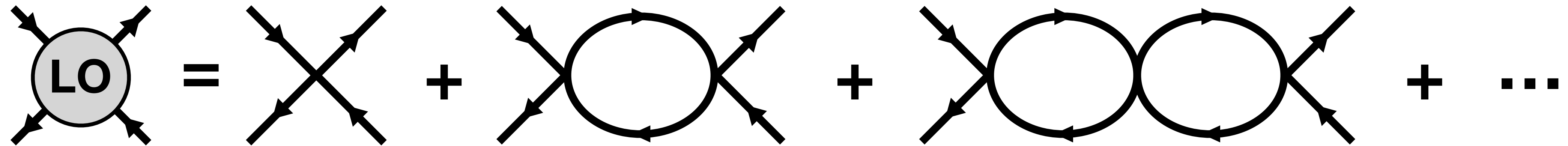


Higher-point functions:

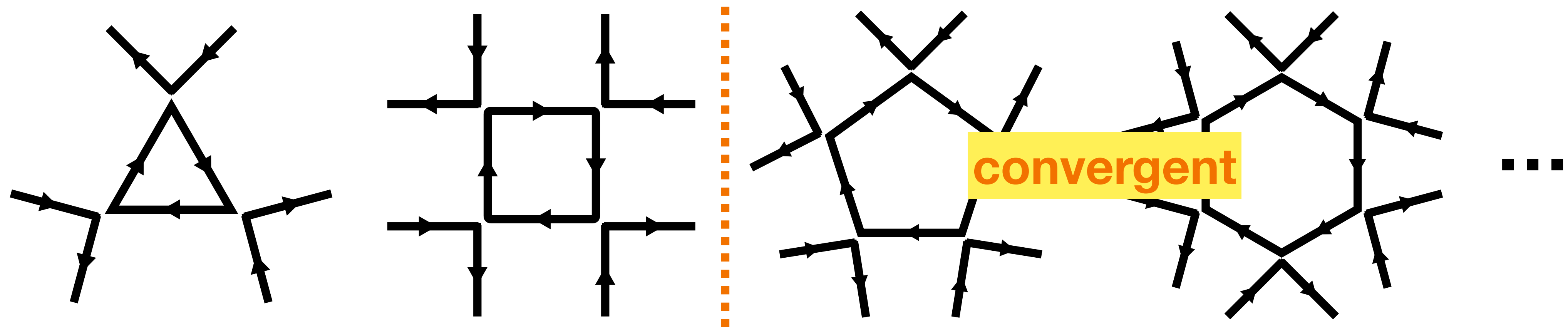


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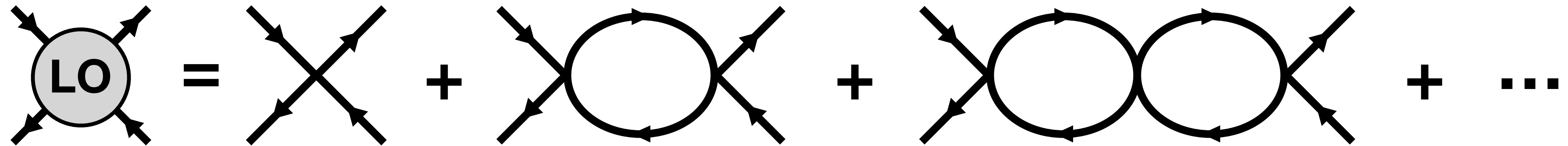


Higher-point functions:

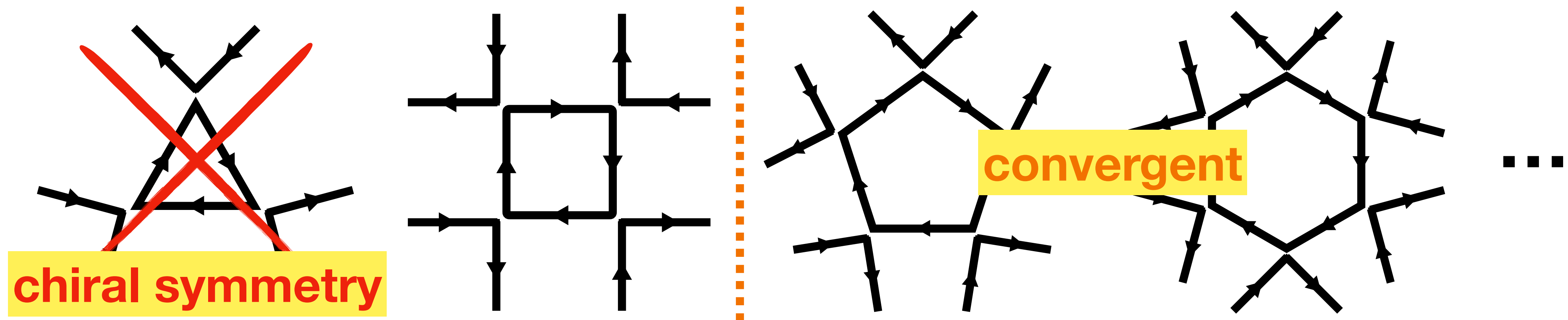


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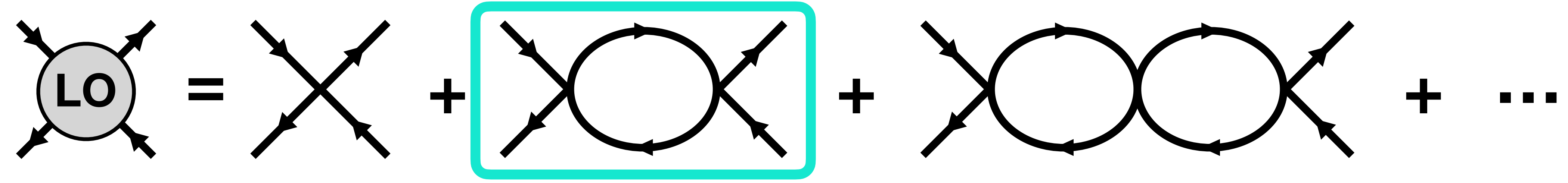


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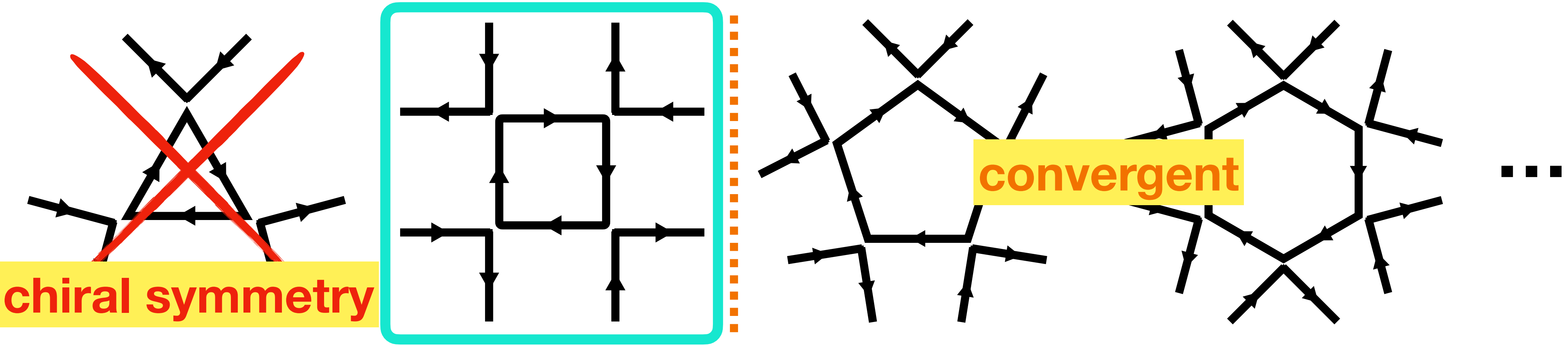
# Large- $N$ diagrammatics

Four-point function:



divergent  
subdiagrams

Higher-point functions:



# Four-point function

$$\text{LO} = \text{tree} + \text{1-loop} + \text{2-loops} + \dots \rightarrow \frac{N^{-1}}{G^{-1} + \text{1-loop}}$$

$$\text{1-loop} \sim \Lambda^2 + p^2 \ln \Lambda \quad \text{infinite tower of higher-derivative divergences}$$

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$$\frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 \rightarrow \frac{1}{2N} (\bar{\psi}_a \psi_a) F(-\partial^2) (\bar{\psi}_b \psi_b)$$

$$F(p^2)^2 \times \text{1-loop}$$

vertices factorise @ LO

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$$F(p^2) = \frac{1}{Z p^2 + G^{-1}} \rightarrow \text{LO} \sim \frac{N^{-1}}{Z p^2 + G^{-1} + \text{1-loop}}$$

