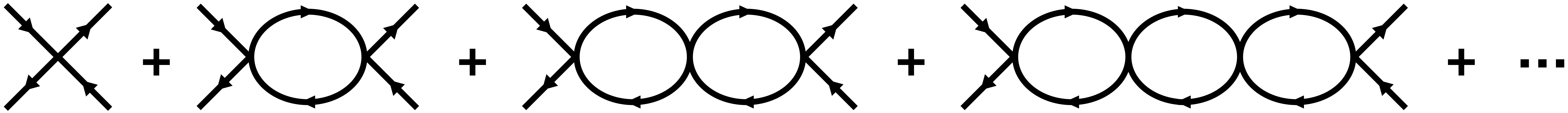
Renormalisable four-fermion interactions in four dimensions

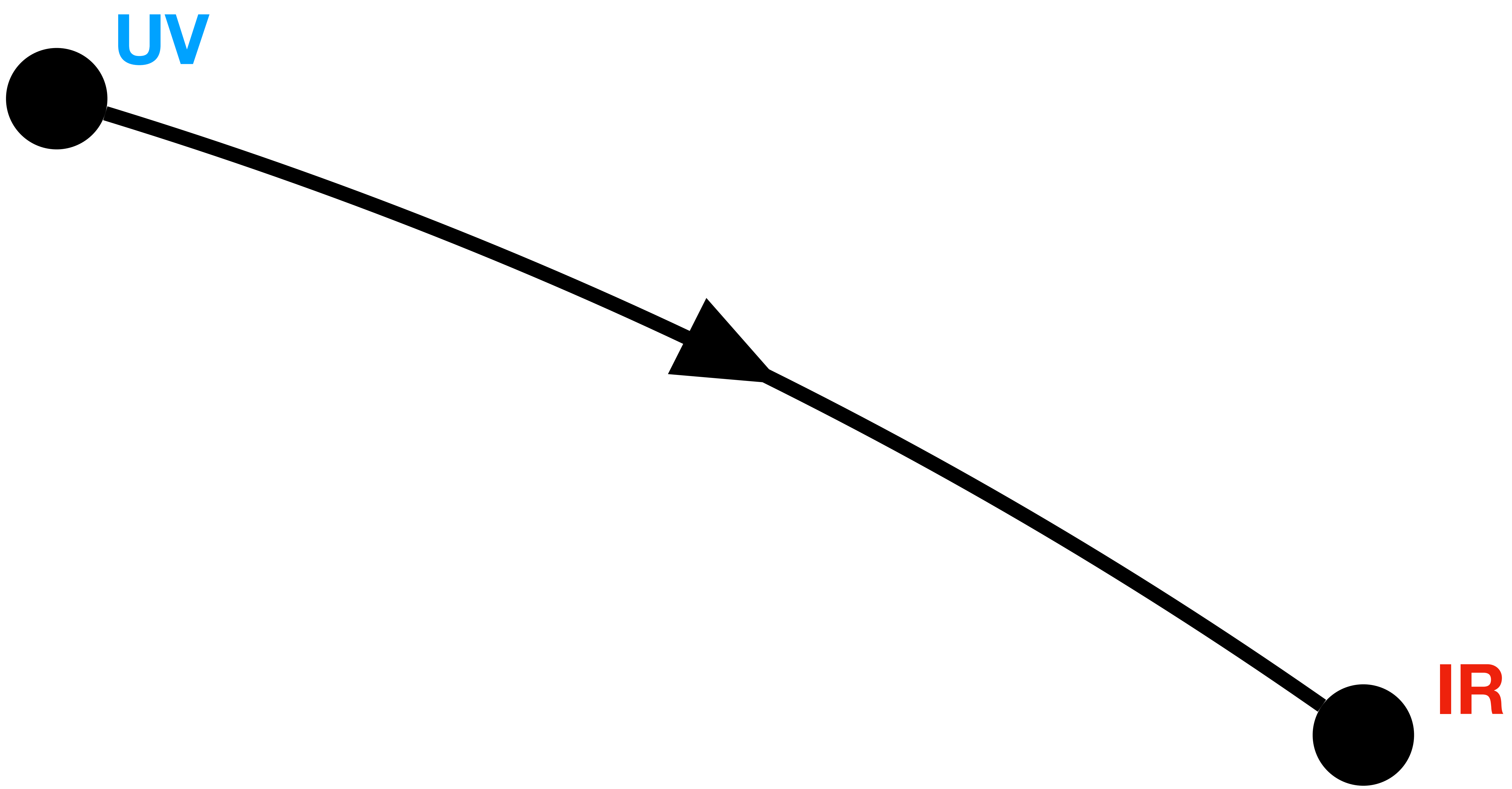


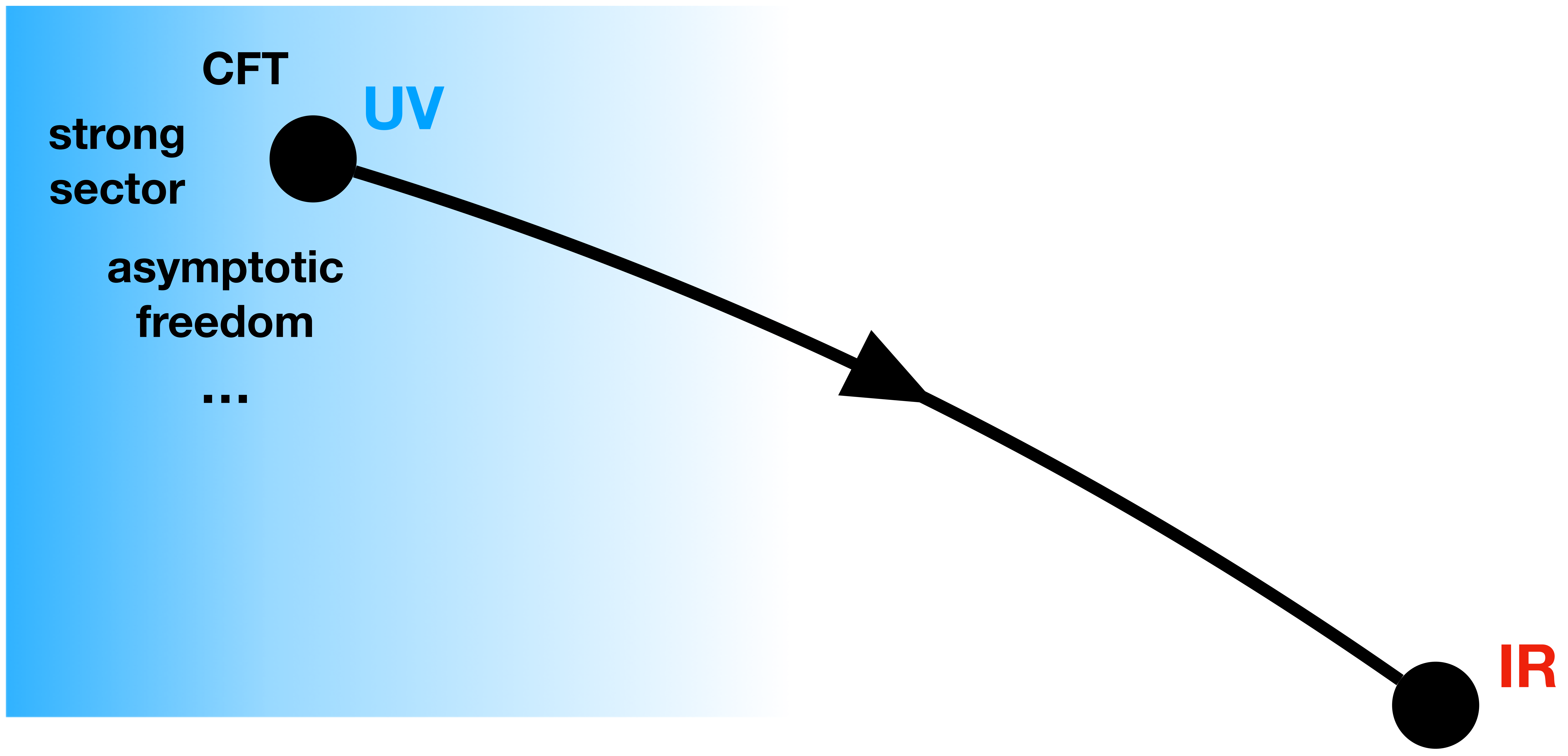
Charlie Cresswell-Hogg
Chalmers University

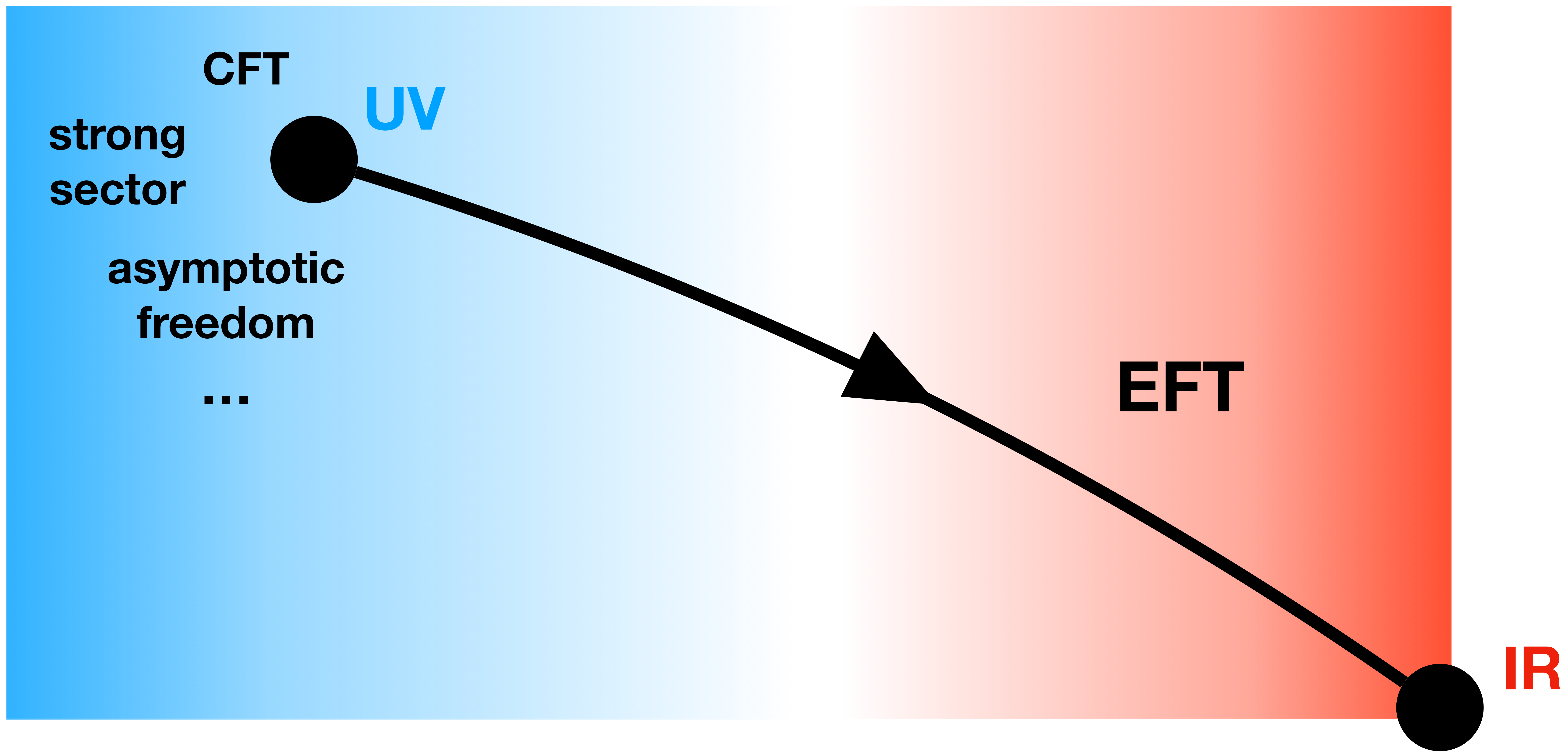
based on ***2603.19357*** *with Daniel Litim @ Sussex U.*