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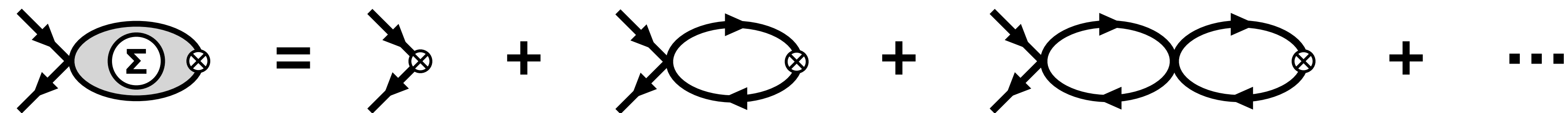
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vertices factorise @ LO

$$F(p^2) = \frac{1}{Z p^2 + G^{-1}} \rightarrow \text{LO} \sim \frac{N^{-1}}{Z p^2 + G^{-1} + \text{1-loop}}$$

subtract infinitely many higher-derivative divergences with one additional parameter

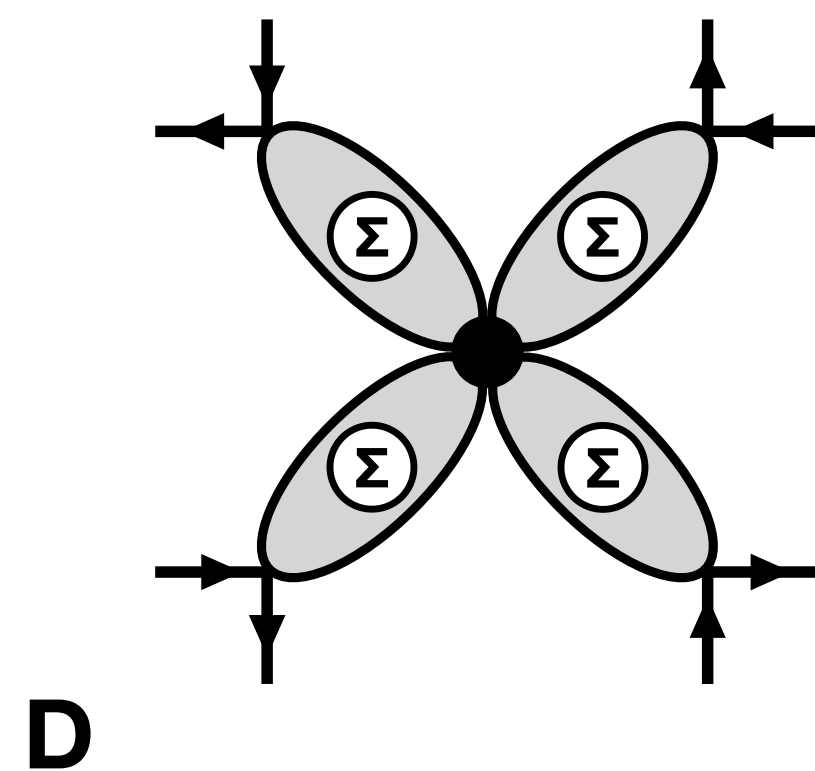
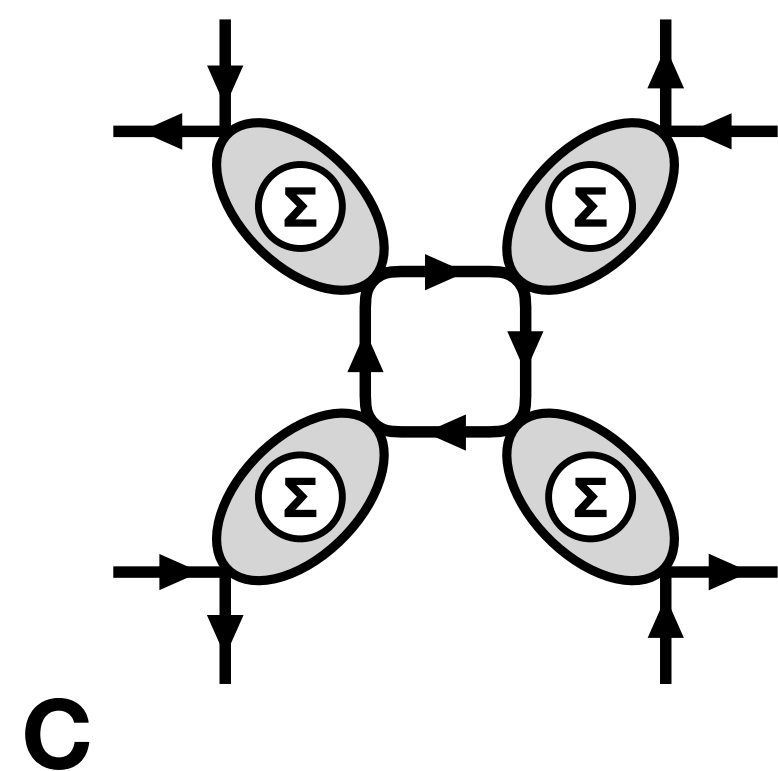
# Eight-point function



The diagram shows an equation for the eight-point function. On the left is a shaded oval with a central circle containing the Greek letter  $\Sigma$ . Two external lines enter from the left, and two exit to the right. The rightmost vertex is marked with a circle containing an 'X'. This is followed by an equals sign, then a series of terms separated by plus signs. The first term is a vertex with two incoming lines from the left and one outgoing line to the right, marked with a circle containing an 'X'. The second term is a loop diagram with two incoming lines from the left and one outgoing line to the right, marked with a circle containing an 'X'. The third term is a diagram with two adjacent loops, two incoming lines from the left, and one outgoing line to the right, marked with a circle containing an 'X'. The equation ends with an ellipsis '...'.

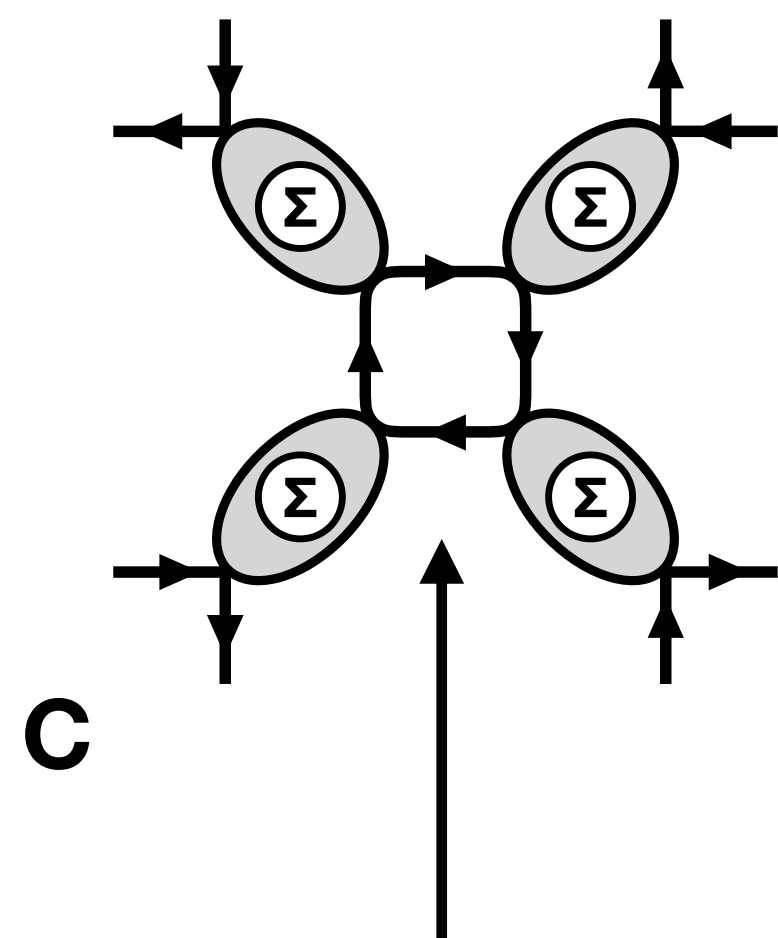
# Eight-point function

A diagrammatic equation showing the expansion of a shaded eight-point function. On the left, a shaded oval with a  $\Sigma$  inside and a crossed circle on the right has two incoming arrows on the left. This is equal to a sum of three terms: 1) a crossed circle with two incoming arrows, 2) a loop with two incoming arrows on the left and a crossed circle on the right, and 3) a chain of two loops with two incoming arrows on the left and a crossed circle on the right. The equation ends with an ellipsis.