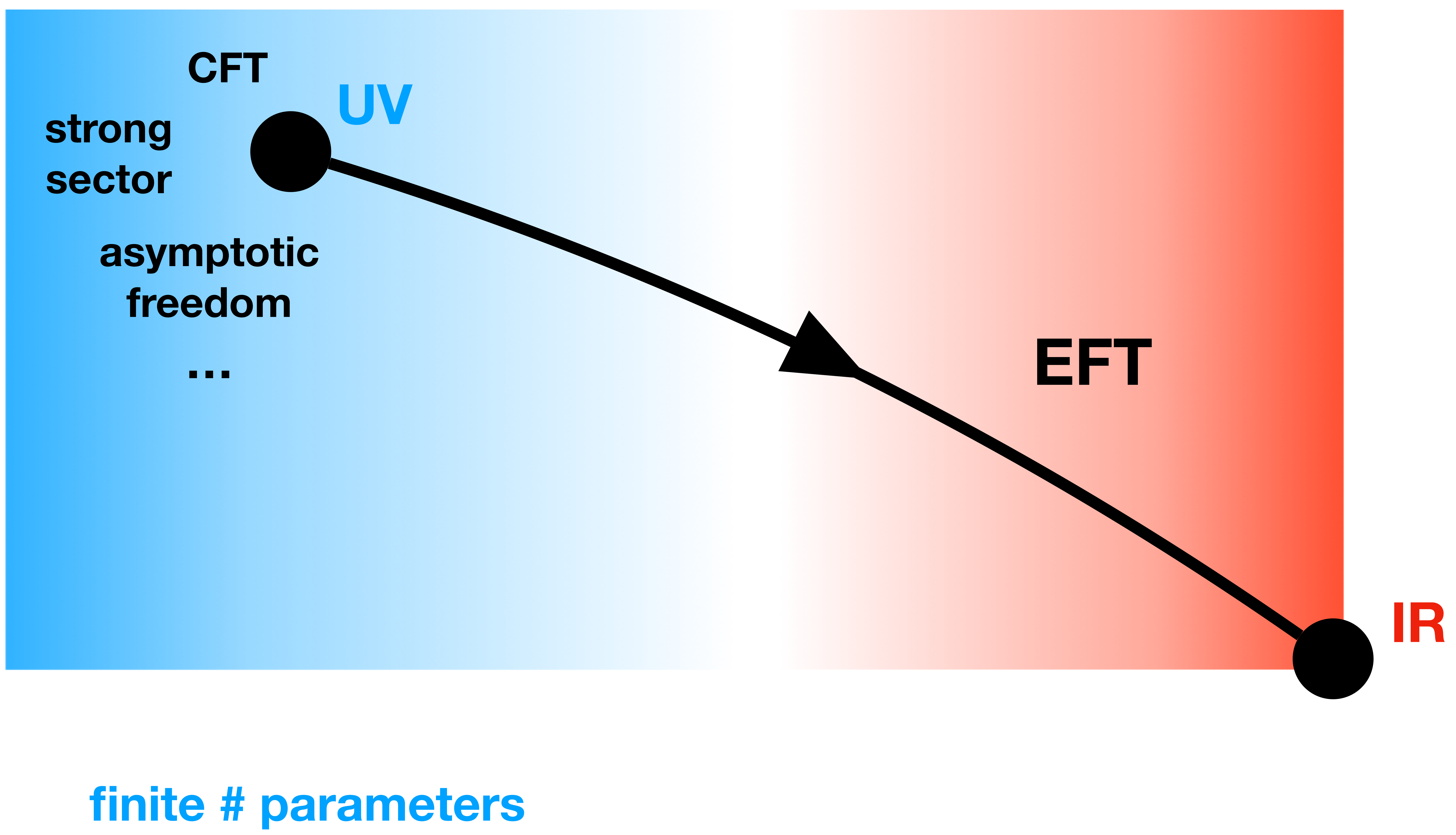
Partikeldagarna 2026 @ Lund

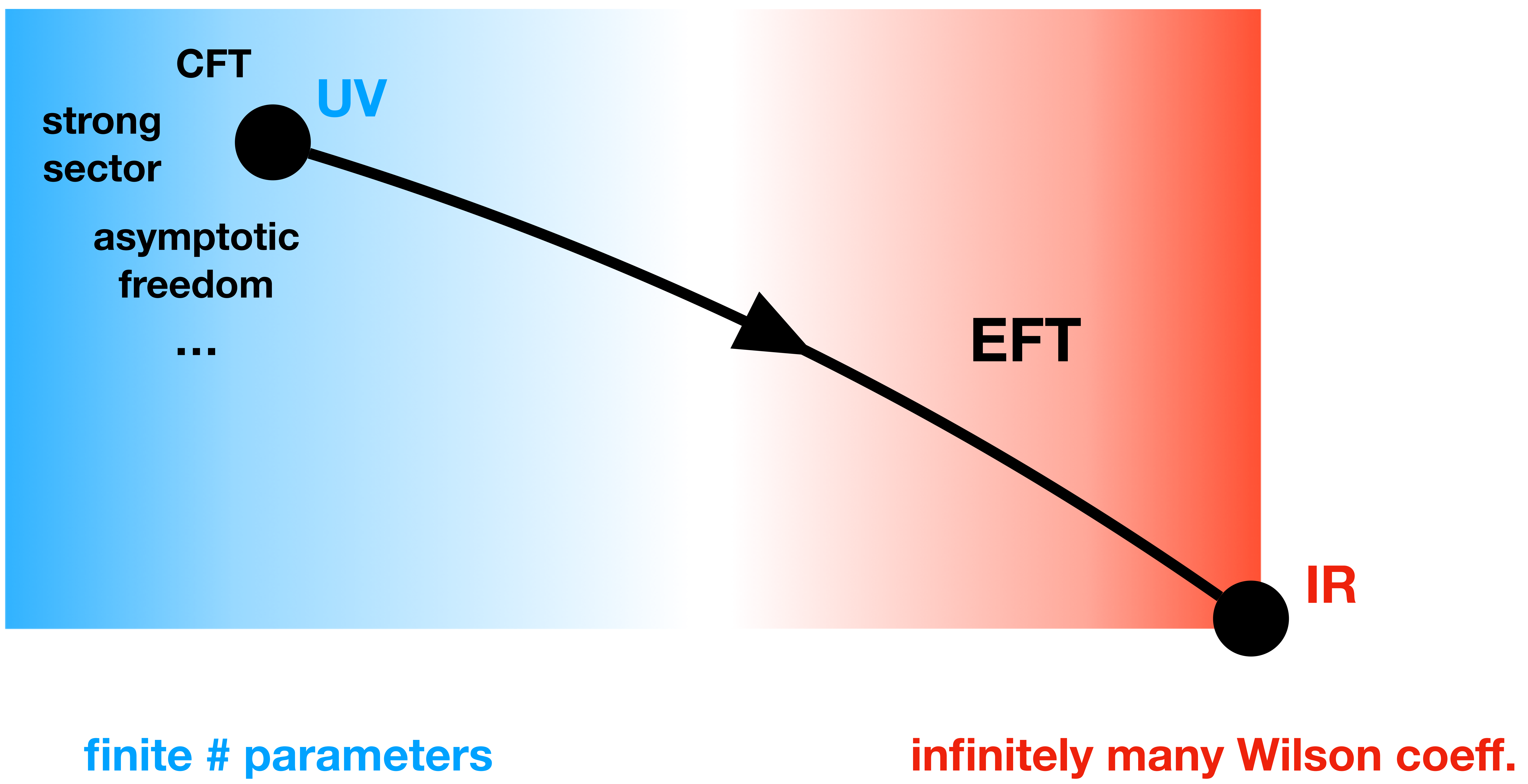
CHALMERS
TEKNISKA HÖGSKOLA

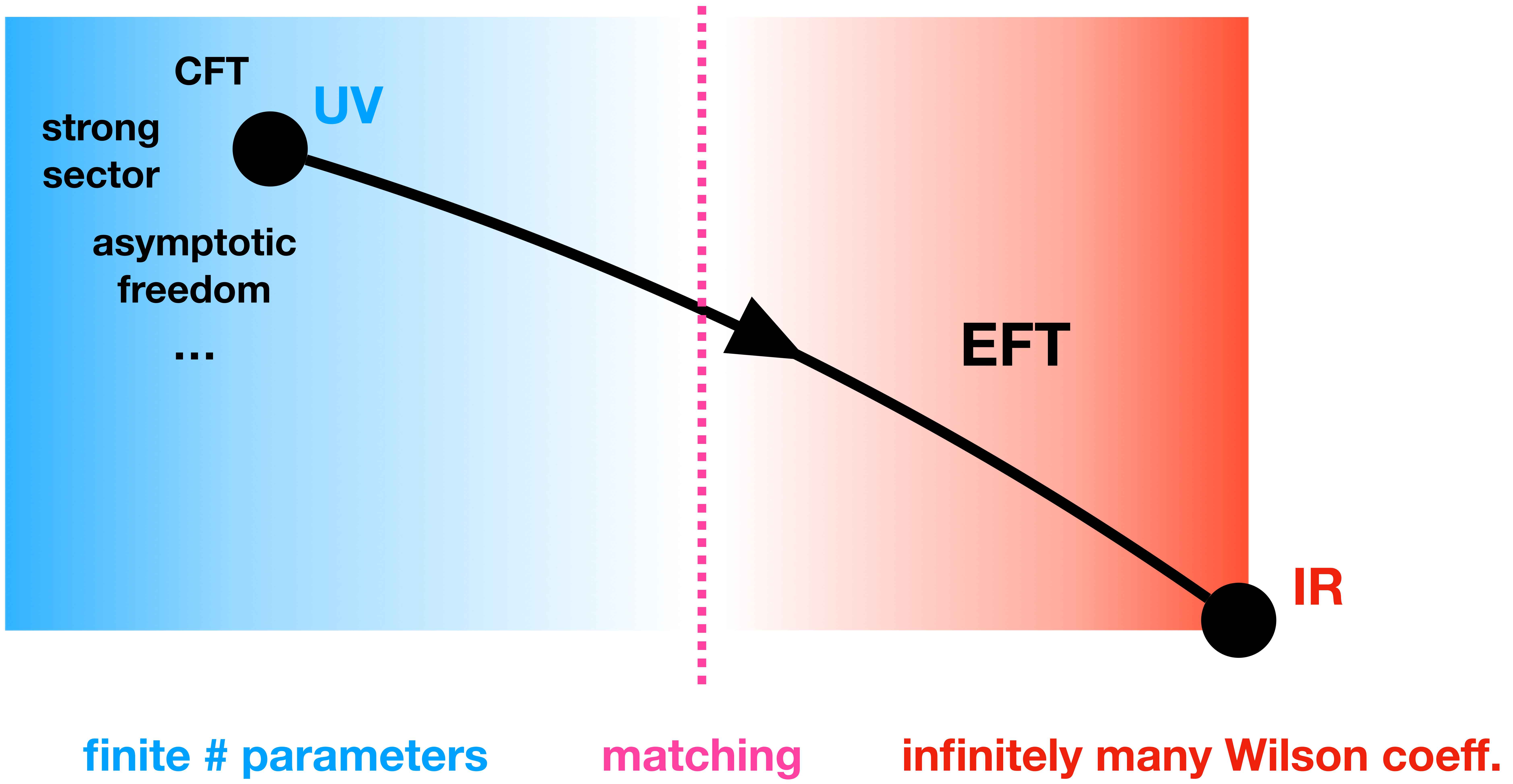

UV
IR


CFT
UV
strong
sector
asymptotic
freedom
...
IR


CFT
UV
strong sector
asymptotic freedom
...
EFT
IR


CFT
UV
strong
sector
asymptotic
freedom
...
EFT
IR
finite # parameters


CFT
UV
strong
sector
asymptotic
freedom
...
EFT
IR
finite # parameters
infinitely many Wilson coeff.


CFT
UV
strong sector
asymptotic freedom
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EFT
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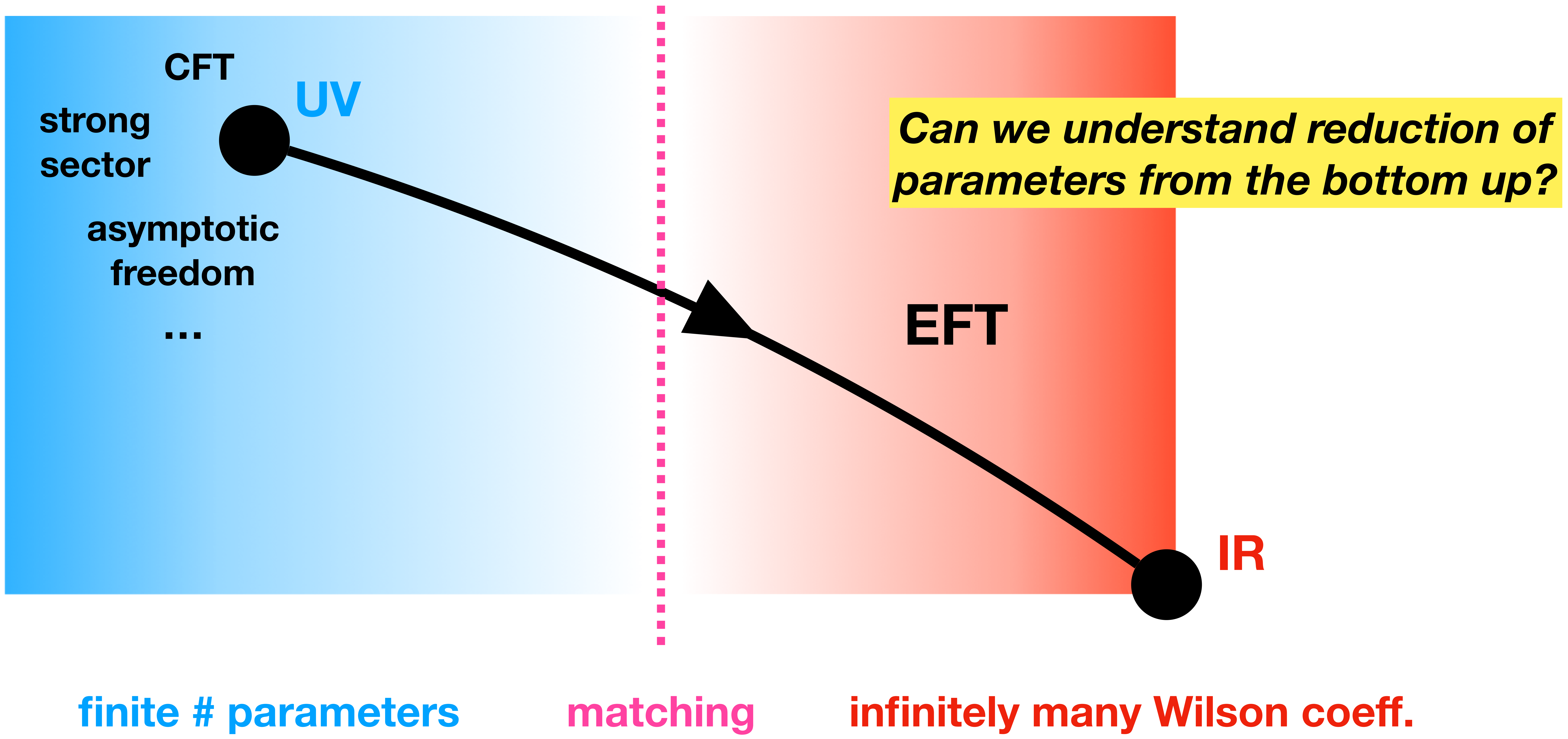
finite # parameters
matching
infinitely many Wilson coeff.


CFT
UV
strong sector
asymptotic freedom
...
Can we understand reduction of parameters from the bottom up?
EFT
IR
finite # parameters
matching
infinitely many Wilson coeff.