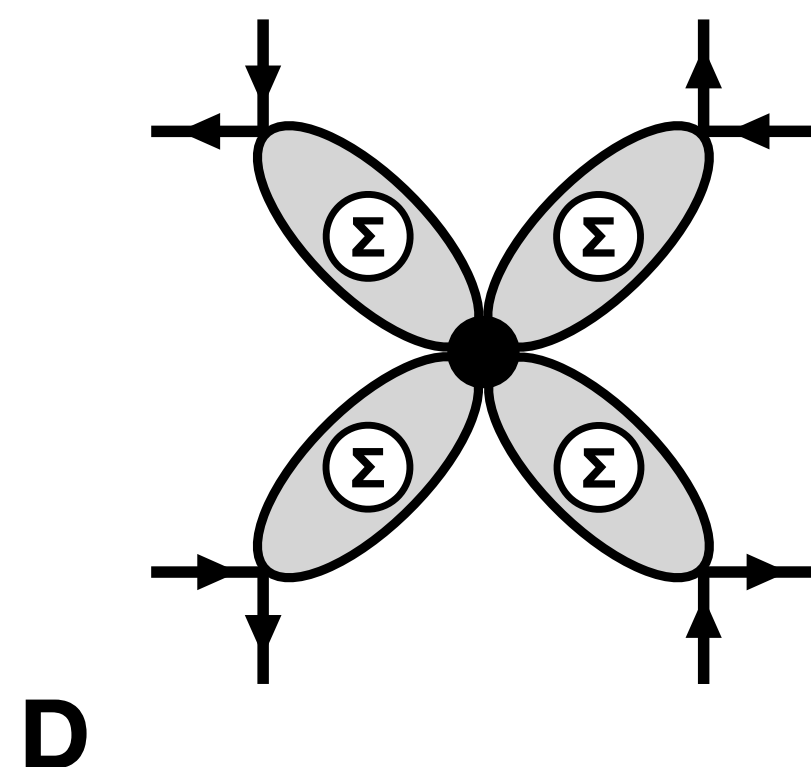


# Eight-point function

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 + \text{Diagram}_3 + \dots$$

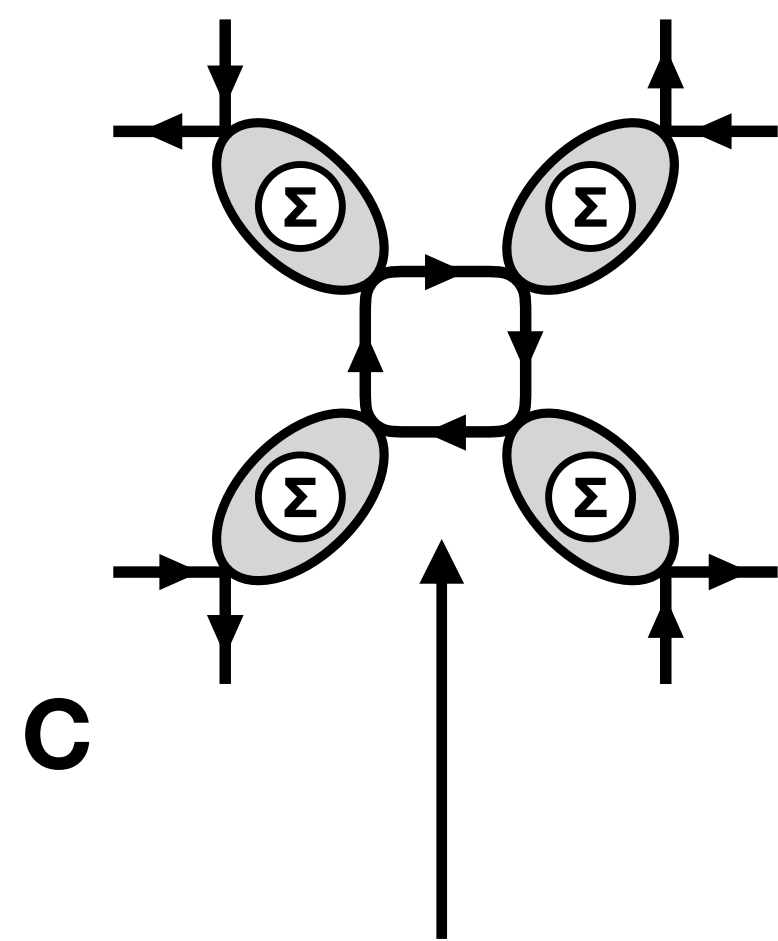


$\sim \ln \Lambda$



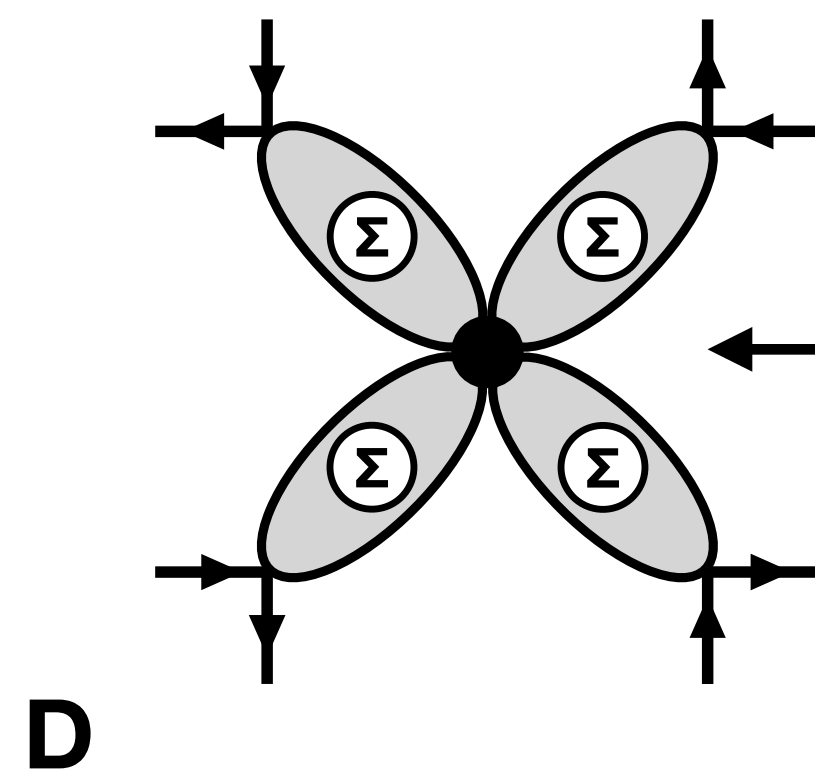
# Eight-point function

A diagrammatic equation showing the expansion of a self-energy loop. On the left, a fermion line with two external arrows enters a shaded oval labeled  $\Sigma$  with a cross on its right side. This is equal to a sum of three terms: 1) a fermion line with a cross on it, 2) a fermion line entering a loop with a shaded  $\Sigma$  loop on the right, and 3) a fermion line entering a chain of two shaded  $\Sigma$  loops on the right. The sequence ends with an ellipsis.



C

$\sim \ln \Lambda$

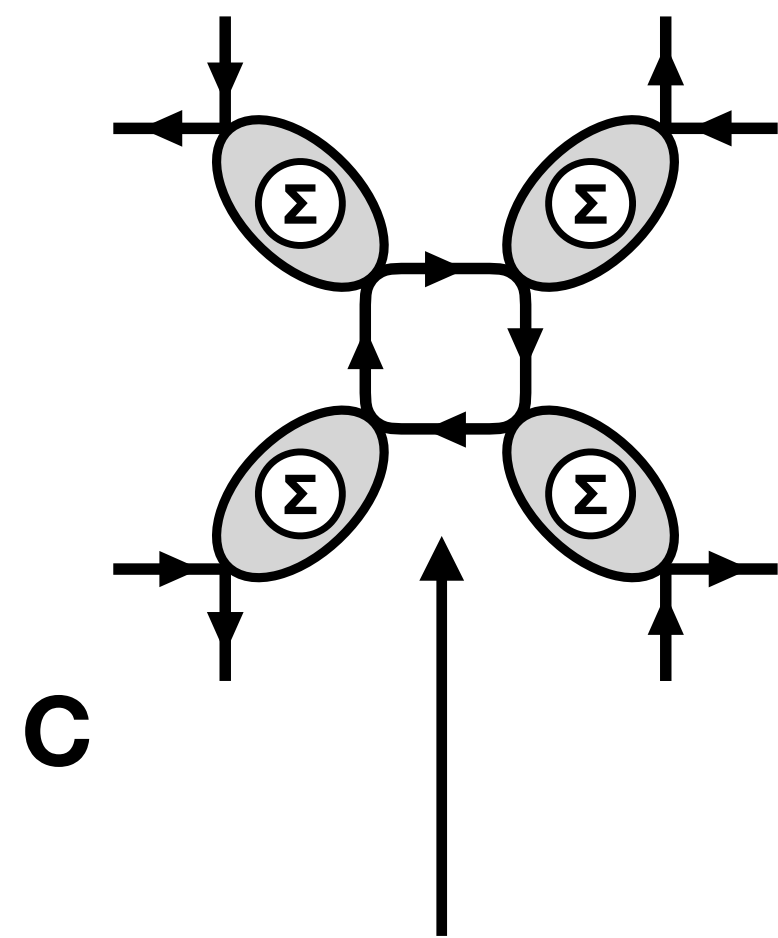


D

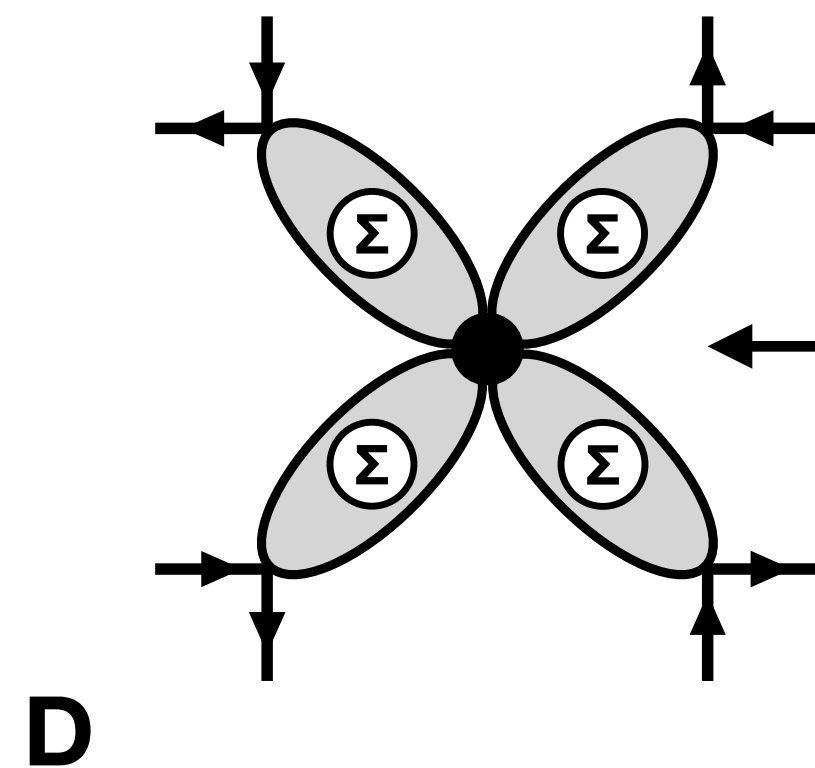
$$\frac{\lambda}{4!N^3} \left[ F(-\partial^2) (\bar{\psi}_a \psi_a) \right]^4$$

# Eight-point function

$$\text{Diagram with } \Sigma \text{ and } \otimes = \text{Diagram with } \otimes + \text{Diagram with bubble} + \text{Diagram with two bubbles} + \dots$$

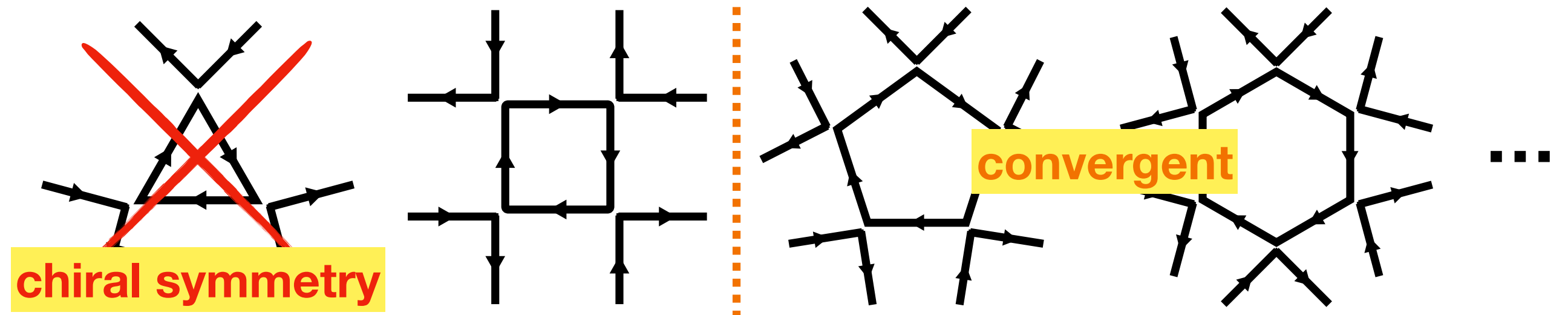


$\sim \ln \Lambda$



$$\frac{\lambda}{4!N^3} \left[ F(-\partial^2) (\bar{\psi}_a \psi_a) \right]^4$$

Everything @ LO = products of bubble sums with:



>> **THEORY RENORMALISED.** <<

# Renormalisable four-fermion theory

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} (\bar{\psi}_a \psi_a) F(-\partial^2) (\bar{\psi}_b \psi_b) + \frac{\lambda}{4!N^3} [F(-\partial^2) (\bar{\psi}_a \psi_a)]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

- **Predictive:** 3 parameters  $Z$ ,  $G$ ,  $\lambda$
- **Tower** of higher derivative  $4F + 8F$
- **NO higher** ( $10F$ ,  $12F$ , ...)

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cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

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- **Tower** of higher derivative  $4F + 8F$
- **NO** higher ( $10F$ ,  $12F$ , ...)