A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \cdots$$

A toy EFT

$$\bar{\psi}_a \partial\!\!\!/ \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \cancel{\cdots}$$

2+1 dimensions: 1/*N* renormalisable & conformal at high energy

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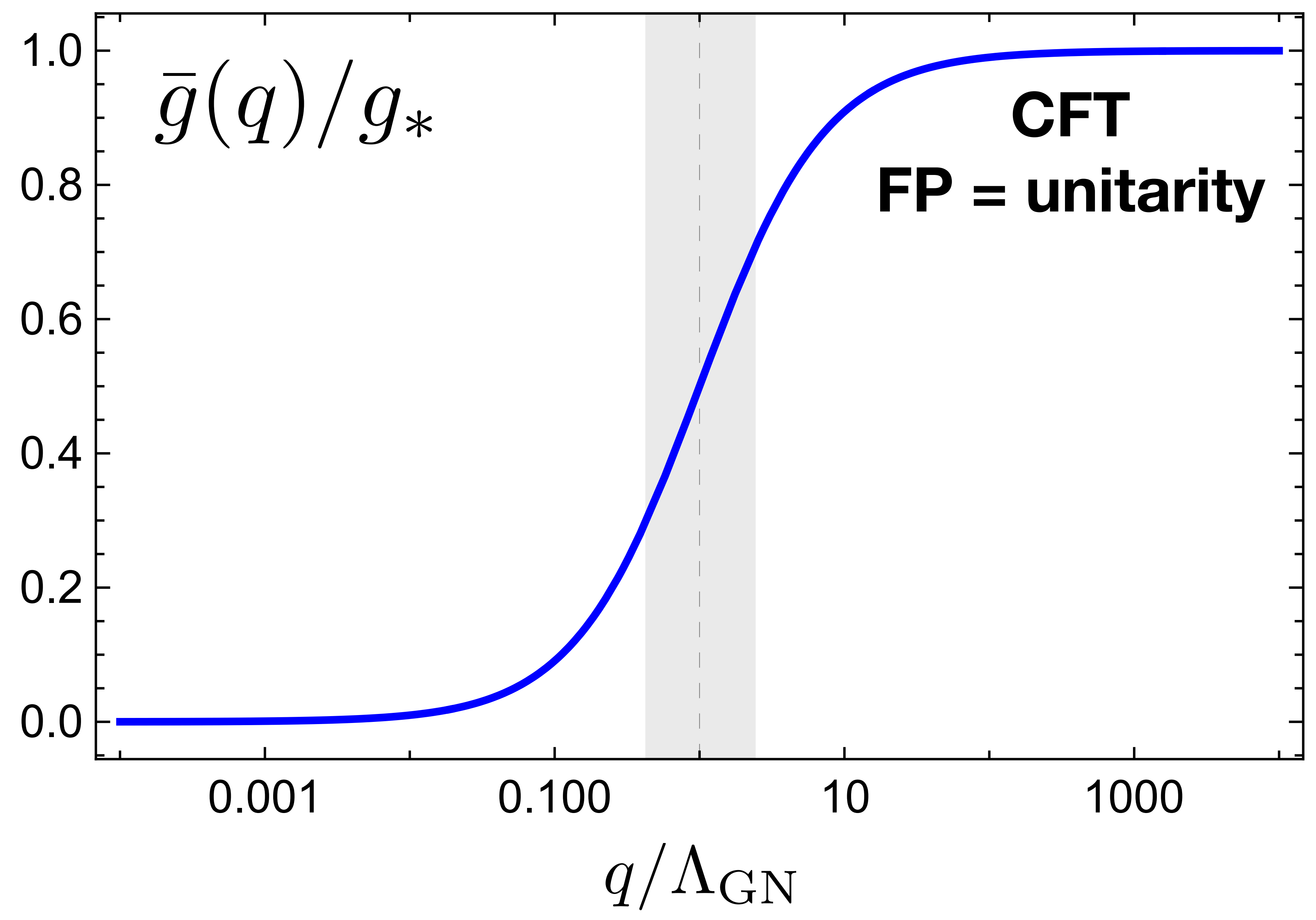
2+1 dimensions: 1/*N* renormalisable & conformal at high energy

Wilson ’73
Gross, Neveu ’74
Parisi ’75
Gawędzki, Kupiainen ’85
TIP! **Rosenstein, Warr, Park ’88 (PRL)**
Rosenstein, Warr, Park ’91 (Phys.Rept.)
Gat, Kovner, Rosenstein, Warr ’90
de Calan, Faria Da Veiga, Magnen, Seneor ’91
+ many more



A toy EFT

$$\bar{\psi}_a \partial\!\!\!/ \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \cdots$$

3+1 dimensions: 1/*N* renormalisable & *near*-conformal at high energy

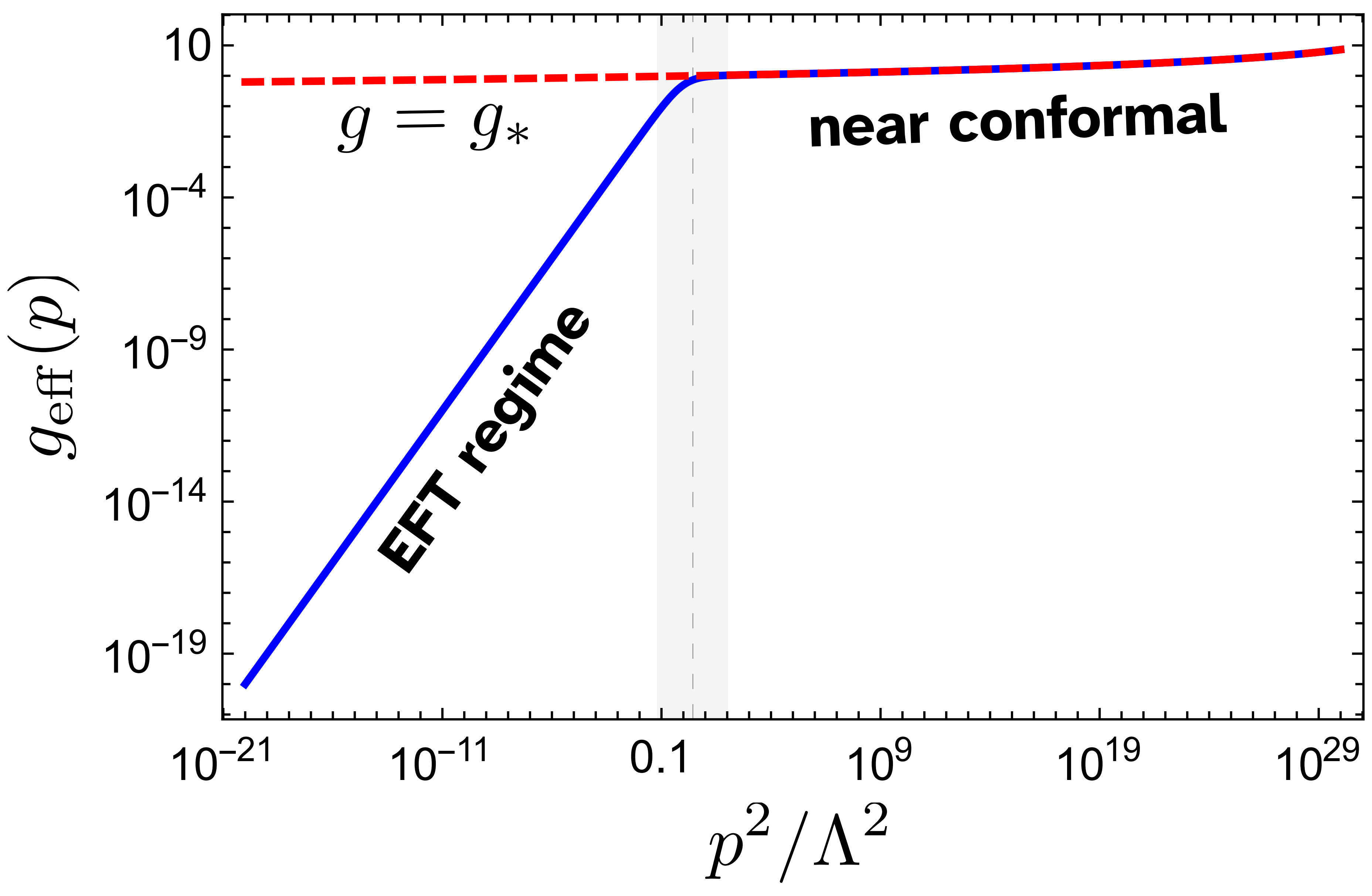
A toy EFT

$$\bar{\psi}_a \partial\!\!\!/ \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \cdots$$

3+1 dimensions: 1/*N* renormalisable & *near*-conformal at high energy

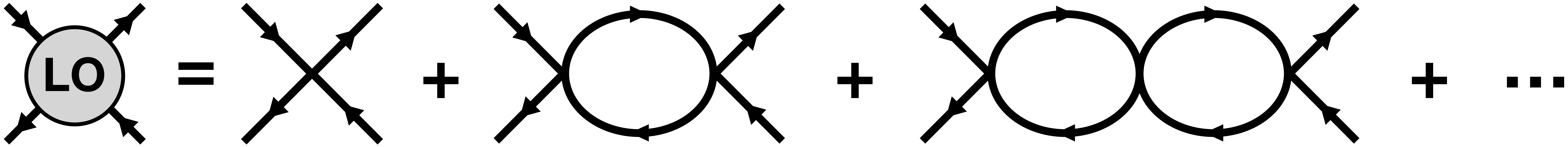
Formal renormalisability in 1/N dictates:

- tower of higher-derivative operators
- eight-fermion interactions (but no higher)
- only three fundamental parameters



Large-*N* diagrammatics

Four-point function:



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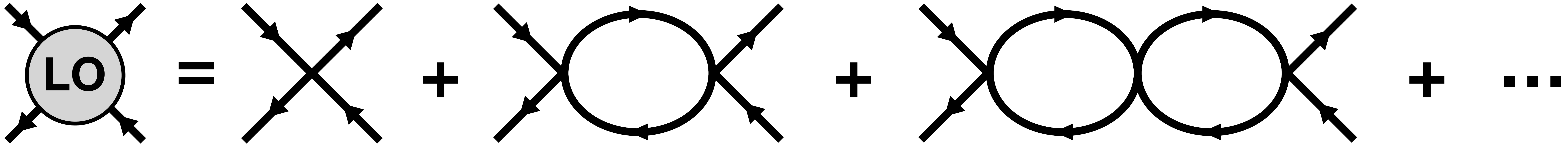
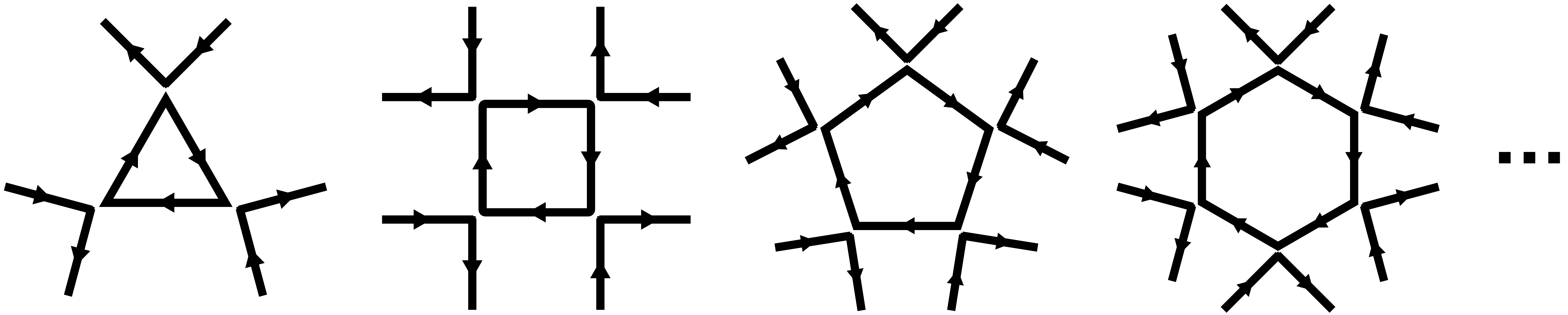


Higher-point functions:



Large-*N* diagrammatics

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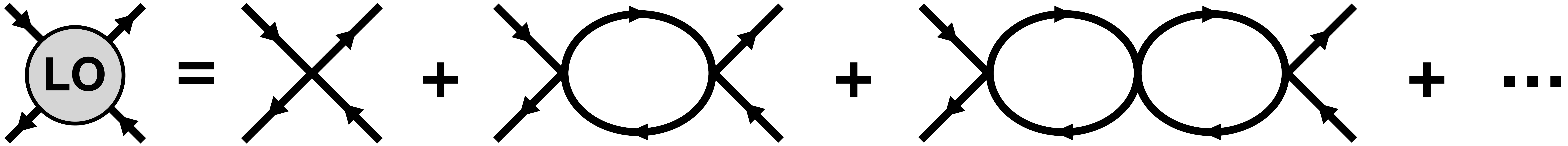
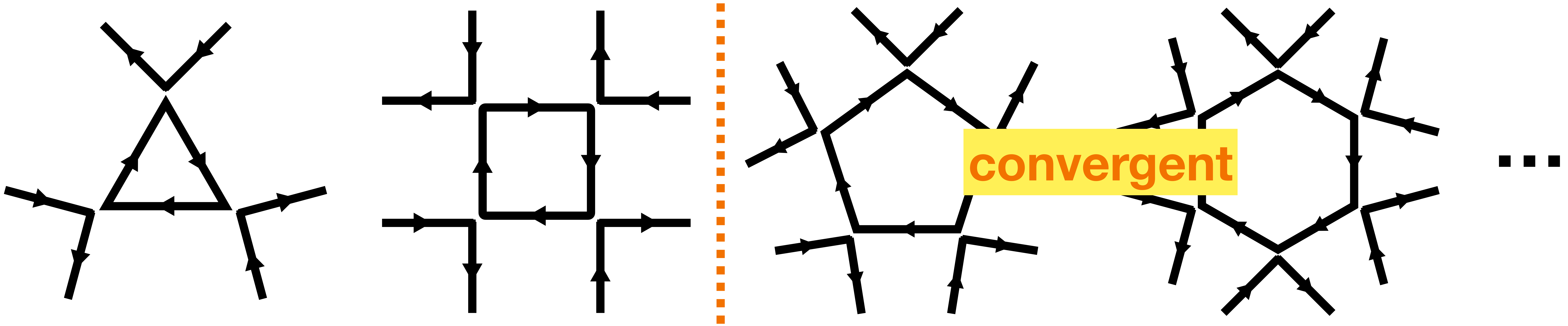


Higher-point functions:



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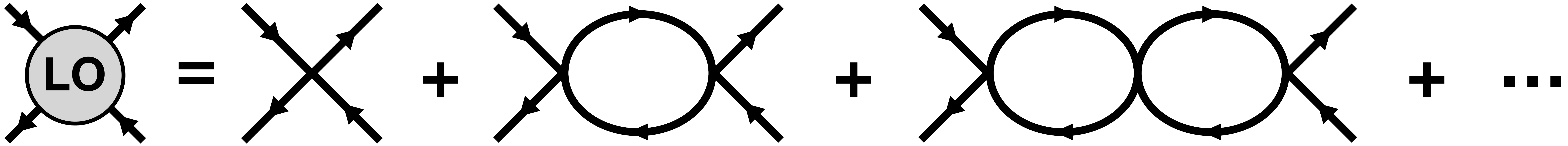

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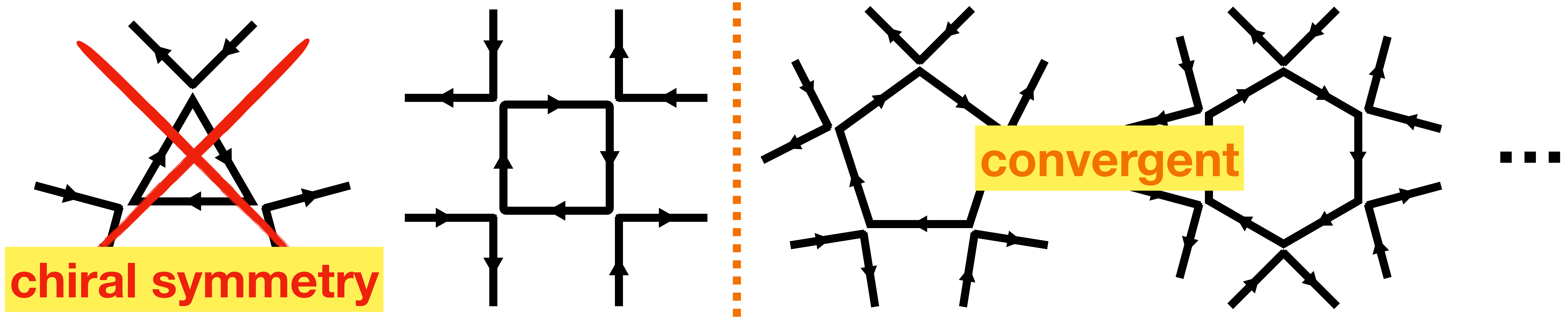
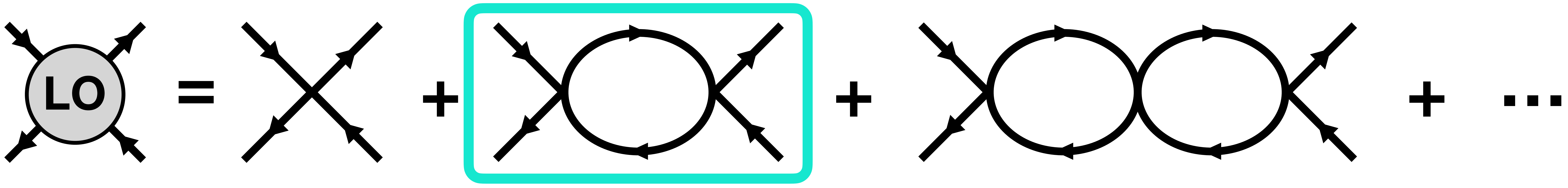


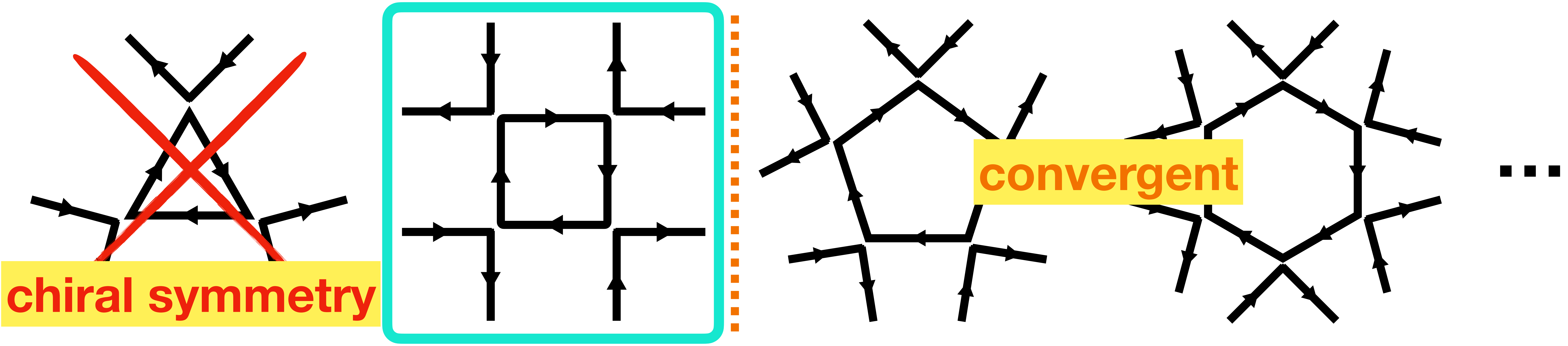
Large-*N* diagrammatics

Four-point function:

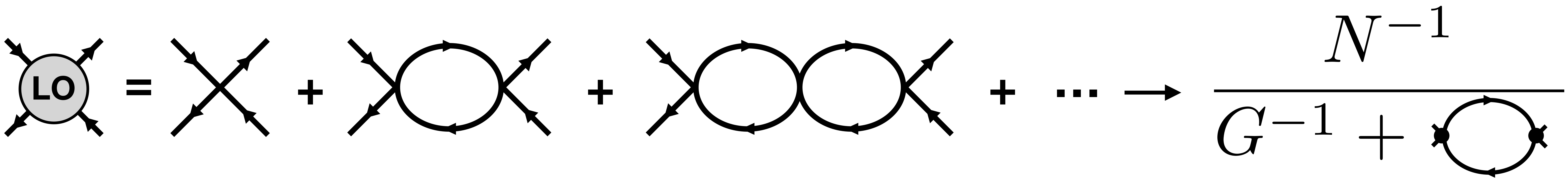


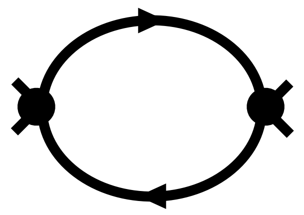
divergent
subdiagrams

Higher-point functions:

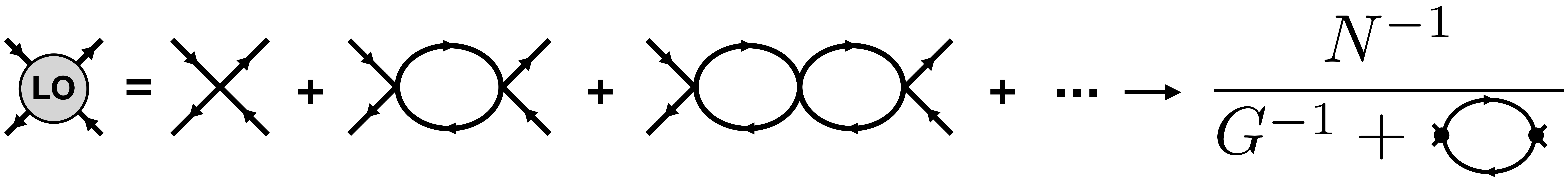


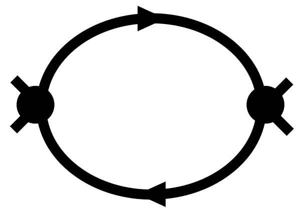
Four-point function



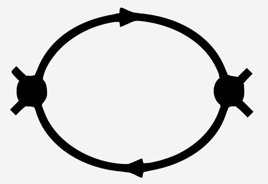
 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

Four-point function



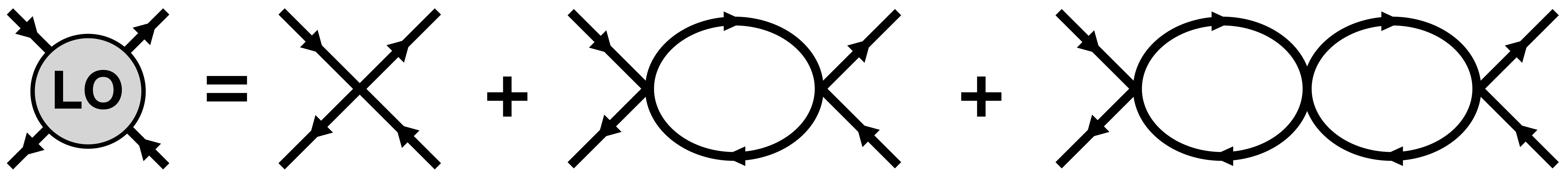
 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

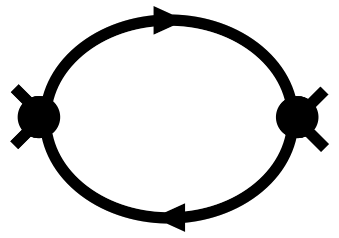
$$\frac{1}{2N} G(\bar{\psi}_a \psi_a)^2 \;\to\; \frac{1}{2N}(\bar{\psi}_a \psi_a) F(-\partial^2)(\bar{\psi}_b \psi_b)$$

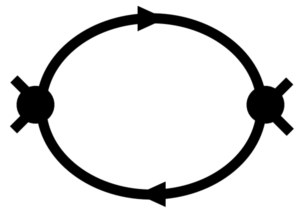
$F(p^2)^2 \times$ 

vertices factorise @ LO

Four-point function



$+ \cdots \longrightarrow \dfrac{N^{-1}}{G^{-1} + }$ 

 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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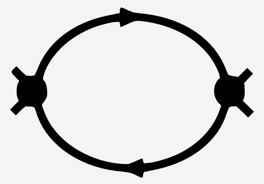
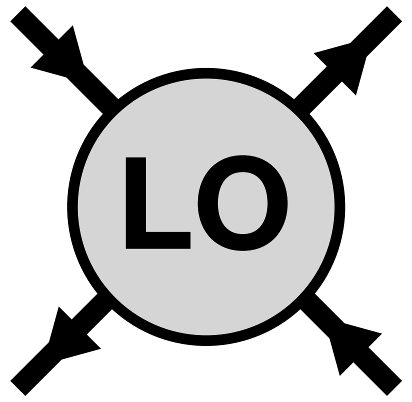
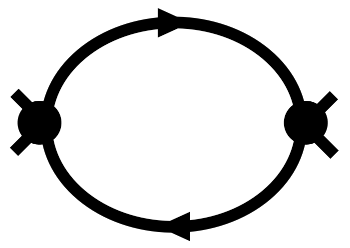
$F(p^2)^2 \times$ 

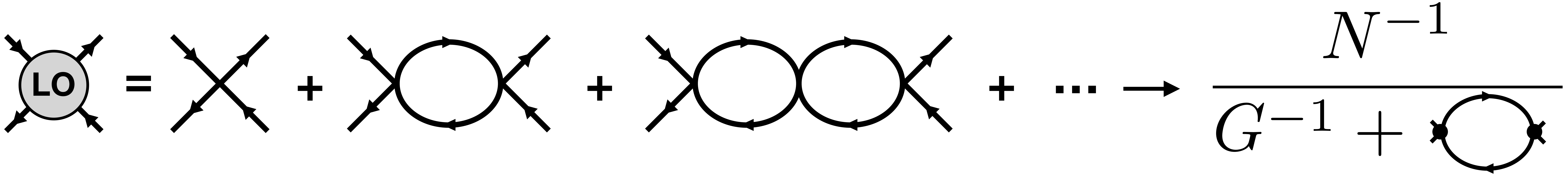
vertices factorise @ LO

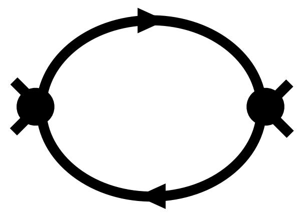
$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \quad \to$$



$\sim \dfrac{N^{-1}}{Zp^2 + G^{-1} + }$ 

Four-point function



 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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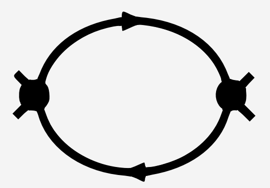
$F(p^2)^2 \times$ 

vertices factorise @ LO

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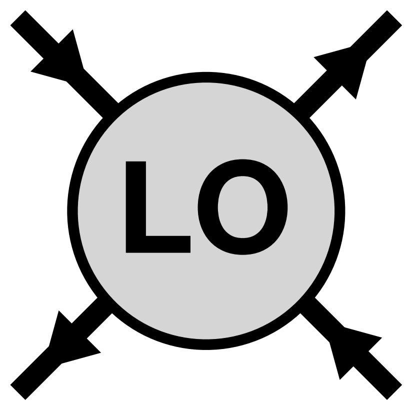
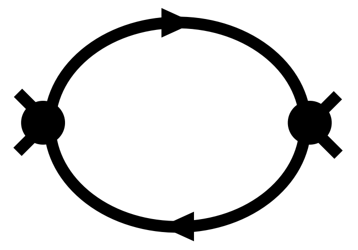
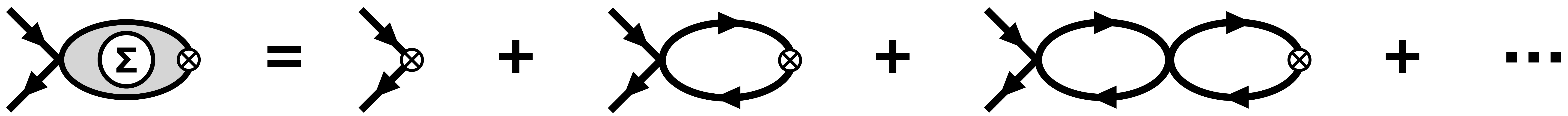


$$\sim \frac{N^{-1}}{Zp^2 + G^{-1} + }$$



subtract infinitely many higher-derivative divergences with one additional parameter