**Renormalised couplings:**

$$g(\mu) = \mu^2 G(\mu)$$

$$Z(\mu) \quad \lambda(\mu)$$

***holds order-by-order in  $1/N$ .***

cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

# Running couplings

**non-universal:**

$$\beta_g(g) = 2g \left( 1 - \frac{g}{g_*} \right)$$

**UV fixed point**

e.g. MOM scheme:  $g_* = 8\pi^2$

**universal:**

$$\beta_Z = -\frac{1}{4\pi^2} \quad \beta_\lambda = -\frac{3}{\pi^2}$$

**logarithmic running**

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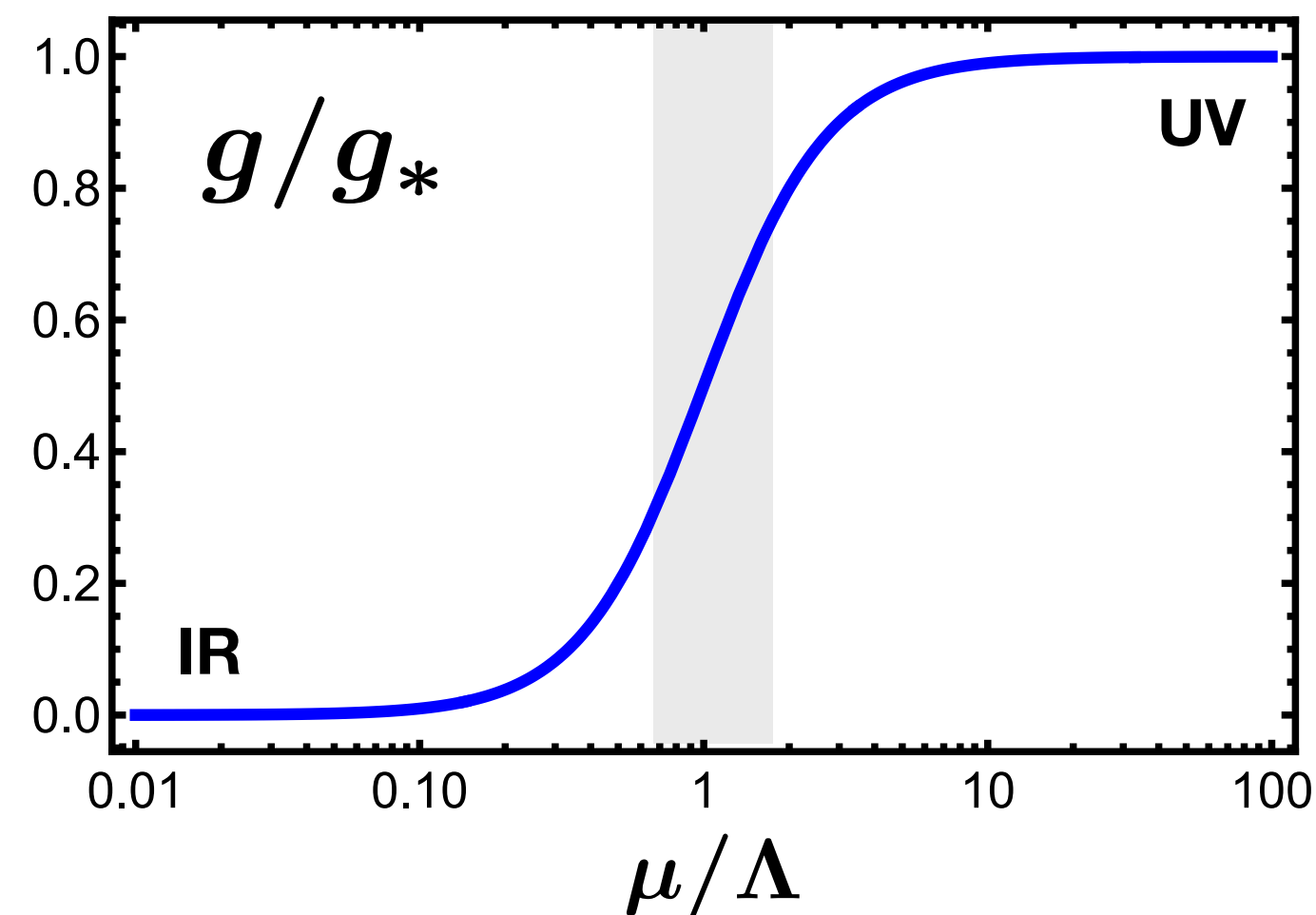
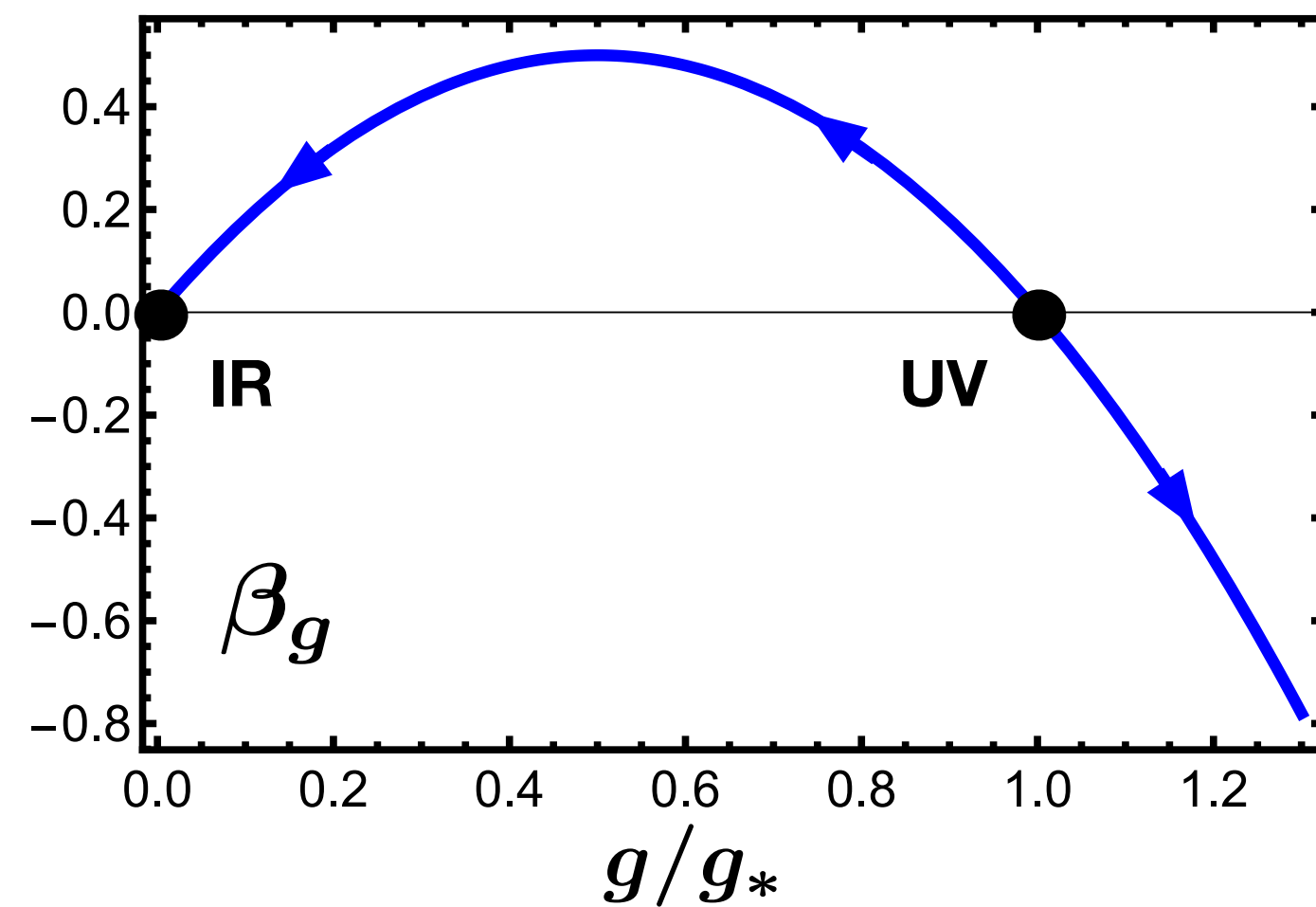
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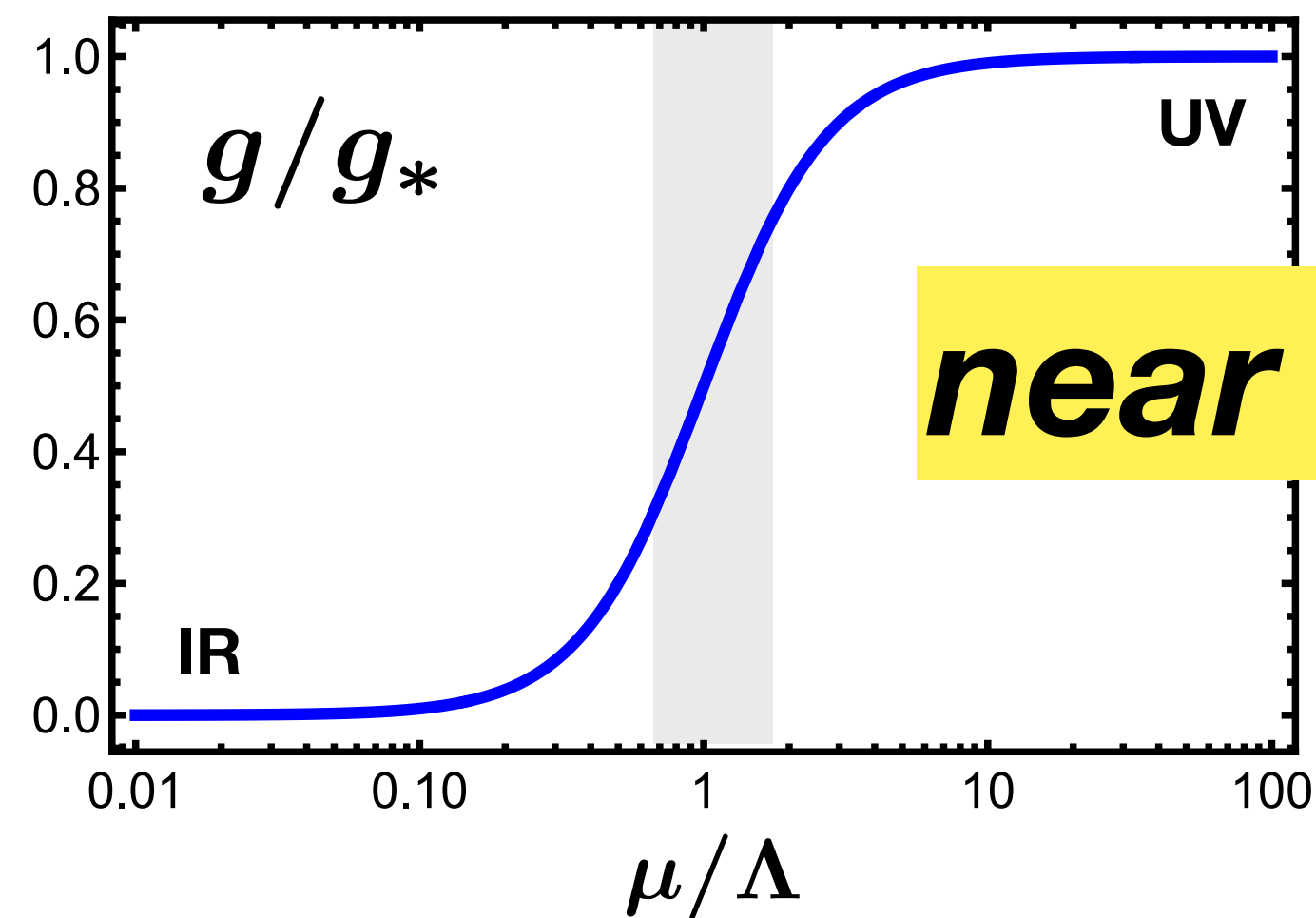
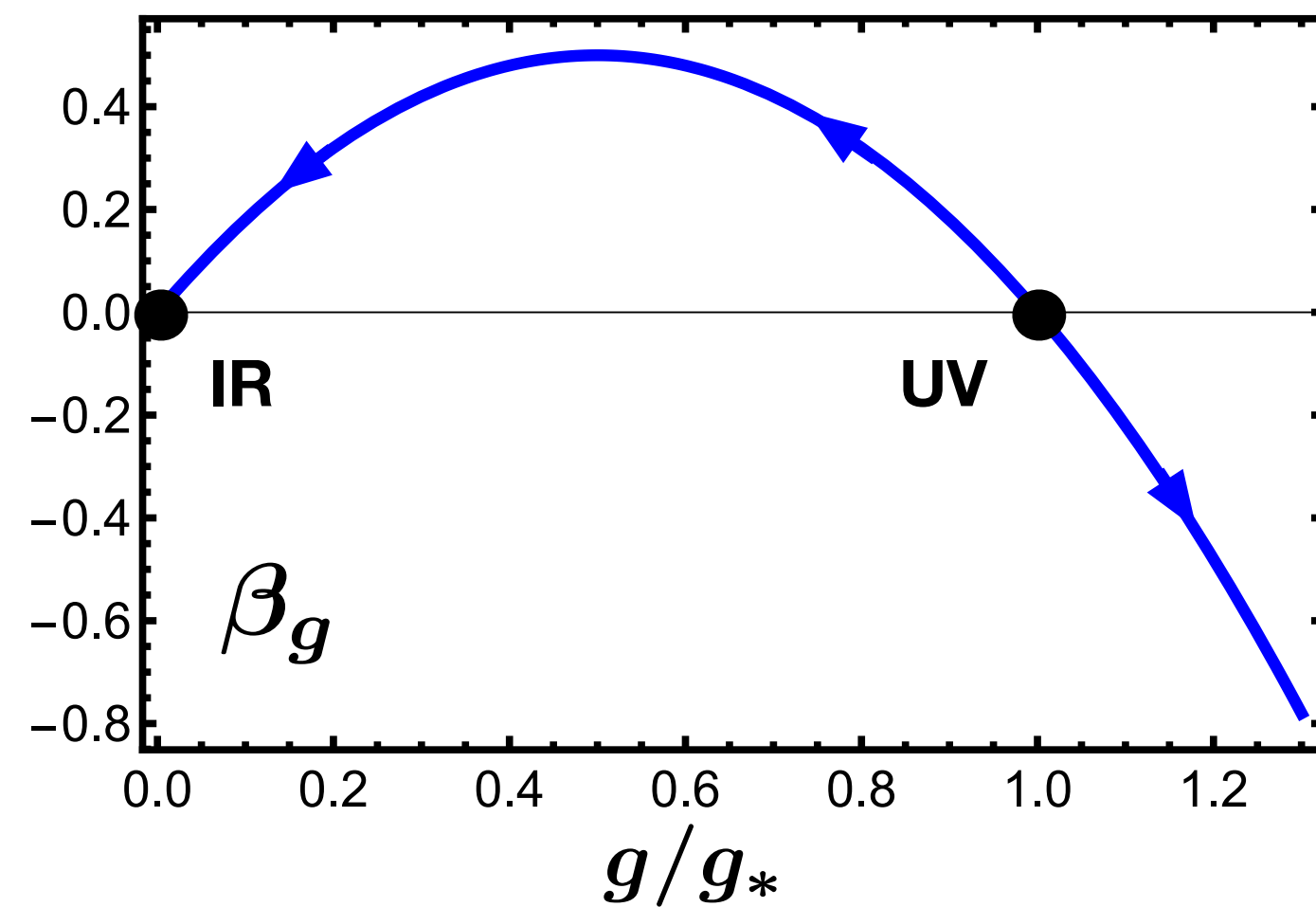
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***near conformal in UV***