Eight-point function



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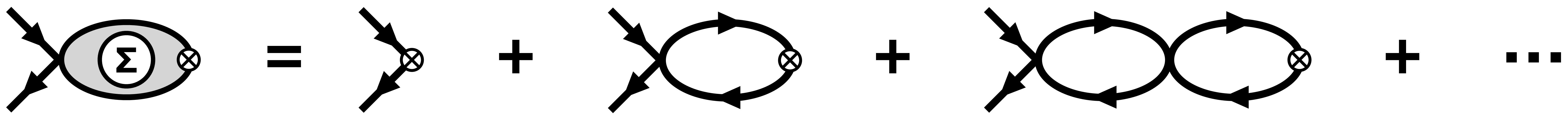
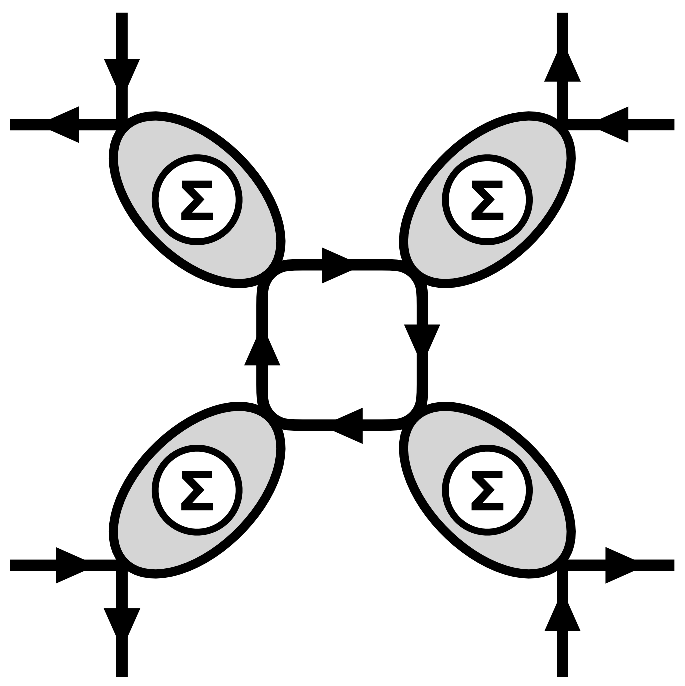
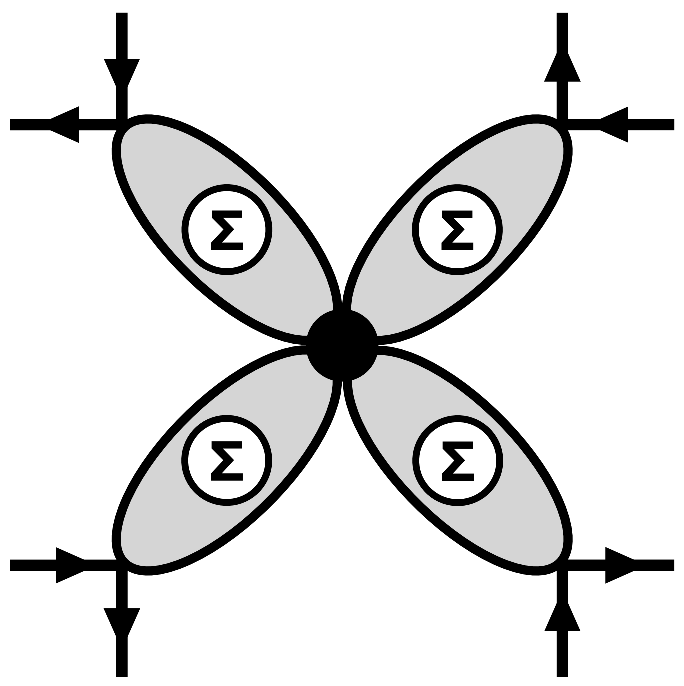


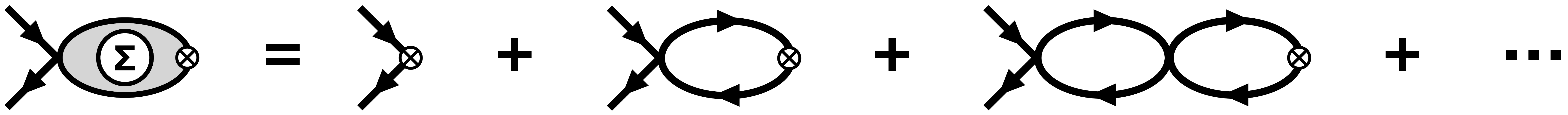


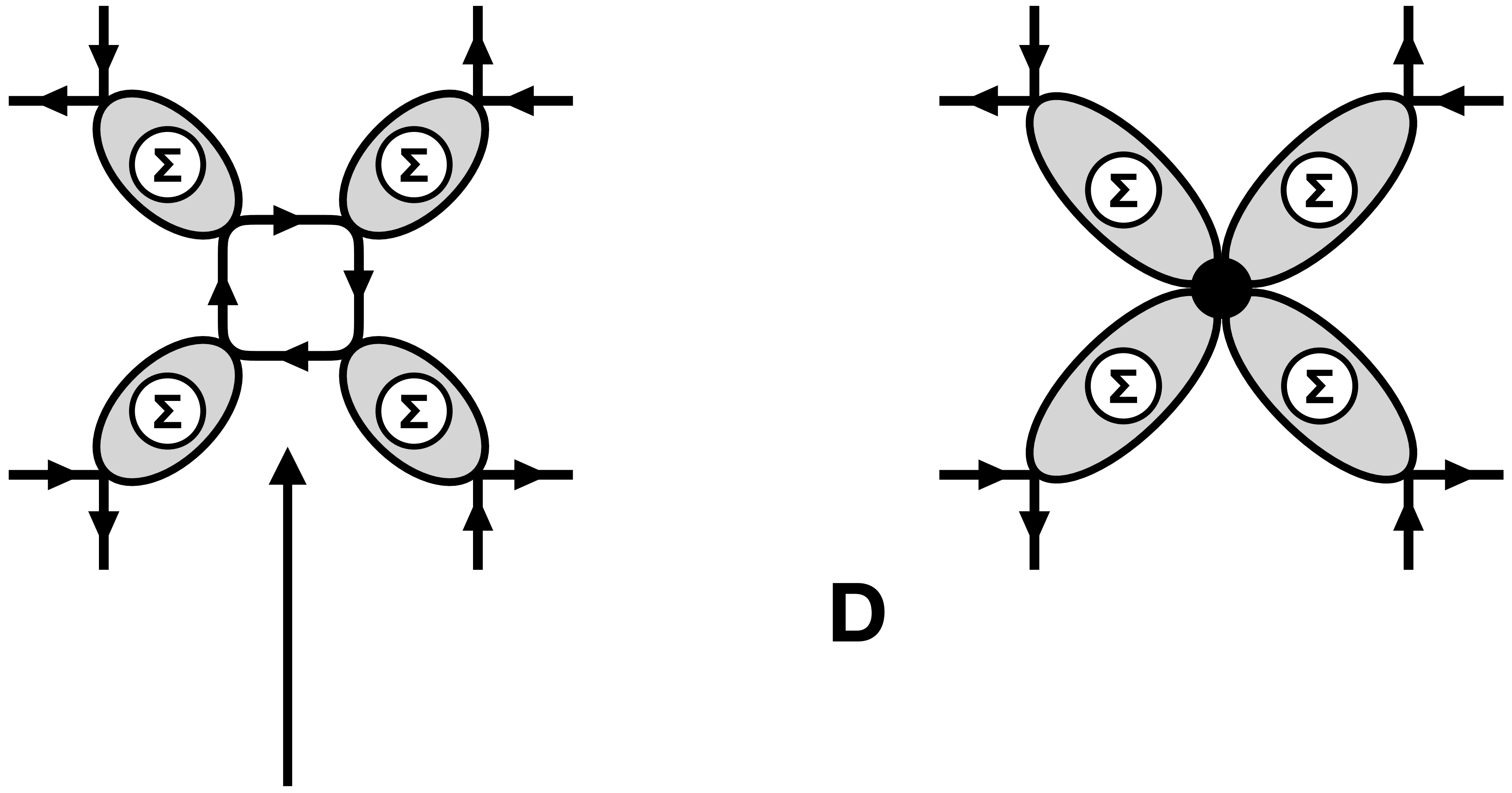
C



D

Eight-point function

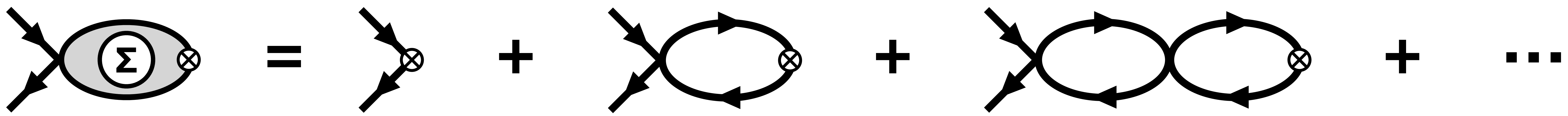


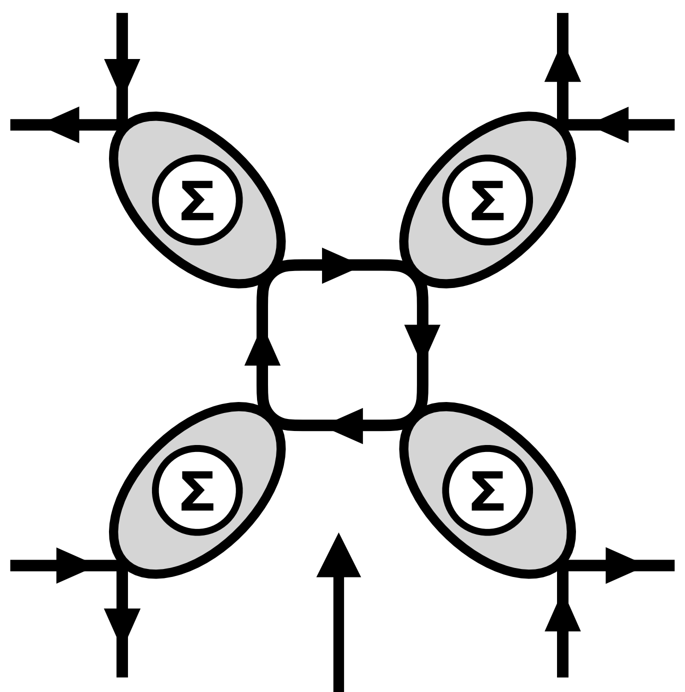


C **D**

$\sim \ln \Lambda$

Eight-point function





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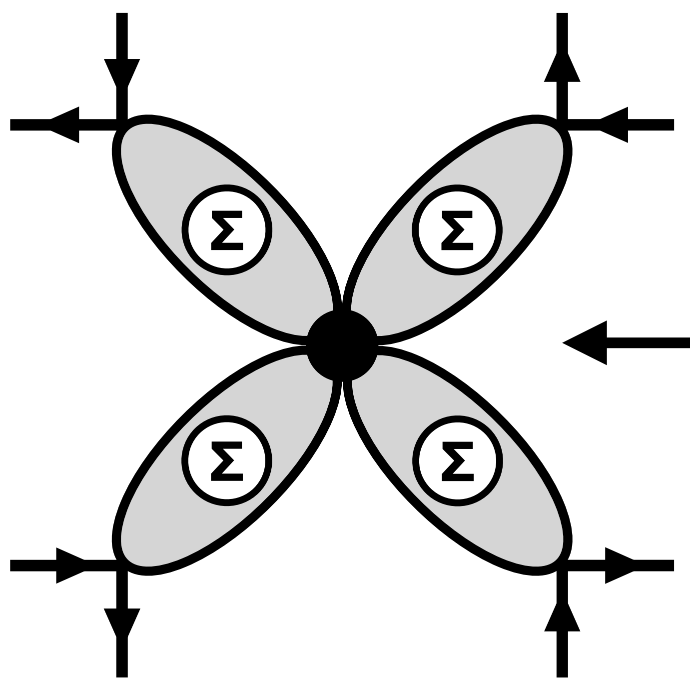
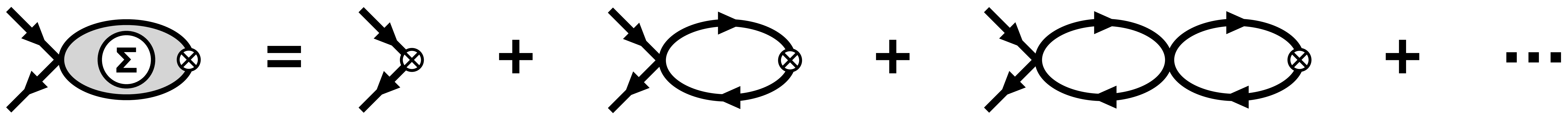


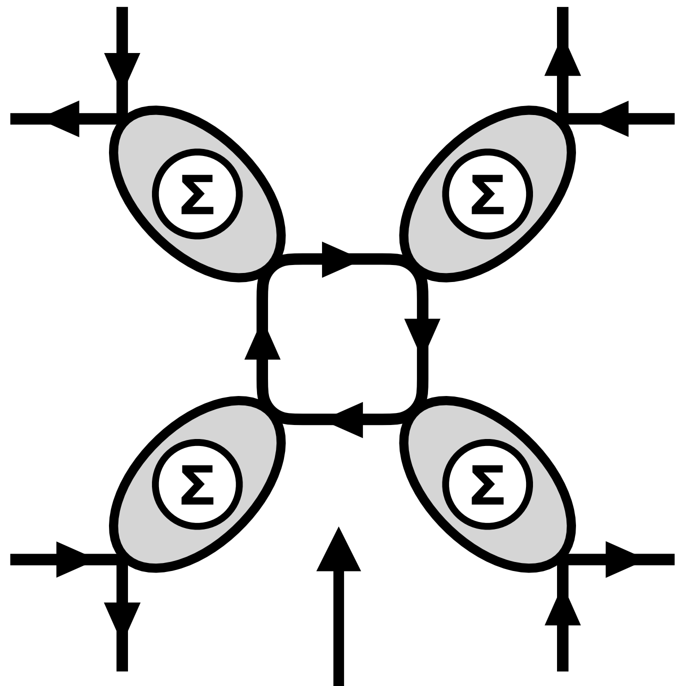
D

$$\frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

$$\sim \ln \Lambda$$

Eight-point function





C

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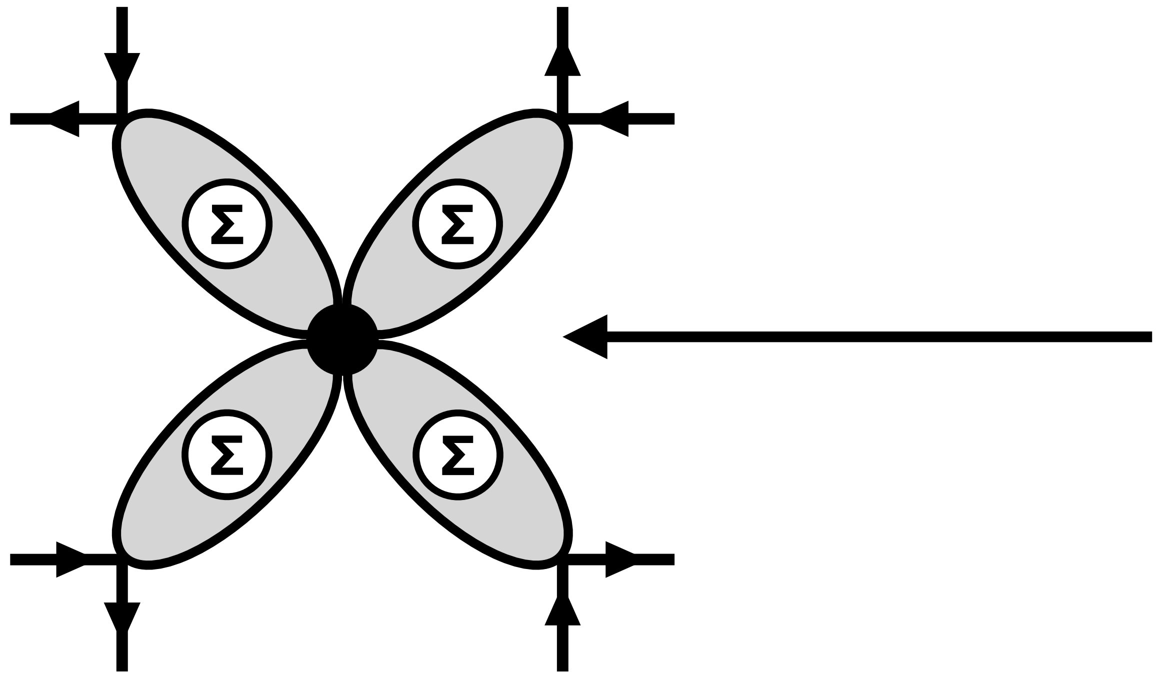

D

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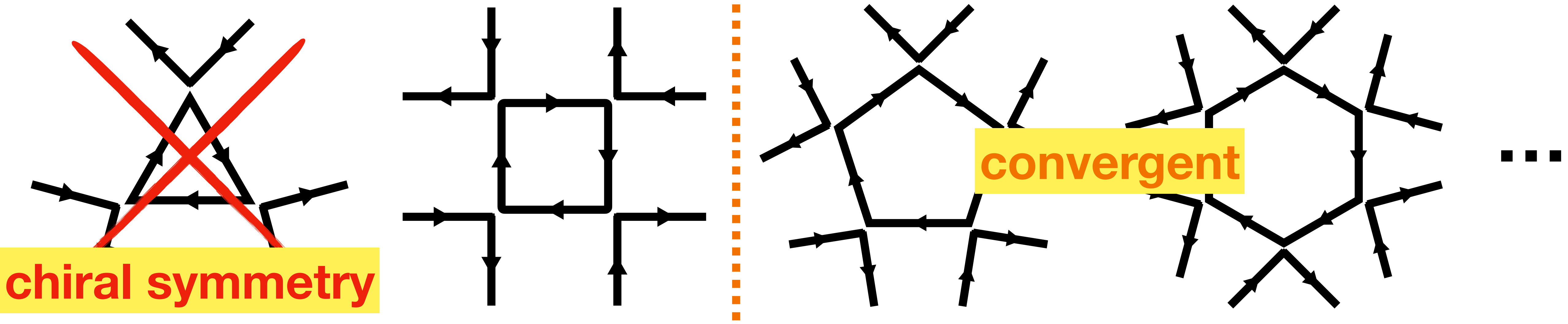
Everything @ LO = products of bubble sums with:



>> THEORY RENORMALISED. <<

Renormalisable four-fermion theory

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N}(\bar{\psi}_a\psi_a)F(-\partial^2)(\bar{\psi}_b\psi_b) + \frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

- **Predictive:** 3 parameters Z, G, λ
- **Tower** of higher derivative 4F + 8F
- **NO higher** (10F, 12F, …)

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holds order-by-order in 1/N.

cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

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- **Predictive:** 3 parameters Z, G, λ
- **Tower** of higher derivative 4F + 8F
- **NO higher** (10F, 12F, …)

Renormalised couplings:

$$g(\mu) = \mu^2\, G(\mu)$$

$$Z(\mu) \qquad \lambda(\mu)$$

holds order-by-order in 1/N.

cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

Running couplings

non-universal:

$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

$$\underbrace{\hspace{10em}}_{\textbf{UV fixed point}}$$

e.g. MOM scheme: $g_* = 8\pi^2$

universal:

$$\beta_Z = -\frac{1}{4\pi^2} \qquad \beta_\lambda = -\frac{3}{\pi^2}$$

$$\underbrace{\hspace{10em}}_{\textbf{logarithmic running}}$$

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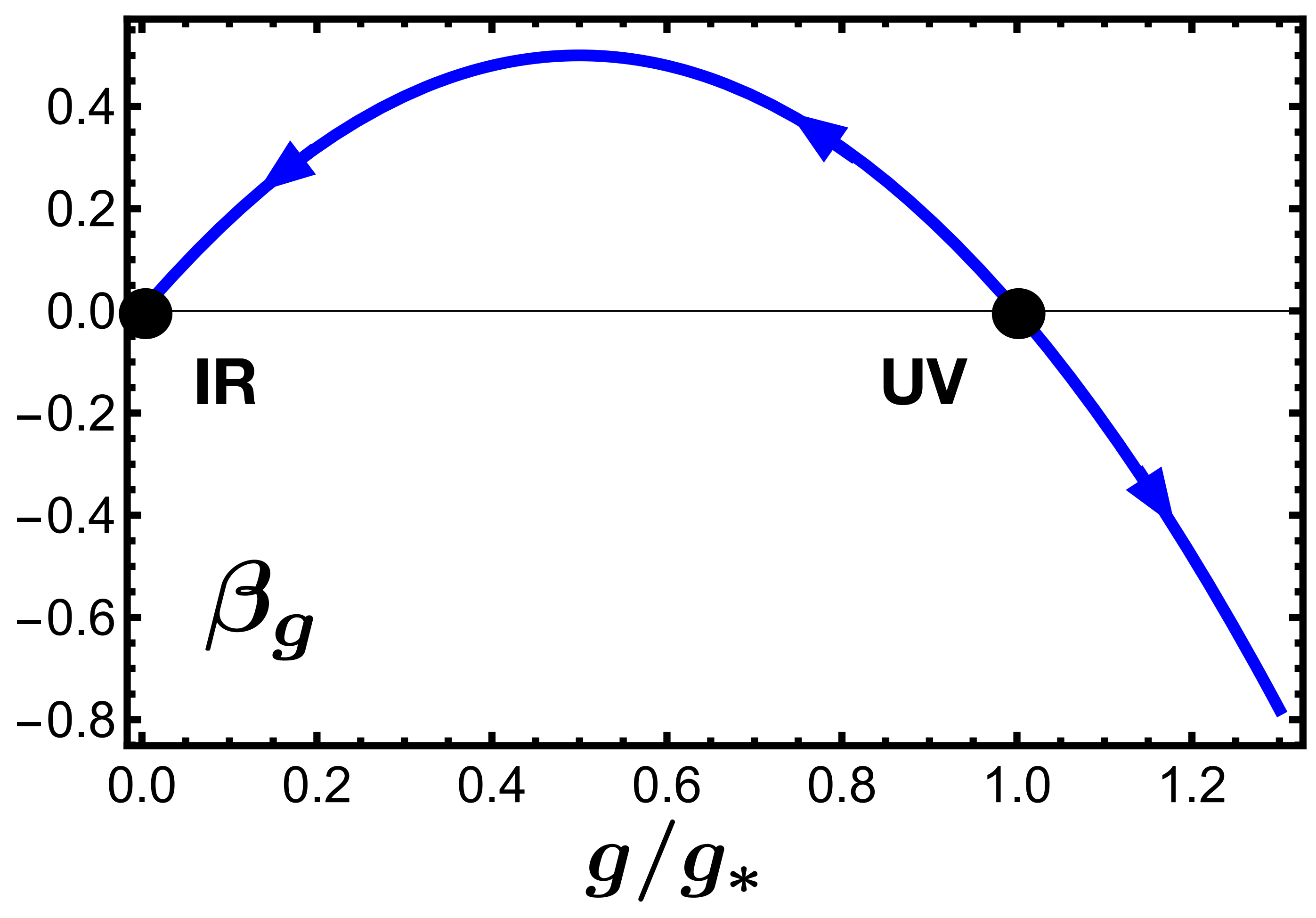