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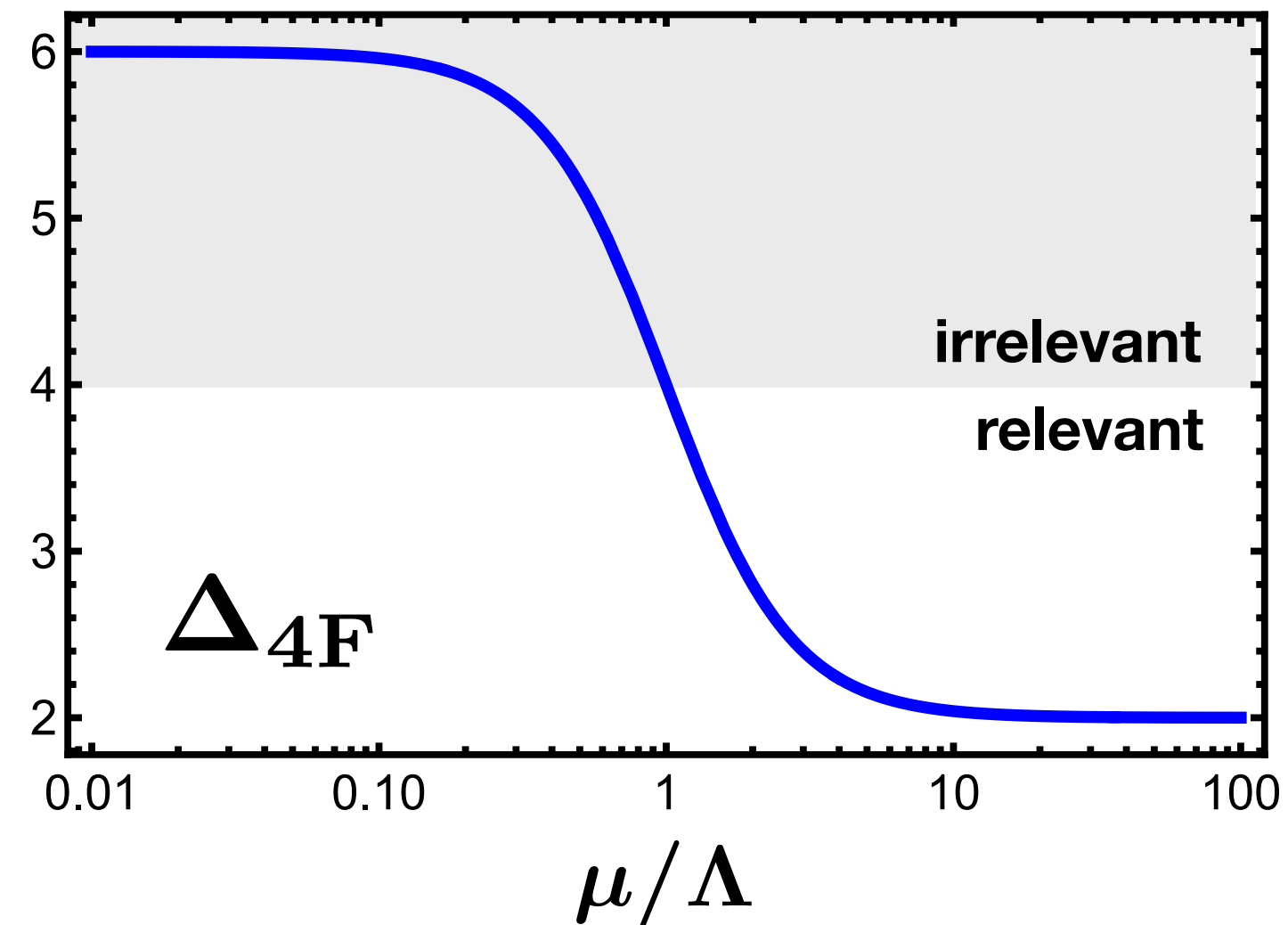
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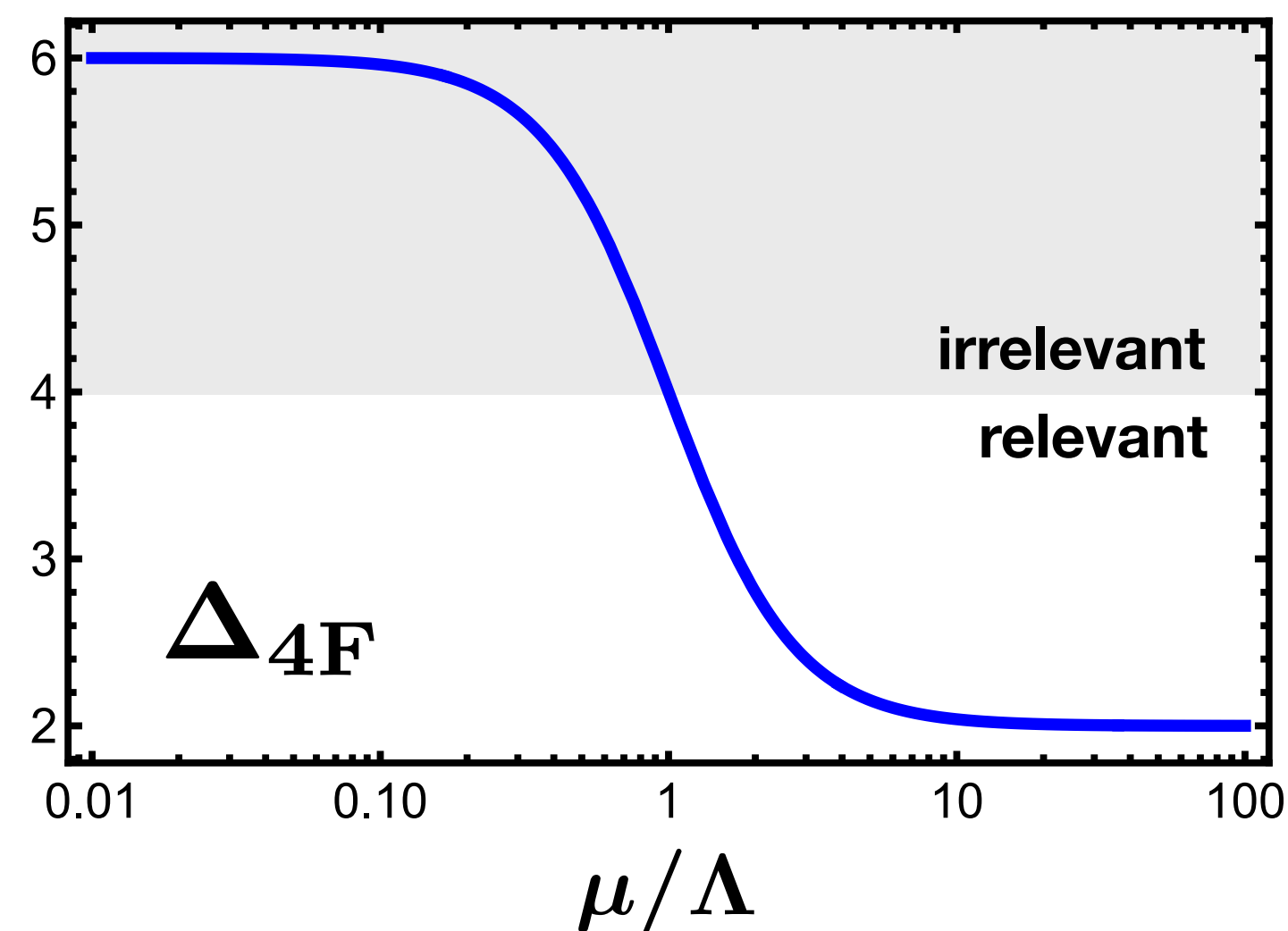
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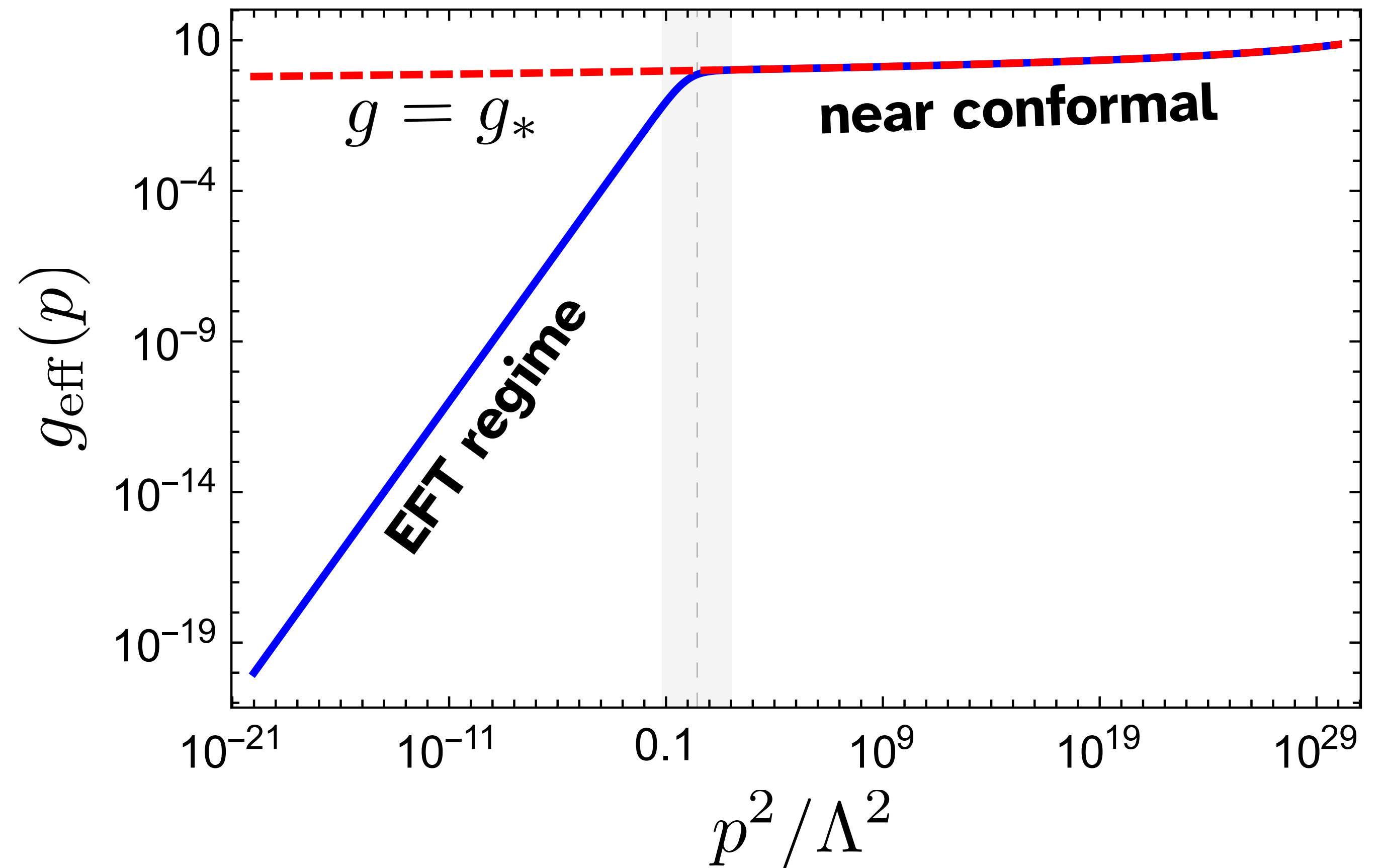
logarithmic running



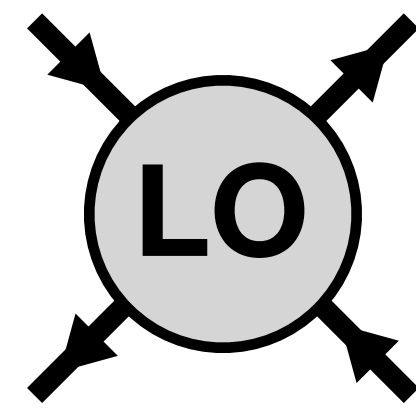
**large anomalous dimensions:**  
*dim-6 becomes dim-2*  
*EFT power counting breaks*

# Near conformality & the EFT crossover

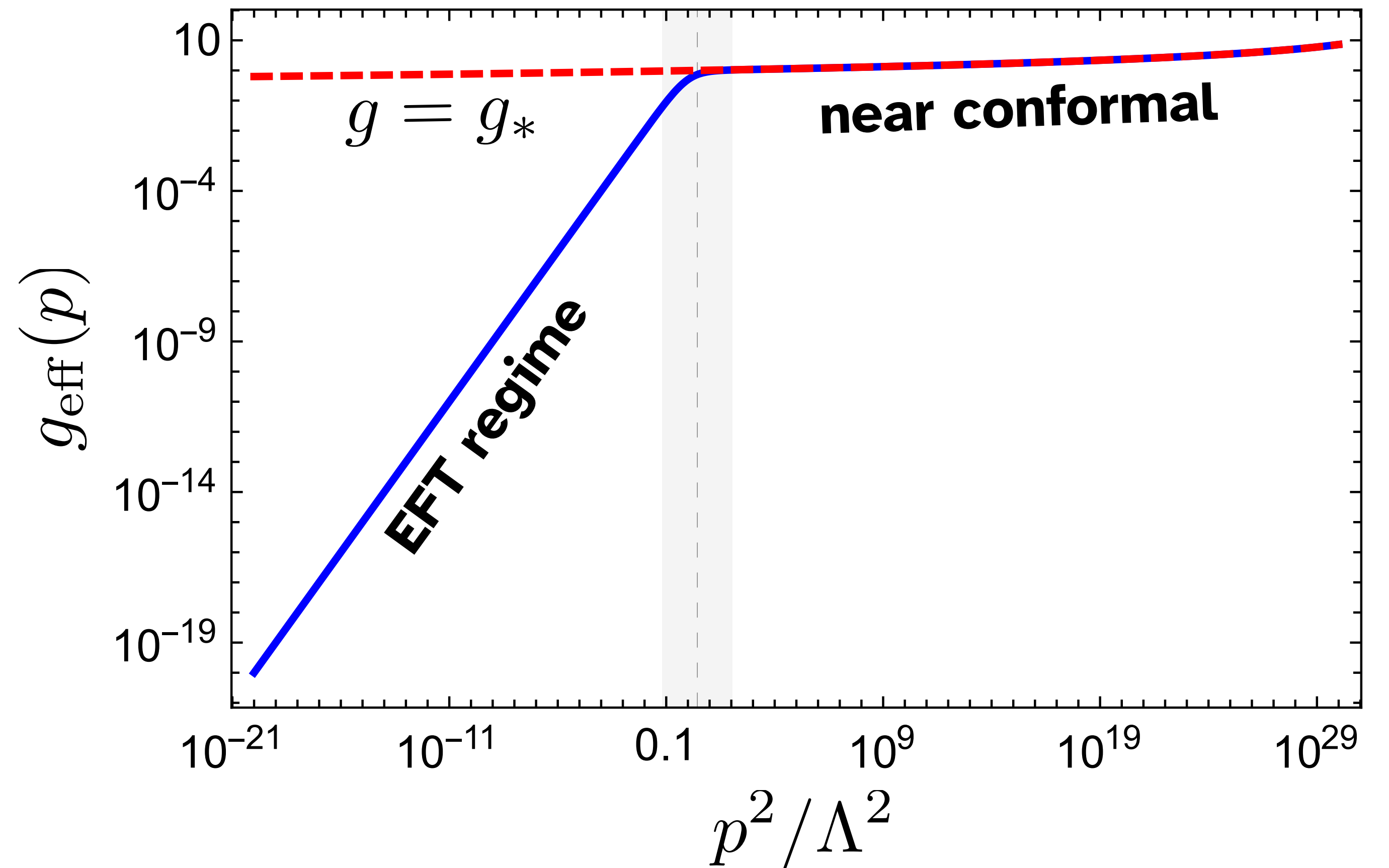
$$\text{LO} \sim \frac{N^{-1}}{p^2} g_{\text{eff}}(p)$$



# Near conformality & the EFT crossover

  $\sim \frac{N^{-1}}{p^2} g_{\text{eff}}(p)$

$$g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}$$

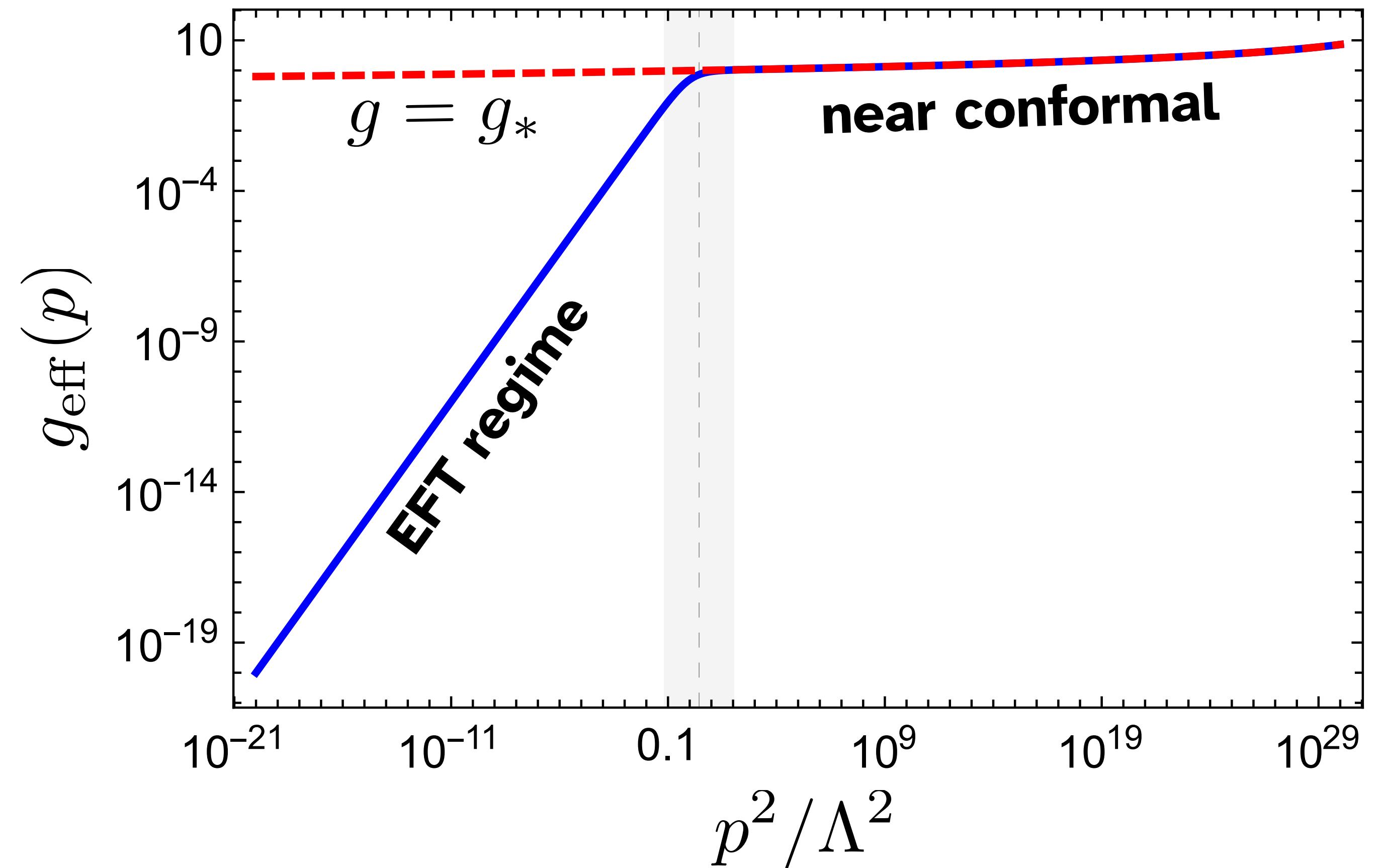


# Near conformality & the EFT crossover

$$\text{LO} \sim \frac{N^{-1}}{p^2} g_{\text{eff}}(p)$$

$$g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}$$

**EFT scale marks crossover:**  
onset of strong coupling, higher  
derivative terms become important



# Summary

## Four-fermion interactions can UV-complete themselves

- Tower of higher derivative 4F + 8F operators, but only **3 parameters**
- **Near scale invariance** in UV with relevant 4F + marginal 8F
- Strong coupling sets in near EFT scale — scheme choice important

## Further aspects

- **Landscape of theories:** symmetries, mom. dependence, ... , SMEFT, ...
- **Unitarity restoration:** (partial) RG fixed points in EFT  $\iff$  UV completions?
- **Constraining power** for Wilson coefficients in EFTs of new physics?