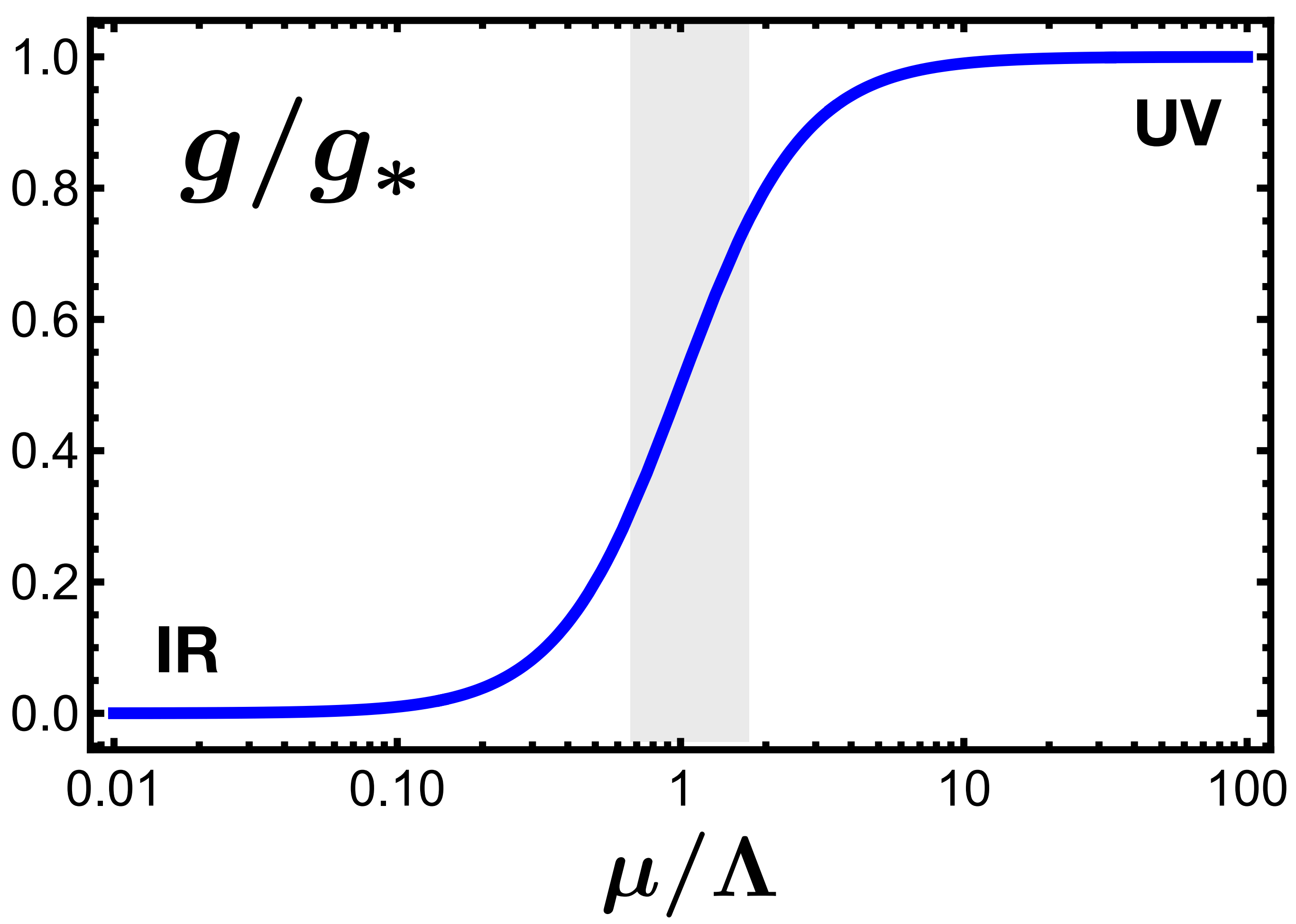

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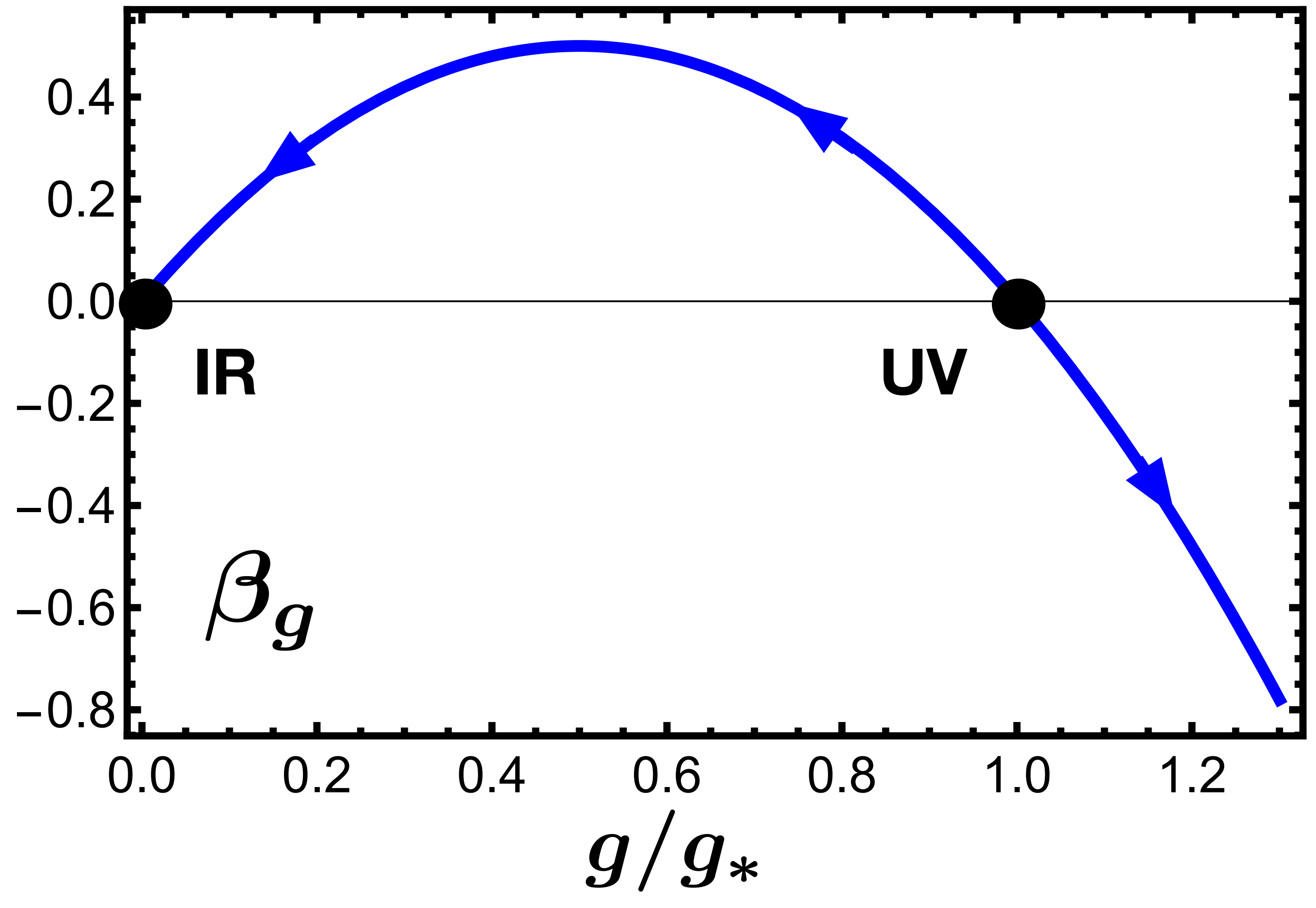

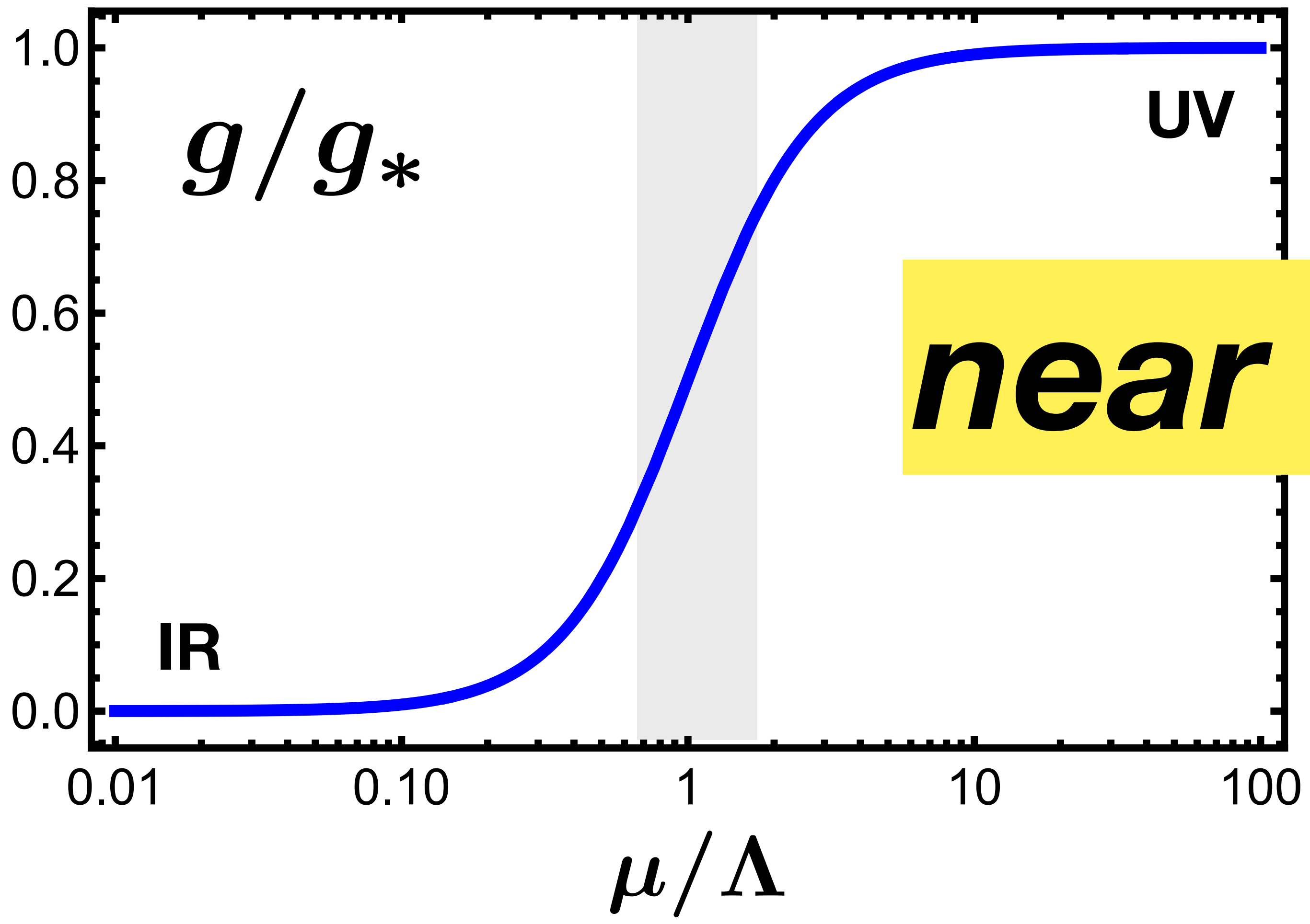

***near* conformal in UV**

Running couplings

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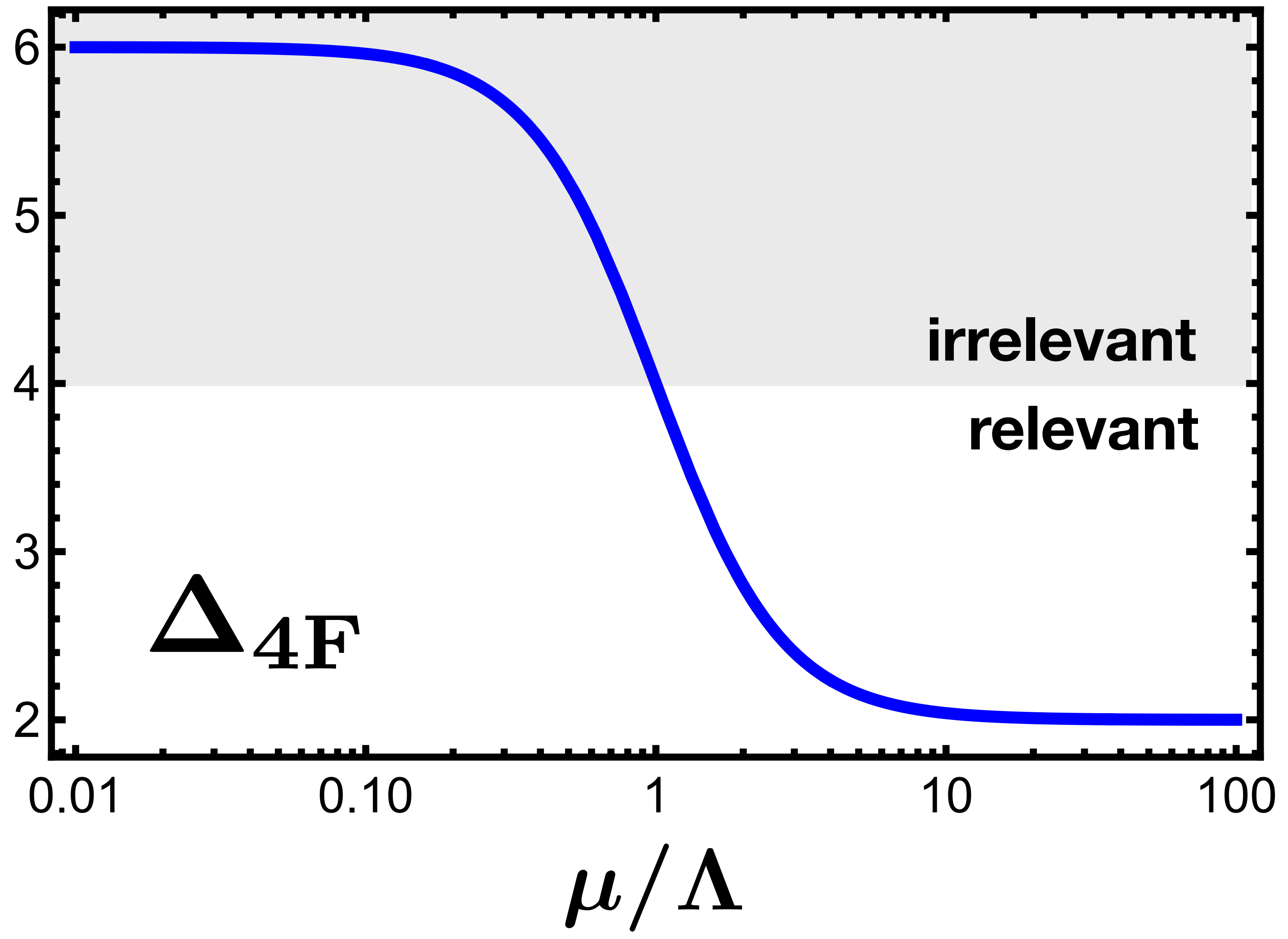
$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

UV fixed point

e.g. MOM scheme: $g_* = 8\pi^2$



universal:

$$\beta_Z = -\frac{1}{4\pi^2} \qquad \beta_\lambda = -\frac{3}{\pi^2}$$

logarithmic running

Running couplings

non-universal:

$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

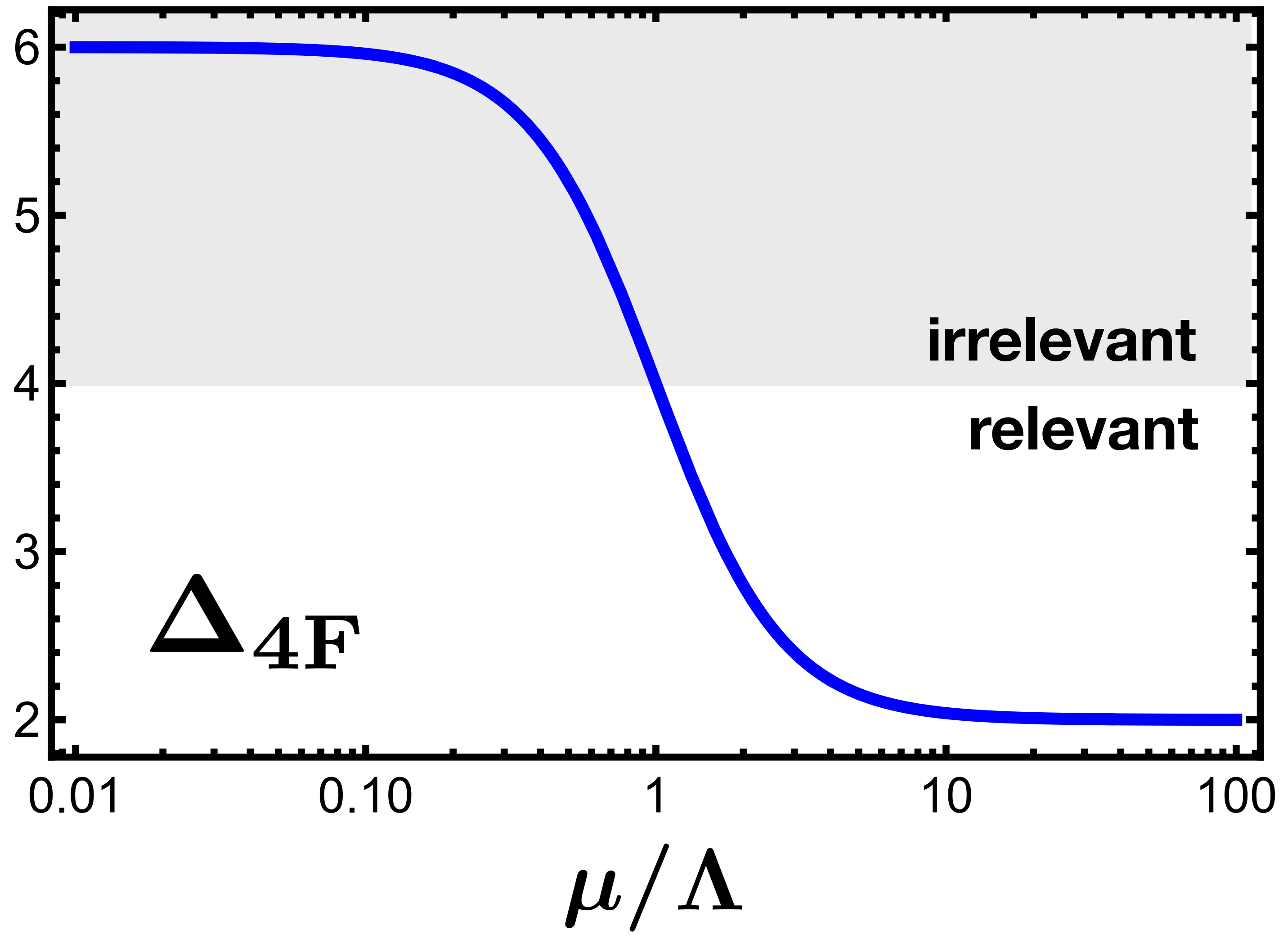
UV fixed point

e.g. MOM scheme: $g_* = 8\pi^2$

universal:

$$\beta_Z = -\frac{1}{4\pi^2} \quad \beta_\lambda = -\frac{3}{\pi^2}$$

logarithmic running

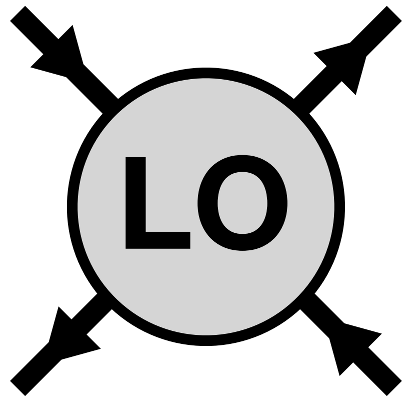


large anomalous dimensions:

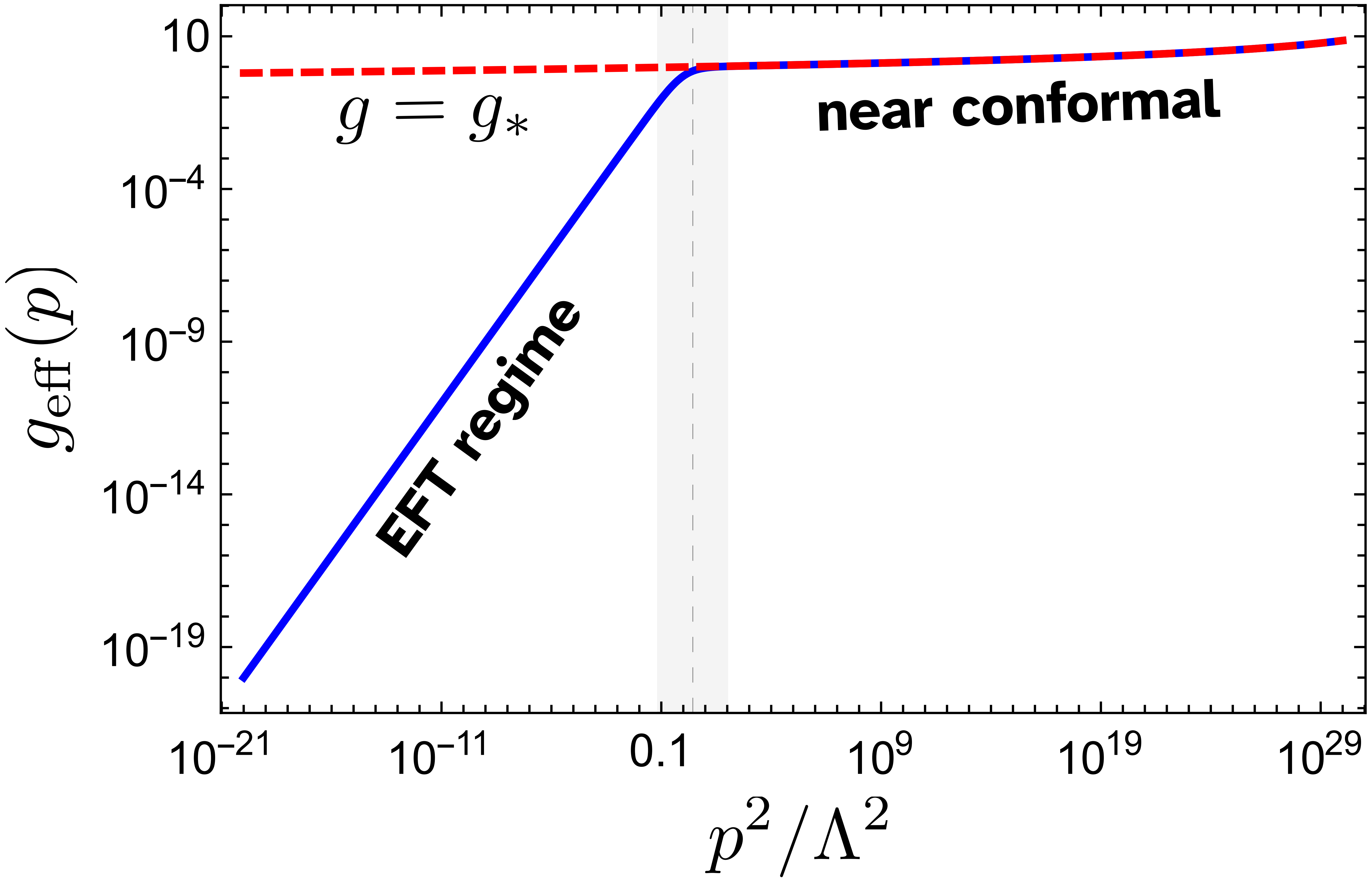
dim-6 becomes dim-2

EFT power counting breaks

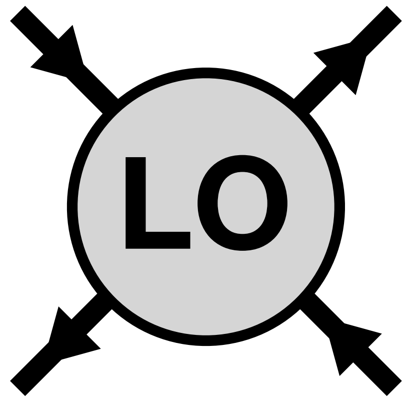
Near conformality & the EFT crossover



$$\sim \frac{N^{-1}}{p^2}\, g_{\text{eff}}(p)$$

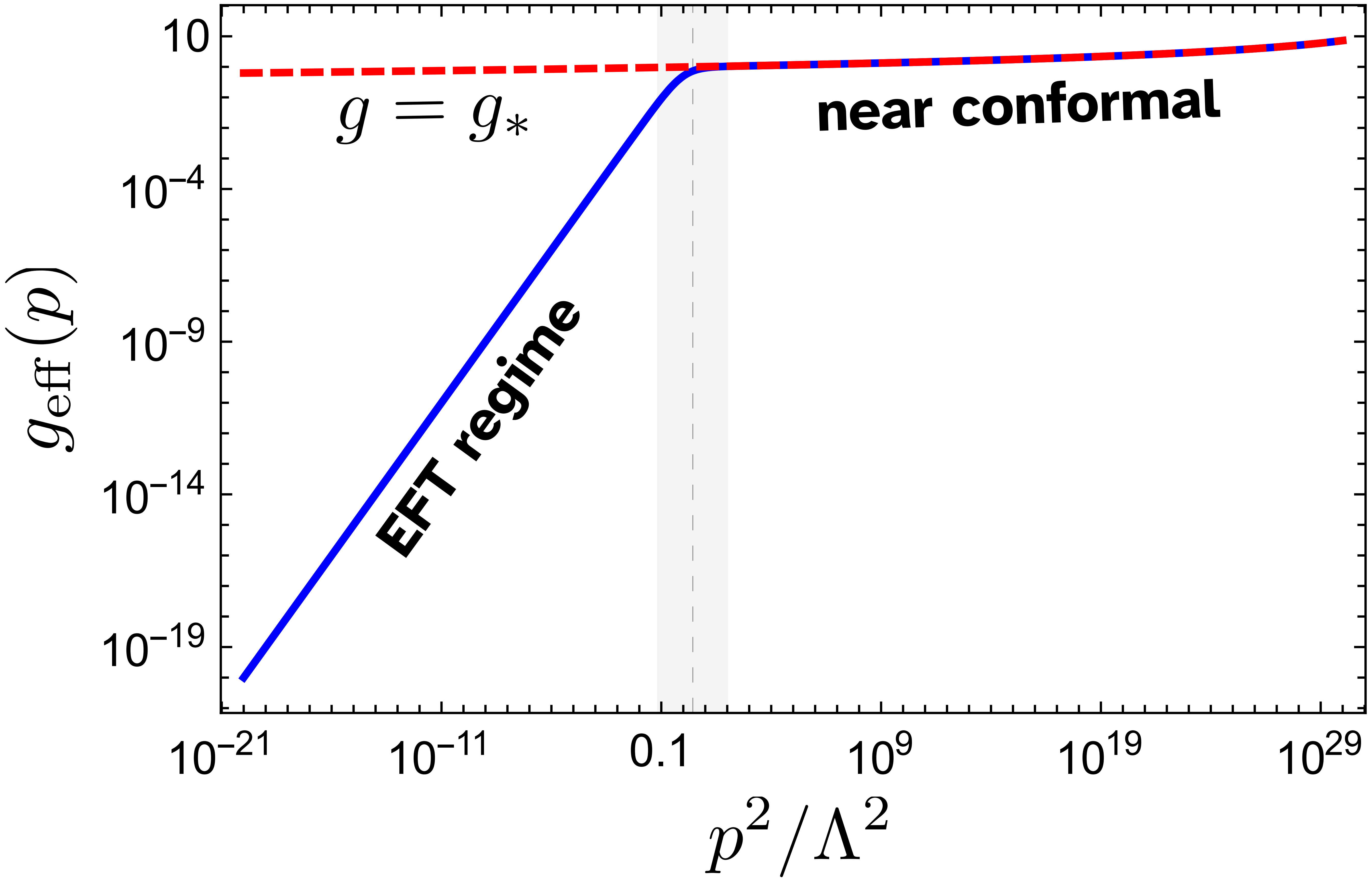


Near conformality & the EFT crossover

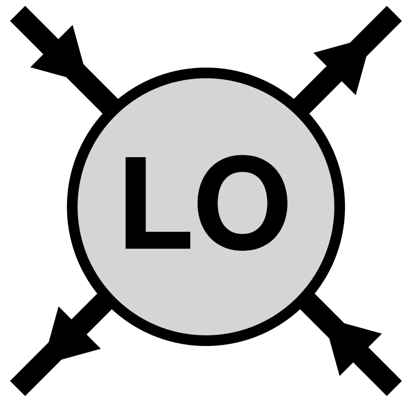


$$\sim \frac{N^{-1}}{p^2} g_{\text{eff}}(p)$$

$$\boxed{g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}}$$



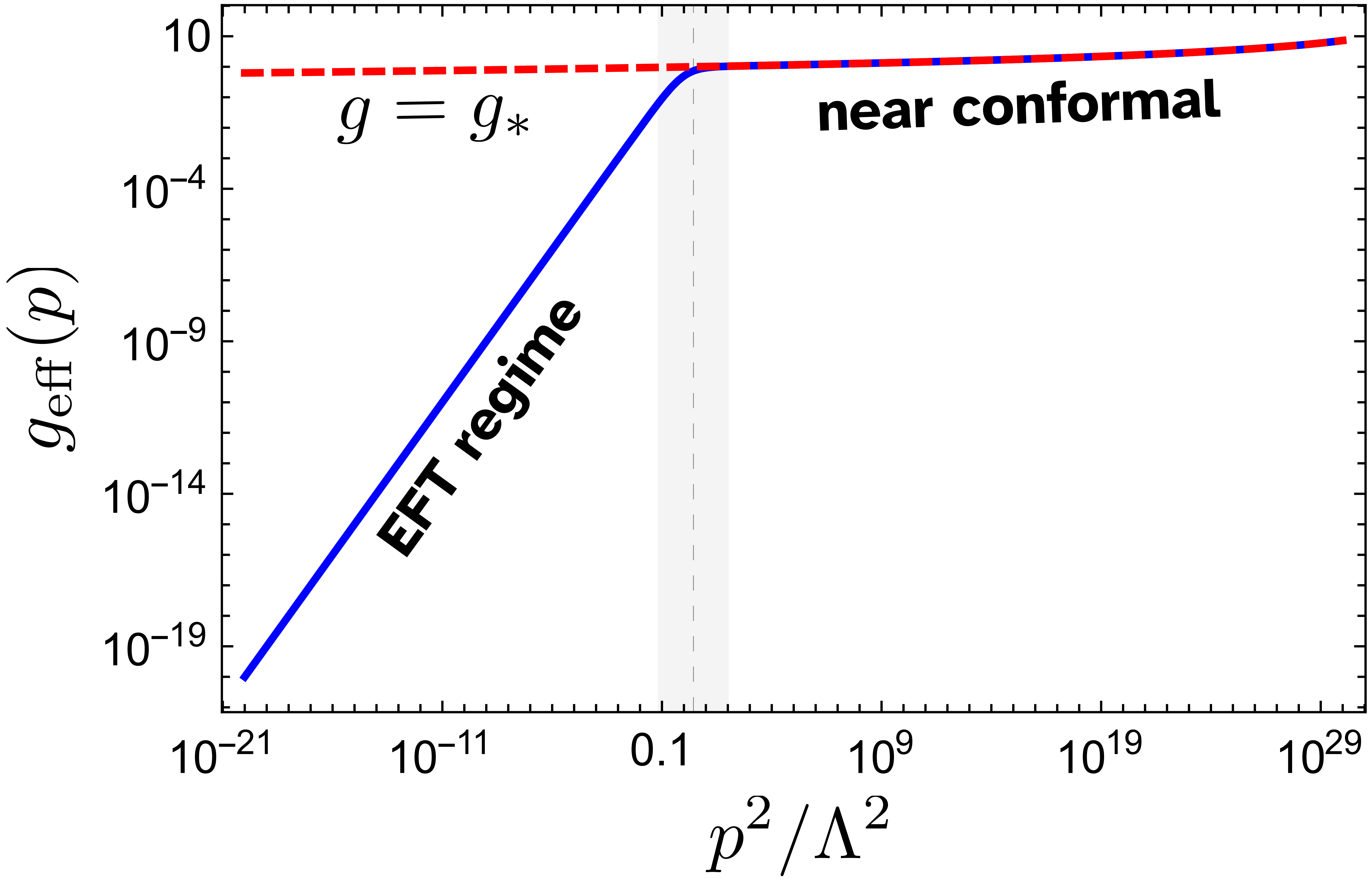
Near conformality & the EFT crossover



$$\sim \frac{N^{-1}}{p^2}\, g_{\text{eff}}(p)$$

$$\boxed{g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}}$$

EFT scale marks crossover:
onset of strong coupling, higher derivative terms become important



Summary

Four-fermion interactions can UV-complete themselves

- Tower of higher derivative 4F + 8F operators, but only **3 parameters**
- **Near scale invariance** in UV with relevant 4F + marginal 8F
- Strong coupling sets in near EFT scale — scheme choice important

Further aspects

- **Landscape of theories:** symmetries, mom. dependence, … , SMEFT, …
- **Unitarity restoration:** (partial) RG fixed points in EFT $\Longleftrightarrow$ UV completions?
- **Constraining power** for Wilson coefficients in EFTs of new physics?